16
The Theory of Cosmic Inflation

The horizon and flatness problems had been recognized since the 1960s, but were rarely discussed—simply because no one had any idea as to what to do about them. These problems could not be resolved without addressing the question of what really happened at the earliest moments of the big bang. With no progress in that direction, physicists grew accustomed to the notion that questions about the initial state of the universe belonged to philosophy, not physics. It therefore came as a total surprise when, in 1980, Alan Guth made his dramatic breakthrough, providing a way to resolve the stubborn cosmological puzzles in one shot.

16.1 Solving the Flatness and Horizon Problems

The origin of the flatness and horizon problems can be traced to the *decelerated expansion* of the universe. In a decelerating universe the density parameter Ω is driven away from one, and thus it is remarkable that it is measured to be so close to unity today. Also, if the expansion decelerates, the horizon grows faster than the separation between regions. This means that if we look backwards in time, the horizon shrinks faster than the separation between any two regions. Thus regions that are not in causal contact now, could never have been in causal contact at any earlier time. Both of these problems can be solved if the universe underwent a period of *accelerated expansion* in its infancy. But *what could have caused such a period*?

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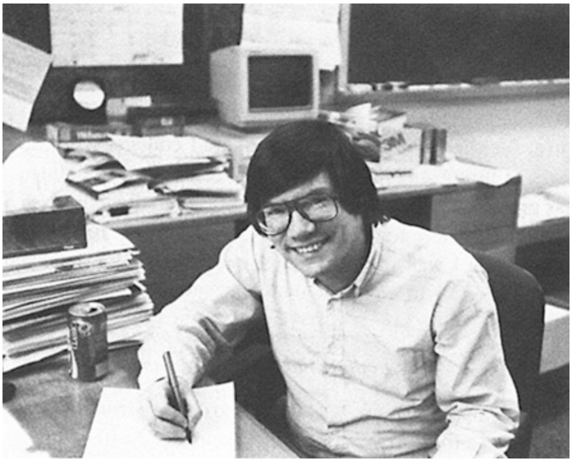

Fig. 16.1 Alan Guth came up with his idea of cosmic inflation while he was in his 9'th year as a temporarily employed postdoctoral fellow. Soon thereafter, he became a tenured Professor at MIT. For his work on inflation Guth won the 2012 Fundamental Physics Prize. Among his other awards, he also won the 2005 contest for the messiest office in the Boston area (organized by the local newspaper *The Boston Globe*)

You might have guessed the answer—vacuum energy! We know that the expansion of the universe is now accelerating, due to the repulsive gravity of the vacuum. However, this accelerated expansion only began at a relatively recent cosmic time, when the density of matter ρ_m dropped below the vacuum energy density ρ_v. At earlier epochs ρ_v was totally negligible so it could not have caused accelerated expansion in the early universe. What we need is a vacuum with a huge energy density at very early times. Fortunately, grand unified theories of particle physics make the existence of such high-energy vacuum states plausible. This led Alan Guth to propose that a large vacuum energy density caused the universe to undergo a period of very fast, accelerated expansion, thereby solving the flatness and horizon problems. Guth also suggested a fitting name for the accelerated expansion epoch: *cosmic inflation* (Fig. 16.1).

16.2 Cosmic Inflation

16.2.1 The False Vacuum

Vacuum is what you end up with when you remove all that can be removed. It is empty space. But according to modern particle physics, vacuum is very

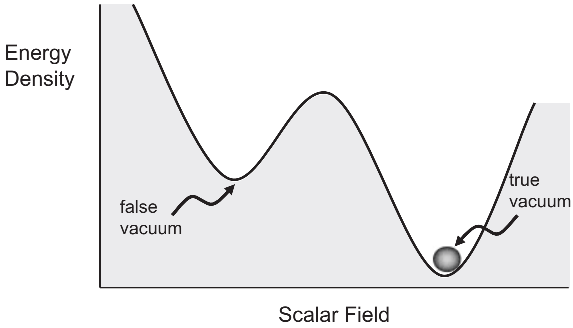

Fig. 16.2 Potential energy density of a scalar field. Here there are two minima, one of which is the true vacuum

different from nothing. At every point in space there is a Higgs field, as well as other scalar fields, responsible for the grand unified symmetry breaking. The vacuum values of these fields determine the masses and interactions of all elementary particles. Symmetry can generally be broken in several different ways, and thus we expect to have a number of vacuum states with different properties. Particle physicists refer to these states as different *vacua*.

As an illustration, let us consider a toy "grand unified theory" with a single scalar field that has a potential energy density curve shown in Fig. 16.2. We can represent the value of the field by a ball that rolls in this energy landscape and comes to rest in one of the two valleys. The valleys represent the two possible vacuum states in this theory. The lowest-energy vacuum is the absolute minimum of energy; it is called the "true vacuum". Any higher-energy vacuum is necessarily unstable; hence it is called a "false vacuum". We know that physical systems tend to minimize their potential energy, so a false vacuum has to decay by converting into true vacuum. (We shall discuss the decay process later in this chapter.)

We are now ready to formulate the idea of cosmic inflation, as it was originally proposed by Guth. Suppose the universe was in a high-energy false vacuum state at some early time in its history. The strong repulsive gravity of the false vacuum would then cause a period of very fast, accelerated expansion. This would solve the horizon and flatness problems of the standard big bang cosmology. The inflationary period ends when the false vacuum decays into the true vacuum. The excess energy of the false vacuum has to go somewhere, and Guth assumed that it gets converted into a hot fireball of particles. The fireball continues to expand by inertia, and the expansion rate gradually slows down due to gravity. The end of inflation plays the role of the big bang in this scenario. At later times, the universe evolves along the lines of the hot big bang cosmology.

How large can we expect the false vacuum energy density to be?

The answer depends on the details of particle physics, but we can make an educated guess, using a trick called "dimensional analysis". Each particle physics model has a characteristic mass, or energy scale, which we shall denote by M. For the electroweak theory, this scale is $M \sim 100$ GeV. The Higgs, W and Z boson masses all have this order of magnitude. The electroweak symmetry breaking energy has a similar magnitude: electromagnetic and weak interactions cannot be distinguished at particle energies much greater than 100 GeV. For grand unified theories the corresponding mass/energy is $M \sim 10^{16}$ GeV. This mass also determines the characteristic scale of the energy density landscape of the theory, that is, the typical height of the hills and valleys in Fig. 16.2. We can thus expect to have a formula expressing the false vacuum energy density ρ_V in terms of M and of the fundamental physics constants—the Planck constant $\hbar$ and the speed of light c. Now, the key point is that only one combination of M, $\hbar$ and c has the dimension of energy density; it is

$$\rho_V \sim \frac{c^5 M^4}{\hbar^3} \tag{16.1}$$

There may also be a numerical coefficient, but it usually does not change the order of magnitude by too much. The above formula can be rewritten as

$$\rho_V \sim 10^{21} M_{\text{GeV}}^4 \frac{\text{kg}}{\text{m}^3}, \tag{16.2}$$

where M_{GeV} is the mass M expressed in units of GeV. For a grand unified theory with $M_{\text{GeV}} \sim 10^{16}$; this gives a truly enormous density of $\rho_V \sim 10^{85}$ kg/m^3. One cubic centimeter of this vacuum contains much more energy than our entire observable universe!

16.2.2 Exponential Expansion

While the universe stays in the false vacuum state, the energy density remains constant. This leads to a very special kind of growth, called exponential expansion. The hallmark of exponential expansion is that in a fixed period of time t_D (the doubling time) the size of a given region will double (and its volume will therefore be increased by a factor of $2^3 = 8$). So, if we start with one cubic nugget of vacuum with length l_0, after one doubling time the cube will have size $2l_0$. After the next doubling time it will have size $2 \times 2 \times l_0 = 4l_0$; and after n doubling times it will have size $2^n l_0$. This is similar to financial inflation at a constant rate. A remarkable property of exponential growth is that the numbers get enormous after relatively few doubling cycles. For example, if a slice of pizza costs \$1 now, then after n doubling cycles it will cost \$$2^n$: so after 10 cycles it will cost \$1024, and after 330 cycles it will cost \$$10^{100}$ (this much money does not even exist) (Fig. 16.3).

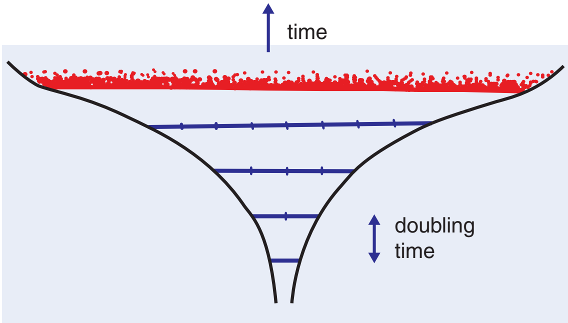

Fig. 16.3 Exponential expansion of the universe. At each time step the universe doubles in size

A universe undergoing exponential expansion is also characterized by a Hubble parameter which does not change in time, $H = const$. This is not difficult to understand, if we recall that an accelerated expansion drives the density of the universe towards the critical value,

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{16.3}$$

During inflation we have $\rho = \rho_v$; then, setting $\rho_c = \rho_v$ and solving for H, we obtain

$$H = \left(\frac{8\pi G\rho_v}{3}\right)^{1/2} = const. \tag{16.4}$$

Any two particles in the inflating universe are driven apart with a velocity given by Hubble's law, $v(t) = Hd(t)$, where $d(t)$ is the distance between the particles. Let's say at some moment they are separated by distance d and receding with speed v (at any later time both the distance and velocity increase). If the particles were to continue separating at this speed, then the distance between them would double in a time interval $t_D = d/v = 1/H$. Since the universe expands with acceleration, the actual doubling time is somewhat shorter, but the relation $t_D \sim 1/H$ still gives a good order of magnitude estimate.[1]

If inflation happened at the GUT-scale, then $H \sim 10^{38}\,\text{s}^{-1}$ and $t_D \sim 10^{-38}$ s. With such an incredibly brief doubling time, the universe would expand by a factor of 10^{100} in less than 10^{-35} s. If, for example, we start with

[1] We show in the Appendix that the doubling time in an inflating spacetime is $t_D = \frac{0.7}{H}$.

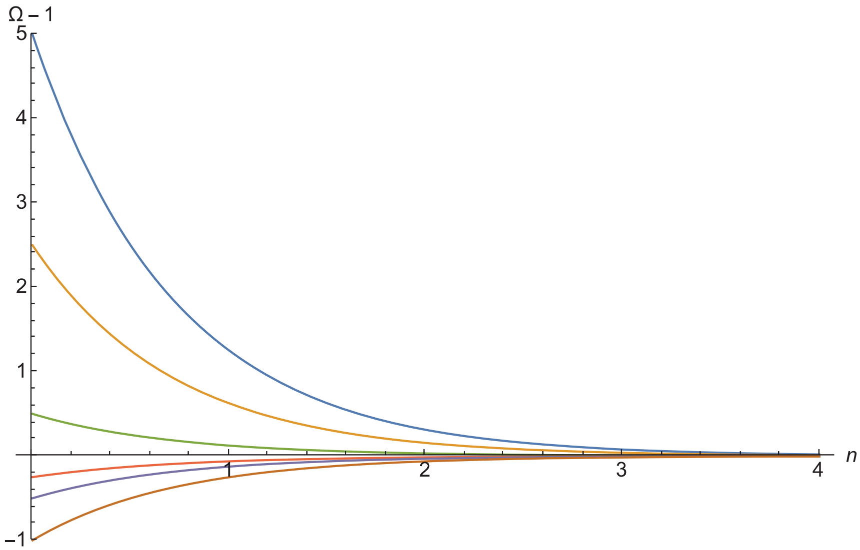

Fig. 16.4 Evolution of the density parameter in an inflationary universe. Here we plot $\Omega - 1 \propto 2^{-2n}$, where n is the number of doubling times (see the end of the appendix for derivation of this equation). Above the x-axis the curves are for closed universes, with different initial values of Ω; below the x-axis the curves are for open universes with different initial values for Ω. Even if the density parameter starts off much larger or smaller than unity, it is quickly driven to unity within several doubling times of inflation

a nugget of false vacuum having the size of a proton, $\sim 10^{-15}$ m, then in less than 10^{-35} s the universe would inflate to a size of 10^{85} m. This is vastly larger than the size of the observable universe, which is "only" 10^{26} m. The exponential expansion of the false vacuum is thus an immensely powerful mechanism that can blow a tiny seed universe up to astronomical dimensions in a very short time.

16.3 Solving the Problems of the Big Bang

Let us now see how inflation helps to explain the puzzling features of the initial state that had to be postulated in the big bang theory.

16.3.1 The Flatness Problem

A period of exponential expansion drives the density parameter towards $\Omega = 1$. Figure 16.4 illustrates that Ω gets extremely close to one in a relatively small number of doubling times. This means that the geometry of the universe gets very close to flat, Euclidean geometry.

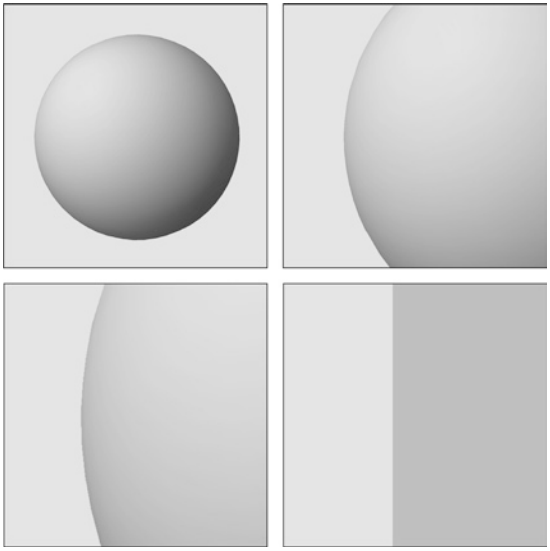

Fig. 16.5 The surface of a huge balloon looks flat, because we can see only a small part of it. Similarly, the universe appears to be flat because we only see a small portion of it after inflation

This effect has a simple intuitive explanation. Imagine a curved surface, like a sphere. Now imagine enlarging this surface by a huge factor. This is what happens to the universe during inflation. We can now see only a tiny part of this big universe. And it appears to be flat, just like the surface of the Earth looks flat when we see a small portion of it (Fig. 16.5).

16.3.2 The Horizon Problem

Consider a spherical region of diameter d much smaller than the Hubble distance d_H at the beginning of inflation. The region is initially expanding at a much smaller speed than light, so there is plenty of time for different parts of the sphere to interact and come to equilibrium. Then the inflationary expansion blows the size of the region up by a factor 2^n, where n is the number of doubling times during inflation. This factor can be enormous, so the present size of the region can easily be much larger than our observable universe. This solves the horizon problem: the CMB temperature is uniform over the sky because all parts of the observable universe were in causal contact at the beginning of inflation.

What is the minimal number n_{min} of doubling times that is necessary to solve the horizon and flatness problems? The answer depends on the energy scale of inflation, M. For M at the GUT scale, one finds n_{min} is about 90.

16.3.3 The Structure Formation Problem

Inflation also offers the most promising explanation for the origin of small density fluctuations that later evolved into galaxies and clusters. We will discuss this shortly.

16.3.4 The Monopole Problem

All monopoles produced before or during inflation get diluted away by the huge inflationary expansion, so that their present density becomes negligible.

16.3.5 The Expansion and High Temperature of the Universe

The hot big bang model assumes that the universe started out in a state of rapid expansion at a very high temperature. But why was the early universe so hot? And why was it expanding? Inflation provides a possible explanation for this initial state. The expansion of the universe is caused by the repulsive gravity of the false vacuum. The vacuum energy density during inflation is expected to be very high, and when the false vacuum decays, this energy gets converted into a hot fireball of particles and radiation; hence the fireball is born with a very high temperature.

We thus see that a period of inflation in the early universe can resolve the perplexing problems of the big bang. But in order for the inflationary model to be complete, we need to understand how inflation begins and how it ends. Inflation ends when the false vacuum decays, so in the next section we shall study the vacuum decay process. The question of the beginning of inflation will be addressed in Chap. 23.

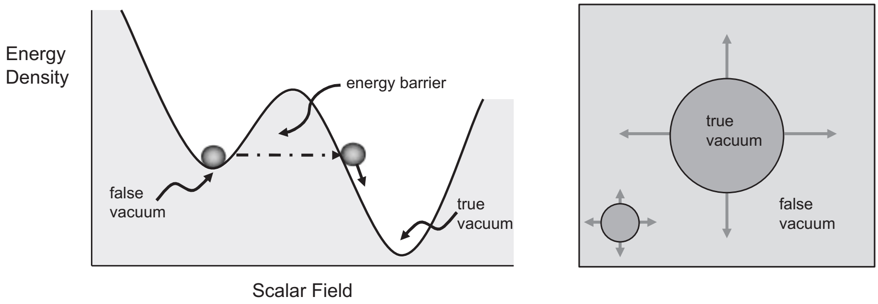

Fig. 16.6 Vacuum decay. When the field tunnels from its false vacuum to its true vacuum value, bubbles of true vacuum nucleate within the false vacuum background. The bubbles then expand at speeds approaching the speed of light

16.4 Vacuum Decay

16.4.1 Boiling of the Vacuum

Consider the energy landscape of a scalar field illustrated in Fig. 16.6. It has a false vacuum and a true vacuum. During inflation the field is in the false vacuum everywhere in space. Now, in order for the false vacuum to decay, the field has to overcome the energy barrier separating the two vacua. As we discussed earlier, the dynamics of the scalar field is similar to that of a ball rolling in the energy landscape. If the ball is located in the valley marked "false vacuum", then, according to classical physics, it will stay there forever, unless someone kicks it upwards, providing the energy needed to go over the barrier. But we learned in Chap. 10 that the ball can quantum-mechanically tunnel through the barrier and emerge on the other side. This is also what happens in vacuum decay.

Quantum tunneling is a probabilistic process, so you cannot predict exactly when and where it is going to happen. You can only calculate the probability for tunneling to occur in a given region of space per interval of time. The probability for a large region of false vacuum to tunnel to the true vacuum is extremely low. Thus tunneling occurs in a tiny microscopic region, resulting in a small true vacuum bubble.[2]

The process of vacuum decay is similar to the boiling of water. Small bubbles of true vacuum pop out (or "nucleate") randomly in the midst of false

[2]Despite the similarity between the tunneling of a ball and that of a scalar field, there is also an important difference. The ball tunnels between two different points in space, while the field tunnels between two different field values at the same location in space.

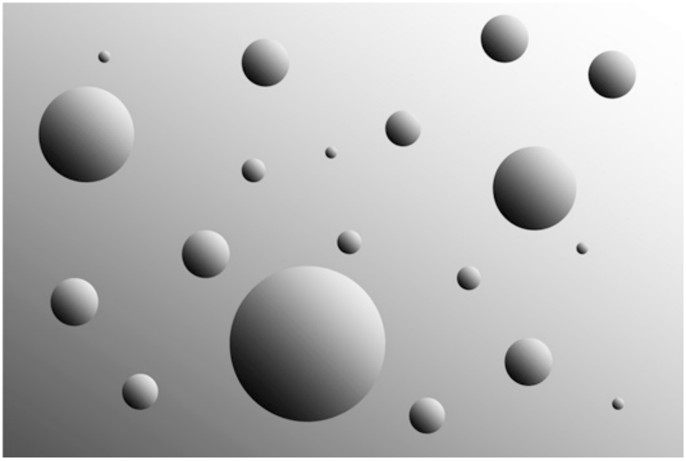

Fig. 16.7 Inflating universe with true vacuum bubbles. Bubbles are driven apart by the expansion of the universe, so they almost never collide

vacuum. The energy released by converting false vacuum into true vacuum gets concentrated in the bubble walls, which expand at a speed approaching the speed of light. When bubbles collide and merge, the walls disintegrate into particles. This is how Guth originally envisioned the end of inflation and the onset of the big bang. But unfortunately this broad-brush scenario has a fatal flaw.

16.4.2 Graceful Exit Problem

The problem is that even though the bubbles expand at nearly the speed of light, we cannot simply assume they will collide, because the space between them is filled with false vacuum and is also rapidly expanding. In fact, any bubbles separated by more than a Hubble distance d_H are driven apart faster than the speed of light and will never collide. The typical distance between the bubbles depends on the rate at which they nucleate. If the nucleation rate is low, then bubbles will form separated by wide stretches of false vacuum and will almost never collide. All the energy of the bubbles will remain concentrated in the expanding bubble walls, and inflation will never end (Fig. 16.7).

To get around this problem, we can consider a model where bubbles nucleate at a very high rate, so that their typical separation is less than d_H. In this case the bubbles will collide and merge, and the whole vacuum decay process will be over in less than a doubling time. But in order to solve the

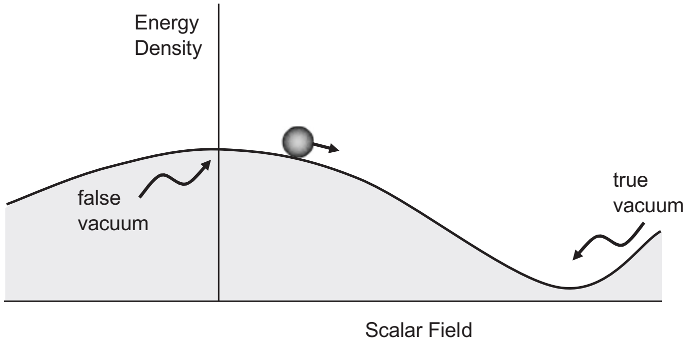

Fig. 16.8 In the slow roll inflation scenario, the role of the false vacuum is played by a very flat plateau at the top of the potential energy density hill

horizon and flatness problems we need inflation to persist for many doubling times (roughly 90 or so, depending on the details of the model). Thus we are faced with an impasse: either inflation does not end at all, or it ends too rapidly to solve the problems it was invented to solve. In the early 80's this became known as *the graceful exit problem.* Guth realized that his theory suffered from this problem soon after he came up with the idea of inflation, so he concluded his landmark paper stating: "I am publishing this paper in the hope that it will encourage others to find some way to avoid the undesirable features of the inflationary scenario."

16.4.3 Slow Roll Inflation

The Russian born cosmologist Andrei Linde was the first to find a solution to the graceful exit problem in 1982. A few months later the same idea was independently proposed by Andreas Albrecht and Paul Steinhardt in the USA. The crucial step was to consider an energy landscape without a barrier, but with a very gentle slope, as shown in Fig. 16.8. Once again, we can represent the scalar field by a ball rolling in this landscape. If we place the ball near the top of the hill, it will start slowly rolling down, and since the slope is so flat, the ball will initially stay at about the same height. For the scalar field this means that its energy density will remain almost constant. But a constant energy density is all that is needed to sustain a constant rate of inflation.

The flat region near the hilltop can be called a "false vacuum". Since the field "rolls" very slowly, it takes a while for it to cross that region, and in the meantime the universe expands exponentially. Once the field reaches the

Fig. 16.9 Andre Linde has been one of the chief architects of inflationary cosmology for over 30 years. Linde began his career in his native Moscow, and has been a Professor at Stanford University since 1989. He often collaborates with his wife Renata Kallosh, who is also a Professor at Stanford. Linde is a flamboyant, entertaining presenter and an outspoken champion of his ideas. He is an excellent artist and occasionally illustrates his lectures with beautifully drawn cartoons. His numerous hobbies include swimming, juggling, card tricks, and photography. *Credit* Vadim Shultz

true vacuum, it oscillates back and forth and eventually comes to rest, with its energy turned into a hot fireball of particles.[3] By this time the universe has expanded by a huge factor.

Note that in this model, the field "rolls" simultaneously at all points in space and produces a fireball at the same time in the entire inflating region. We thus have a large, hot, homogeneous, expanding universe. The graceful exit problem has been solved! (Fig. 16.9).

Like the Higgs field of the Standard Model, the rolling scalar field must have some particle associated with it. Particle physicists have suggested a number of candidates, but none of them is particularly compelling. For now, the particle goes by the generic name "inflaton", and the field is called the "inflaton field".

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[3]A ball rolling on a similarly curved surface would also oscillate about the lowest point, would gradually slow down due to friction, and would come to rest, with all its mechanical energy turned into heat. Similarly, analysis shows that an oscillating field loses its energy by particle production, creating a fireball.

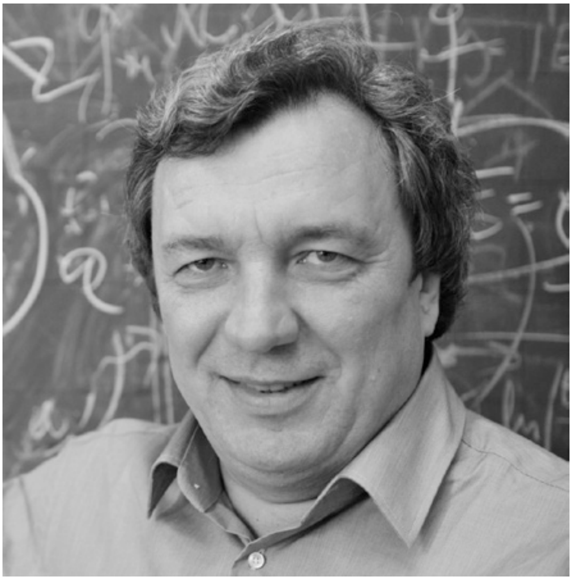

Fig. 16.10 Viatcheslav Mukhanov suggested (with Chibisov) that cosmological density fluctuations could have a quantum origin and later did seminal work developing the details of this scenario. He is known among cosmologists for his flamboyant personality and politically incorrect sense of humor. *Credit* PR Image iau1304a, Viatcheslav Mukhanov, recipient of the 2013 Gruber Prize (https://www.iau.org/news/pressreleases/detail/iau1304/)

16.5 Origin of Small Density Fluctuations

As we discussed in Chap. 12, galaxies and galaxy clusters arise from gravitational collapse in a universe that begins with small density variations from one location to the next. But where do these initial density variations come from?

If inflation gives us a perfectly homogeneous universe, then it works *too* well. Russian physicists Viatcheslav Mukhanov and Gennady Chibisov proposed in 1981 that density fluctuations in the early universe could arise due to random quantum fluctuations. This means that quantum effects, which are usually only important in the microworld, could be ultimately responsible for the existence of the largest structures in the universe! (Fig. 16.10).

Let's see how this is possible. In addition to classical motion, the inflaton field is subject to quantum mechanical effects (see Fig. 16.11). As the field rolls downhill, it experiences quantum fluctuations, which randomly kick the field *up or down* the hill. The directions of these small kicks are not the same in different spatial regions of the universe. Thus the field arrives at the bottom of the hill and produces a fireball at different times in different spatial locations.

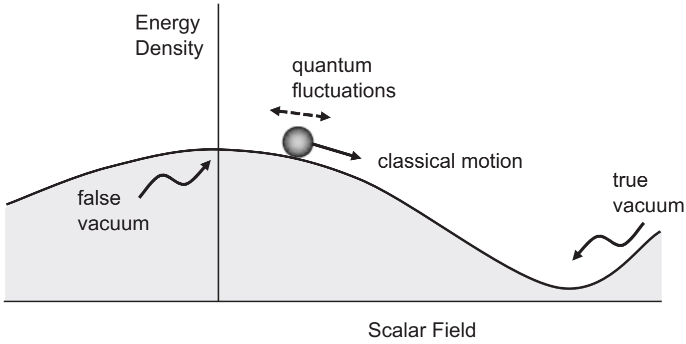

Fig. 16.11 The evolution of the inflaton field is a combination of its deterministic classical motion down the hill and its random quantum mechanical jumps up and down the hill

In regions where the field took a little longer to reach the true vacuum, inflation lasts a bit longer, and the matter density will be slightly higher. Why? Because during inflation, the energy density stays roughly constant even as the universe expands. But once the fireball is produced, matter and radiation get diluted. So parts of the universe that exit the inflating stage sooner, get a bit diluted by the time nearby lagging regions start their hot big bang evolution. The upshot of inflation ending at slightly different times is that the very early universe is imbued with small differences in density from one region to another. These are the density fluctuations that could be responsible for the formation of cosmic structure (Fig. 16.12).

All density fluctuations originate as quantum kicks in tiny regions which have a size roughly given by the Hubble distance d_H.[4] But then they are stretched by the expansion to a much greater size. Fluctuations produced earlier are stretched for a longer time and encompass a larger region. The magnitude of fluctuations is set by the initial quantum kick and is about the same for all distance scales. This leads to a scale-invariant spectrum of density fluctuations.

To clarify what a scale invariant spectrum means, imagine that we divide the universe into cubic regions of size 100 light years and measure the aver-

[4] Quantum fluctuations occur on smaller scales as well, but upward and downward kicks alternate in rapid succession, so their overall effect is nil. But once the fluctuation region is stretched to a size larger than d_H, its different parts become causally disconnected, and coherent fluctuations in such a region are no longer possible. The surviving fluctuations are the ones produced in regions of size $\sim d_H$. The region is then immediately stretched to a larger size, and the fluctuation "freezes".

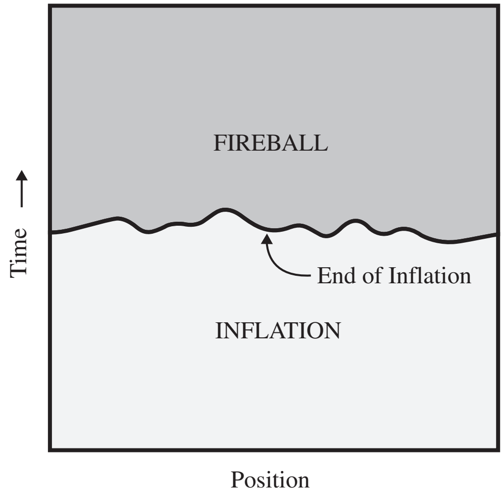

Fig. 16.12 Different ending times for inflation result in small density fluctuations. Regions where inflation ends later have a higher matter density

age density in each cube. Let's say we find the density fluctuation (that is, the typical variation from one cube to another) to be 1%. Now we can repeat this experiment with cubic regions of different size (say, 1000 light years, 10 light years, etc.). If the density fluctuation is the same for any choice of the size, we say that the spectrum of density fluctuations is scale invariant.

The scale invariance of density fluctuations from inflation is only approximate. The magnitude of quantum kicks decreases slightly as the field rolls downhill. As a result, the density fluctuations on greater distance scales, which were produced when the field was at a higher altitude, are slightly larger than the smaller-scale fluctuations. The form of the spectrum of primordial density fluctuations is one of the most important observational predictions of inflation, as we will discuss in Chap. 17.

16.6 More About Inflation

16.6.1 Communication in the Inflating Universe

Let us imagine two comoving observers in an inflating universe, who communicate by exchanging light signals. The observers are moving apart at the speed $v = Hd$, where d is the distance between them. Suppose the observers begin their signal exchange when d is very small, so that the separation speed v is small compared to the speed of light. Then the Hubble expansion has almost no effect on light propagation between the observers, and they can exchange many signals before their distance is appreciably enlarged. But as the observers move apart, their separation speed gradually increases and becomes equal to the speed of light at the Hubble distance

$$d_H = \frac{c}{H} \tag{16.5}$$

When d gets close to d_H, light signals take longer and longer to propagate and arrive more and more redshifted. Once d becomes greater than d_H, any communication between the observers becomes impossible, since they are now moving apart faster than the speed of light. At later times v will get only larger, so light signals sent by one observer will never catch up with the other one. We thus see that comoving observers who are in causal contact in an inflating universe will necessarily fall out of causal contact at later times (assuming that inflation continues).

A spherical surface of radius d_H surrounding an observer is called the observer's Hubble sphere; its properties in an exponentially expanding universe are similar to those of the Schwarzschild horizon of a black hole. Events that occur beyond the Hubble sphere cannot be detected by the observer. For GUT-scale inflation, the Hubble distance is tiny, $d_H \sim 10^{-30}$ m, hardly enough to contain an observer. But note that today our universe is vacuum-dominated and is once again undergoing a stage of exponential expansion. The present vacuum energy density is much lower than it was in the early inflationary phase, and the Hubble distance is now astronomically large: $d_H \sim 10^{10}$ ly. Galaxies in the observable universe are driven towards d_H. As they approach the Hubble sphere, they become more and more redshifted and gradually fade away.

16.6.2 Energy Conservation

A short period of inflation can blow a tiny subatomic-size region up to dimensions much greater than the entire observable universe. On the face of it, this seems to be in conflict with energy conservation. The false vacuum has a constant energy density ρ_v, so its energy is proportional to the volume V that it occupies, $E_v = \rho_v V$. At the end of inflation the volume is enormous, and so is the energy. The question is: where did all this energy come from?

To see what is going on here, let us first note that the total energy must include the contribution of the gravitational potential energy. Furthermore, let us recall that the gravitational energy is always negative and that it also gets large when the mass is large. Hence it is conceivable that as the mass/energy of the false vacuum grows during inflation, its negative gravitational energy grows at the same rate, so the total energy remains constant.

An analogous situation arises in Newtonian theory when a small particle falls towards a massive star of mass M. The energy of the particle in this case is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}, \tag{16.6}$$

where m is the particle's mass, v is its velocity, and r is its distance from the center of the star. The first term is the particle's kinetic energy and the second is the gravitational potential energy. Suppose the particle is initially at rest ($v = 0$) at a large distance from the star, so the energy is very small. As it falls, the particle accelerates, and its kinetic energy can get very large as it approaches the star. On the other hand, as r gets smaller, the potential energy gets large and negative. But the two contributions nearly cancel one another, so the total energy is conserved and is close to zero, as it was from the start.

A detailed analysis, based on general relativity, shows that the energetics of cosmic inflation is very similar. The total energy of the huge false vacuum region at the end of inflation is very tiny; it is the same as the energy of the initial nugget from which this volume originated.

Summary

The horizon and flatness problems can be solved if the universe underwent a period of *accelerated* expansion, called inflation. The theory of inflation assumes that the universe originated in a state of a high-energy false vacuum. The repulsive gravity of that vacuum causes a super-fast, exponential expansion of the universe. Regardless of its initial size, the universe very quickly becomes huge. The false vacuum eventually decays, producing a hot fireball, marking the end of inflation. The fireball continues to expand by inertia and evolves along the lines of hot big bang cosmology. Decay of the false vacuum plays the role of the big bang in this scenario.

The theory of inflation explains the expansion of the universe (it is due to the repulsive gravity of the false vacuum), its high temperature (due to the high energy density of the false vacuum), and its observed homogeneity (false vacuum has an almost constant energy density). It also predicts a nearly scale invariant spectrum of density fluctuations, which can serve as seeds for structure formation.

Questions

1. What is cosmic inflation? How is it different from a rapid expansion of the early universe in the hot big bang cosmology?
2. What properties of the false vacuum are responsible for the inflationary expansion?
3. If inflation occurred at the electroweak energy scale, the doubling time would be $t_D \sim 10^{-10}$ s. Roughly how long would it then take for the universe to grow by a factor of 1000?
4. How does inflation solve the horizon and flatness problems?
5. Does the theory of inflation replace the big bang theory? Explain.
6. Name the key features of the universe that a brief period of inflation explains.
7. In the context of Guth's original inflationary model: (a) In what sense is vacuum decay like the boiling of water? (b) When false vacuum is converted into a true vacuum bubble, energy is released. Where is this energy stored?
8. The original version of inflation had the so-called "graceful exit problem". Even though bubbles of true vacuum expand with almost the speed of light, it was difficult to get the bubbles to collide and release their energy. What prevented the bubbles from colliding?
9. Suppose we have a potential energy landscape with a steep slope and no barrier. If the scalar field starts somewhere on the slope, it will roll down very quickly. Would this give a satisfactory inflationary scenario?
10. Consider the potential energy density diagram in Fig. 16.8. What key feature of this potential solved the "graceful exit problem". Briefly explain how this potential gives rise to a large, hot, homogeneous, expanding universe.
11. How does inflation explain the origin of small density fluctuations?
12. As the inflaton field rolls down the energy hill, it experiences random quantum fluctuations in different directions. Thus the field arrives at the bottom at different times, in different regions. For those regions where inflation lasted a little longer, will the matter density be slightly higher or lower than average? Why?

13. Quantum fluctuations take place in tiny regions of space. They are then stretched to macroscopic sizes by the expansion of space. Early fluctuations undergo more stretching and thus encompass larger regions than those produced later on. Is the magnitude of the resulting density fluctuations on large scales less than, greater than or the same as the magnitude of fluctuations on much smaller scales?