

Chapter 2

Global Properties of Nuclei

The discovery of the electron and of radioactivity marked the beginning of a new era in the investigation of matter. At that time, some signs of the atomic structure of matter were already clearly visible: e.g. the integer stoichiometric proportions of chemistry, the thermodynamics of gases, the periodic system of the elements or Brownian motion. However, the existence of atoms was not yet generally accepted. The reason was simple: nobody was able to really picture these building blocks of matter, the atoms. The new discoveries showed for the first time “particles” emerging from matter which had to be interpreted as its constituents.

It now became possible to use the particles produced by radioactive decay to bombard other elements in order to study the constituents of the latter. This experimental ansatz is the basis of modern nuclear and particle physics. Systematic studies of nuclei became possible by the late 1930s with the availability of modern particle accelerators. But the fundamental building blocks of atoms – the electron, proton and neutron – were detected beforehand. A pre-condition for these discoveries were important technical developments in vacuum techniques and in particle detection. Before we turn to the global properties of nuclei from a modern viewpoint, we will briefly discuss these historical experiments.

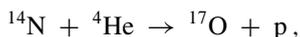
2.1 The Atom and Its Constituents

The electron The first building block of the atom to be identified was the electron. In 1897 Thomson was able to produce electrons as beams of free particles in discharge tubes. By deflecting them in electric and magnetic fields, he could determine their velocity and the ratio of their mass and charge. The results turned out to be independent of the kind of cathode and gas used. He had in other words found a universal constituent of matter. He then measured the charge of the electron

independently – using a method that was in 1910 significantly refined by Millikan (the drop method) – this of course also fixed the electron mass.

The atomic nucleus Subsequently, different models of the atom were discussed, one of them being the model of Thomson. In this model, the electrons, and an equivalent number of positively charged particles are uniformly distributed throughout the atom. The resulting atom is electrically neutral. Rutherford, Geiger and Marsden succeeded in disproving this picture. In their famous experiments, where they scattered α -particles off heavy atoms, they were able to show that the positively charged particles are closely packed together. They reached this conclusion from the angular distribution of the scattered α -particles. The angular distribution showed α -particle scattering at large scattering angles which was incompatible with a homogeneous charge distribution. The explanation of the scattering data was a central Coulomb field caused by a massive, positively charged nucleus. The method of extracting the properties of the scattering potential from the angular distribution of the scattered projectiles is still of great importance in nuclear and particle physics, and we will encounter it repeatedly in the following chapters. These experiments established the existence of the atom as a positively charged, small, massive nucleus with negatively charged electrons orbiting it.

The proton Rutherford also bombarded light nuclei with α -particles which themselves were identified as ionised helium atoms. In these reactions, he was looking for a conversion of elements, i.e., for a sort of inverse reaction to radioactive α -decay, which itself is a conversion of elements. While bombarding nitrogen with α -particles, he observed positively charged particles with an unusually long range, which must have been ejected from the atom as well. From this he concluded that the nitrogen atom had been destroyed in these reactions, and a light constituent of the nucleus had been ejected. He had already discovered similar long-ranged particles when bombarding hydrogen. From this he concluded that these particles were hydrogen nuclei which, therefore, had to be constituents of nitrogen as well. He had indeed observed the reaction



in which the nitrogen nucleus is converted into an oxygen nucleus, by the loss of a proton. The hydrogen nucleus could therefore be regarded as an elementary constituent of atomic nuclei. Rutherford also assumed that it would be possible to disintegrate additional atomic nuclei by using α -particles with higher energies than those available to him. He so paved the way for modern nuclear physics.

The neutron The neutron was also detected by bombarding nuclei with α -particles. Rutherford's method of visually detecting and counting particles by their scintillation on a zinc sulphide screen is not applicable to neutral particles. The development of ionisation and cloud chambers significantly simplified the detection of charged particles, but did not help here. Neutral particles could only be detected indirectly. Chadwick in 1932 found an appropriate experimental approach. He used

the irradiation of beryllium with α -particles from a polonium source, and thereby established the neutron as a fundamental constituent of nuclei. Previously, a “neutral radiation” had been observed in similar experiments, but its origin and identity was not understood. Chadwick arranged for this neutral radiation to collide with hydrogen, helium and nitrogen, and measured the recoil energies of these nuclei in a ionisation chamber. He deduced from the laws of collision that the mass of the neutral radiation particle was similar to that of the proton. Chadwick named this particle the “neutron”.

Nuclear force and binding With these discoveries, the building blocks of the atom had been found. The development of ion sources and mass spectrographs now permitted the investigation of the forces binding the nuclear constituents, i.e., the proton and the neutron. These forces were evidently much stronger than the electromagnetic forces holding the atom together, since atomic nuclei could only be broken up by bombarding them with highly energetic α -particles.

The binding energy of a system gives information about its binding and stability. This energy is the difference between the mass of a system and the sum of the masses of its constituents. It turns out that for nuclei this difference is close to 1 % of the nuclear mass. This phenomenon, historically called the mass defect, was one of the first experimental proofs of the mass-energy relation $E = mc^2$. The mass defect is of fundamental importance in the study of strongly interacting bound systems. We will therefore describe nuclear masses and their systematics in this chapter at some length.

2.2 Nuclides

The atomic number The atomic number Z gives the number of protons in the nucleus. The charge of the nucleus is, therefore, $Q = Ze$, the elementary charge being $e = 1.6 \cdot 10^{-19}$ C. In a neutral atom, there are Z electrons, which balance the charge of the nucleus, in the electron cloud. The atomic number of a given nucleus determines its chemical properties.

The classical method of determining the charge of the nucleus is the measurement of the characteristic X-rays of the atom to be studied. For this purpose the atom is excited by electrons, protons or synchrotron radiation. Moseley’s law says that the energy of the K_α -line is proportional to $(Z - 1)^2$. Nowadays, the detection of these characteristic X-rays is used to identify elements in material analysis.

Atoms are electrically neutral, which shows the equality of the absolute values of the positive charge of the proton and the negative charge of the electron. Experiments measuring the deflection of molecular beams in electric fields yield an upper limit for the difference between the proton and electron charges [4]:

$$|e_p + e_e| \leq 10^{-21} e . \quad (2.1)$$

Today's cosmological estimates give an even smaller upper limit for any difference between these charges.

The mass number In addition to the Z protons, N neutrons are found in the nucleus. The mass number A gives the number of nucleons in the nucleus, where $A = Z + N$. Different combinations of Z and N (or Z and A) are called *nuclides*.

- Nuclides with the same mass number A are called *isobars*.
- Nuclides with the same atomic number Z are called *isotopes*.
- Nuclides with the same neutron number N are called *isotones*.

The binding energy B is usually determined from atomic masses [1], since they can be measured to a considerably higher precision than nuclear masses. We have:

$$B(Z, A) = [ZM(^1\text{H}) + (A - Z)M_n - M(A, Z)] \cdot c^2. \quad (2.2)$$

Here, $M(^1\text{H}) = M_p + m_e$ is the mass of the hydrogen atom (the 13.6 eV binding energy of the H-atom is negligible), M_n is the mass of the neutron and $M(A, Z)$ is the mass of an atom with Z electrons whose nucleus contains A nucleons. The rest masses of these particles are:

$$\begin{aligned} M_p &= 938.272 \text{ MeV}/c^2 = 1,836.153 m_e \\ M_n &= 939.565 \text{ MeV}/c^2 = 1,838.684 m_e \\ m_e &= 0.511 \text{ MeV}/c^2. \end{aligned}$$

The conversion factor into SI units is $1.783 \cdot 10^{-30} \text{ kg}/(\text{MeV}/c^2)$.

In nuclear physics, nuclides are denoted by ${}^A\text{X}$, X being the chemical symbol of the element. For example, the stable carbon isotopes are labelled ${}^{12}\text{C}$ and ${}^{13}\text{C}$; while the radioactive carbon isotope frequently used for isotopic dating is labelled ${}^{14}\text{C}$. Sometimes the notations ${}^A_Z\text{X}$ or ${}^A_Z\text{X}_N$ are used, whereby the atomic number Z and possibly the neutron number N are explicitly added.

Determining masses from mass spectroscopy The binding energy of an atomic nucleus can be calculated if the atomic mass is accurately known. At the start of the twentieth century, the method of mass spectrometry was developed for precision determinations of atomic masses (and nucleon binding energies). The deflection of an ion with charge Q in an electric and magnetic field allows the simultaneous measurement of its momentum $p = Mv$ and its kinetic energy $E_{\text{kin}} = Mv^2/2$. From these, its mass can be determined. This is how most mass spectrometers work.

While the radius of curvature r_E of the ionic path in an electrical sector field is proportional to the energy,

$$r_E = \frac{M}{Q} \cdot \frac{v^2}{E}, \quad (2.3)$$

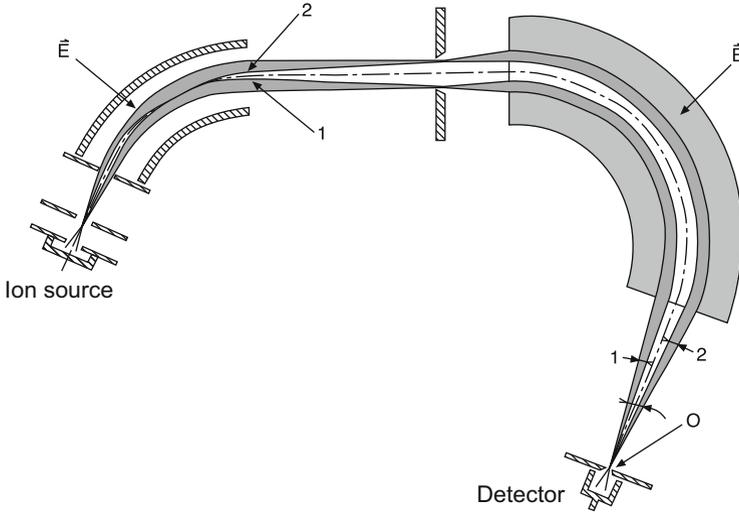


Fig. 2.1 Doubly focusing mass spectrometer [5]. The spectrometer focuses ions of a certain specific charge to mass ratio Q/M . For clarity, only the trajectories of particles at the edges of the beam are drawn (*1 and 2*). The electric and magnetic sector fields draw the ions from the ion source into the collector. Ions with a different Q/M ratio are separated from the beam in the magnetic field and do not pass through the slit *O*

in a magnetic field B the radius of curvature r_M of the ion is proportional to its momentum:

$$r_M = \frac{M}{Q} \cdot \frac{v}{B}. \quad (2.4)$$

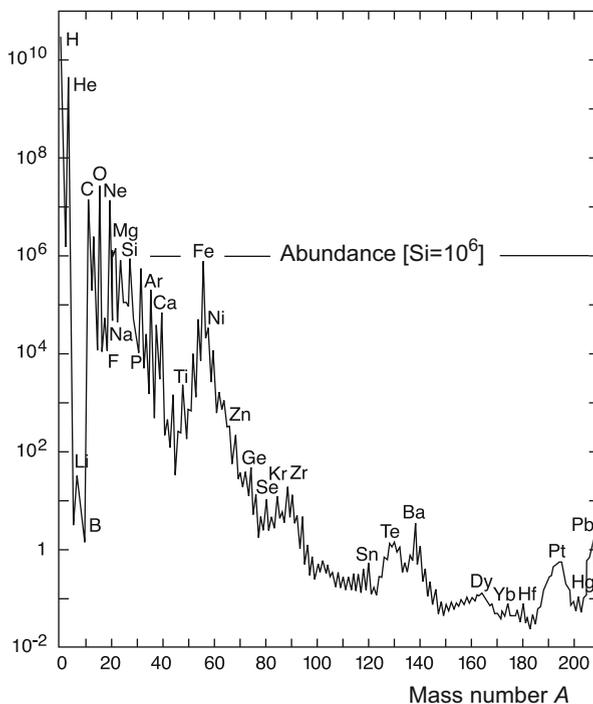
Figure 2.1 shows a common spectrometer design. After leaving the ion source, the ions are accelerated in an electric field to about 40 keV. In an electric field, they are then separated according to their energy and, in a magnetic field, according to their momentum. By careful design of the magnetic fields, ions with identical Q/M ratios leaving the ion source at various angles are focused at a point at the end of the spectrometer where a detector can be placed.

For technical reasons, it is very convenient to use the ^{12}C nuclide as the reference mass. Carbon and its many compounds are always present in a spectrometer and are well suited for mass calibration. An atomic mass unit u was therefore defined as $1/12$ of the atomic mass of the ^{12}C nuclide. We have:

$$1u = \frac{1}{12} M_{^{12}\text{C}} = 931.494 \text{ MeV}/c^2 = 1.66054 \cdot 10^{-27} \text{ kg}.$$

Mass spectrometers are still widely used both in research and industry.

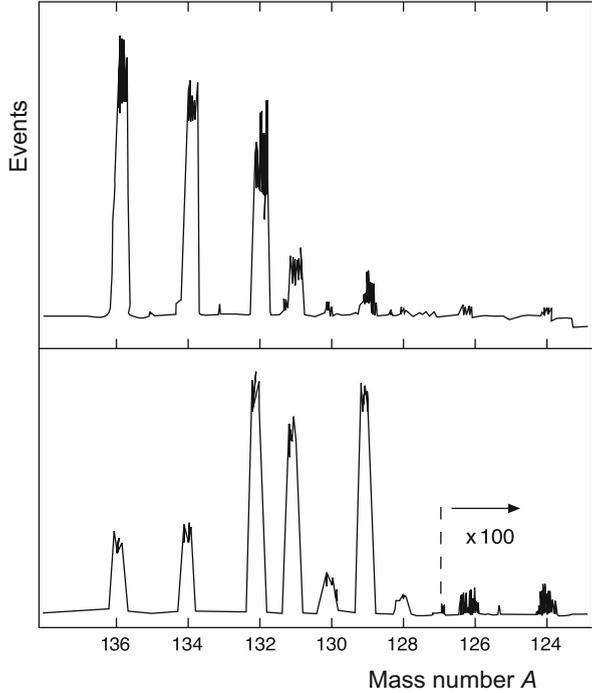
Fig. 2.2 Abundance of the elements in the solar system as a function of their mass number A , normalised to the abundance of silicon ($=10^6$)



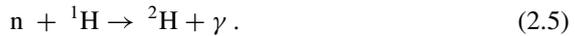
Nuclear abundance A current application of mass spectroscopy in fundamental research is the determination of isotope abundances in the solar system. The relative abundance of the various nuclides as a function of their mass number A is shown in Fig. 2.2. The relative abundances of isotopes in terrestrial, lunar, and meteoritic probes are, with few exceptions, identical and coincide with the nuclide abundances in cosmic rays from outside the solar system. According to current thinking, the synthesis of the presently existing deuterium and helium from hydrogen fusion mainly took place at the beginning of the universe (minutes after the big bang [2]). Nuclei up to ^{56}Fe , the most stable nucleus, were produced by nuclear fusion in stars. Nuclei heavier than this last were created in the explosion of very heavy stars (supernovae) [6].

Deviations from the universal abundance of isotopes occur locally when nuclides are formed in radioactive decays. Figure 2.3 shows the abundances of various xenon isotopes in a drill core which was found at a depth of 10 km. The isotope distribution strongly deviates from that which is found in the Earth's atmosphere. This deviation is a result of the atmospheric xenon being, for the most part, already present when the Earth came into existence, while the xenon isotopes from the core come from radioactive decays (spontaneous fission of uranium isotopes).

Fig. 2.3 Mass spectrum of xenon isotopes, found in a roughly $2.7 \cdot 10^9$ year old gneiss sample from a drill core produced in the Kola peninsula (*top*) and, for comparison, the spectrum of Xe-isotopes as they occur in the atmosphere (*bottom*). The Xe-isotopes in the gneiss were produced by spontaneous fission of uranium (Picture courtesy of Klaus Schäfer, Max-Planck-Institut für Kernphysik)



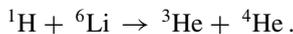
Determining masses from nuclear reactions Binding energies may also be determined from systematic studies of nuclear reactions. Consider, as an example, the capture of thermal neutrons ($E_{\text{kin}} \approx 1/40$ eV) by hydrogen,



The energy of the emitted photon is directly related to the binding energy B of the deuterium nucleus ${}^2\text{H}$:

$$B = (M_n + M_{{}^1\text{H}} - M_{{}^2\text{H}}) \cdot c^2 = E_\gamma + \frac{E_\gamma^2}{2M_{{}^2\text{H}}c^2} = 2.225 \text{ MeV}, \tag{2.6}$$

where the last term takes into account the recoil energy of the deuteron. As a further example, we consider the reaction



The energy balance of this reaction is given by

$$E_{{}^1\text{H}} + E_{{}^6\text{Li}} = E_{{}^3\text{He}} + E_{{}^4\text{He}}, \tag{2.7}$$

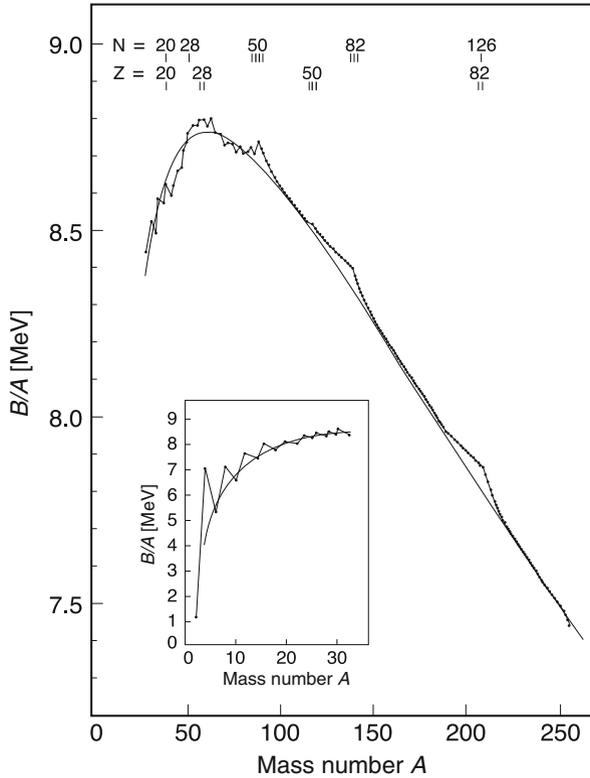


Fig. 2.4 Binding energy per nucleon of nuclei with even mass number A . The *solid line* corresponds to the Weizsäcker mass formula (2.8). Nuclei with a small number of nucleons display relatively large deviations from the general trend, and should be considered on an individual basis. For heavy nuclei deviations in the form of a somewhat stronger binding per nucleon are also observed for certain proton and neutron numbers. These so-called “magic numbers” will be discussed in Sect. 18.3

where the energies E_X each represent the total energy of the nuclide X , i.e., the sum of its rest mass and kinetic energy. If three of these nuclide masses are known, and if all of the kinetic energies have been measured, then the binding energy of the fourth nuclide can be determined.

The measurement of binding energies from nuclear reactions was mainly accomplished using low-energy (van de Graaff, cyclotron, betatron) accelerators. Following two decades of measurements in the 1950s and 1960s, the systematic errors of both methods, mass spectrometry and the energy balance of nuclear reactions, have been considerably reduced and both now provide high precision results which are consistent with each other. Figure 2.4 shows schematically the results of the binding energies per nucleon measured for stable nuclei. Nuclear reactions even provide mass determinations for nuclei which are so short-lived that they cannot be studied by mass spectroscopy.

2.3 Parametrisation of Binding Energies

Apart from the lightest elements, the binding energy per nucleon for most nuclei is about 8–9 MeV. Depending only weakly on the mass number, it can be described with the help of just a few parameters. The parametrisation of nuclear masses as a function of A and Z , which is known as the *Weizsäcker formula* or the *semi-empirical mass formula*, was first introduced in 1935 [3, 7]. It allows the calculation of the binding energy according to (2.2). The mass of an atom with Z protons and N neutrons is given by the following phenomenological formula:

$$\begin{aligned}
 M(A, Z) &= NM_n + ZM_p + Zm_e - a_v A + a_s A^{2/3} \\
 &+ a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{4A} + \frac{\delta}{A^{1/2}} \quad (2.8) \\
 &\text{with } N = A - Z.
 \end{aligned}$$

The exact values of the parameters a_v , a_s , a_c , a_a and δ depend on the range of masses for which they are optimised. One possible set of parameters is given below:

$$\begin{aligned}
 a_v &= 15.67 \text{ MeV}/c^2 \\
 a_s &= 17.23 \text{ MeV}/c^2 \\
 a_c &= 0.714 \text{ MeV}/c^2 \\
 a_a &= 93.15 \text{ MeV}/c^2 \\
 \delta &= \begin{cases} -11.2 \text{ MeV}/c^2 & \text{for even } Z \text{ and } N \text{ (even-even nuclei)} \\ 0 \text{ MeV}/c^2 & \text{for odd } A \text{ (odd-even nuclei)} \\ +11.2 \text{ MeV}/c^2 & \text{for odd } Z \text{ and } N \text{ (odd-odd nuclei)}. \end{cases}
 \end{aligned}$$

To a great extent the mass of an atom is given by the sum of the masses of its constituents (protons, neutrons and electrons). The nuclear binding responsible for the deviation from this sum is reflected in five additional terms. The physical meaning of these five terms can be understood by recalling that the nuclear radius R and mass number A are connected by the relation

$$R \propto A^{1/3}. \quad (2.9)$$

The experimental proof of this relation and a quantitative determination of the coefficient of proportionality will be discussed in Sect. 5.4. The individual terms can be interpreted as follows:

Volume term This term, which dominates the binding energy, is proportional to the number of nucleons. Each nucleon in the interior of a (large) nucleus contributes an energy of about 16 MeV. From this we deduce that the nuclear force has a short range, corresponding approximately to the distance between two nucleons. This

phenomenon is called saturation. If each nucleon would interact with each of the other nucleons in the nucleus, the total binding energy would be proportional to $A(A - 1)$ or approximately to A^2 . Due to saturation, the central density of nucleons is the same for all nuclei, with few exceptions. The central density is

$$\rho_0 \approx 0.17 \text{ nucleons/fm}^3 = 3 \cdot 10^{17} \text{ kg/m}^3. \quad (2.10)$$

The average nuclear density, which can be deduced from the mass and radius (see (5.56)), is smaller ($0.13 \text{ nucleons/fm}^3$). The average inter-nucleon distance in the nucleus is about 1.8 fm.

Surface term For nucleons at the surface of the nucleus, which are surrounded by fewer nucleons, the above binding energy is reduced. This contribution is proportional to the surface area of the nucleus (R^2 or $A^{2/3}$).

Coulomb term The electrical repulsive force acting between the protons in the nucleus further reduces the binding energy. This term is calculated to be

$$E_{\text{Coulomb}} = \frac{3}{5} \frac{Z(Z - 1) \alpha \hbar c}{R}. \quad (2.11)$$

This is approximately proportional to $Z^2/A^{1/3}$.

Asymmetry term As long as mass numbers are small, nuclei tend to have the same number of protons and neutrons. Heavier nuclei accumulate more and more neutrons, to partly compensate for the increasing Coulomb repulsion by increasing the nuclear force. This creates an asymmetry in the number of neutrons and protons. For, e.g., ^{208}Pb it amounts to $N - Z = 44$. The dependence of the nuclear force on the surplus of neutrons is described by the asymmetry term $(N - Z)^2/(4A)$. This shows that the symmetry decreases as the nuclear mass increases. We will further discuss this point in Sect. 18.1. The dependence of the above terms on A is shown in Fig. 2.5.

Pairing term A systematic study of nuclear masses shows that nuclei are more stable when they have an even number of protons and/or neutrons. This observation is interpreted as a coupling of protons and neutrons in pairs. The pairing energy depends on the mass number, as the overlap of the wave functions of these nucleons is smaller in larger nuclei. Empirically this is described by the term $\delta \cdot A^{-1/2}$ in (2.8).

All in all, the global properties of the nuclear force are rather well described by the mass formula (2.8). However, the details of nuclear structure which we will discuss later (mainly in Chap. 18) are not accounted for by this formula.

The Weizsäcker formula is often mentioned in connection with the *liquid drop model*. In fact, the formula is based on some properties known from liquid drops: constant density, short-range forces, saturation, deformability and surface tension. An essential difference, however, is found in the mean free path of the particles. For molecules in liquid drops, this is far smaller than the size of the drop; but for nucleons in the nucleus, it is large. Therefore, the nucleus has to be treated as a

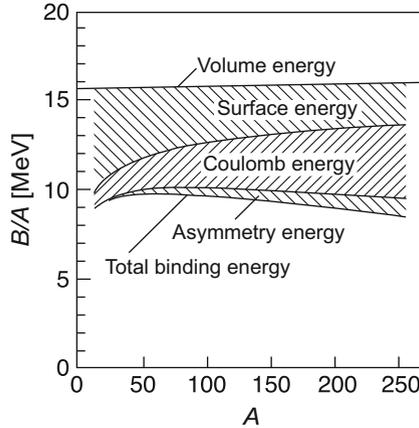


Fig. 2.5 The different contributions to the binding energy per nucleon versus mass number A . The horizontal line at ≈ 16 MeV represents the contribution of the volume energy. This is reduced by the surface energy, the asymmetry energy and the Coulomb energy to the effective binding energy of ≈ 8 MeV (lower line). The contributions of the asymmetry and Coulomb terms increase rapidly with A , while the contribution of the surface term decreases

quantum liquid, and not as a classical one. At low excitation energies, the nucleus may be even more simply described as a Fermi gas; i.e., as a system of free particles only weakly interacting with each other. This model will be discussed in more detail in Sect. 18.1.

2.4 Charge Independence of the Nuclear Force and Isospin

Protons and neutrons not only have nearly equal masses, they also have similar nuclear interactions. This is particularly visible in the study of *mirror nuclei*. Mirror nuclei are pairs of isobars, in which the proton number of one of the nuclides equals the neutron number of the other and vice versa.

Figure 2.6 shows the lowest energy levels of the mirror nuclei ${}^{14}_6\text{C}_8$ and ${}^{14}_8\text{O}_6$, together with those of ${}^{14}_7\text{N}_7$. The energy-level diagrams of ${}^{14}_6\text{C}_8$ and ${}^{14}_8\text{O}_6$ are very similar with respect to the quantum numbers J^P of the levels as well as with respect to the distances between them. The small differences and the global shift of the levels as a whole in ${}^{14}_6\text{C}_8$, as compared to ${}^{14}_8\text{O}_6$ can be explained by differences in the Coulomb energy. Further examples of mirror nuclei will be discussed in Sect. 18.3 (Fig. 18.8). The energy levels of ${}^{14}_6\text{C}_8$ and ${}^{14}_8\text{O}_6$ are also found in the isobaric nucleus ${}^{14}_7\text{N}_7$. Other states in ${}^{14}_7\text{N}_7$ have no analogy in the two neighbouring nuclei. We therefore can distinguish between triplet and singlet states.

These multiplets of states are reminiscent of the multiplets known from the coupling of angular momenta (spins). The symmetry between protons and neutrons

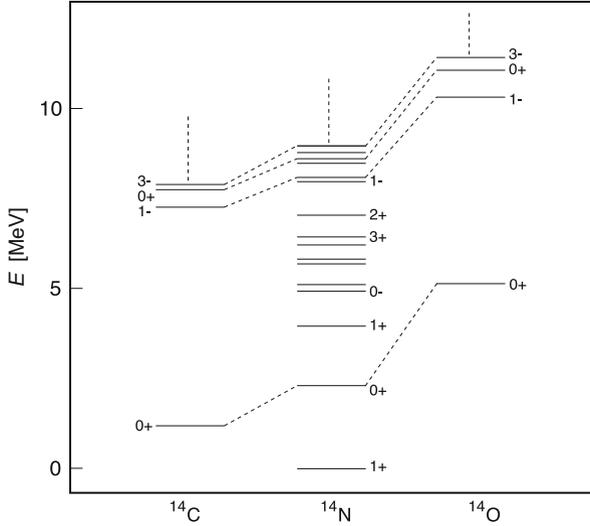


Fig. 2.6 Low-lying energy levels of the three most stable $A = 14$ isobars. Angular momentum J and parity P are shown for the most important levels. The analogous states of the three nuclei are joined by *dashed lines*. The zero of the energy scale is set to the ground state of $^{14}_7\text{N}$.

may therefore be described by a similar formalism, called *isospin* I . The proton and neutron are treated as two states of the nucleon which form a doublet ($I = 1/2$).

$$\text{Nucleon : } I = 1/2 \quad \left\{ \begin{array}{l} \text{proton : } I_3 = +1/2 \\ \text{neutron : } I_3 = -1/2. \end{array} \right. \quad (2.12)$$

Formally, isospin is treated as a quantum mechanical angular momentum. For example, a proton-neutron pair can be in a state of total isospin 1 or 0. The third (z -) component of isospin is additive:

$$I_3^{\text{nucleus}} = \sum I_3^{\text{nucleon}} = \frac{Z-N}{2}. \quad (2.13)$$

This enables us to describe the appearance of similar states in Fig. 2.6: $^{14}_6\text{C}_8$ and $^{14}_8\text{O}_6$, have respectively $I_3 = -1$ and $I_3 = +1$. Therefore, their isospin cannot be less than $I = 1$. The states in these nuclei thus necessarily belong to a triplet of similar states in $^{14}_6\text{C}_8$, $^{14}_7\text{N}_7$ and $^{14}_8\text{O}_6$. The I_3 component of the nuclide $^{14}_7\text{N}_7$, however, is 0. This nuclide can, therefore, have additional states with isospin $I = 0$.

Since $^{14}_7\text{N}_7$ is the most stable $A = 14$ isobar, its ground state is necessarily an isospin singlet since otherwise $^{14}_6\text{C}_8$ would possess an analogous state, which, with less Coulomb repulsion, would be lower in energy and so more stable. States with $I = 2$ are not shown in Fig. 2.6. Such states would have analogous states in $^{14}_5\text{B}_9$ and in $^{14}_9\text{F}_5$. These nuclides, however, are very unstable (i.e., highly energetic), and

lie above the energy range of the diagram. The $A = 14$ isobars are rather light nuclei in which the Coulomb energy is not strongly felt. In heavier nuclei, the influence of the Coulomb energy grows, which increasingly disturbs the isospin symmetry.

The concept of isospin is of great importance not only in nuclear physics, but also in particle physics. As we will see quarks, and particles composed of quarks, can be classified by isospin into isospin multiplets. In dynamical processes of the strong-interaction type, the isospin of the system is conserved.

Problem

1. Isospin symmetry

One could naively imagine the three nucleons in the ${}^3\text{H}$ and ${}^3\text{He}$ nuclei as being rigid spheres. If one solely attributes the difference in the binding energies of these two nuclei to the electrostatic repulsion of the protons in ${}^3\text{He}$, how large must the separation of the protons be? (The maximal energy of the electron in the β^- -decay of ${}^3\text{H}$ is 18.6 keV.)

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