2

Newton's Universe

In his monumental Principia, Newton formulated the general laws of motion and the law of universal gravitation. He then applied these laws to explain the motion of planets and comets, projectile trajectories, and the marine tides, among other things. In so doing, he showed how natural phenomena could be understood using a handful of physical laws, which hold just as well for the "heavenly Moon" as for the "Earthly apple" (Fig. 2.1).

2.1   Newton's Laws of Motion

Newton's first law states that a body that is at rest will stay at rest, and a body that is moving with a constant velocity will maintain that constant velocity, unless it is acted upon by a force.

What does this mean? Let's imagine we are at an ice rink and there is a hockey puck which has been carefully placed at rest on the ice. Now we stand and watch the puck. What happens? According to Newton, the puck will stay where it is unless someone comes by and gives it a push—that is, applies a force.[1]

Now imagine we have given our little puck a push, so that it is sliding along the surface of the ice. We will assume that our ice rink has no friction. The puck will then continue to move at a constant speed in the same direction,

---

[1]Even a motionless puck on frictionless ice is subject to forces. Gravity pulls the puck downwards, but the surface of the ice pushes back with equal and opposite force, so the total force on the puck is zero.
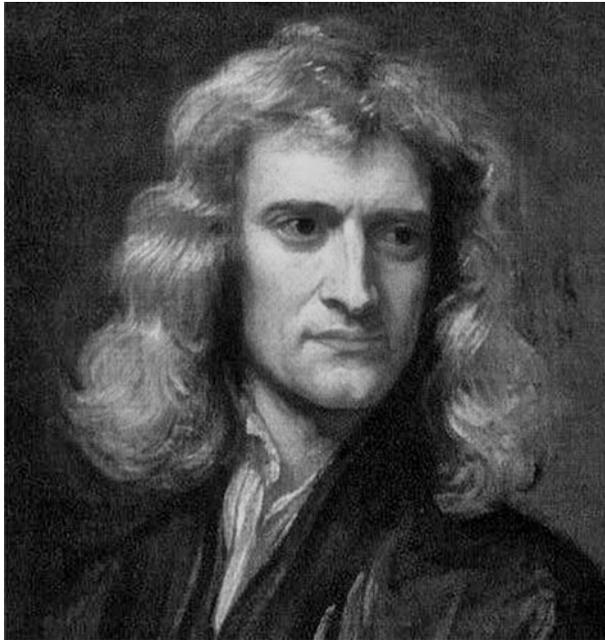
Fig. 2.1 Isaac Newton (1642–1726) made most of his major discoveries in 1665–1667, shortly after receiving his Bachelor's degree from the University of Cambridge. Although Newton earned financial support for further study, the University closed because of the plague, and he had to return to his family home in Lincolnshire for 18 months. It was during this time that he discovered his theory of colors, the law of gravitation, and calculus. In later years, apart from pursuing research in physics and mathematics, Newton devoted much effort to alchemy and to Scriptural studies. *Credit* Copy of a painting by Sir Godfrey Kneller (1689), painted by Barrington Bramley

unless it hits the wall of the rink, or bumps into someone or something along its way. These obstructions would provide a force that would alter the puck's uniform state of motion. If our imaginary frictionless ice rink were also infinite and devoid of other obstacles, the puck would coast along at the same velocity for eternity.

Newton's first law also goes by the name of *The Law of Inertia*.[2] A spaceship traveling with its engines turned off in interstellar space glides along

---

[2]The law of inertia was actually discovered by Galileo and was adopted by Newton as one of his laws of motion.

with a constant velocity, and provides yet another example of a body undergoing "inertial" motion.

Newton's second law tells us that if a force is applied to a body, the body accelerates—meaning its velocity changes. The law can be stated mathematically as

$$\vec{a} = \vec{F}/m \tag{2.1}$$

where $\vec{a}$ is the acceleration of the body, m is its mass, and $\vec{F}$ is the applied force. The acceleration is defined as the rate at which the velocity changes. For example, if in one second the velocity changes by one meter per second, then the acceleration is one meter per second per second, or one meter per second squared (m/s^2). In general, if the velocity is in m/s, the acceleration is measured in m/s^2.

The overhead arrows indicate that force and acceleration are vector quantities, which means they each have a magnitude and direction. Another example of a vector is velocity. The magnitude of a car's velocity is its speed, but very often we also need to know the direction in which the car is traveling. In Newton's first law, when we say that in the absence of forces a body moves at a constant velocity, this means that both the magnitude and direction of the velocity remain constant. When we want to refer only to the magnitude of a vector quantity, we drop the overhead arrow. For example, F is the magnitude of $\vec{F}$ and $a = F/m$ means that the magnitude of the acceleration is given by the magnitude of the force divided by the mass.

We can arrange an experiment in which the same force is applied to two different masses. Equation (2.1) tells us that the acceleration of the larger mass will be less than the acceleration of the smaller mass. Thus mass is a measure of a body's resistance to acceleration. More massive objects are harder to accelerate.

Force is measured in Newtons, which can be expressed in terms of other units as: $1\,N = 1\,kg\,m/s^2$. One Newton is the force required to accelerate a one kilogram (1 kg) mass at $1\,m/s^2$. It is important to remember that physical quantities only have meaning when we specify units. For example, if someone asks you how old you are and you reply 240, they would think you're crazy. However, if you said 240 months, they would probably convert that to 20 years, and think it just a little odd that you chose to measure your age in months instead of years. It is also essential to use consistent units throughout any calculation.

A common misconception is to think that the direction of an applied force is always the same as the direction of motion. We need to remember

that a net force acting on an object produces an *acceleration* in the same direction as the force, but the *velocity* of the object might be in a different direction. For example, suppose you are traveling in your car at a uniform speed, and then you apply the brakes. The force your brakes apply is in the opposite direction to motion, although your declining velocity is still in the original direction.

We have been discussing Newton's laws governing the motion of objects.[3] Although we are all familiar with velocities and accelerations from our everyday experience, it is important to point out that when we say an object is moving, we need to specify what it is moving with respect to. This defines a "reference frame". For example, during dinner on an airplane, your food tray is motionless relative to your lap, although relative to the ground it is traveling as fast as the plane. We can call your lap a "frame of reference" (the one in which the tray is still) and the ground is another, different frame of reference (relative to this frame the tray is moving very fast). So, a reference frame is an object relative to which we measure the locations and motions of other objects.

An *inertial frame of reference* is a frame associated with an object that is not acted upon by any net force and is moving by inertia. Once we specify one inertial frame of reference, any other frame that is moving with a constant velocity relative to the chosen frame, is also an inertial frame of reference. For example, the room you are in now is an inertial frame of reference (approximately).[4] Any train outside that is moving with a constant speed relative to the room is also an inertial reference frame. Newton's laws apply in all inertial frames of reference, thus any experiment you do in your room will yield the same results as the identical experiment performed by a friend on one of those trains.

2.2   Newtonian Gravity

Every day we experience the force of gravity. Gravity is an attractive force—it brings objects together. Every atom in our bodies is attracted to the Earth. Furthermore, every atom in the Earth is attracted to us. In fact any

---

[3]Newton also formulated a third law, which states that in every interaction between two bodies, the force the first body exerts on the second body is equal and opposite to the force the second body exerts on the first. If you push your friend facing you on an ice-rink, she will coast backwards, but so will you.

[4]The Earth is not exactly an inertial frame because of its rotation about its axis, which can be observed with a Foucault pendulum.
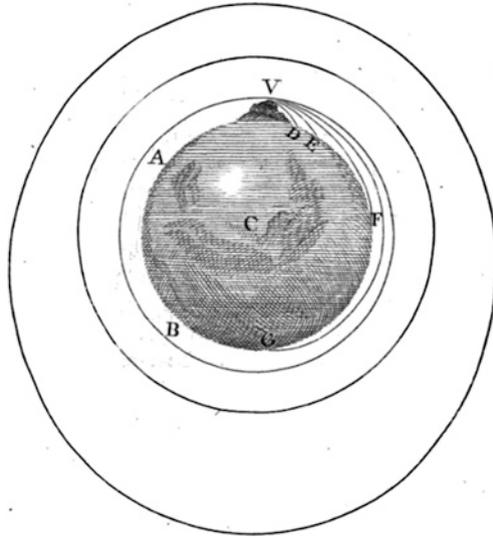
Fig. 2.2 Newton's thought experiment. Suppose a cannon is placed on top of a mountain and is fired with a moderate muzzle velocity. What will happen to the projectile? It will fall to Earth as shown at point *D*. If the muzzle velocity is increased it will fall a little farther away, as shown at points *E*, *F*, and *B*. Newton deduced that if the projectile is launched with progressively larger velocities, eventually, at just the right launch velocity, it will travel all the way around the Earth in a circular path, always falling in towards the Earth, but never reaching it, as indicated at *A*. Newton concluded that the Moon's orbit was of the same nature, with the Moon constantly falling toward the Earth. He also realized that if the launch velocity got higher, then elliptical orbits would be possible as shown. *Credit* Philosophae Naturalis Principia Mathematica

two objects in the Universe exert a gravitational attraction on one another. Newton realized that the same kind of force responsible for an apple falling from a tree was also responsible for the revolution of the Moon around the Earth, and the Earth around the Sun (see Fig. 2.2). Thus his law of gravity is sometimes called *The Law of Universal Gravitation*, applying both to the Earthly and the heavenly realm.

Newton's law of gravity states that any two objects are attracted to one another with a force
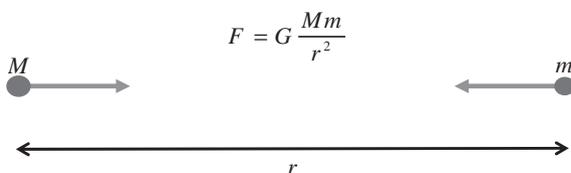
$$F = \frac{GMm}{r^2} \tag{2.2}$$

$$F = G \frac{Mm}{r^2}$$

Fig. 2.3 Gravitational force of attraction between two point masses a distance *r* apart

where *M* and *m* are the masses of the two objects and *r* is the distance between them. The force acting on mass *m* is directed towards the mass *M* and vice versa (see Fig. 2.3). We have also introduced Newton's gravitational constant *G*, which has a measured value of $G = 6.67 \times 10^{-11} \, \mathrm{Nm^2/kg^2}$.

Newton's law of gravity is an "inverse square law", because in Eq. (2.2) the gravitational force is inversely proportional to the square of the distance between the two objects. For example, let *M* be the mass of the Earth and *m* the mass of the Moon. If the Moon were placed twice as far away from the Earth as its actual distance, then the Earth would exert a force of gravity on the Moon that is one quarter as strong as it currently is.

The masses in Eq. (2.2) are assumed to be "point masses"; that is, we assume that their sizes can be neglected, so we can imagine that each mass is located at a point. This is a good approximation for the Earth—Moon system: the sizes of the Earth and the Moon are much smaller than the distance between them, so they can be approximated as point masses located at their centers. Then, to calculate the gravitational force of attraction, we use the distance from the center of the Earth to the center of the Moon. The same logic applies to the Earth orbiting the Sun.

Furthermore, Newton proved the "shell theorem", which states two important facts: (1) A uniform spherical shell of matter attracts an outside object as if all of the shell's mass were concentrated at its center. This applies to any uniform spherically symmetric object, like a solid sphere, since the object can be thought of as consisting of shells. (2) The gravitational force exerted on an object that is *inside* a uniform spherical shell of matter is zero. This result is surprising. The object doesn't even have to be at the center of the spherical shell—it can be anywhere inside the shell, and it will still feel no force.[5]

---

[5]To prove the shell theorem, Newton represented the shell as consisting of a large number of point masses and added together the forces produced by all of these masses. He had to invent calculus to perform this calculation!
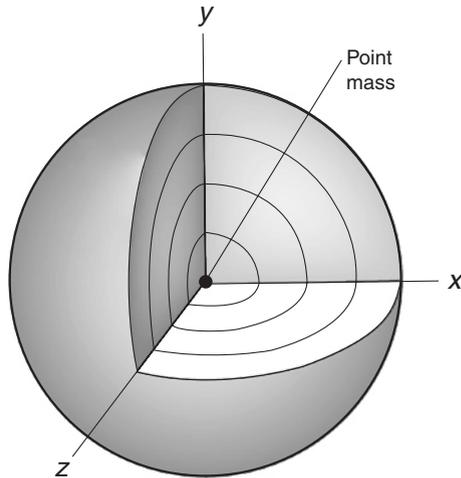
Fig. 2.4 Sphere divided into concentric shells. Each shell acts as if its mass is located at the center

To find the force of gravity acting on a small object near the surface of the Earth, we can imagine that the Earth (which is nearly spherical) is composed of a large number of thin concentric shells. Each shell will act as if all its mass is localized at the center, so the overall effect will be as if the entire mass of the Earth is localized at its center. Note that we do not have to assume that the mass density is uniform throughout the volume: each individual shell must have a uniform density, but the density can vary from one shell to the next. (In fact, the density of Earth is much greater near the center than near the surface.) (Fig. 2.4).

2.3 Acceleration of Free Fall

In everyday terms we often confuse weight and mass. When we get on a scale, we measure our weight—this is the force of gravity which pulls us towards the center of the Earth. For a small object of mass m near the surface of the Earth, the weight is given by

$$F = \frac{GM_E m}{r_E^2} \tag{2.3}$$

where $M_E = 6 \times 10^{24}$ kg is the Earth's mass, and $r_E = 6.4 \times 10^6$ m is its radius. On the Moon, we would weigh about 1/6 of our Earthly weight, even though our bodies would have the exact same amount of mass. The force of gravity on any object on the surface of the Moon is weaker than the gravitational force on that same object on the surface of the Earth. This is because the Earth has so much more mass than the Moon, that $\left(M/r^2\right)_{Earth} > \left(M/r^2\right)_{Moon}$, despite the fact that the Earth's radius is larger than the Moon's radius.

Now let's consider what happens if we have an object of mass m, close to the surface of the Earth, and we let it go. It will fall with acceleration $a = F/m$, which becomes (using Eq. (2.3) for F),

$$a = \frac{GM_E m}{r_E^2} \frac{1}{m} = \frac{GM_E}{r_E^2}. \tag{2.4}$$

This does not depend on the mass m, which means that all bodies close to the surface of the Earth fall with the same acceleration, independent of their mass (as long as we ignore air resistance). This remarkable fact was established by Galileo. The acceleration of free fall is denoted by the letter g; its measured value is $g = 9.8$ m/s^2. So, if we drop any object off a building, it will fall down with a velocity which increases by 9.8 m/s every second. Thus, after the first second the object will have a velocity of 9.8 m/s; after the second, it will have a velocity of 19.6 m/s and so on (assuming the object is simply let go, with an initial velocity of zero). From Eq. (2.4) we see that

$$g = \frac{GM_E}{r_E^2} \tag{2.5}$$

Substituting the values of M_E, r_E and G (Newton's constant), you can verify that indeed $g = 9.8$ m/s^2.

2.4   Circular Motion and Planetary Orbits

Velocity characterizes how fast the position of a body changes with time, and acceleration is the rate of change of velocity with time. When we travel at a constant speed and in a constant direction, our acceleration is zero. What happens if we travel around a large circular track at a constant speed? Do we accelerate? Yes, we do. Even though we maintain a strictly uniform speed, we constantly have to change the direction in which we are traveling. This
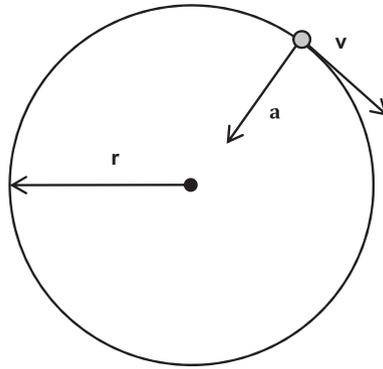
Fig. 2.5 Direction of acceleration for uniform circular motion

change in direction indicates that there is an acceleration. For uniform circular motion, the magnitude of the acceleration is

$$a = \frac{v^2}{r} \qquad (2.6)$$

where v is the speed of the object undergoing the motion and r is the radius of the circle. The direction of the acceleration is radially inward, towards the center of the circle (see Fig. 2.5); it is called the centripetal acceleration.[6] If you have ever swirled an object attached to a string above your head, you know that the tension in the string keeps the object from flying off at a tangent to its orbit. The string thus provides a force directed towards your hand that results in the object undergoing centripetal acceleration.

Newton showed in his Principia that the inverse square law implies that celestial bodies like planets and comets should move in elliptical orbits, in agreement with Kepler. While comets often move in highly eccentric orbits, for planets the two focal points of the ellipse almost coincide, so the orbit is approximately a circle. If the radius of the orbit is r, its circumference is $2\pi r$, and the velocity of the planet is

$$v = \frac{2\pi r}{T} \qquad (2.7)$$

---

[6]The derivation of this formula relies on some simple geometry and can be found in any basic physics textbook.

where T is the time it takes to complete one revolution.

We can now apply what we have learned about Newton's laws to weigh the Sun. Let's do it!

We know that the force keeping the Earth in motion around the Sun is gravity; thus Eq. (2.2) holds, with m the Earth's mass, and M the Sun's mass. We also know that to a good approximation the Earth orbits the Sun with uniform velocity in a circle and thus undergoes centripetal acceleration (gravity is the force responsible for this centripetal acceleration). Then, substituting Eq. (2.6) into Eq. (2.1) and equating this with Eq. (2.2) we find $F = m\frac{v^2}{r} = GmM/r^2$.

Rearranging, the Sun's mass is given by

$$M = v^2 r/G \approx 2 \times 10^{30}\, \text{kg}, \tag{2.8}$$

where the Earth's orbital velocity $v \approx 30$ km/s can be calculated from Eq. (2.7) using our knowledge that it takes the Earth one year ($T \approx 3 \times 10^7$ s) to complete one orbit at a distance of $r \approx 1.5 \times 10^8$ km. This method is often used in astronomy to measure the masses of stars, galaxies, and even clusters of galaxies.

2.5   Energy Conservation and Escape Velocity

Energy is nature's ultimate currency—it comes in several different forms, and can be converted from one form to another. For example, to launch a rocket into space, chemical energy must be converted into kinetic energy of motion. In general, the conservation of energy is one of the most fundamental laws of nature.

Here we will focus on mechanical energy. Mechanical energy can be divided into two types: kinetic energy and potential energy. Kinetic energy is the energy an object has by virtue of its motion. An object of mass m traveling with speed v has kinetic energy

$$K = \frac{1}{2}mv^2 \tag{2.9}$$

Potential energy is the energy a system has due to interactions between its parts. It can be thought of as stored energy that has the capacity to be unleashed and turned into kinetic energy. There is no universal formula for the potential energy; it depends on the kind of interaction. In the case of gravitational interaction between two spherical masses, it is given by

$$U = -\frac{GMm}{r} \qquad (2.10)$$

where r is the distance between the centers of the spheres.[7] If there are more than two masses, one simply has to add the potential energies for all pairs.

For a small object close to the Earth's surface, the potential energy (Eq. 2.10) can be approximated by the following useful formula:

$$U = mgh + \text{const} \qquad (2.11)$$

Here, m is the object's mass, h is its height above the ground ($h = r - r_E$ where r is its distance from the center of the Earth, and r_E is the Earth's radius), and $g = 9.8 \, \text{m/s}^2$ is the gravitational acceleration close to the Earth's surface.

The constant in Eq. (21.1) is $-GM_E m/r_E$, where M_E is the mass of the Earth. Such constant additions to the energy are unimportant for most purposes and are often omitted.

The total energy of the system is the sum of its kinetic and potential energies,

$$E = K + U \qquad (2.12)$$

In an isolated system, to which no external forces are applied, the total energy is conserved—that is, it does not change with time. This is an immensely useful property, which makes the solution of many problems much easier than it would otherwise be.

A ball of mass m in a frictionless U-shaped track provides a classic example of the interplay between potential and kinetic energy (see Fig. 2.6). Let's place the ball on the left arm of the track, so that it is at a height h above the bottom of the track. We will let go of the ball, and it will start rolling. The ball's initial speed will be zero, but as it rolls down the track it picks up speed, attaining its maximum velocity at the bottom of the track (technically the ball has a rotational velocity in addition to its translational velocity, which we will ignore here for clarity. In other words, we will treat the ball as though it is "sliding" down the track). It will then rise up the right arm of

---

[7]This formula can also be used for a small object (like a human) interacting with a large spherical body (like the Earth). In this case, the small object does not have to be spherical, and the distance r is the distance from any point in the object to the Earth's center.
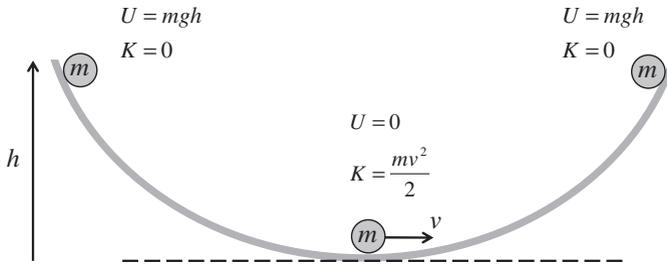
At the top of the track, all the energy is in the form of potential energy. At the bottom, all potential energy has been converted to kinetic energy, and the ball has its maximum velocity. On its way up or down the track there is a mix of potential and kinetic energy, but the total mechanical energy is the same at every point along the ball's trajectory

Fig. 2.6 At the *top* of the track, all the energy is in the form of potential energy. At the *bottom*, all potential energy has been converted to kinetic energy, and the ball has its maximum velocity. On its way up or down the track there is a mix of potential and kinetic energy, but the total mechanical energy is the same at every point along the ball's trajectory

the "U", until it reaches a maximum height and momentarily comes to rest before rolling down the right arm.[8] How high up the right arm does it get?

The answer is easy. By conservation of energy, it must reach the same height h as it started with. When the ball is at its starting point it has no kinetic energy (it is released from rest), but it has gravitational potential energy $U = mgh$. When it reaches the maximum position on the right arm, it also has no kinetic energy, since it is momentarily at rest. So it must have the same amount of potential energy, thus it must reach the same height h.

How fast will the ball be moving at the bottom of the "U"? At the bottom, the ball has no potential energy because it has zero height above the reference ground level. Thus all its initial potential energy is converted into kinetic energy and we have $\frac{1}{2}mv^2 = mgh$, which can be solved to yield the velocity at the bottom if we know h. In fact, we can find the velocity of the ball at any point along its motion if we know the height at that point.

Similar interchange between kinetic and potential energies occurs when planets move around the Sun. The expression for the potential energy U to use in this case is Eq. (2.10). This formula can be a little tricky to deal with, so it is useful to consider a plot of U versus r, as shown in Fig. 2.7.

We see that U approaches a maximum value of zero, when two objects are separated by larger and larger distances. But because the sign of the potential energy is negative, the gravitational potential energy decreases as objects are brought closer to each other (watch out for the minus sign!). Hence the

---

[8]You probably remember what happens when you are pulled up and released on a swing: you start from rest, reach a maximum speed at the bottom of your trajectory and then slow down as you swing up, momentarily coming to rest before going backwards, and so on.
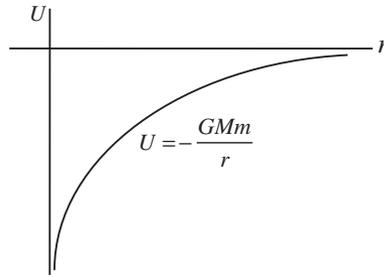
Fig. 2.7   Gravitational potential energy as a function of distance

kinetic energy should grow and the objects should move faster as they get closer. A planet moving along its elliptic orbit speeds up as it gets closer to the Sun and slows down as it gets further away.

The gravitational potential energy between two orbiting bodies can be thought of as a binding energy. The closer the two bodies are, the more negative is the potential energy, and thus we would have to work harder, or put in more energy, to separate them.

Since mechanical energy is the sum of kinetic and potential energy, it is possible for a pair of orbiting objects to have negative, zero or positive total mechanical energy. When the total energy is negative, it simply means that the system has less kinetic energy than the magnitude of the gravitational potential energy. This is in fact the case for all bound orbits, like the Earth-Moon system, the comets that orbit the Sun, or even the man-made satellites that orbit the Earth. Orbits with zero or positive total mechanical energy are said to be unbound. For example there are currently five spacecraft, Voyager 1 and 2, Pioneer 10 and 11, and the New Horizons Spacecraft, which are heading out of our Solar System on unbound orbits (or escape trajectories).

You might be wondering, how do we control whether a satellite we launch goes into orbit around the Earth, or goes off into the Solar System? The answer is very simple. There is a minimum initial speed, called the escape speed, with which the object must be launched in order for it to escape from the Earth. So, how do we calculate this escape speed? We use the principle of energy conservation and the fact that for the object to escape, its total mechanical energy must be greater than or equal to zero. Let's say we launch a spacecraft of mass m with speed v. Its total initial mechanical energy as it leaves the Earth is

$$E_i = \frac{1}{2}mv^2 - \frac{GM_E m}{r_E}, \tag{2.13}$$

which must equal its total final energy E_f when it has escaped.[9] If we take the marginal case of zero total energy, the launch speed is by definition the escape velocity, and we have

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_Em}{r_E} = 0. \tag{2.14}$$

This can be solved to find the escape speed

$$v_{esc} = \sqrt{\frac{2GM_E}{r_E}}. \tag{2.15}$$

Substituting in values for the Earth's mass and radius, we find $v_{esc} \approx 11.2\,\text{km/s}$. So if we launch a satellite with a speed slightly greater than 11.2 km/s, it will leave the Earth's gravitational clutches. If we launch it with less than this speed, it will fall back to Earth. And if we launch it at exactly the escape speed, it will barely escape, with its velocity getting smaller and smaller as it moves away and approaches zero in the limit.

Note that although we have derived the escape velocity for an object launched from the Earth, this formula holds in general, with the Earth's mass and radius replaced by whatever body you are considering. In later chapters we will apply similar considerations to the entire universe. Note also that the escape (or no escape) outcome depends only on the magnitude of the velocity, not on its direction.

2.6   Newtonian Cosmology

Newton's cosmological ideas developed during a correspondence with the Cambridge theologian Richard Bentley. Bentley was preparing to give public lectures titled "A confutation of atheism" and wrote to Newton asking him how his theory of gravity applied to the universe as a whole. During the winter of 1692–93, Newton sent a series of four letters to Bentley, in which he described a universe that is infinite and static: "The fixed stars, being equally spread out in all points of the heavens, cancel out their mutual pulls by opposite attractions."

---

[9]Note that the final energy is purely kinetic and must therefore be positive. This says that only objects with positive (or zero in the marginal case) total energy can escape.

However, as Newton was acutely aware, there is a problem with this line of reasoning. If a region of the universe has a slight excess of matter, then that region will begin to attract material from its surroundings. The region will become denser, and it will attract more and more matter. Thus a uniform distribution of stars is unstable due to gravity: it would be destroyed by an arbitrarily small perturbation. Newton's solution was to invoke a supernatural intervention, stating "…this frame of things could not always subsist without a divine power to conserve it."

2.7   Olbers' Paradox

What do you think the Sun would look like if it were at twice its present distance from the Earth? The total brightness of the Sun would be four times smaller because the brightness of an object decreases as the inverse square of the distance to the object.[10] The area of the Sun's disc on the sky would also be four times smaller. This means that the brightness per unit area (called the surface brightness) remains the same. So what? Well, in an infinite universe that is uniformly sprinkled with stars, every line of sight should eventually hit a star, and each star should have roughly the same surface brightness as the Sun. This implies that the whole sky should be glowing with the same intensity as the Sun's surface. So why is the sky dark at night? This paradox is known as Olbers' paradox or the "dark night sky paradox". It indicates that Newton's picture of the universe cannot be right (Fig. 2.8).

An infinite static universe has other problems in addition to the gravitational instability and Olbers' paradox. We shall return to this issue in Chap. 5. For now, we just note that the problems of Newtonian cosmology give a foretaste of things to come: it is not so easy to come up with a cosmological model that makes any sense at all.

While Newton showed that his universal law of gravity could explain a vast scope of natural phenomena, he was at a loss to explain how it could be that the force of gravity acts instantaneously, between every pair of particles, across the vastness of space. This mysterious action-at-a-distance fueled Newton's critics. At the end of his Principia, Newton conceded: "*Thus far I have explained the phenomena of the heavens and our sea by the force of gravity, but I have not yet*
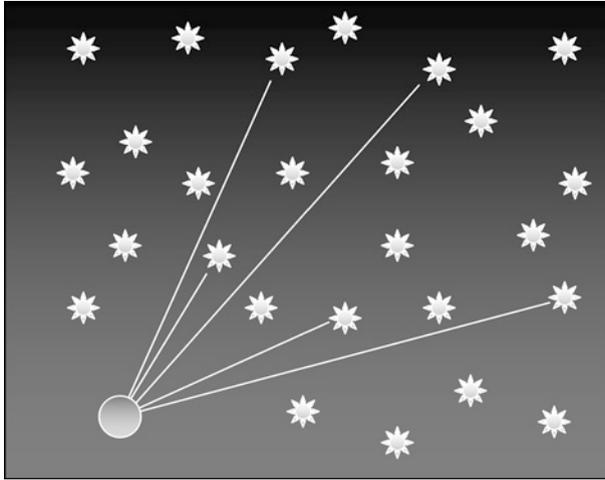
---

[10]This will be discussed in Chaps. 4 and 6.

Fig. 2.8 Olbers' paradox. The entire sky should be as bright as the Sun. The astronomer Heinrich Wilhelm Olbers popularized the paradox, although he was not the first person to formulate it

assigned a cause to gravity.... *I have not as yet been able to deduce from phenomena the reason for these properties of gravity and I do not feign hypotheses,*" but he then went on to declare "*And it is enough that gravity really exists and acts according to the laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies and of our sea.*" Despite these rumblings, Newton's description of gravity held sway for two hundred years—until Einstein's general theory of relativity revolutionized our understanding of gravity once again, as we shall later learn.

Summary

Newtonian mechanics forms the foundation for our understanding of the physical universe. We used Newton's laws of motion and his universal law of gravity to explore planetary orbits, energy conservation and escape velocities. We then discussed Newton's cosmological picture of an infinite, static universe uniformly filled with stars. Amongst other problems, this picture is incompatible with the observation of a dark night sky—this is known as Olbers' paradox. The problems of Newton's static universe give us an inkling of how difficult it is to develop a sensible cosmology.

Questions

1. Can you give an example, from everyday life (that is not mentioned in the text), where the force applied to an object is not in the direction of motion?
2. Is every frame of reference an inertial frame of reference?
3. In what ways are the laws of Nature, like Newton's laws, different from criminal laws?
4. If you apply the same force to two boxes, one of which is twice as heavy as the other, how will their accelerations compare? (Assume there is no friction)
5. What would happen to the force exerted by the Moon on the Earth if the Moon were placed twice as far away? And if it were brought into a third of its current distance? In which direction does the Earth pull on the Moon? In which direction does the Moon pull on the Earth?
6. Suppose you weigh 150 lbs. What would you weigh if the Earth was shrunk to half its current radius? (Assume that you and the Earth have the same mass before and after the contraction.)
7. Suppose we dig a tunnel radially through the Earth. If you weigh 150 lbs on the surface of the Earth, what would your weight be if you descend half way towards the Earth's center? (Assume that the Earth has uniform density throughout its volume. Also note that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ where r is the radius, and that density is $\rho = M/V$ where M is mass, and V is volume.)
8. (a) Find the Earth-Moon distance given that it takes a radio signal 1.3 s to travel to the Moon. Note: radio signals travel at the speed of light which is denoted c and is approximately 3×10^8 m/s.
   (b) It takes roughly 27.3 days for the Moon to orbit the Earth (sidereal month). Calculate the velocity of the Moon around the Earth.
   (c) Use your results to help you calculate the mass of the Earth.
9. A space probe is launched from Earth with twice the escape velocity, $v = 2v_{esc}$. What will the velocity of the probe be when it gets far away from Earth? Express your answer in terms of v_{esc}.
10. A ball is released from rest at height h in a landscape shown in the figure. Assuming no friction, where will the ball reach its maximum speed? Indicate on the figure where it will come momentarily to rest (Fig. 2.9).
11. What is Olbers' paradox? Does it prove that the universe cannot be infinite? If not, what does it prove?
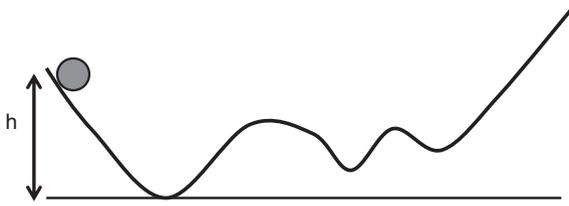
Fig. 2.9   Ball on a landscape

12. To explain Olbers' paradox, we argued that the brightness of a star and the area it subtends on the sky are both inversely proportional to the square of the distance to the star. Can you justify these statements?
13. Can you think of two ways to resolve Olbers' paradox?