10
The Quantum World

Modern physics began with two revolutions at the turn of the 20th century. The first revolution, which radically changed our concepts of space and time, was single handedly accomplished by Einstein with his special and general theories of relativity. The development of quantum mechanics by a number of physicists ushered in the second revolution, which shook the foundations of physics even more than the first. Quantum mechanics was developed as a theory of the microworld but as we shall see, quantum effects are essential in the early universe and even play a role on the largest cosmic scales.

10.1 Quantum Discreteness

According to quantum mechanics, at the microscopic level electromagnetic waves consist of *photons*—small bundles (or quanta) of electromagnetic energy. Photons always travel at the speed of light and have zero rest mass. The energy of a photon is inversely proportional to its wavelength λ, and is given by

$$E = \frac{hc}{\lambda}, \tag{10.1}$$

where h is Planck's constant: in SI units $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$. Scientists often use the reduced Planck constant, $\hbar \equiv h/2\pi$—we will use both. Because h is

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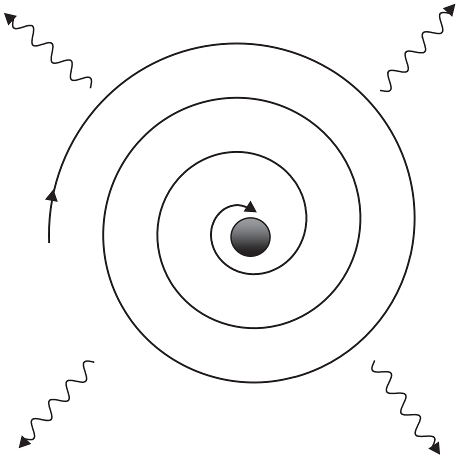

Fig. 10.1 Classical mechanics and electromagnetism predict that orbiting electrons will radiate electromagnetic waves, lose energy and spiral in toward the nucleus

such a tiny number, a photon typically carries a tiny amount of energy. The classical wave description of light is accurate when we have a large number of photons; for example, a 100 W light bulb produces about 10^{19} photons per second.

Quantum discreteness is also manifested in atomic structure. The early 20th century "planetary" model of the atom consisted of negatively charged electrons orbiting around a positively charged nucleus, much like the planets orbit the Sun. However, Maxwell's theory of electromagnetism predicts that charged particles moving along curved trajectories will radiate electromagnetic waves. Thus physicists were puzzled by how the electrons could maintain their stable orbits, and avoid continuously radiating away energy that would cause them to spiral into the nucleus (Fig. 10.1).

In quantum theory, atomic electrons are allowed to occupy only a discrete set of orbits, with each orbit having a specific energy. An electron can emit a photon and jump to a lower orbit, as shown schematically in Fig. 10.2.[1] This process must conserve energy, so the energy of the photon must be equal to the energy difference between the two orbits. The inverse process is also possible—an electron can absorb a photon and jump to a higher-energy orbit. Thus atoms can emit and absorb photons of only specific energies (and wavelengths). The existence of discrete energy levels is essential for spectroscopic measurements, which provide much of the information that we have about the universe.

[1]Electron orbits are actually somewhat fuzzy and are more accurately described by wave functions; see Sect. 10.3.

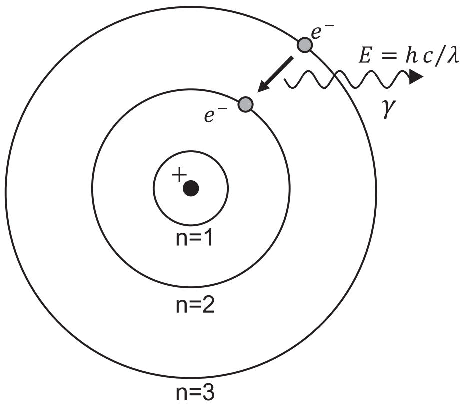

Fig. 10.2 Quantum model of the atom. *Larger* orbits have higher energy. n = 1 is the lowest energy level of the atom. Here, a photon is emitted when an electron jumps from the n = 3 level to the n = 2 level. The energy of the photon is equal to the energy difference between the levels, so the total energy is conserved

10.2 Quantum Indeterminism

The quantum world is fundamentally unpredictable. We can never know for certain where a given particle will be or how fast it will move; the best we can do is to predict probabilities for possible future positions and velocities. This is in contrast to classical, Newtonian physics, where the entire future history of a particle can be predicted from its position and velocity at some initial moment (Fig. 10.3).

At the core of quantum physics is the uncertainty principle, discovered by Werner Heisenberg in 1927. It states that the position and velocity of a particle cannot be simultaneously determined. The more precisely we measure the position, the greater is the uncertainty in the velocity, and vice versa. This is encoded in the equation,

$$\Delta x \cdot \Delta v > \frac{h}{4\pi m} \tag{10.2}$$

where Δx and Δv are respectively the uncertainties in the particle's position and velocity, and m is the particle's mass (this equation applies only to non-relativistc particles). If we make Δx very small, then Δv will get large—in a sense, the more we try to localize the particle, the more it tries to "escape". A quantum particle is thus inherently fuzzy and cannot be assigned a definite trajectory.

Macroscopic objects, like planets or billiard balls, follow their classical trajectories with very high probability, which is why the motion of planets can

Fig. 10.3 Werner Heisenberg. *Credit* AIP Emilio Segre Visual Archives, Gift of Jost Lemmerich

be predicted for many centuries to come. But for small particles, like electrons, deviations from classical motion can be very large. Such unpredictable deviations are called quantum fluctuations.

One of the most striking examples of quantum fluctuations is illustrated in Fig. 10.4. A ball lies at a low point in a one-dimensional landscape, separated by a hill from a still lower valley. In the world of classical physics, the ball would stay where it is, unless someone kicks it, providing the energy necessary to get over the hill. But in the quantum world, there is a non-zero probability that the ball will suddenly and discontinuously emerge on the other side of the hill and start rolling down. This process is called "quantum tunneling". The larger the energy barrier (or height and/or width of the hill) that needs to be surmounted, the smaller is the tunneling probability.

While tunneling might sound like an exotic quantum effect, it has many real world consequences and applications. It explains, for example, the phenomenon of alpha radioactivity, where an alpha particle (consisting of two

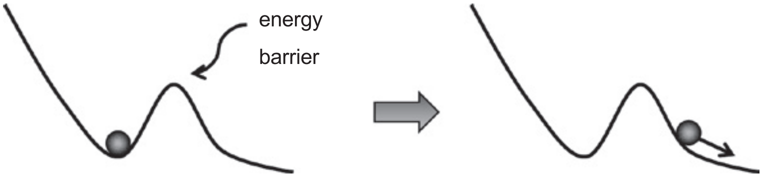

Fig. 10.4 Quantum tunneling through an energy barrier

Fig. 10.5 Erwin Schrodinger. *Credit* Photograph by Francis Simon, *courtesy* AIP Emilio Segre Visual Archives

protons and two neutrons) is emitted from inside a nucleus, despite the energy barrier produced by attractive nuclear forces. Also, the scanning tunneling electron microscope can be used to see individual atoms on the surface of a material. When a sharp conducting probe is scanned above a surface, the distance between the probe and the surface will vary slightly, depending on the arrangement of the surface atoms. When the surface atoms are closer to the probe it will be easier for electrons to tunnel from the surface to the probe, which then registers a current. Thus, by measuring the rate at which electrons tunnel from the surface to the probe, one can image the individual bumps and depressions of atoms on the surface (Fig. 10.5).

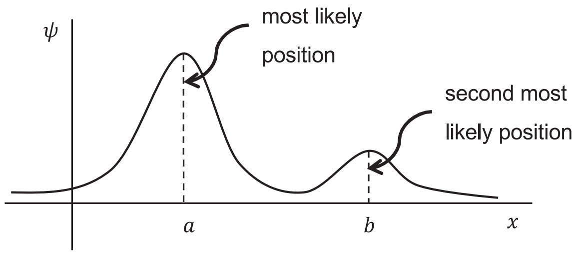

Fig. 10.6 Wave function of a particle

10.3 The Wave Function

In quantum theory, a particle is mathematically described by a wave function $\psi(x, t)$, which is a function of position x and time t. It contains all the information we can have about the particle. The shape and time evolution of the wave functions are determined by the so-called Schrodinger equation, derived by Erwin Schrodinger in 1927. The wave function does not tell us where the particle is located; it only determines the *probability* to find it in one location or another.[2] Suppose at some moment of time an electron is described by the wave function shown in Fig. 10.6. If we measure the position of the electron, we are most likely to find it near position a, where ψ has the largest magnitude. The second most likely possibility is to find it near position b, and there is some non-zero probability that it is at any other location where the wave function is not zero.

If we perform many identical measurements, their outcomes will be distributed according to the probabilities predicted by the wave function.

Prior to a measurement, an electron described by the wave function in Fig. 10.6 does not have any definite position. We say it is in a superposition of states corresponding to different positions. Once we perform a measurement, we know where the electron is at that moment, so the wave function "collapses" to a peaked shape around that point, as in Fig. 10.7. The electron will not generally stay localized, and the peak of the wave function will immediately start to spread. Once again, its time evolution can be found by solving the Schrodinger equation, and the resulting wave function can be used to determine the probabilities of future measurements.

If we prepare a large number of electrons in the same quantum state (described by the same wave function) and perform identical experiments

[2]More precisely, the probability distribution is given by the square of the wave function.

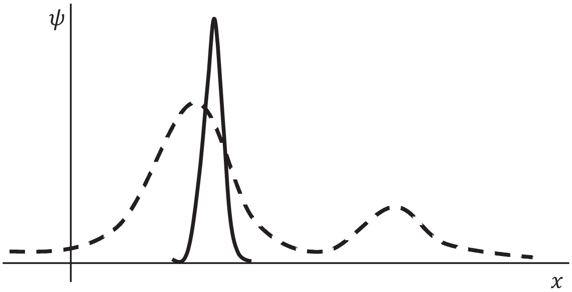

Fig. 10.7 Collapse of the wave function. The *solid line* represents the wave function after measurement

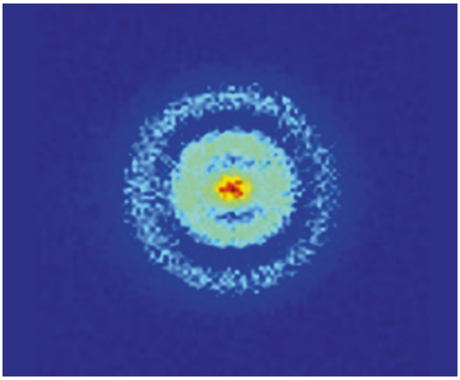

Fig. 10.8 The measured probability distribution for one of the energy levels in a hydrogen atom. Each *dot* on this image represents the measured location of an electron relative to the nucleus. By using a large collection of hydrogen atoms, whose electrons are all in the same energy level, this image represents the probability distribution, or wave function, of the electron. *Credit Stodolna et al. Phys. Rev. Lett. 110, 213001*

measuring the positions of the electrons, the data points will provide an image of the probability distribution, as shown in Fig. 10.8.

The wave function description is not limited to the positions of particles; it can be applied to any quantum system. As another example, consider a radioactive atom, whose nucleus can decay by emitting an alpha particle. Radioactive decay is a fundamentally random process, so you cannot predict the time of decay. You can only determine the decay probability per unit time (say, per hour). Suppose you checked that the atom is intact at some initial moment and placed it in a sealed box, so you cannot observe it. Then, at a later time, the wave function of the atom will be a superposition of decayed and un-decayed states. Just like the electron in the previous example

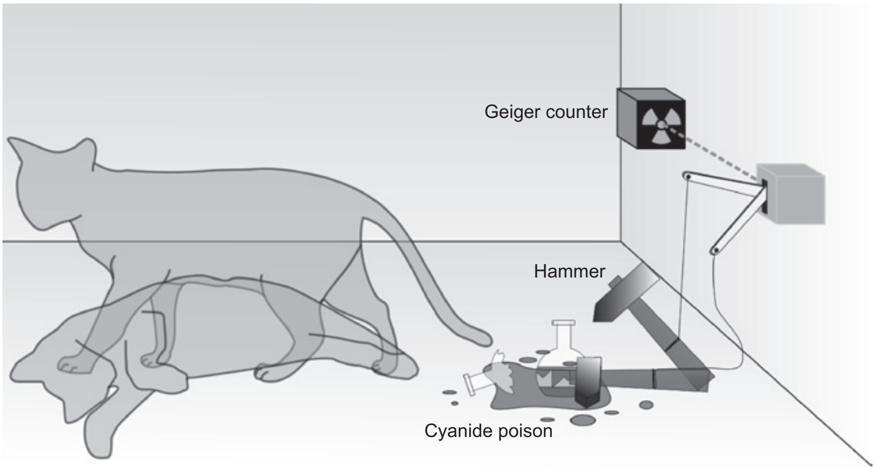

Fig. 10.9 Schrodinger's cat. *Credit* Dhatfield, wikipedia

did not have any definite position, the atom has no definite state of decay. You can open the box and inspect the atom; then the wave function will collapse to either a decayed or un-decayed state, with probabilities that you can calculate. It appears that the atom "makes up its mind" at the last moment, when the measurement is performed.

To highlight just how bizarre this is, Erwin Schrodinger proposed the following thought experiment. Imagine there is a cat in a perfectly sealed box containing a radioactive atom and a Geiger counter. There is also a flask of cyanide poison in the box. If the radioactive atom decays, the Geiger counter detects a signal that triggers a hammer to smash open the poison, instantaneously killing the cat. We should now describe the entire content of the box by a wave function, and it will be a superposition of two states—an intact atom plus a living cat and a decayed atom plus a dead cat. The cat is thus in a superposition of "dead" and "alive" states! If we were to open the box and look inside, the cat would suddenly become either "alive" or "dead"—its wave function would "collapse" (Fig. 10.9).

If you are scratching your head, rest assured you are not alone. The probabilistic interpretation of the wave function that we outlined above was developed by Max Born and by Niels Bohr and his colleagues at his institute in Copenhagen; it is called the Copenhagen interpretation. But some of the founders of quantum mechanics never accepted quantum indeterminism. Most notable amongst them was Einstein who quipped: "God does not play dice with the universe".

10.4 Many Worlds Interpretation

In 1957 a Princeton graduate student, Hugh Everett, proposed an alternative interpretation of quantum mechanics which postulates that the wave function never collapses. Instead, all possible outcomes of any measurement do occur, but they occur in "parallel" universes, which have no contact with one another.

With every measurement of a particle's position, the universe branches into multiple copies of itself, where the particle is found to be in all possible places.

The branching process is described by the Schrodinger equation and is fully deterministic. But we cannot predict which of the parallel universes we will find ourselves in, and thus the outcomes of *our* measurements can still be determined only probabilistically. Everett showed that the probabilities come out exactly the same as when one uses the Copenhagen interpretation. Everett's approach is now called "the many worlds interpretation" (Fig. 10.10).

Debate about the meaning of the wave function still continues. But despite this uncertainty about its philosophical foundations, quantum mechanics is a tremendously successful theory which has been crucial for

Fig. 10.10 Hugh Everett circa 1964. *Credit* Courtesy of Mark Everett. Hugh Everett III Manuscript Archive, UCISpace@the Libraries Permanent url: http://hdl.handle.net/10575/1060

our understanding of atomic structure, chemistry, biochemistry, particle physics and so on. All of its predictions have been borne out by experiments with incredible precision. It is also the theoretical framework that underpins the technology of transistors, atomic clocks, lasers, superconductivity, etc.

Since the choice of interpretation does not affect any predictions of the theory, most physicists simply disregard the philosophical problems and follow the dictum "*Shut up and calculate!*" This attitude works fine, except in cosmology, where one might want to apply quantum theory to the entire universe. The Copenhagen interpretation, which requires an external observer to perform measurements on the system, cannot even be formulated in this case: there are no observers external to the universe. Cosmologists, therefore, tend to favor the many worlds interpretation.

Summary

The physics of the microworld is governed by the inherently discrete quantum mechanics. In particular, the classical picture of electromagnetic waves gives way to a quantum description in which light consists of photons that carry discrete amounts of energy. Atomic electrons also have quantized energies that can increase or decrease only by discrete amounts via the absorption or emission of photons. This gives rise to the spectroscopic absorption and emission lines.

In contrast to the classical, deterministic universe, the quantum world is fundamentally unpredictable. Even if we have complete information about a quantum system, we can only make probabilistic predictions about its future evolution. Macroscopic bodies, like cars or tennis balls, behave nearly classically, but in the microworld, unpredictable deviations from the classical motion, called quantum fluctuations, are typically large.

In quantum physics a particle is described by a *wave function,* which determines the probability for the particle to be in various locations. According to the Copenhagen interpretation of quantum mechanics, once we perform a measurement, the wave function "collapses" and the particle then momentarily has the measured position. An alternative interpretation of quantum mechanics, called "the many worlds interpretation", asserts that all possible outcomes of the measurement occur in disconnected "parallel" universes. We cannot determine which universe we are in, thus the future events we expect to observe can only be predicted probabilistically. Regardless of how we interpret quantum mechanics, its predictions remain the same.

Questions

1. Einstein was one of the founders of quantum mechanics. Yet he still felt uneasy about the probabilistic nature of quantum mechanics. Do you find the probabilistic world of quantum mechanics more or less appealing than the deterministic classical universe?
2. Epicurus asserted that atoms move deterministically, but occasionally experience random "swerves". He thought the swerves were necessary to explain the existence of free will. Quantum mechanics appears to provide something very similar to the swerves. Do you think this helps to explain free will?
3. Can you think of examples where it is better to think of light as a wave, and light as a particle?
4. Can you explain why atoms have specific absorption and emission spectral lines?
5. Do you think that with improved technology we will be able to overcome Heisenberg's uncertainty principle, and measure the exact position and velocity of an electron?
6. Could a stationary grape in a glass bowl spontaneously appear outside the bowl? Compare the classical and quantum mechanical "answers".
7. What is a quantum fluctuation?
8. What do physicists mean when they talk about the "collapse of the wave function"?
9. Discuss and compare the Copenhagen and many-worlds interpretations of quantum mechanics. Which one do you prefer?
10. Do you think the many worlds interpretation can ever be disproved?