Chapter 7
Deep-Inelastic Scattering

Verlockend ist der äußre Schein der Weise dringet tiefer ein.

Wilhelm Busch
Der Geburtstag

In the present chapter we will discuss deep-inelastic scattering of charged leptons off nucleons and demonstrate that these nucleons are not fundamental particles but that they have a substructure of quarks and gluons. To resolve the nucleon's constituents experimentally, the wavelength of the exchanged virtual photon has to be small compared to the nucleon's radius, $\lambda \ll R$, and consequently high beam energies are required. The first generation of such experiments was carried out in the late 1960s and in the 1970s at SLAC using a linear electron accelerator with a maximum energy of 25 GeV. The second generation was performed in the 1980s and 1990s at CERN and FNAL using beams of muons instead of electrons. Like electrons, muons are point-like charged particles; the scattering processes are completely analogous and the cross-sections are the same. Muon beams have the advantage that they can be produced at much higher energies than electron beams. In order to make those muon beams, protons with energies of several hundred GeV impinge on a target producing a large number of pions. On a several hundred metre long decay line, a fraction of these pions decays in flight into muons (cf. Sect. 10.1) which are then momentum-selected and focused by a series of magnetic lenses to form a beam. At CERN (FNAL) average beam energies of up to 280 GeV (490 GeV) and Q^2-values of several hundred $(\text{GeV}/c)^2$ have been achieved. The last generation of such experiments has been performed in the years 1992–2007 at the electron-proton collider HERA located at DESY. Here electrons or positrons with 27.6 GeV and protons with a maximum beam energy of 920 GeV circulated in two separate storage rings in opposite directions and were brought to collision at two crossing points. The resulting kinematic region extended to Q^2-values of several $10^4(\text{GeV}/c)^2$.

The basic properties of the quark and gluon structure of the hadrons were established by the experiments at SLAC, which will be discussed and interpreted in this chapter. The second and the third generations of experiments served for detailed studies of this structure and tests of Quantum Chromodynamics (QCD), the theory of the strong interaction, which we will discuss in the subsequent chapter.

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7.1 Excited States of the Nucleons

In Fig. 5.10 of Sect. 5.5 we presented the spectrum observed in electron scattering off the ^{12}C nucleus where, in addition to the sharp peak due to elastic scattering off the whole nucleus, further peaks appeared associated with nuclear excitations. Similar spectra are observed for electron-nucleon scattering.

Figure 7.1 shows a spectrum from electron-proton scattering. It was obtained at an electron energy $E = 4.9$ GeV and at a scattering angle of $\theta = 10°$ by varying the accepted scattering energy of a magnetic spectrometer in small steps. Besides the sharp elastic scattering peak (scaled down by a factor of 15 for clarity), peaks at lower scattering energies are observed associated with inelastic excitations of the proton. These peaks correspond to excited states of the nucleon which we call *nucleon resonances*. The existence of these excited states of the proton already indicates that the proton is a composite system. In Chap. 16 we will explain the structure of these resonances in the framework of the quark model.

The invariant mass of these states is denoted by W. It is calculated from the four-momenta of the exchanged photon (q) and of the incoming proton (P) according to

$$W^2c^2 = P'^2 = (P+q)^2 = M^2c^2 + 2Pq + q^2 = M^2c^2 + 2M\nu - Q^2 \, . \qquad (7.1)$$

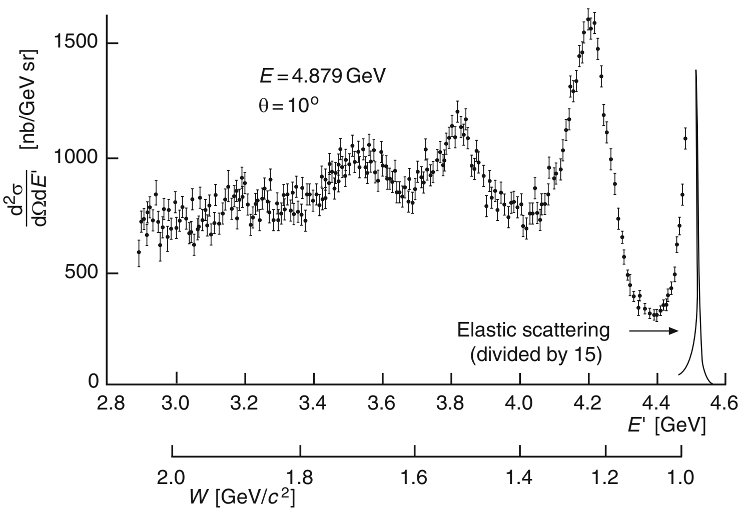

Fig. 7.1 Spectrum of scattered electrons from electron-proton scattering at an electron energy of $E = 4.9$ GeV and a scattering angle of $\theta = 10°$ (From [4])

Here the Lorentz-invariant quantity ν is defined as

$$\nu = \frac{Pq}{M} . \tag{7.2}$$

The target proton is at rest in the laboratory system. This corresponds to $P = (Mc, \mathbf{0})$ and $q = ((E - E')/c, \boldsymbol{q})$. Therefore the energy transferred by the virtual photon from the electron to the proton in the laboratory frame is:

$$\nu = E - E' . \tag{7.3}$$

For the following discussion it is useful to introduce two additional dimensionless Lorentz-invariant quantities. These are the variable

$$y := \frac{Pq}{Pp} \stackrel{\text{Lab.}}{=} 1 - \frac{E'}{E} \tag{7.4}$$

and the *Bjorken scaling variable*

$$x := \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu} . \tag{7.5}$$

We will interpret the latter quantity in more detail further down in Sect. 7.3. It is a measure for the inelasticity of the process. For elastic scattering the invariant mass W is equal to the nucleon mass M and therefore we get with (7.1)

$$2M\nu - Q^2 = 0 \quad \Longrightarrow \quad x = 1 , \tag{7.6}$$

while for inelastic processes W is larger than M and we get

$$2M\nu - Q^2 > 0 \quad \Longrightarrow \quad 0 < x < 1 . \tag{7.7}$$

The $\boldsymbol{\Delta}$(1232) resonance The nucleon resonance $\Delta(1232)$, which appears in Fig. 7.1 at about $E' = 4.2$ GeV, has a mass $W = 1{,}232\,\text{MeV}/c^2$. As we will see in Chap. 16, this resonance exists in four different charge states: Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-}. In Fig. 7.1, the Δ^{+} excitation is observed since charge is not transferred in the reaction (Fig. 7.2).

The width observed for the elastic peak is a result of the finite resolution of the spectrometer, but resonances have a real width[1] of typically $\Gamma \approx 100$ MeV. The uncertainty principle then implies that such resonances have very short lifetimes. The $\Delta(1232)$ resonance has a width of approximately 120 MeV and thus a lifetime of

$$\tau = \frac{\hbar}{\Gamma} = \frac{6.6 \cdot 10^{-22}\,\text{MeV s}}{120\,\text{MeV}} = 5.5 \cdot 10^{-24}\,\text{s} .$$

[1] The exact meaning of "width" will be discussed in Sect. 9.2.

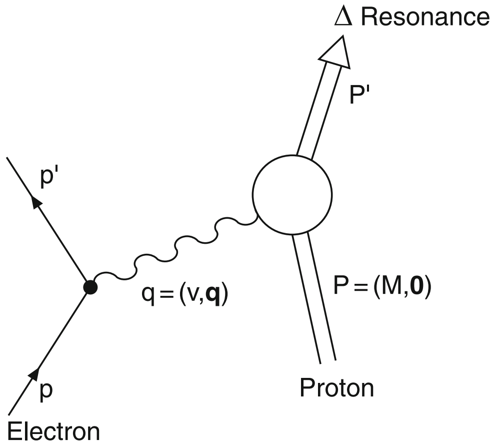

Fig. 7.2 Inelastic electron-nucleon scattering with the excitement of the nucleon to a Δ^+ resonance

This is the typical time scale for strong interaction processes. The Δ^+ resonance decays by:

$$\Delta^+ \to \mathrm{p} + \pi^0$$
$$\Delta^+ \to \mathrm{n} + \pi^+ \,.$$

A light particle, the π-meson (or pion) is produced in such decays in addition to the nucleon.

7.2 Structure Functions

Individual resonances cannot be distinguished in the excitation spectrum for invariant masses $W \gtrsim 2.5\ \mathrm{GeV}/c^2$. Instead, one observes that many further strongly interacting particles (hadrons) are produced (Fig. 7.3).

Electron scattering in the kinematic region where W, $\sqrt{Q^2}/c$ and ν/c^2 are much larger than the nucleon mass M, we denote as *deep-inelastic scattering*. The dynamics of such production processes may be, similar to the case of elastic scattering, described in terms of form factors. In the inelastic case they are usually termed *structure functions* W_1 and W_2, or F_1 and F_2, respectively.

In *elastic* scattering, at a given beam energy E, only *one* of the kinematical parameters may vary freely. For example, if the scattering angle θ is fixed, kinematics requires that the squared four-momentum transfer Q^2, the energy transfer ν, the energy of the scattered electron E' etc. are also fixed. In *inelastic* scattering, however, the excitation energy of the proton adds a further degree of freedom. Hence these structure functions and cross-sections are functions of *two* independent, free parameters, e.g., (E', θ), (Q^2, ν) or (Q^2, x).

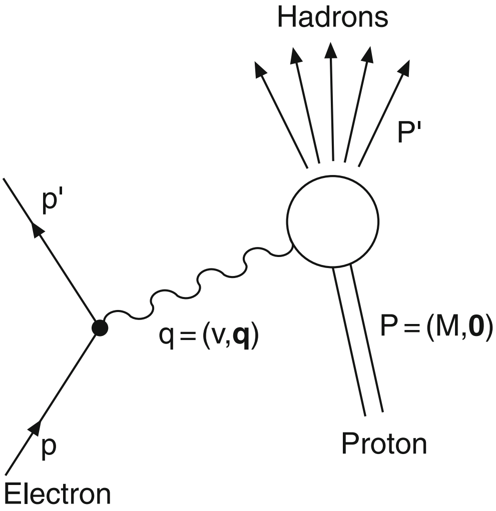

Fig. 7.3 Inelastic electron-nucleon scattering leading to several hadrons in the final state

The Rosenbluth formula (6.10) is now replaced by the cross-section:

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left[W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \frac{\theta}{2}\right]. \tag{7.8}$$

The second term again stems from the magnetic interaction.

This notation of the cross-section is mainly used for didactic and historical purposes. Instead of the two structure functions $W_1(Q^2, \nu)$ and $W_2(Q^2, \nu)$ usually the two dimensionless structure functions

$$\begin{aligned} F_1(x, Q^2) &= Mc^2\, W_1(Q^2, \nu)\ , \\ F_2(x, Q^2) &= \nu\, W_2(Q^2, \nu) \end{aligned} \tag{7.9}$$

are used and the differential cross-section is expressed in terms of the two variables x and Q^2:

$$\frac{d^2\sigma}{dQ^2\, dx} = \frac{4\pi\alpha^2\hbar^2}{Q^4} \left[\left(\frac{1-y}{x} - \frac{My}{2E}\right) F_2(x, Q^2) + y^2 F_1(x, Q^2)\right]. \tag{7.10}$$

Measurements of the deep-inelastic cross-section at fixed values of x and Q^2 but several values of y, i.e., several beam energies E, are required for the determination of both structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$.

The first deep-inelastic scattering experiments were carried out in the late 1960s at SLAC [5, 6]. Figure 7.4 shows one of the results of these experiments that came as a surprise. Displayed is the structure function $F_2(x, Q^2)$ as a function of x, for

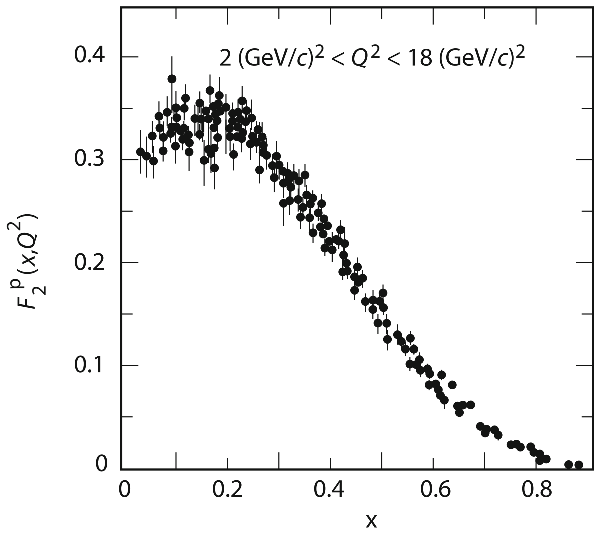

Fig. 7.4 The structure function F_2 of the proton as a function of x, for Q^2 between 2 $(\text{GeV}/c)^2$ and 18 $(\text{GeV}/c)^2$ [3].

data covering a range of Q^2 between 2 $(\text{GeV}/c)^2$ and 18 $(\text{GeV}/c)^2$. At fixed values of x the structure function depends only weakly, if at all, on Q^2.

The fact that the structure functions are independent of Q^2 means, according to our previous discussion, that the electrons are scattered off a point charge (cf. Fig. 5.7). Since nucleons are extended objects, it follows from the above result that:

Nucleons have a sub-structure made up of point-like constituents.

The F_1 structure function results from the magnetic interaction. It vanishes for scattering off spin-zero particles. For spin-1/2 Dirac particles (6.5) and (7.8) imply the so called *Callan-Gross relation* [7] (see the exercises)

$$2xF_1(x) = F_2(x)\,. \tag{7.11}$$

The ratio $2xF_1/F_2$ is shown in Fig. 7.5 as a function of x. It can be seen that the ratio is consistent with unity within experimental uncertainties. Hence we can further conclude that:

The point-like constituents of the nucleon have spin 1/2.

7.3 The Parton Model

The interpretation of deep-inelastic scattering off protons may be considerably simplified if the reference frame is chosen judiciously. The physics of the process is, of course, independent of this choice. If one looks at the proton in a fast moving

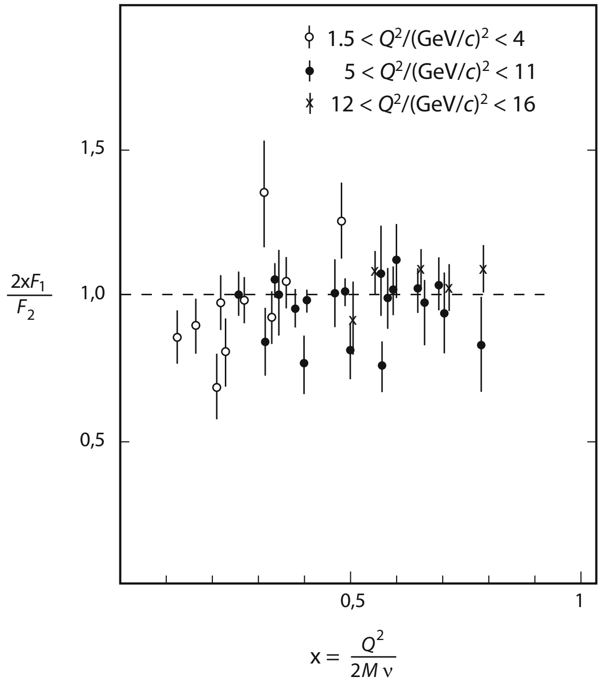

Fig. 7.5 Ratio of the structure functions $2xF_1(x)$ and $F_2(x)$. The data are from experiments at SLAC (From [9])

system, then the transverse momenta and the rest masses of the proton constituents can be neglected. The structure of the proton is then given to a first approximation by the longitudinal momenta of its constituents. This is the basis of the *parton model* of Feynman and Bjorken. In this model the constituents of the proton are called *partons*. Today the charged partons are identified with the quarks and the electrically neutral ones with the gluons – the field quanta of the strong interaction.

Decomposing the proton into independently moving partons, the interaction of the electron with the proton can be viewed as the incoherent sum of its interactions with the individual partons. These interactions in turn can be regarded as elastic scattering. This approximation is valid as long as the duration of the photon-parton interaction is so short that the interaction between the partons themselves can be safely neglected (Fig. 7.6). This is the *impulse approximation* which we have already met in quasi-elastic scattering (p. 82). In deep-inelastic scattering this approximation is valid because the interaction between partons at short distances is weak, as we will see in Sect. 8.2.

If we make this approximation and assume both that the parton masses can be safely neglected and that $Q^2 \gg M^2c^2$, we obtain a direct interpretation of the Bjorken scaling variable $x = Q^2/2M\nu$ which we defined in (7.5). It is that fraction of the four-momentum of the proton which is carried by the struck parton. A photon

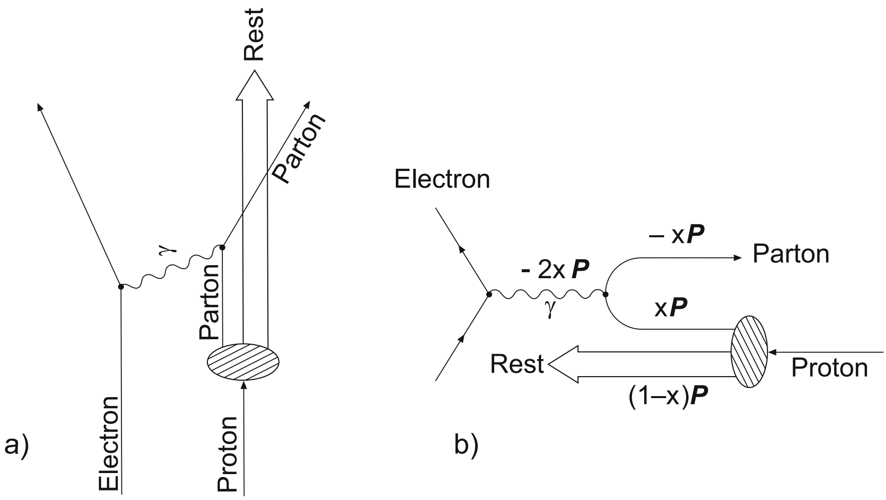

Fig. 7.6 Schematic representation of deep-inelastic electron-proton scattering according to the parton model, in the laboratory system (**a**) and in a fast moving system (**b**). This diagram shows the process in two spatial dimensions. The *arrows* indicate the directions of the momenta. Diagram (**b**) depicts the scattering process in the Breit frame in which the energy transferred by the virtual photon is zero. Hence the momentum of the struck parton is turned around but its magnitude is unchanged

which, in the laboratory system, has four-momentum $q = (\nu/c, \boldsymbol{q})$ interacts with a parton carrying the four-momentum xP. We emphasise that this interpretation of x is only valid in the impulse approximation, and then only if we neglect transverse momenta and the rest mass of the parton; i.e. in a very fast moving system.

A popular reference frame satisfying these conditions is the *Breit frame* (Fig. 7.6b), where the photon does not transfer any energy ($q_0 = 0$). In this system x is the three-momentum fraction of the parton.

The spatial resolution of deep-inelastic scattering is given by the reduced wavelength $\lambdabar$ of the virtual photon. This quantity is not Lorentz-invariant but depends upon the reference frame. In the laboratory system ($q_0 = \nu/c$) it is:

$$\lambdabar = \frac{\hbar}{|\boldsymbol{q}|} = \frac{\hbar c}{\sqrt{\nu^2 + Q^2c^2}} \approx \frac{\hbar c}{\nu} = \frac{2Mx\hbar c}{Q^2} . \tag{7.12}$$

For example, if $x = 0.1$ and $Q^2 = 4\ (\text{GeV}/c)^2$ one finds $\lambdabar \simeq 10^{-17}$ m in the laboratory system. In the Breit frame, the equation simplifies to

$$\lambdabar = \frac{\hbar}{|\boldsymbol{q}|} = \frac{\hbar}{\sqrt{Q^2}} . \tag{7.13}$$

The quantity Q^2, therefore, has an obvious interpretation in the Breit frame: it is a measure for the spatial resolution with which structures can be studied.

7.4 The Quark Structure of Nucleons

Quarks The quark model was conceived in the mid-1960s of the last century in order to systematise the great diversity of strongly interacting particles (hadrons) which had been discovered up to then. By means of deep-inelastic scattering, we found that nucleons consist of electrically charged, point-like particles. We now identify them with the *quarks*. It should be possible to reconstruct and to explain the properties of the nucleons (charge, mass, magnetic moment, isospin, etc.) from the quantum numbers of these constituents. For this purpose, we need at least two different types of quarks, which are designated by u (*up*) and d (*down*). The quarks have spin 1/2 and, in the naive quark model, their spins must combine to give the total spin 1/2 of the nucleon. Hence nucleons are built up out of at least 3 quarks. The proton has two u-quarks and one d-quark, while the neutron has two d-quarks and one u-quark.

		u	d	p (uud)	n (udd)
Charge number	z_q	+2/3	−1/3	1	0
Isospin	I	1/2		1/2	
	I_3	+1/2	−1/2	+1/2	−1/2
Spin	s	1/2	1/2	1/2	1/2

Formally, the proton and the neutron maybe transformed to each other by interchanging the u- and d-quarks. They form an isospin doublet with $I = 1/2$ (cf. (2.12)). This is attributed to the fact that u- and d-quarks form an isospin doublet as well. The charges of proton and neutron are obtained by assigning charges to the quarks that are multiples of $e/3$, the charge of the u-quark being $e_u = z_u \cdot e = 2e/3$ and the charge of the d-quark being $e_d = z_d \cdot e = -1e/3$. These charges of the quarks are not unequivocally fixed by the charges of the proton and the neutron. This assignment is rather related to other clues; such as the fact that the maximum positive charge found in hadrons is two (e.g., Δ^{++}), and the maximum negative charge is one (e.g., Δ^-). Hence the charges of these hadrons are attributed to 3 u-quarks (charge: $3 \cdot (2e/3) = 2e$) and 3 d-quarks (charge: $3 \cdot (-1e/3) = -1e$) respectively.

Valence quarks and sea quarks The three quarks that determine the quantum numbers of the nucleons are called *valence quarks*. As well as these there also exist quark-antiquark pairs in the nucleon. They are produced and annihilated as virtual particles in the field of the strong interaction (cf. Sect. 8.2). This process is analogous to the production of virtual electron-positron pairs in the Coulomb field. These quark-antiquark pairs are called *sea quarks*. Their effective quantum numbers average out to zero and do not alter those of the nucleon. Because of their electrical charge, they are "visible" in deep-inelastic scattering, too. However, they carry only very small fractions x of the nucleon's momentum.

As well as u- and d-quarks, further types of quark-antiquark pairs are found in the "sea"; they will be discussed in more detail in Chap. 9. The different types of quarks are called "flavours". The additional quarks were named s (*strange*), c (*charm*), b (*bottom*) and t (*top*). As we will see later, the six quark types can be arranged in doublets (called *families* or *generations*), according to their increasing mass:

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix} .$$

The quarks of the top row have charge number $z_\mathrm{q} = +2/3$, those of the bottom row $z_\mathrm{q} = -1/3$. The c-, b- and t- quarks are so heavy that they play a very minor role at Q^2-values attainable in experiments with stationary targets. We will therefore neglect them in what follows.

7.5 Interpretation of Structure Functions in the Parton Model

Structure functions describe the internal composition of the nucleon. We now assume the nucleon to be built from different types of quarks q carrying an electrical charge $z_\mathrm{q} \cdot e$. The cross-section for electromagnetic scattering from a quark is proportional to the square of its charge, and hence to z_q^2.

We denote the distribution function of the quark momenta by $\mathrm{q}(x)$, i.e., $\mathrm{q}(x)\mathrm{d}x$ is the expectation value of the number of quarks of type $\mathrm{q} = \mathrm{u, d, s}$ in the nucleon whose momentum fraction lies within the interval $[x, x + \mathrm{d}x]$. The momentum distribution of the valence quarks we denote by $\mathrm{q_v}(x)$ and correspondingly the distribution of the antiquarks in the "sea" by $\bar{\mathrm{q}}_\mathrm{s}(x)$. The proton consists of two valence u-quarks and one valence d-quark. Therefore we have

$$\int_0^1 \mathrm{u_v}(x)\,\mathrm{d}x = 2, \qquad \int_0^1 \mathrm{d_v}(x)\,\mathrm{d}x = 1\ . \tag{7.14}$$

The structure function F_2 is the sum of the momentum distributions weighted by x and z_q^2. Here the sum is over all types of quarks and antiquarks:

$$F_2(x) = x \cdot \sum_{\mathrm{q=u,d,s}} z_\mathrm{q}^2 \left[\mathrm{q}(x) + \bar{\mathrm{q}}_\mathrm{s}(x)\right] , \tag{7.15}$$

with $\mathrm{q}(x) = \mathrm{q_v}(x) + \mathrm{q_s}(x)$ for u- and d-quarks and $\mathrm{q}(x) = \mathrm{q_s}(x)$ for s-quarks.

Structure functions of proton and neutron Much detailed information about the distribution functions of quarks can be obtained by the study of combinations of the structure functions F_2^p and F_2^n of proton and neutron. In the absence of free neutron targets, information about F_2^n must be obtained from deep-inelastic scattering of deuterons. By convention in scattering off nuclei, the structure function is always given per nucleon. Except for small corrections due to the Fermi motion of the nucleons, the structure function of the deuteron F_2^D is equal to the proton-neutron average structure function F_2^N

$$F_2^\mathrm{D} \approx \frac{F_2^\mathrm{p} + F_2^\mathrm{n}}{2} =: F_2^\mathrm{N} \,, \tag{7.16}$$

and hence we have $F_2^\mathrm{n} \approx 2F_2^\mathrm{D} - F_2^\mathrm{p}$.

According to (7.15), the structure functions F_2 of the proton and the neutron are given by

$$\begin{aligned} F_2^\mathrm{p}(x) &= x \cdot \left[\frac{4}{9}\left(u_\mathrm{v}^\mathrm{p} + u_\mathrm{s}^\mathrm{p} + \bar{u}_\mathrm{s}^\mathrm{p}\right) + \frac{1}{9}\left(d_\mathrm{v}^\mathrm{p} + d_\mathrm{s}^\mathrm{p} + \bar{d}_\mathrm{s}^\mathrm{p}\right) + \frac{1}{9}\left(s_\mathrm{s}^\mathrm{p} + \bar{s}_\mathrm{s}^\mathrm{p}\right)\right] \\ F_2^\mathrm{n}(x) &= x \cdot \left[\frac{4}{9}\left(u_\mathrm{v}^\mathrm{n} + u_\mathrm{s}^\mathrm{n} + \bar{u}_\mathrm{s}^\mathrm{n}\right) + \frac{1}{9}\left(d_\mathrm{v}^\mathrm{n} + d_\mathrm{s}^\mathrm{n} + \bar{d}_\mathrm{s}^\mathrm{n}\right) + \frac{1}{9}\left(s_\mathrm{s}^\mathrm{n} + \bar{s}_\mathrm{s}^\mathrm{n}\right)\right] , \end{aligned} \tag{7.17}$$

where $\mathrm{u}_\mathrm{v}^\mathrm{p,n}(x)$ denotes the distribution of valence u-quarks in the proton and the neutron, respectively, and $\mathrm{u}_\mathrm{s}(x)^\mathrm{p,n}$ the distribution of the sea u-quarks etc.

From *isospin symmetry* we obtain for the quark distributions

$$\begin{aligned} \mathrm{u}_\mathrm{v,s}^\mathrm{p}(x) &= \mathrm{d}_\mathrm{v,s}^\mathrm{n}(x) =: \mathrm{u}_\mathrm{v,s}(x) \,, \\ \mathrm{d}_\mathrm{v,s}^\mathrm{p}(x) &= \mathrm{u}_\mathrm{v,s}^\mathrm{n}(x) =: \mathrm{d}_\mathrm{v,s}(x) \,. \end{aligned} \tag{7.18}$$

Ratio of the neutron and proton structure functions The effective quantum numbers of the sea quarks average out to zero and we therefore have $\mathrm{q}_\mathrm{s}(x) = \bar{\mathrm{q}}_\mathrm{s}(x)$. We assume that the distributions of s-quarks in the proton and the neutron are identical ($\bar{\mathrm{s}}_\mathrm{s}^\mathrm{p}(x) = \bar{\mathrm{s}}_\mathrm{s}^\mathrm{n}(x)$), and also that the contributions of the two light u- and d-quarks to the “sea” are equal ($\bar{\mathrm{u}}_\mathrm{s}(x) = \bar{\mathrm{d}}_\mathrm{s}(x)$). (Below we will see that this relation is only approximately true.) Because of the larger mass of s-quarks, fluctuations into quark-antiquark pairs of this flavour have a smaller probability and we have $\bar{\mathrm{u}}_\mathrm{s}(x) > \bar{\mathrm{s}}_\mathrm{s}(x)$.

Summing up the z_q^2-weighted contributions of all sea quarks we can define

$$S(x) = 10\,\bar{\mathrm{u}}_s(x) + 2\,\bar{\mathrm{s}}_s(x) \,. \tag{7.19}$$

Then we obtain for the ratio of the neutron and proton structure functions:

$$\frac{F_2^\mathrm{n}(x)}{F_2^\mathrm{p}(x)} = \frac{[\mathrm{u}_\mathrm{v}(x) + 4\mathrm{d}_\mathrm{v}(x) + S(x)]}{[4\mathrm{u}_\mathrm{v}(x) + \mathrm{d}_\mathrm{v}(x) + S(x)]} \,. \tag{7.20}$$

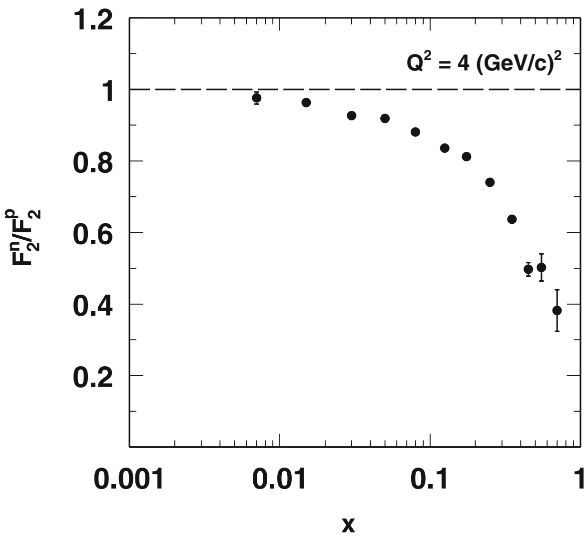

Fig. 7.7 The structure function ratio F_2^n/F_2^p as a function of x [2]. The data were obtained from muon scattering with beam energies of 90 and 280 GeV. Shown are results at $Q^2 = 4\ (\text{GeV}/c)^2$

Figure 7.7 shows the ratio F_2^n/F_2^p as a function of x. Plotted are data of one of the second-generation muon experiments [1, 2]. This experiment has a beam energy that is more than an order of magnitude higher than the experiments done at SLAC, and therefore the data cover much smaller values of x. Since the proton is composed of two valence u-quarks and one valence d-quark we could assume that their distributions are related by $\text{u}_\text{v}(x) = 2\text{d}_\text{v}(x)$. For a vanishing contribution of the sea quarks ($S(x) = 0$), F_2^n/F_2^p would obtain the value $2/3$ independent of x. In reality, however, the ratio approaches unity for $x \to 0$ and decreases with x down to a value of approximately $1/4$ for $x \to 1$. We can interpret this behaviour as follows: for small values of x the distribution of sea quarks $S(x)$ is much larger than the two valence quark distributions, the ratio is mainly determined by the last term in the numerator and denominator of (7.20). As $x \to 1$, the situation is reversed: the sea quarks no longer play a role and we obtain the value $1/4$ for the ratio by neglecting in (7.20) both $S(x)$ and $\text{d}_\text{v}(x)$ compared to $\text{u}_\text{v}(x)$. The distribution of d-quarks drops much faster with x than the u-quark distribution. This implies that large momentum fractions in the proton (neutron) are carried by u-quarks (d-quarks).

Difference of the proton and neutron structure functions The difference of the proton and neutron structure functions is given by

$$F_2^p(x) - F_2^n(x) = x \cdot \left[\frac{1}{3}(\text{u}_\text{v}(x) - \text{d}_\text{v}(x)) + \frac{2}{3}(\bar{\text{u}}_\text{s}(x) - \bar{\text{d}}_\text{s}(x))\right]. \tag{7.21}$$

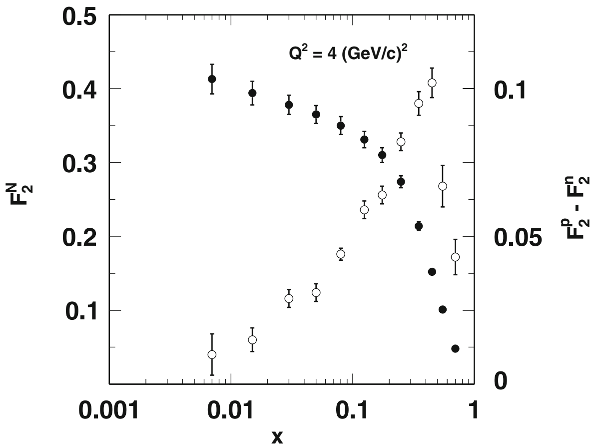

Fig. 7.8 The structure function F_2^{N} for an "average" nucleon (*closed symbols*, *left scale*) and the difference of the proton and neutron structure functions $F_2^{\mathrm{p}} - F_2^{\mathrm{n}}$ (*open symbols*, *right scale*) as function of x [2]. The data were obtained from muon scattering with beam energies of 90 and 280 GeV. Shown are results at $Q^2 = 4\ (\mathrm{GeV}/c)^2$

Thus if and only if the "sea" is symmetric in the two light-quark flavours, i.e., $\bar{\mathrm{u}}_{\mathrm{s}}(x) = \bar{\mathrm{d}}_{\mathrm{s}}(x)$, then the contributions from sea quarks drop out and the difference (7.21) is a pure valence quark distribution. In Fig. 7.8 data from the same muon experiment are shown for $F_2^{\mathrm{p}} - F_2^{\mathrm{n}}$ (open symbols, right scale) as a function of x. The distribution has a maximum near $x \approx 1/3$ and drops down to zero for $x \to 0$ and $x \to 1$. This supports our assumption, made above, that at low values of x mainly sea quarks contribute to the structure function. Also at large values of x the distribution becomes very small. Thus it is very unlikely that *one* quark alone carries the major part of the momentum of the nucleon.

The observed behaviour has often been interpreted as resulting from three valence quarks, each of them carrying on average one third of the nucleon's momentum and the sharply defined momentum at $x = 1/3$ is then washed out by the Fermi motion of the quarks inside the nucleon. This interpretation is incorrect. As we will see below, quarks carry only about half of the nucleon's momentum. The distributions $\mathrm{u_v}(x)$ and $\mathrm{d_v}(x)$ both have a maximum near $x \approx 0.17$ and the maximum of $F_2^{\mathrm{p}} - F_2^{\mathrm{n}}$ near $x = 1/3$ accidentally arises from the different x dependencies of these two distributions.

When we divide (7.21) by x and integrate over x, we obtain

$$S_G = \int_0^1 \frac{1}{x}\left[F_2^{\mathrm{p}}(x) - F_2^{\mathrm{n}}(x)\right] \mathrm{d}x = \frac{1}{3}(2-1) + \frac{2}{3}\int_0^1 (\bar{\mathrm{u}}_{\mathrm{s}}(x) - \bar{\mathrm{d}}_{\mathrm{s}}(x))\, \mathrm{d}x\,. \qquad (7.22)$$

For $\bar{u}_s(x) = \bar{d}_s(x)$ the last term drops out and we get $S_G = \frac{1}{3}$. This is the *Gottfried Sum Rule* [8]. Experimentally, however, the integral amounts to [2]

$$S_G = 0.235 \pm 0.026\,. \tag{7.23}$$

This leads to the conclusion that $\bar{d}_s(x) > \bar{u}_s(x)$ and that consequently the quark-antiquark "sea" is not symmetric in the two light-quark flavours. We will come back to this finding in Sect. 8.4.

Quark charges All of the quantitative statements made in the present chapter confirm the assignment of the fractional quark charges $e_u = 2e/3$ and $e_d = -1e/3$. In addition, a convincing confirmation comes from the comparison of the nucleon structure functions measured in deep-inelastic scattering of electrons or muons and of neutrinos that we will discuss in Sect. 10.6. Thus we can conclude:

Quarks carry fractional charges of $2e/3$ *and* $-1e/3$.

Structure function for an "average" nucleon Finally, after having discussed the ratio and the difference of the proton and neutron structure functions, we can get another important information about the structure of the nucleon by looking at their average. The structure function for an "average" nucleon (7.16) reads:

$$\begin{aligned} F_2^{\mathrm{N}}(x) &= \frac{5}{18}\,x \cdot \sum_{q=d,u} [q(x) + \bar{q}_s(x)] \;+\; \frac{1}{9}\,x \cdot \left[s_s(x) + \bar{s}_s(x)\right] \\ &= \frac{5}{18}\,x \cdot \sum_{q=d,u,s} [q(x) + \bar{q}_s(x)] \;-\; \frac{1}{3}\,x \cdot \bar{s}_s(x)\,. \end{aligned} \tag{7.24}$$

The last term in the equation is small, since s-quarks occur only as sea quarks. To a good approximation, F_2^{N} is therefore given by the product of the average squared charges 5/18 of u- and d-quarks (in units of e^2) and the sum over all quark distributions.

The integral of $F_2^{\mathrm{N}}(x)$ is taken over all quark momenta weighted by their distribution functions and the average squared quark charges. Therefore, the integral should yield the value 5/18, provided that the whole nucleon momentum is carried by its charged constituents, the quarks.

However, integration of the data shown in Fig. 7.8 only yields the value

$$\int_0^1 F_2^{\mathrm{N}}(x)\,\mathrm{d}x \;\approx\; 0.55 \cdot \frac{5}{18}\,. \tag{7.25}$$

Thus we have to conclude:

Quarks carry only about half of the nucleon's momentum.

The other half must be carried by uncharged particles interacting neither electromagnetically nor weakly. This finding was the starting point for the development of

QCD, the field theory of the strong interaction. The electrically neutral constituents have been identified with the field quanta of this interaction, the *gluons*.

Problems

1. **Deep-inelastic scattering**
   Derive the Callan-Gross relation (7.11). Which value for the mass of the target must be used?
2. **Parton momentum fractions and x**
   Show that in the parton model of deep-inelastic scattering, if we do **not** neglect the masses of the nucleon M and of the parton m, the momentum fraction ξ of the scattered parton in a nucleon with momentum P is given by

$$\xi = x\left[1 + \frac{m^2c^2 - M^2c^2x^2}{Q^2}\right].$$

   In the deep-inelastic domain $\frac{x^2M^2c^2}{Q^2} \ll 1$ and $\frac{m^2c^2}{Q^2} \ll 1$. (Hint: for small ε, ε' we can approximate $\sqrt{1+\varepsilon(1+\varepsilon')} \approx 1 + \frac{\varepsilon}{2}(1 + \varepsilon' - \frac{\varepsilon}{4})$.)

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