21
The Principle of Mediocrity

According to the multiverse worldview, the constants of nature vary from one part of the universe to another. In some regions the constants allow for the existence of life, and that is where observers will evolve and those constants will be measured. Observers in different regions will generally measure different values of the constants. We don't know a priori what kind of region we live in, so we cannot predict the local values of the constants with certainty. However, it may be possible to make *statistical* predictions. Some region types may be more numerous or more densely populated than others, and we are more likely to find ourselves in one of these more populous regions.

21.1 The Bell Curve

If you have ever taken a large introductory college course, you have probably wondered if you are being graded on a "curve". The curve of course, is the so-called "bell curve" (see Fig. 21.1). What does the bell curve represent? Let's suppose the same final exam is given to a class of 300 students, every year for 20 years. If you were to randomly pick a name from a hat (containing the names of all students who have taken the class), what grade do you expect that student to have attained on their final exam? You would be surprised if the student's grade were, say, in the top or bottom 1% of the class. If the teacher supplied you with a set of data plotting the results of students who took that final for the last 20 years, as shown in Fig. 21.1, you

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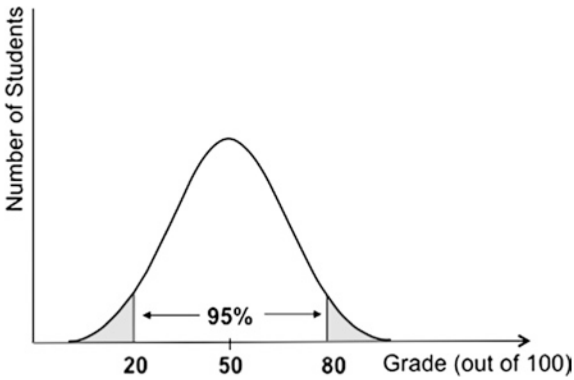

Fig. 21.1 Grade distribution of students in a large class. The number of students whose grades are within a specific range is proportional to the area under the corresponding piece of the *curve*. The median grade is 50 points. This means that half of the students got grades above, and the other half below, this value. The *shaded tails* represent the lowest and highest 2.5%. The range of grades between the two *shaded areas* is predicted at 95% confidence level

would be able to make more accurate predictions. If you discard the 2.5% of students who got the highest and lowest grades, then with 95% confidence you can say that a randomly picked student got a grade in the remaining 95% range (that is, between 20 and 80 out of 100 in the example shown in Fig. 21.1). This means that if you were to pick name after name, then 95% of the time you would pick students who scored in the aforementioned range. This is called a prediction at 95% confidence level. In order to make a 99% confidence prediction, you would have to discard 0.5% at both ends of the distribution. As the confidence level is increased, your chances of being wrong get smaller, but the predicted range of grades gets wider and the prediction less interesting.

21.2 The Principle of Mediocrity

A similar technique can be applied to make predictions for the constants of nature. Suppose for a moment that there is a Universal Super Observer who can survey all of spacetime and measure anything she wants. The USO looks around and sees many different regions of the universe with different values for the constants of nature. She decides to count the number of observers who live in regions that have different values of a certain constant, call it X. To isolate the effect of varying X, she focuses on regions where the other constants have nearly the same values and only X changes from one region to another.[1] The USO finds that some of the regions contain many observers, some only a few, and others have none. She can then plot the number of observers who will measure various values of X. The resulting distribution will most likely be similar to a bell curve. If the USO graciously gives us the distribution, we could discard 2.5% at both of its ends and make a 95% confidence level prediction for the value of X measured by a randomly selected observer.

What would be the use of such a prediction? Obviously, we would not be able to test it directly—we can't "pick up the phone" and ask randomly selected observers to disclose their measurements—because all regions with different values of X are beyond our horizon. What we can do, though, is to think of ourselves as having been randomly selected. Since we have no a priori reason to believe that the values of the constants in our region are unusually large or small, or otherwise very special, it makes sense to assume that we are typical, or unexceptional observers. This assumption is called the *principle of mediocrity*. If there are some constants of nature that we have not yet measured, and if we have somehow obtained the statistical distribution for their values measured by all the observers in the multiverse, we can use the principle of mediocrity to predict that the values of the constants in our local region should correspond to the range around the peak of the distribution.

But where are we going to get the distribution? In lieu of a cooperative USO, we will have to derive it from our theory of the multiverse. If the resulting predictions agree with our measurements, this would provide evidence supporting the theory; if not, the theory can be ruled out at a specified confidence level.

21.3 Obtaining the Distribution by Counting Observers

Let us now discuss how the distribution of the constants measured by randomly picked observers can be derived from the theory. We have to count the number of observers in regions with different constants. In order to do

[1] If the vacuum landscape is indeed as rich as string theory suggests, it will include vacua with practically any values of the constants. So the USO will have no problem finding regions where X varies while the other constants are nearly fixed.

so, we need to know the density of observers in each environment, and the corresponding volume. The volume factor can in principle be calculated from the theory of inflation (we'll discuss this in Sect. 21.5). However, since we are mostly ignorant about the evolution of life and intelligence, how can we hope to calculate the density of observers that may arise in different environments?

For starters, we should note that some constants of nature are "life-changing" and some are "life-neutral". A "life-changing" constant is one that directly affects the physics and chemistry of life, and thus directly impinges on the ability of life to evolve and thrive within a galaxy. Examples of life-changing constants include the electron mass and the gravitational constant. On the other hand, a "life-neutral" constant is one that, as long as there are galaxies around, does not directly influence the ability of life to emerge. For example, the cosmological constant and the magnitude of the primordial density fluctuations are life-neutral constants. As long as their values lie within the windows that permit the formation of galaxies in a region, they do not influence the density of observers within any galaxy.[2]

At the present level of understanding, we can only attempt to calculate the distributions of life-neutral constants. Furthermore, our ignorance about the emergence of life can be factored out if we focus on those regions where the life-changing constants have the same values as in our neighborhood, and only the life-neutral constants are different. All galaxies in such regions will have about the same number of observers, and thus to compare the density of observers in different regions, we only need to compare the density of galaxies. In other words, *we can use the density of galaxies as a proxy for the density of observers*. We shall now discuss how this approach was used to predict the value of the cosmological constant.

21.4 Predicting the Cosmological Constant

The anthropic bound on the cosmological constant ρ_v specifies the value above which the vacuum energy would dominate too soon for any viable galaxies to form. In regions where ρ_v is near this bound, galaxy formation is barely possible, and the density of galaxies is very low. But most observers

[2]This is a bit of an oversimplification. Some properties of galaxies may in fact change due to variation of life-neutral constants. For example, if the density fluctuations get larger, galaxies form earlier and have a higher density of matter. As a result, close encounters between stars, which can disrupt planetary orbits and extinguish life, become more common.

will not live in these lonely places; they will live in regions that are teeming with galaxies. Thus, if we assume that we are typical observers, we should expect to live in one of the galaxy-rich regions and to measure a cosmological constant that is significantly lower than the anthropic bound.

21.4.1 Rough Estimate

A rough estimate of the expected value of ρ_v can be obtained as follows. Let us consider a large ensemble of regions where ρ_v takes a variety of values, while the other constants are very close to what they are in our local neighborhood. Depending on the value of ρ_v, the vacuum energy in these regions will start dominating at different times t_v, or different redshifts z_v. Once the vacuum energy dominates, galaxy formation comes to a halt, so regions where the vacuum domination occurs after only a few galaxies have had a chance to form, will have sparse observers.

As we discussed in Chap. 12, galaxy formation proceeds in a hierarchical manner, with smaller clumps merging to form larger and larger structures. Large galaxies like ours, massive enough to efficiently form stars and to retain the heavy elements dispersed in supernova explosions, are formed at redshifts $z \sim 2$ or later. In galaxy-rich regions the vacuum domination should occur at a later time, and thus we must have $z_v < 2$ (Remember: smaller redshifts correspond to later times.). Now, the density of matter at $z = 2$ is $\rho_m = (1+z)^3 \rho_{m0} = 27\rho_{m0}$, where ρ_{m0} is the present value of ρ_m.

Requiring that the vacuum energy does not dominate at this epoch, we obtain (see Sect. 5.1 in Chap. 5)

$$\rho_v < \frac{\rho_m}{2} \approx 14\rho_{m0} \tag{21.1}$$

The values of ρ_v measured by most observers in the multiverse living in environments similar to ours are expected to satisfy this condition.

21.4.2 The Distribution

The probability distribution for the values of ρ_v measured by randomly picked observers requires a more careful calculation. The result of such a calculation is plotted in Fig. 21.2. The distribution is peaked at $\rho_v \sim 3\rho_{v0}$, where $\rho_{v0} \sim 2\rho_{m0}$ is the observed value, and the 95% confidence range is between $\sim 0.1\rho_{v0}$ and $\sim 20\rho_{v0}$. Values of $\rho_v > 20\rho_{v0}$ are not likely to be observed because there are very few galaxies in the corresponding regions. Very small

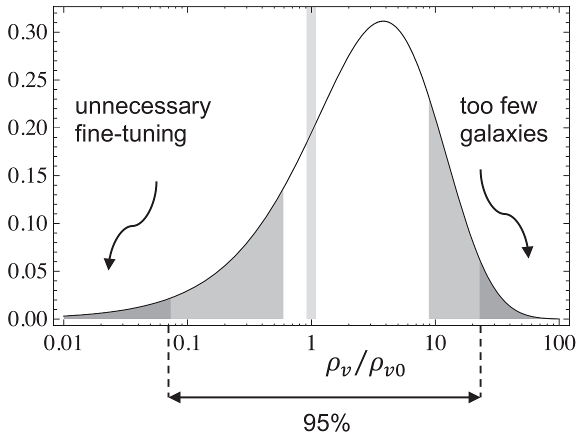

Fig. 21.2 Probability for a randomly picked observer to measure a given value of ρ_v. The *dark grey* and *light grey areas* mark the values excluded at 95 and 67% confidence levels, respectively. The *vertical bar* marks the observed value. From A. De Simone, A. Guth, M. Salem and A. Vilenkin, Phys. Rev. D78, 063520 (2008)

values of $\rho_v < 0.1\rho_{v0}$ are also unlikely, simply because this range of values is so narrow. A value in this range would amount to unnecessary fine-tuning; that is, a fine-tuning even more severe than required by the anthropic considerations.

Throughout this section, we implicitly assumed that $\rho_v > 0$. A similar analysis can be performed for negative values of ρ_v, with very similar conclusions. The main difference is that the bound on large negative values of $\rho_v < -20\rho_{v0}$ comes from requiring that the universe does not collapse to a big crunch before some galaxies manage to form.

Anthropic bounds on ρ_v were first derived in 1987 by Steven Weinberg and by Andrei Linde. A prediction based on the principle of mediocrity was made by Vilenkin in 1995 and was later refined by George Efstathiou (1995) and by Hugo Martel, Paul Shapiro and Weinberg (1998). At the time anthropic arguments were highly unpopular,[3] and it came as a complete surprise to most physicists when a vacuum energy density of roughly

[3]The referee of the Astrophysical Journal objected to publishing papers based on anthropic reasoning, so in order for Martel Shapiro and Weinberg to get their 1998 paper accepted, they had to convince the editor, that if ρ_v was ever measured to be below a certain value, this would show that anthropic reasoning could not explain it. Of course, the value of ρ_v turned out to be just in the sweet spot for an anthropic explanation to make perfect sense.

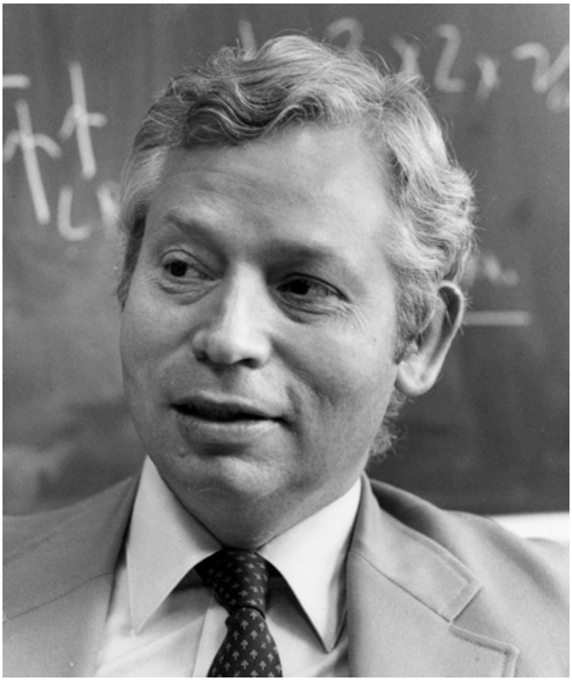

Fig. 21.3 Steven Weinberg won the 1979 Nobel Prize for his work on the Standard model of particle physics. He also made seminal contributions to cosmology. *Credit* AIP Emilio Segre Visual Archives

the expected magnitude was detected in supernova observations in 1998. As of now, no viable alternative explanations for the observed value of ρ_v have been proposed. This may be our first observational evidence for the existence of a multiverse (Fig. 21.3).

21.5 The Measure Problem

In order to calculate the probability distribution for values of a certain constant measured by randomly picked observers, we need to know the fraction[4] of volume of the universe where various values of the constants are realized, and also the density of observers in each of these environments. For life-neutral constants the density of observers is proportional to the density

[4]See question 4 to convince yourself that it is sufficient to use volume fractions, instead of actual volumes, to calculate probabilities.

of galaxies, which can be calculated in a relatively straightforward way. However, the calculation of the volume fraction presents a serious problem.

The problem arises because the volumes of all kinds of environments in the multiverse grow without bound and become infinite in the limit. So, to find the fraction of the volume occupied by a given environment we have to compare infinities, and this is a mathematically ambiguous task. This can be illustrated by a simple example of an infinite sequence of integers:

$$1, 2, 3, 4, 5, 6, 7, 8, \ldots$$

We can ask: What fraction of the integers are odd? You probably guessed ½. Indeed, if we take N numbers in a row, the fraction of odd numbers will be close to ½ for large N and will exactly equal ½ in the limit of infinite N.

But if you reorder the sequence so that each odd integer is followed by two even integers,

$$1, 2, 4, 3, 6, 8, 5, \ldots$$

then the answer would be 1/3 (even though this sequence contains all the same integers as the natural ordering). In fact, by reordering the sequence one can obtain any answer to this question between 0 and 1.

In this particular example the ambiguity can be avoided by requiring that the natural order of integers should be used. The answer is then ½, as one intuitively expects. We could try to adopt a similar prescription for the volume fraction in the multiverse, using the natural ordering of events in time. This would amount to including only the regions (e.g., bubble universes) that were formed prior to a certain time t. The problem is, however, that the result depends on how time is defined in different places. There is no unique, or preferred way to do so in general relativity. One can use, for example, the time measured by the clocks of local observers; this is called the *proper time measure*. Alternatively, one could use the expansion of the universe as a measure of time. Equal times would then correspond to equal values of the scale factor; this is referred to as the *scale factor measure*. There is an infinity of possible choices, and thus the volume fraction remains ambiguous. This ambiguity is known as *the measure problem.*

Cosmologists have studied different measure prescriptions and found that some of them lead to paradoxes or to a conflict with the data and should therefore be discarded. For instance, the proper time measure performed rather poorly, while the scale factor measure has successfully passed all tests so far.[5] It is unlikely, however, that this kind of analysis will yield a unique prescription for the probabilities. This suggests that some important element may be missing in our understanding of cosmic inflation.

Some people feel the measure problem is so grave that it puts the validity of the theory of inflation seriously in doubt. But this is the view of only a small minority of cosmologists. The situation with the theory of inflation is similar to that with Darwin's theory of evolution some 100 years ago. Both theories greatly expanded the range of scientific inquiry, proposing an explanation for something that was previously believed impossible to explain. In both cases, the explanation was compelling, and no viable alternatives have been suggested. Darwin's theory was widely accepted, even though some important aspects remained unclear before the discovery of the genetic code. The theory of inflation may be similarly incomplete and may require additional new ideas. But it also has a similar air of inevitability.

21.6 The Doomsday Argument and the Future of Our Civilization

The principle of mediocrity has been used in many different contexts. As an example, suppose you are presented with a bag containing N cards. You know the cards are labeled 1 through N, but you don't know what N is. Now you draw one card at random and see number 15 written on it. Based on this, how would you estimate the total number of cards N?

The principle of mediocrity suggests that your card is not likely to be from the very beginning or the very end of the list and comes most likely from somewhere in the middle. Then your best estimate is $N = 30$. If you want to make a 90% confidence prediction, this would be $16 < N < 300$. (Can you figure out how we obtained these numbers?). You can make a more accurate prediction if you draw more than one card. The Allied forces used a similar method during World War II to estimate the total number of German tanks based on the serial numbers of the tanks that they captured.

If we imagine giving a "serial number" to every person at birth, we can use the same reasoning to predict the total number of humans who will ever live. The number of people who have lived on Earth since the origin of our

[5] The distribution for the cosmological constant in Fig. 21.2 was calculated using the scale factor measure. In fact, analysis shows that this distribution is not very sensitive to the choice of measure, so the prediction for the cosmological constant is rather robust and is not expected to change much when the measure problem is finally resolved.

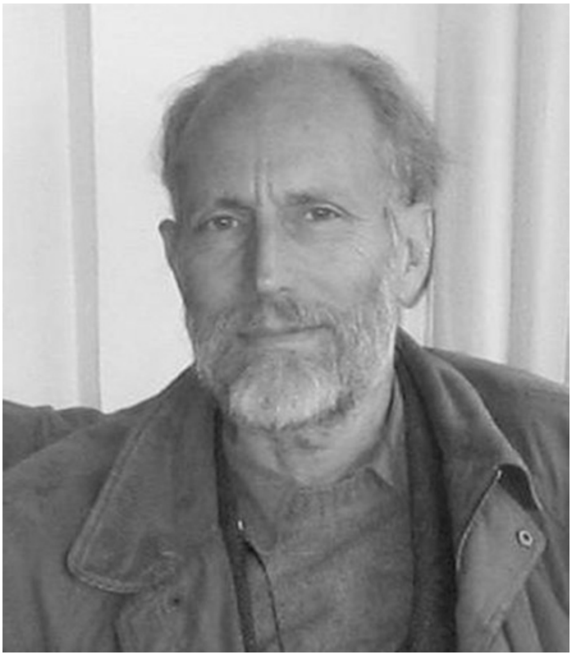

Fig. 21.4 Brandon Carter is known for his important work on the properties of black holes. He also introduced the anthropic principle and the doomsday argument. *Credit* Courtesy Brandon Carter

species is about 100 billion, so our best estimate is that 200 billion people will ever live. If the world birth rate stabilizes at its current value (130 million births per year), this number will be reached in less than 800 years.

This is the notorious "doomsday argument", first presented by Brandon Carter in 1983. The argument becomes more subtle and the prediction less gloomy if one takes into account the existence of multiple other civilizations in the universe. It should be noted that the doomsday argument is rather controversial and many people believe that the principle of mediocrity should not be used in this context (Fig. 21.4).

21.6.1 Large and Small Civilizations

For any civilization confined to a single planet, the prospects of long-term survival are rather bleak. It can be destroyed by an asteroid impact or a nearby supernova explosion, or it can self-destruct in a nuclear war. It is not a matter of *if* but rather of *when* the disaster will strike, and the only sure way for the civilization to survive in the long run is to spread beyond its native planet and colonize space.

An advanced civilization may colonize nearby planetary systems. The colonies may then spread further, until the entire galaxy is colonized. It is conceivable that civilizations could expand even beyond their native galaxies.

The probability for a civilization to survive the existential challenges and colonize its galaxy may be small, but it is non-zero, and in a vast universe such civilizations should certainly exist. We shall call them *large* civilizations. There will also be *small* civilizations which die out before they spread much beyond their native planets.

For the sake of argument, let us assume that small civilizations do not grow much larger than ours and die soon after they reach their maximum size. The total number of individuals who lived in such a civilization throughout its entire history is then comparable to the number of people who ever lived on Earth, about 100 billion.

A large civilization contains a much greater number of individuals. A galaxy like ours has about 100 billion stars and about 20% of the stars have habitable planets (see Sect. 13.5). This amounts to 20 billion habitable planets per galaxy. Assuming that each planet will reach a population similar to that of the Earth, we get $\sim 2 \times 10^{21}$ individuals. The numbers can be much higher if the civilization spreads well beyond its galaxy. The crucial question is: what is the probability P for a civilization to become large?

It takes more than 10^{10} small civilizations to provide the same number of individuals as a single large civilization. Thus, unless P is extremely small (less than 10^{-10}), individuals live predominantly in large civilizations. That's where we should expect to find ourselves if we are typical inhabitants of the universe. Furthermore, a typical member of a large civilization should expect to live at a time when the civilization is close to its maximum size, since that is when most of its inhabitants are going to live.

These expectations are in glaring conflict with what we actually observe: we either live in a small civilization or at the very beginning of a large civilization. With the assumption that P is not very small, both of these options are very unlikely—which indicates that the assumption is probably wrong. If indeed we are typical observers in the universe, then we have to conclude that the probability P for a civilization to survive long enough to become large must be very tiny. In our example, it cannot be much more than 10^{-10}.

21.6.2 Beating the Odds

The Doomsday argument is statistical in nature. It does not predict anything about our civilization in particular. All it says is that the odds for any given civilization to grow large are very low. At the same time, some rare civilizations do beat the odds.

What distinguishes these exceptional civilizations? Apart from pure luck, civilizations that dedicate a substantial part of their resources to space colonization, start the colonization process early, and do not stop, stand a better chance of long-term survival. With many other diverse and pressing needs, this strategy may be difficult to implement, but this may be one of the reasons why large civilizations are so rare.

One question that needs to be addressed is: why is our Galaxy not yet colonized? There are stars in the Galaxy that are billions of years older than our Sun, and it may take less than a million years for an advanced civilization to colonize the entire Galaxy. So, we are faced with Enrico Fermi's famous question: Where are they? A possible answer is that we may be the only intelligent civilization in our Galaxy and maybe even in the entire observable universe. Evolution of life and intelligence may require some extremely improbable events, as we discussed in Chap. 13. Their probability may be so low that the nearest planet with intelligent life could be far beyond our horizon.

Summary
If the constants of nature vary from one part of the universe to another, their local values cannot be predicted with certainty, but we can still make *statistical* predictions.

We have no a priori reason to believe that the values of the constants in our region are particularly special, so it makes sense to assume that we are typical, or unexceptional observers. This assumption is called the *principle of mediocrity*. It suggests that the probability for us to measure certain values of the constants in our local region is the same as for a randomly picked observer in the multiverse. Even though our understanding of the multiverse is rather incomplete, in some special cases we can calculate probabilities from the theory of eternal inflation.

This strategy was applied to the cosmological constant and led to a prediction, which was later confirmed by the 1998 Supernova observations. This could be our first evidence for the existence of the multiverse.

In general, making statistical predictions in the multiverse is notoriously difficult. The problem is that an eternally inflating spacetime will produce an infinite number of all kinds of regions. So comparing the relative likelihood of one type versus another involves a comparison of infinite numbers. Cosmologists have attempted to regulate these infinities, in order to make sensible predictions, but in general the issue is still unresolved.

Questions

1. At the end of Chap. 9 we noted the remarkable fact that we live at a very special epoch when the density of matter is comparable to that of the vacuum: $\rho_m \sim \rho_v$. At much earlier times ρ_m was much greater and in the distant future it will be much smaller than ρ_v. Can you explain this fact using anthropic arguments? (For a complete explanation, you may need to know that the lifetime of a star like our Sun is about 10 Gyr.)
2. Which of the following parameters are life-changing and which are life-neutral: the electron charge, the neutrino mass, the strength of strong nuclear force, the large-scale curvature of the universe.
3. In some region of the multiverse the density of matter is measured to be the same as in our neighborhood, while the vacuum energy density is $\rho_v = 8\rho_{m0}$. At what redshift z_v did the vacuum dominated era begin in this region?
4. A population survey on some planet revealed that 5% of its territory is occupied by cities, 75% by rural areas, and the remaining 20% is not suitable for living. The population density (that is, the number of inhabitants per square kilometer) is 50 times higher in cities than in rural areas. What is the probability that a randomly picked inhabitant lives in a city? (This problem illustrates how volume fractions can be used to calculate probabilities.)
5. What ordering of integers would yield the fraction of odd integers equal to 2/3? Is this ordering unique, or can you find other orderings that give the same answer?
6. Considering that the number of people who have ever lived is presently about 100 billion, use the doomsday argument to estimate the number of people who will ever live on Earth at 90% confidence level.
7. (a) Are you persuaded by the doomsday argument? If not, what do you think is wrong with it?

   (b) Suppose humans will use bioengineering to evolve into some more advanced species within a few hundred years from now. Would the doomsday argument still apply?