17

Testing Inflation: Predictions and Observations

We have seen how cosmic inflation can create an enormous universe from a tiny seed, while solving many problems that plagued pre-inflation models. Most cosmologists have embraced the inflationary scenario, but how can we know that inflation actually happened? Fortunately, the theory of inflation has made several testable predictions, three of which we now discuss.

17.1 Flatness

As we learned in the previous chapter, accelerated expansion during inflation rapidly drives the density parameter towards $\Omega = 1$ and the geometry of the universe to flatness. Thus, inflation predicts that, on the largest observable scales, the universe should be accurately described by a flat geometry with $\Omega = 1$. When Alan Guth made this prediction in the early 80's, astronomers viewed it with a high degree of skepticism. All the evidence at that time pointed to an open hyperbolic universe. Even including dark matter, observation favored $\Omega_m \sim 0.3$.

Then, quite unexpectedly, nearly 20 years after Guth's prediction, dark energy was discovered. Today we know from CMB and supernovae measurements that its contribution to the cosmic energy balance is $\Omega_{vac} \approx 0.69$, so that

$$\Omega_{tot} = \Omega_m + \Omega_{vac} = 1 \pm 0.01, \qquad (17.1)$$

The universe is thus very close to being flat, in excellent agreement with the inflationary prediction.

17.2 Density Fluctuations

Perhaps the most impressive triumph of inflation has been the explanation of primordial density fluctuations. The theory of inflation specifically predicts that the magnitude of fluctuations is about the same for all observable distance scales: the initial density fluctuations have a scale-invariant spectrum. Also, the different components of the early universe—dark matter, electrons, protons and photons—all start out with the same density perturbations. Thus, regions that have an initial over-density of photons, say, by 0.01%, also have a 0.01% initial over-density of dark matter, and so on. (However, as we shall soon see, the dark matter does not evolve in tandem with the other components during times of interest in this chapter.)

To test this prediction, cosmologists used a computer code to follow the evolution of scale-invariant fluctuations up until recombination. The resulting pattern of evolved density fluctuations was then converted into a pattern of temperature anisotropies, and these were compared to the CMB observations. The agreement between theory and experiment, as most accurately measured by the Planck satellite (see Fig. 17.1), is striking.[1]

Let us now discuss the CMB temperature anisotropies depicted in Fig. 17.1 in more detail. The horizon distance at recombination corresponds to about 1.5° on the sky (see Question 9). Matter could not have moved over distances exceeding the horizon, and thus on angular scales significantly larger than 1.5° the temperature anisotropies represent the density fluctuations in their pristine form, as they came out of the inflationary epoch. As expected, the magnitude of the anisotropies, and hence the density fluctuations, is about the same for all angles in this range.

Photons that propagate to us from higher density regions start out a little hotter than average, and those that travel towards us from less dense regions are initially cooler than average. On the other hand, photons from denser regions lose more energy as they climb out of stronger gravitational fields produced by those regions. This gravitational redshift turns out to be the dominant effect, so the net result is that, surprisingly, hot patches in

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[1]Recall, the scale invariance of density fluctuations from inflation is only approximate: fluctuations on greater distance scales are slightly larger than the smaller-scale fluctuations. The experimental data are consistent with these details.
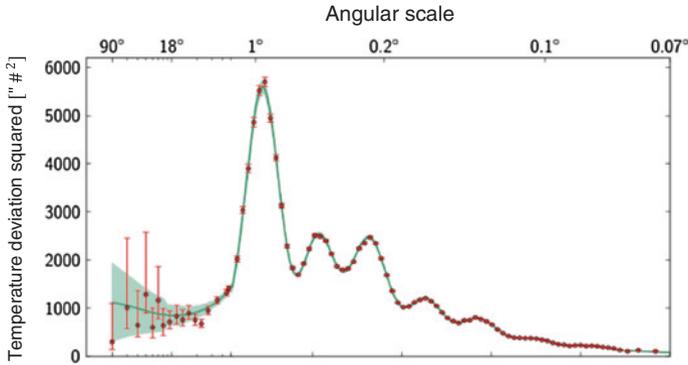
Fig. 17.1 Planck satellite temperature anisotropies. The squares of temperature deviations are plotted (in micro Kelvins squared) versus the angular scale on the sky subtended by hot or cold spots. The *red dots* are data points, and the *green line* is the theoretical prediction. On large angular scales, there are only a few cold or hot patches that can fit in the sky. This results in a large statistical uncertainty, indicated by the *light green band* in the figure. *Credit* ESA and the Planck Collaboration

the CMB sky indicate under-dense, cooler regions in the early universe at recombination.

On angular scales of about 1° and smaller, the data in Fig. 17.1 exhibit a number of peaks. These peaks are signatures of primordial sound waves, as we shall now explain. Prior to recombination, protons, electrons and photons are tightly coupled together by electromagnetic forces and act as a single proton-electron-photon gas. Such a mixture of charged particles and radiation is called a *plasma*. In denser regions the plasma has a higher temperature and a higher pressure. The difference in pressure pushes the plasma into neighboring low-density regions. The pressure momentarily equalizes, but the plasma keeps moving by inertia, so the regions that were initially hotter and denser become cooler and vice versa. Now the pressure difference works in the opposite direction and the plasma rushes back to the initially over-dense regions. The resulting oscillations of compression and rarefaction are simply sound waves in the primordial plasma. Dark matter particles interact very weakly with ordinary matter, so they do not participate in plasma oscillations.

Like sound waves in the air, the plasma sound waves are characterized by an oscillation period P and a wavelength λ, equal to the distance traveled by sound in one period. The wavelength of the plasma waves is determined by the size of the over-dense and under-dense regions. Since these regions come in a variety of sizes, many different waves are "sounding" at the same time. The speed of sound v_s in a gas is comparable to the typical particle velocity.

The cosmic plasma consists predominantly of photons, so v_s is not much different from the speed of light: $v_s \approx 0.6c$. When charged particles eventually recombine to form neutral atoms, the oscillations stop, and photons stream freely through the universe, bringing us the pattern of primordial sound, frozen in time at recombination.

The waves that reach their maximum amplitude at t_{rec} contribute the most to the observed temperature fluctuations. The longest waves of this kind have period $2t_{rec}$. Their wavelength λ_f is called the fundamental wavelength. A simple calculation shows that $\lambda_f \approx 1.4 \times 10^6 lyrs$ (see Question 10). By the time of recombination, such a wave completes half a period of oscillation, so a region that started out at maximal rarefaction after inflation will have reached maximal compression at t_{rec}, and vice versa. The highest peak in Fig. 17.1 is generated by sound waves with the fundamental wavelength. Other peaks come from sound waves having wavelengths that are integer fractions of λ_f. The second peak is due to waves with half the fundamental wavelength. In this case, a maximally rarefied region has had time to reach maximum compression and then rebound, again becoming maximally rarefied by the time of recombination. The third peak is due to a sound wave with a third of the fundamental wavelength, and so on. It is also evident in the figure that peaks at smaller angular scales drop off in magnitude. This is due to the dissipation of sound waves.

Mining the CMB

There is much that can be learned about our universe from the CMB anisotropies. The angle of the fundamental peak in Fig. 17.1 allows cosmologists to directly measure the curvature of the universe. This angle (approximately 1°) gives us the angular size of the most intense temperature variations on the sky.

It is the angle subtended by half of the fundamental wavelength λ_f (because the full wavelength includes both a cold and a hot patch). Since we know both the distance from us to the surface of last scattering (this distance is very close to the horizon distance; see Sect. 7.7.) and the physical size of λ_f, cosmologists can determine if the observed angle is consistent with a flat, open or closed geometry (see Fig. 17.2). It turns out that to a very high accuracy, the universe is flat, in full agreement with the measurements of the energy density of the universe.

As we already mentioned, on large angular scales the temperature fluctuations in the plasma and the gravitational redshift work in opposite directions, so the observed fluctuations are due to the difference between the two effects. On the other hand, in the fundamental peak, the hot and cold plasma regions switch places, while the high and low density dark matter regions do not move. As a result the plasma temperature fluctuations and the gravitational redshift due to dark matter are now added together. This is why the first peak is so high. In the second peak the two effects are again opposite. By comparing the
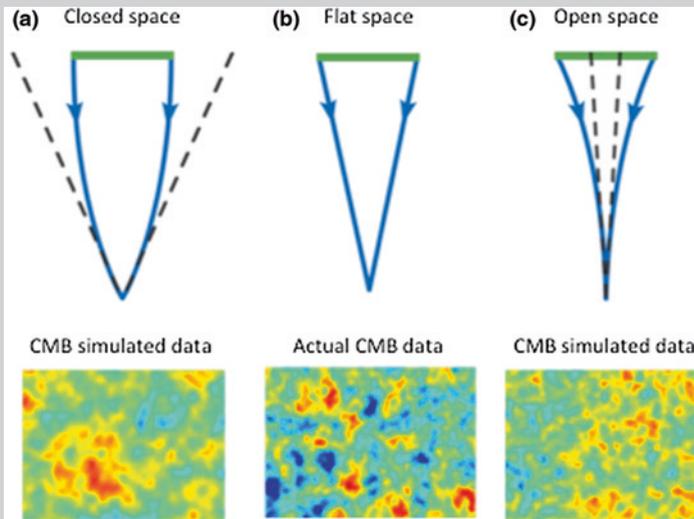
Fig. 17.2 CMB measurements show that the universe is flat. In the *top* figures, the solid *green lines* represent the physical size of the fundamental wavelength and the *blue lines* represent light propagation in closed, flat and open universes. The fundamental wavelength and the light rays emanating from its two ends form a giant triangle in space (as discussed in Chap. 4, the sum of angles in a triangle can be more than or less than 180° depending on the geometry of space). The *black dotted lines* indicate the angle subtended by the fundamental wavelength for each geometry. In a closed universe the fundamental wavelength subtends a larger angle than in flat space, and thus hot and cold spots would appear larger, as shown in the simulated CMB data in part (**a**). In an open universe the fundamental wavelength subtends a smaller angle than in flat space, and thus hot and cold spots would appear to be smaller, as shown in the CMB simulation in part (**c**). The actual CMB data in part (**b**) is consistent with a flat universe. *Credit* NASA

heights of the first and second peaks, cosmologists have been able to determine the relative amounts of atomic and dark matter. The results are perfectly consistent with calculations based on nucleosynthesis. Such consistency checks provide reassurance that our understanding of the early universe is on the right track.

The prediction of a scale invariant primordial spectrum is one of inflation's most important features. In addition to the CMB data which favor inflation, numerical simulations have been very successful in reproducing observations

of the large-scale structure of the universe—starting from the primordial scale-invariant spectrum (see Chap. 12).

17.3 Gravitational Waves

Another key prediction is that the tumultuous epoch of inflation generated gravitational waves that should also have a scale invariant spectrum (Fig. 17.3). As we discussed in Chap. 4, Einstein predicted the existence of gravitational waves nearly one hundred years ago. Gravitational waves can be detected because they stretch and squeeze space as they pass through it (without changing the volume), and this can cause distances between objects to change (see Fig. 17.4). However, this is a miniscule effect: for example, if a gravitational wave produced by a close pair of neutron stars rotating about one another were to pass between you and your friend on the other side of the room, the distance between you would be altered by less than the size of a proton! Although extremely challenging, gravitational waves from astrophysical sources have been recently detected (as discussed in Chap. 4).

Fig. 17.3 The Russian physicist Alexei Starobinsky was the first to show that gravitational waves would be generated during an inflationary period. He did so in 1979 in the context of the "Starobinsky model", which predated Guth's version of inflation. *Credit* PR Image iau1304b, Alexei Starobinsky, recipient of the 2013 Gruber Prize (https://www.iau.org/news/pressreleases/detail/iau1304/)

Fig. 17.4 A gravitational wave alternately stretches and squeezes a ring of freely floating test particles as it passes by

The origin of gravitational waves from inflation is similar to that of the primordial density fluctuations. The waves are produced by quantum fluctuations in the geometry of space and time. They originate in tiny regions of the Hubble size d_H, and then their wavelengths are stretched to astronomical sizes by the rapid inflationary expansion. The magnitude of the fluctuations is set by the Hubble parameter H, which is in turn determined by the false vacuum energy density [see Eq. (16.2)]. Since H remains nearly constant during inflation, the amplitude of gravitational waves is about the same for all wavelengths. In other words, the predicted gravitational wave spectrum is scale invariant.

Once created, primordial gravitational waves propagate through the universe. The predicted amplitude of the waves is too small to be directly detected with instruments like LIGO. However, primordial gravitational waves are expected to leave an imprint on the CMB radiation, both by impacting temperature fluctuations and by causing specific polarization patterns.

The reason gravitational waves can cause temperature fluctuations is because as the waves pass through the plasma at recombination, in some spots they stretch the plasma in our direction—that is, in the direction where our galaxy will eventually emerge, causing photons from those regions to be somewhat blue-shifted and thus hotter. In other spots the gravitational waves cause regions of plasma to be compressed away from us, and such regions appear red-shifted and thus cooler. It is difficult, however, to distinguish these temperature variations from those caused by primordial density fluctuations. But the polarization patterns induced by gravitational waves have a unique signal.

When photons scatter off electrons in the cosmic plasma, they get polarized—which means that the electric field of the photon gets oriented in a certain way (determined by the direction of motion of the incoming and scattered photons). With a large number of photons undergoing multiple
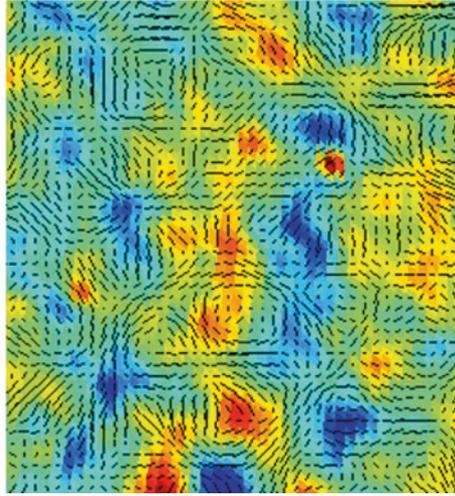
Fig. 17.5 Temperature hot and cold spots, plus polarization (*black line* segments) as measured by the Boomerang detector. The direction of polarization (that is, the direction of the electric field) in a region of the sky is indicated by line segments. *Credit* BOOMERanG experiment

scatterings, there is no net polarization. But at the epoch of recombination, just before the universe became transparent to radiation, the CMB photons scattered for the last time. The photons that we see come mostly from denser plasma regions, and we can see only photons that scattered in our direction. As a result, the observed CMB radiation is polarized (see Fig. 17.5). Primordial density fluctuations produce an E-mode pattern, consisting of radial and ring-like structures (see Fig. 17.6). In addition to E-modes, polarization caused by gravitational waves displays a swirl-like pattern that can be clockwise or anticlockwise; such patterns are called *B-modes*.

Many collaborations around the world have been searching for traces of primordial B-mode polarization. In March 2014, the BICEP 2 team[2] announced that they had found a pattern of polarization that is consistent with gravitational waves from inflation (see Fig. 17.7). Unfortunately,

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[2]BICEP stands for Background Imaging of Cosmic Extragalactic Polarization.
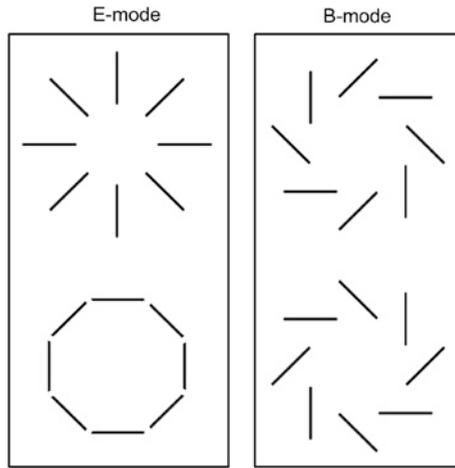
Fig. 17.6 Polarization pattern for E and B-modes. B-modes have a "curl" or a swirl-like pattern, and are produced by primordial gravitational radiation
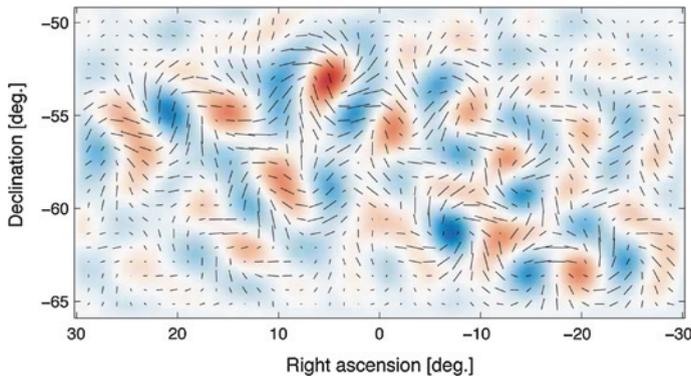
Fig. 17.7 BICEP 2 polarization data. The segments indicate the direction of polarization, after the E-mode pattern has been removed. The *blue* and *red spots* indicate whether the B-modes are clockwise or anticlockwise, respectively. Tightly wound spots have higher color intensity. *Credit* From: BICEP2 collaboration, Detection of B-Mode Polarization at Degree Angular Scales by BICEP2. PRL 112, 241101 (2014)

subsequent analysis and collaboration with other scientific teams showed that their detected B-mode polarization is probably due to galactic dust.

The search for B-modes continues in next generation experiments, and researchers are hopeful that they will detect signs of primordial gravitational
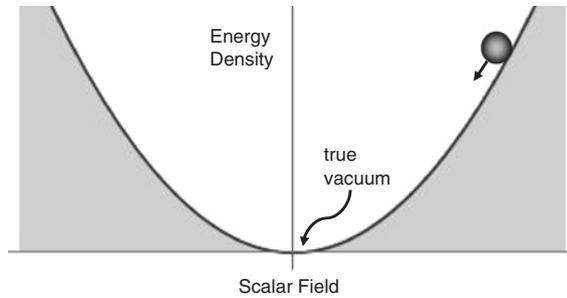
Fig. 17.8   "Topless" inflation model

waves soon. The stakes are very high, for two main reasons: Firstly, inflation predicts that the amplitude of the gravitational waves is proportional to the energy scale at which inflation takes place. Thus, if we can measure the magnitude of gravitational wave perturbations, we stand to learn about the physics behind inflation and about physics at energies that are far too high to be studied in accelerators. And secondly, these gravitational waves are thought to be produced via quantum mechanical effects, thus their existence could shed light on the unification of gravity and quantum mechanics.

17.4 Open Questions

The theory of inflation explains many puzzling features of the big bang and makes observational predictions that have been beautifully confirmed by the data. We thus have good reasons to believe that a period of accelerated inflationary expansion did occur in the early universe. This does not mean, however, that the problem of the origin of the universe has been solved.

First, we should emphasize that inflation is not a specific model, like the Standard Model of particle physics, but rather a paradigm encompassing a wide class of models. The main differences between the models are in the choice of the inflaton potential energy landscape. Linde's 1982 model assumed an energy hill with a flat hilltop, as illustrated in Fig. 16.8. A few years later, Linde proposed another model, where the energy hill keeps rising without limit in both directions (Fig. 17.8). Such a "topless" hill has a true vacuum at the bottom and no definite location for the false vacuum. The role of the false vacuum can be played by some point on the slope, where the inflaton field starts its downward roll. If the slope is sufficiently gentle, the field will roll slowly, and inflation will occur. Yet another possibility is a hybrid of

Linde's hilltop model with Guth's original scenario (we shall discuss this in more detail in Chap. 18). Cosmologists have also studied models of inflation including several scalar fields and models where the inflaton field is incorporated in a particular particle physics theory.[3]

Despite the variety of models, the predictions of inflation are rather robust. All models predict a nearly flat universe and an almost scale-invariant spectrum of density fluctuations. There are, however, some differences in the details. For example, the predicted small deviations from scale invariance are different for different models. The Planck satellite observations disfavor the "topless" models of Fig. 17.8, and as the empirical data continue to pour in, we can expect a further reduction in the number of viable models.

Even if we converge on a single model of inflation, that will not be the end of the story. All models assume that at the onset of inflation the universe was in a state of false vacuum. Why was it so? Only a tiny nugget of false vacuum is required, but even a small initial nugget calls for an explanation. Where did it come from? We shall discuss this and other questions raised by the theory of inflation in subsequent chapters.

Summary

Inflation makes several predictions, three of which we discussed here:

The universe is flat on the largest observable scales; density fluctuations have a nearly scale-invariant spectrum; and a scale-invariant spectrum of gravitational waves should also be present. The first two predictions have been observationally confirmed and the search for primordial gravity waves is currently underway. The idea of inflation appears to be on the right track and has by now become the leading cosmological paradigm.

Questions
1. Why was the discovery of dark energy such a boost for the theory of inflation?
2. Why are CMB temperature anisotropies so important?

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[3]Alexei Starobinsky suggested a model of inflation without scalar fields. In this model, the accelerated expansion of the universe is due to a quantum modification of Einstein's equations. Starobinsky introduced his model in 1979, before Guth published his first paper on inflation. But he did not realize that an accelerated expansion period explains the puzzling features of the big bang, so Guth is generally credited with the idea of inflation.

3. Why do temperature anisotropies on large angular scales (greater than 2°) represent density fluctuations as they emerged immediately following inflation?

4. Do hot patches in the CMB on large angular scales emerge from under- or over-dense regions? Explain your answer.

5. From the data in Fig. 17.1, estimate the magnitude of CMB temperature anisotropies on large angular scales. Can you tell from the figure that the spectrum of primordial density perturbations is approximately scale invariant?

6. What is a primordial sound wave?

7. Why are sound waves with varying wavelengths present in the early universe?

8. What physical processes give rise to the peaks in Fig. 17.1? Why is the fundamental peak higher than the other peaks?

9. Calculate the angle θ subtended by the horizon distance at recombination. *Hint:* in this calculation you can go through the following steps. First recall from Chap. 7 that the horizon distance at time t in the matter era is $d_{hor}(t) \approx 3ct$. Once you calculated $d_{hor}(t_{rec})$, find its present size, d, by accounting for the expansion of the universe from t_{rec} to present. The angle θ subtended by a distance d on the surface of last scattering can be found from the formula[4] $\theta = d/d_{ls}$ where the current distance to the surface of last scattering is $d_{ls} \approx 46 \times 10^9 lyrs$. (In Sect. 15.2 of Chap. 15 we explained that, because the CMB photons were last scattered so early in the universe's history, the distance to the surface of last scattering is approximately equal to the present horizon distance).

10. Find the physical size of the fundamental wavelength λ_f. *Hint:* λ_f should be equal to twice the distance traveled by sound from the big bang to the time t_{rec} (because the period of these waves is $2\ t_{rec}$). You can use the horizon distance $d_{hor}(t_{rec})$ calculated in Question 9 and the fact that sound waves propagate at 0.6 of the speed of light.

11. Explain how the CMB data is used to measure spatial curvature.

12. How could primordial gravitational waves cause temperature fluctuations in the CMB?

13. What do physicists mean when they say that light is polarized?

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[4]This formula assumes a flat geometry and gives the angular size in radians. If you want to express θ in degrees, you can use $2\pi\ radians = 360°$.

14. If scientists detect polarization in the CMB that is caused by primordial gravitational waves, what information can we learn about the universe?
15. Is inflation a specific theory or is it more like a general framework? In your opinion, is this good or bad? Do you think gravitational wave physics might help to hone in on a specific theory of inflation?
16. Does the theory of inflation fully explain the origin of the universe? If not, what questions does it leave unanswered?