Chapter 3
Nuclear Stability

Stable nuclei only occur in a very narrow band in the $Z - N$ plane (Fig. 3.1). All other nuclei are unstable and decay spontaneously in various ways. Isobars with a large surplus of neutrons gain energy by converting a neutron into a proton. In the case of a surplus of protons, the inverse reaction may occur: i.e., the conversion of a proton into a neutron. These transformations are called *β-decays* and they are manifestations of the weak interaction. After dealing with the weak interaction in Chap. 10, we will discuss these decays in more detail in Sects. 16.6 and 18.6. In the present chapter, we will merely survey certain general properties, paying particular attention to the energy balance of β-decays.

Iron and nickel isotopes possess the maximum binding energy per nucleon and they are therefore the most stable nuclides. In heavier nuclei the binding energy is smaller because of the larger Coulomb repulsion. For still heavier masses nuclei become unstable to fission and decay spontaneously into two or more lighter nuclei should the mass of the original atom be larger than the sum of the masses of the daughter atoms. For a two-body decay, this condition has the form

$$M(A,Z) \; > \; M(A-A',Z-Z') + M(A',Z') \, . \tag{3.1}$$

This relation takes into account the conservation of the number of protons and neutrons. However, it does not give any information about the probability of such a decay. An isotope is said to be stable if its lifetime is considerably longer than the age of the solar system. We will not consider many-body decays any further since they are much rarer than two-body decays. It is very often the case that one of the daughter nuclei is a ^{4}He nucleus, i.e., $A' = 4$, $Z' = 2$. This decay mode is called *α-decay*, and the Helium nucleus is called an *α-particle*. If a heavy nucleus decays into two similarly massive daughter nuclei we speak of *spontaneous fission*. The probability of spontaneous fission exceeds that of α-decay only for nuclei with $Z \gtrsim 110$ and is a fairly unimportant process for the naturally occurring heavy elements.

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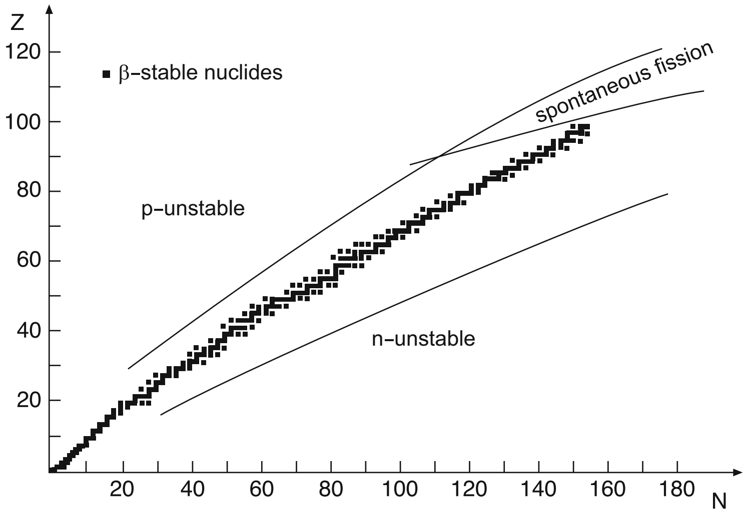

Fig. 3.1 Beta-stable nuclei in the $Z - N$ plane (From [1])

Decay constants The probability per unit time for a radioactive nucleus to decay is known as the *decay constant* λ. It is related to the *lifetime* τ and the *half-life* $t_{1/2}$ by

$$\tau = \frac{1}{\lambda} \qquad \text{and} \qquad t_{1/2} = \frac{\ln 2}{\lambda} \,. \tag{3.2}$$

The measurement of the decay constants of radioactive nuclei is based upon finding the *activity* (the number of decays per unit time)

$$A = -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N \,, \tag{3.3}$$

where N is the number of radioactive nuclei in the sample. The unit of activity is defined to be

$$1 \text{ Bq [Becquerel]} = 1 \text{ decay/s} \,. \tag{3.4}$$

For short-lived nuclides, the fall-off over time of the activity

$$A(t) = \lambda N(t) = \lambda N_0 \, \mathrm{e}^{-\lambda t} \,, \qquad \text{where} \quad N_0 = N(t = 0) \,, \tag{3.5}$$

may be measured using fast electronic counters. This method of measuring is not suitable for lifetimes larger than about a year. For longer-lived nuclei both the number of nuclei in the sample and the activity must be measured in order to obtain the decay constant from (3.3).

3.1 Beta Decay

Let us consider nuclei with equal mass number A (isobars). Equation (2.8) can be transformed into

$$M(A,Z) = \alpha \cdot A - \beta \cdot Z + \gamma \cdot Z^2 + \frac{\delta}{A^{1/2}}\,, \tag{3.6}$$

where
$$\begin{aligned}
\alpha &= M_\mathrm{n} - a_\mathrm{v} + a_\mathrm{s}A^{-1/3} + \frac{a_\mathrm{a}}{4}\,,\\
\beta &= a_\mathrm{a} + (M_\mathrm{n} - M_\mathrm{p} - m_\mathrm{e})\,,\\
\gamma &= \frac{a_\mathrm{a}}{A} + \frac{a_\mathrm{c}}{A^{1/3}}\,,\\
\delta &= \text{as in (2.8)}\,.
\end{aligned}$$

The nuclear mass is now a quadratic function of the charge number Z. A plot of such nuclear masses, for constant mass number A, as a function of Z yields a parabola for odd A. For even A, the masses of the even-even and the odd-odd nuclei are found to lie on two vertically shifted parabolas. The odd-odd parabola lies at twice the pairing energy ($2\delta/\sqrt{A}$) above the even-even one. The minimum of the parabolas is found at $Z = \beta/2\gamma$. The nucleus with the smallest mass in an isobaric spectrum is stable with respect to β-decay.

Beta decay in odd mass nuclei In what follows we wish to discuss the different kinds of β-decay, using the example of the $A = 101$ isobars. For this mass number, the parabola minimum is at the isobar ^{101}Ru which has $Z = 44$. Isobars with more neutrons, such as $^{101}_{42}$Mo and $^{101}_{43}$Tc, decay through the conversion

$$\mathrm{n} \;\rightarrow\; \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,. \tag{3.7}$$

The charge number of the daughter nucleus is one unit larger than that of the parent nucleus (Fig. 3.2). An electron and an electron-antineutrino are also produced:

$$\begin{aligned}
^{101}_{42}\mathrm{Mo} &\rightarrow {}^{101}_{43}\mathrm{Tc} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,,\\
^{101}_{43}\mathrm{Tc} &\rightarrow {}^{101}_{44}\mathrm{Ru} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}\,.
\end{aligned}$$

Historically such decays where a negative electron is emitted are called β^--decays. Energetically, β^--decay is possible whenever the mass of the daughter atom $M(A, Z+1)$ is smaller than the mass of its isobaric neighbour:

$$M(A,Z) > M(A,Z+1)\,. \tag{3.8}$$

We consider here the mass of the whole atom and not just that of the nucleus alone and so the rest mass of the electron created in the decay is automatically taken into

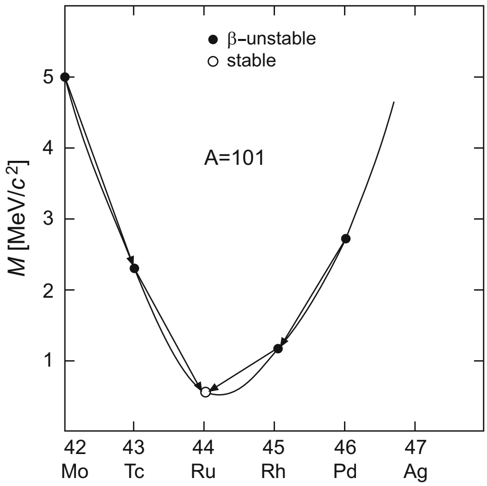

Fig. 3.2 Mass parabola of the $A = 101$ isobars (From [4]). Possible β-decays are shown by *arrows*. The abscissa co-ordinate is the atomic number, Z. The zero point of the mass scale was chosen arbitrarily

account. The tiny mass of the (anti-)neutrino ($<2\,\mathrm{eV}/c^2$) [3] is negligible in the mass balance.

Isobars with a proton excess, compared to $^{101}_{44}\mathrm{Ru}$, decay through proton conversion

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}\,. \tag{3.9}$$

The stable isobar $^{101}_{44}\mathrm{Ru}$ is eventually produced via

$$^{101}_{46}\mathrm{Pd} \rightarrow {}^{101}_{45}\mathrm{Rh} + \mathrm{e}^+ + \nu_\mathrm{e}\,, \quad \text{and}$$
$$^{101}_{45}\mathrm{Rh} \rightarrow {}^{101}_{44}\mathrm{Ru} + \mathrm{e}^+ + \nu_\mathrm{e}\,.$$

Such decays are called β^+-decays. Since the mass of a free neutron is larger than the proton mass, the process (3.9) is only possible inside a nucleus. By contrast, neutrons outside nuclei can and do decay via (3.7). Energetically, β^+-decay is possible whenever the following relationship between the masses $M(A, Z)$ and $M(A, Z-1)$ (of the parent and daughter atoms respectively) is satisfied:

$$M(A, Z) > M(A, Z-1) + 2m_\mathrm{e}\,. \tag{3.10}$$

This relationship takes into account the creation of a positron and the existence of an excess electron in the parent atom.

Beta decay in even nuclei Even mass-number isobars form, as we described above, two separate (one for even-even and one for odd-odd nuclei) parabolas which are split by an amount equal to twice the pairing energy.

Often there is more than one β-stable isobar, especially in the range $A > 70$. Let us consider the example of the nuclides with $A = 106$ (Fig. 3.3). The even-even

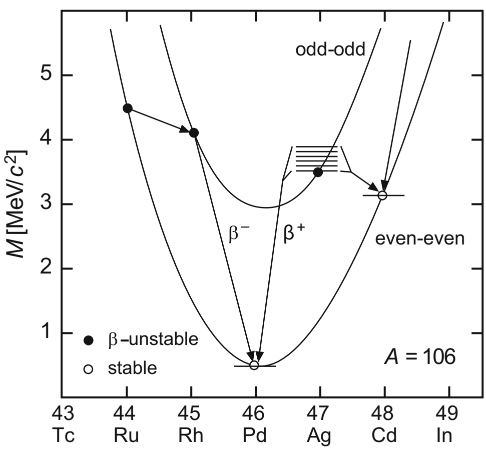

Fig. 3.3 Mass parabolas of the $A = 106$-isobars (From [4]). Possible β-decays are indicated by *arrows*. The abscissa coordinate is the charge number Z. The zero point of the mass scale was chosen arbitrarily

${}^{106}_{46}$Pd and ${}^{106}_{48}$Cd isobars are on the lower parabola, and ${}^{106}_{46}$Pd is the stablest. ${}^{106}_{48}$Cd is β-stable, since its two odd-odd neighbours both lie above it. The conversion of ${}^{106}_{48}$Cd is thus only possible through a double β-decay into ${}^{106}_{46}$Pd:

$$ {}^{106}_{48}\mathrm{Cd} \;\rightarrow\; {}^{106}_{46}\mathrm{Pd} + 2\mathrm{e}^{+} + 2\nu_{\mathrm{e}}\,. $$

The probability for such a process is so small that ${}^{106}_{48}$Cd may be considered to be a stable nuclide. Details of double β-decay will be discussed in Sect. 18.7.

Odd-odd nuclei always have at least one more strongly bound even-even neighbour nucleus in the isobaric spectrum. They are therefore unstable. The only exceptions to this rule are the very light nuclei ${}^{2}_{1}$H, ${}^{6}_{3}$Li, ${}^{10}_{5}$B and ${}^{14}_{7}$N, which are stable to β-decay, since the increase in the asymmetry energy would exceed the decrease in pairing energy. Some odd-odd nuclei can undergo both β^--decay and β^+-decay. Well-known examples of this are ${}^{40}_{19}$K (Fig. 3.4) and ${}^{64}_{29}$Cu.

Electron capture Another possible decay process is the capture of an electron from the cloud surrounding the atom. There is a finite probability of finding such an electron inside the nucleus. In such circumstances it can combine with a proton to form a neutron and a neutrino in the following way:

$$ \mathrm{p} + \mathrm{e}^{-} \rightarrow \mathrm{n} + \nu_{\mathrm{e}}\,. \tag{3.11} $$

This reaction occurs mainly in heavy nuclei where the nuclear radii are larger and the electronic orbits are more compact. Usually the electrons that are captured are from the innermost (the "K") shell since such K-electrons are closest to the nucleus and their radial wave function has a maximum at the centre of the nucleus. Since an electron is missing from the K-shell after such a *K-capture*, electrons from higher energy levels will successively cascade downwards and in so doing they emit characteristic X-rays.

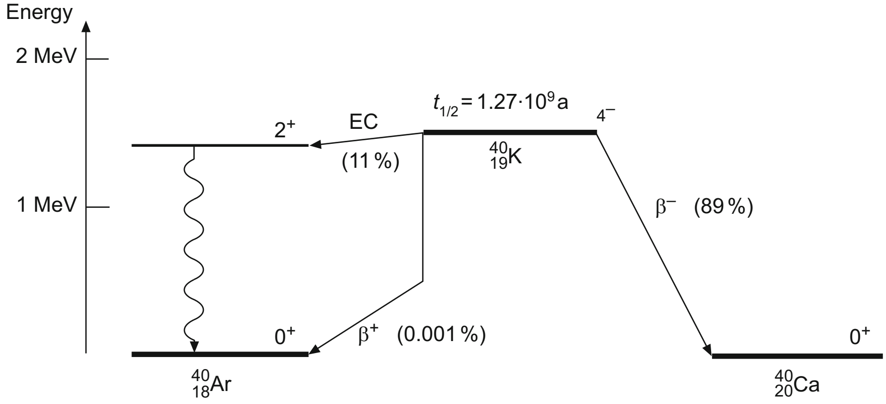

Fig. 3.4 The β-decay of ^{40}K. In this nuclear conversion, β^-- and β^+-decay as well as electron capture (EC) compete with each other. The relative frequency of these decays is given in parentheses. The bent arrow in β^+-decay indicates that the production of an e^+ and the presence of the surplus electron in the ^{40}Ar atom requires 1.022 MeV, and the remainder is carried off as kinetic energy by the positron and the neutrino. The excited state of ^{40}Ar produced in the electron capture reaction decays by photon emission into its ground state

Electron-capture reactions compete with β^+-decay. The following condition is a consequence of energy conservation

$$M(A, Z) > M(A, Z-1) + \varepsilon\,, \tag{3.12}$$

where ε is the excitation energy of the atomic shell of the daughter nucleus (electron capture always leads to a hole in the electron shell). This process has, compared to β^+-decay, more kinetic energy ($2m_ec^2 - \varepsilon$ more) available to it and so there are some cases where the mass difference between the initial and final atoms is too small for conversion to proceed via β^+-decay and yet K-capture can take place.

Lifetimes The lifetimes τ of β-unstable nuclei vary between a few ms and 10^{16} years. They strongly depend upon both the energy E which is released ($1/\tau \propto E^5$) and upon the nuclear properties of the mother and daughter nuclei. The decay of a free neutron into a proton, an electron and an antineutrino releases 0.78 MeV and this particle has a lifetime of $\tau = 880.1 \pm 1.1$ s [3]. No two neighbouring isobars are known to be β-stable.[1]

A well-known example of a long-lived β-emitter is the nuclide ^{40}K. It transforms into other isobars by both β^-- and β^+-decay. Electron capture in ^{40}K also competes

[1] In some cases, however, one of two neighbouring isobars is stable and the other is extremely long-lived. The most common isotopes of indium (^{115}In, 96 %) and rhenium (^{187}Re, 63 %) β^--decay into stable nuclei (^{115}Sn and ^{187}Os), but they are so long-lived ($\tau = 3 \cdot 10^{14}$ years and $\tau = 3 \cdot 10^{11}$ years respectively) that they may also be considered stable.

here with β^+-decay. The stable daughter nuclei are ^{40}Ar and ^{40}Ca respectively, which is a case of two stable nuclei having the same mass number A (Fig. 3.4).

The ^{40}K nuclide was chosen here because it contributes considerably to the radiation exposure of human beings and other biological systems. Potassium is an essential element: for example, signal transmission in the nervous system functions by an exchange of potassium ions. The fraction of radioactive ^{40}K in natural potassium is 0.01 %, and the decay of ^{40}K in the human body contributes about 16 % of the total natural radiation which we are exposed to.

3.2 Alpha Decay

Protons and neutrons have binding energies, even in heavy nuclei, of about 8 MeV (Fig. 2.4) and cannot generally escape from the nucleus. In many cases, however, it is energetically possible for a bound system of a group of nucleons to be emitted, since the binding energy of this system increases the total energy available to the process. The probability for such a system to be formed in a nucleus decreases rapidly with the number of nucleons required. In practice the most significant decay process is the emission of a ^{4}He nucleus; i.e., a system of 2 protons and 2 neutrons. Contrary to systems of 2 or 3 nucleons, this so-called *α-particle* is extraordinarily strongly bound – 7 MeV/nucleon (cf. Fig. 2.4). Such decays are called *α-decays*.

Figure 3.5 shows the potential energy of an α-particle as a function of its separation from the centre of the nucleus. Beyond the nuclear force range, the α-particle feels only the Coulomb potential $V_C(r) = 2(Z-2)\alpha\hbar c/r$, which increases closer to the nucleus. Within the nuclear force range a strongly attractive nuclear potential prevails. Its strength is characterised by the depth of the potential well. Since we are considering α-particles which are energetically allowed to escape from the nuclear potential, the total energy of this α-particle is positive. This energy is released in the decay.

The range of lifetimes for the α-decay of heavy nuclei is extremely large. Experimentally, lifetimes have been measured between 10 ns and 10^{17} years. These

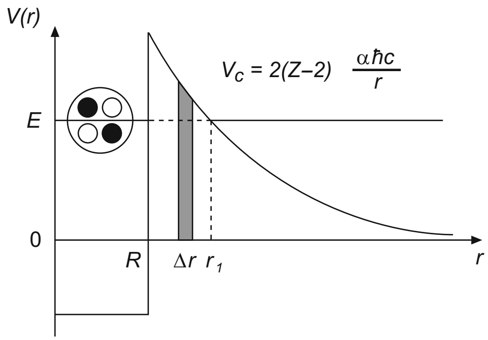

Fig. 3.5 Potential energy of an α-particle as a function of its separation from the centre of the nucleus. The probability that it tunnels through the Coulomb barrier can be calculated as the superposition of tunnelling processes through thin potential walls of thickness Δr (cf. Fig. 3.6)

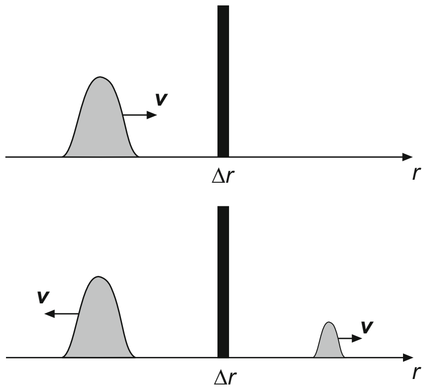

Fig. 3.6 Illustration of the tunnelling probability of a wave packet with energy E and velocity v faced with a potential barrier of height V and thickness Δr

lifetimes can be calculated in quantum mechanics by treating the α-particle as a wave packet. The probability for the α-particle to escape from the nucleus is given by the probability for its penetrating the *Coulomb barrier* (the tunnel effect). If we divide the Coulomb barrier into thin potential walls and look at the probability of the α-particle tunnelling through one of these (Fig. 3.6), then the transmission T is given by

$$T \approx \mathrm{e}^{-2\kappa \Delta r}, \qquad \text{where} \quad \kappa = \sqrt{2m|E-V|}/\hbar\,, \tag{3.13}$$

and Δr is the thickness of the barrier and V is its height. E is the energy of the α-particle. A Coulomb barrier can be thought of as a barrier composed of a large number of thin potential walls of different heights. The transmission can be described accordingly by

$$T = \mathrm{e}^{-2G}\,. \tag{3.14}$$

The *Gamow factor G* can be approximated by the integral [4]

$$G = \frac{1}{\hbar}\int_R^{r_1} \sqrt{2m|E-V|}\,\mathrm{d}r \approx \frac{\pi \cdot 2 \cdot (Z-2) \cdot \alpha}{\beta}\,, \tag{3.15}$$

where $\beta = v/c$ is the velocity of the outgoing α-particle and R is the nuclear radius.

The probability per unit time λ for an α-particle to escape from the nucleus is therefore proportional to: the probability $w(\alpha)$ of finding such an α-particle in the nucleus, the number of collisions ($\propto v_0/2R$) of the α-particle with the barrier and the transmission probability:

$$\lambda = w(\alpha)\,\frac{v_0}{2R}\,\mathrm{e}^{-2G}\,, \tag{3.16}$$

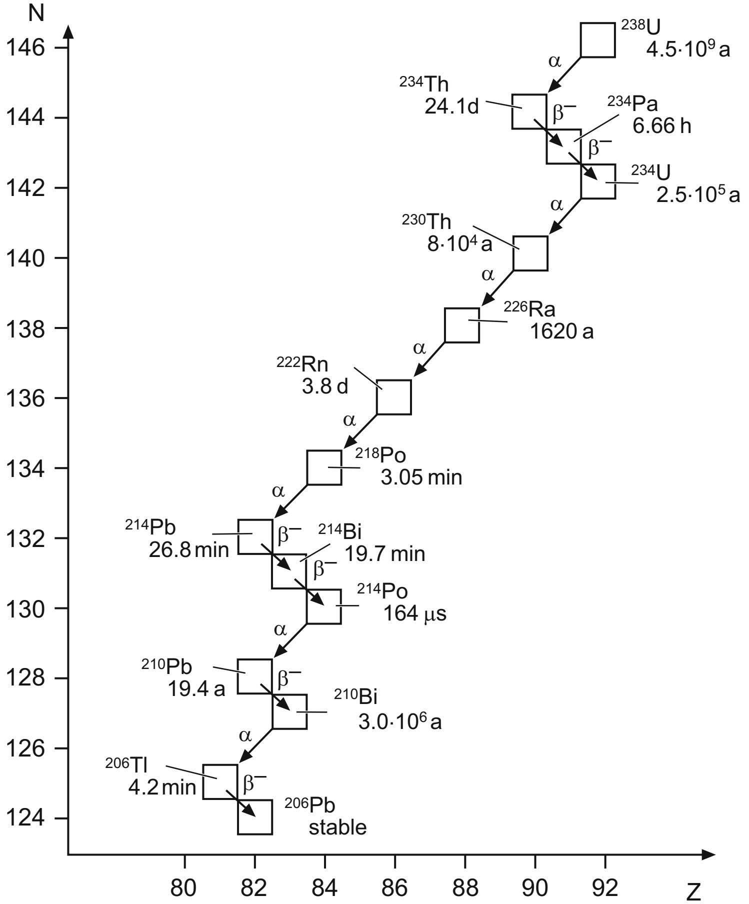

Fig. 3.7 Illustration of the ^{238}U decay chain in the *N*-*Z* plane. The half-life of each of the nuclides is given together with its decay mode

where v_0 is the velocity of the α-particle in the nucleus ($v_0 \approx 0.1\,c$). The large variation in the lifetimes is explained by the Gamow factor in the exponent: since $G \propto Z/\beta \propto Z/\sqrt{E}$, small differences in the energy of the α-particle have a strong effect on the lifetime.

Most α-emitting nuclei are heavier than lead. For lighter nuclei with $A \lesssim 140$, α-decay is energetically possible, but the energy released is extremely small. Therefore, their nuclear lifetimes are so long that decays are usually not observable.

An example of an α-unstable nuclide with a long lifetime, ^{238}U, is shown in Fig. 3.7. Since uranium compounds are common in granite, uranium and its radioactive daughters are a part of the stone walls of buildings. They therefore contribute to the environmental radiation background. This is particularly true of the inert gas ^{222}Rn, which escapes from the walls and is inhaled into the lungs. The α-decay of ^{222}Rn is responsible for about 40 % of the average natural human radiation exposure.

3.3 Nuclear Fission

Spontaneous fission The largest binding energy per nucleon is found in those nuclei in the region of ^{56}Fe. For heavier nuclei, it decreases as the nuclear mass increases (Fig. 2.4). A nucleus with $Z > 40$ can thus, in principle, split into two lighter nuclei. The potential barrier which must be tunnelled through is, however, so large that such spontaneous fission reactions are generally speaking extremely unlikely.

The lightest nuclides where the probability of spontaneous fission is comparable to that of α-decay are certain uranium isotopes. The shape of the fission barrier is shown in Fig. 3.8.

It is interesting to find the charge number Z above which nuclei become fission unstable, i.e., the point from which the mutual Coulombic repulsion of the protons outweighs the attractive nature of the nuclear force. An estimate can be obtained by considering the surface and the Coulomb energies during the fission deformation. As the nucleus is deformed the surface energy increases, while the Coulomb energy decreases. If the deformation leads to an energetically more favourable configuration, the nucleus is unstable. Quantitatively, this can be calculated as follows: keeping the volume of the nucleus constant, we deform its spherical shape into an ellipsoid with axes $a = R(1+\varepsilon)$ and $b = R(1+\varepsilon)^{-1/2} \approx R(1-\varepsilon/2)$ (Fig. 3.9).

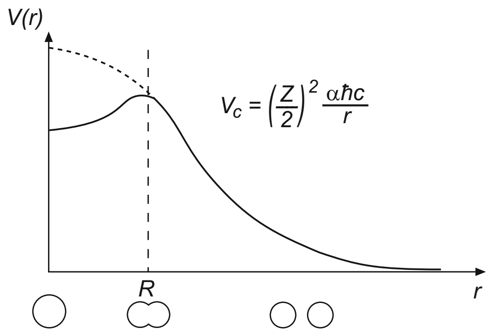

Fig. 3.8 Potential energy during different stages of a fission reaction. A nucleus with charge Z decays spontaneously into two daughter nuclei. The *solid line* corresponds to the shape of the potential in the parent nucleus. The height of the barrier for fission determines the probability of spontaneous fission. The fission barrier disappears for nuclei with $Z^2/A \gtrsim 48$ and the shape of the potential then corresponds to the *dashed line*

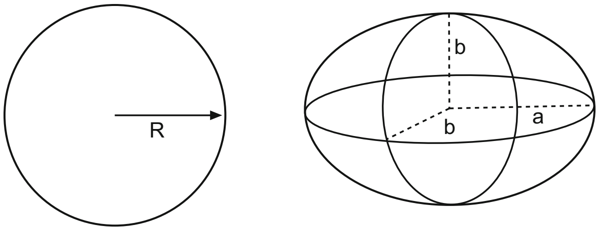

Fig. 3.9 Deformation of a heavy nucleus. For a constant volume V ($V = 4\pi R^3/3 = 4\pi ab^2/3$), the surface energy of the nucleus increases and its Coulomb energy decreases

The surface energy then has the form

$$E_s = a_s A^{2/3}\left(1 + \frac{2}{5}\varepsilon^2 + \ldots\right), \tag{3.17}$$

while the Coulomb energy is given by

$$E_c = a_c Z^2 A^{-1/3}\left(1 - \frac{1}{5}\varepsilon^2 + \ldots\right). \tag{3.18}$$

Hence a deformation ε changes the total energy by

$$\Delta E = \frac{\varepsilon^2}{5}\left(2a_s A^{2/3} - a_c Z^2 A^{-1/3}\right). \tag{3.19}$$

If ΔE is negative, a deformation is energetically favoured. The fission barrier disappears for

$$\frac{Z^2}{A} \geq \frac{2a_s}{a_c} \approx 48\,. \tag{3.20}$$

This is the case for nuclei with $Z > 114$ and $A > 270$.

Induced fission For very heavy nuclei ($Z \approx 92$) the fission barrier is only about 6 MeV. This energy may be supplied if one uses a flow of low energy neutrons to induce neutron capture reactions. These push the nucleus into an excited state above the fission barrier and it splits up. This process is known as *induced nuclear fission*.

Neutron capture by nuclei with an odd neutron number releases not just some binding energy but also a pairing energy. This small extra contribution to the energy balance makes a decisive difference to nuclide fission properties: in neutron capture by ^{238}U, for example, 4.9 MeV binding energy is released, which is below the threshold energy of 5.5 MeV for nuclear fission of ^{239}U. Neutron capture by ^{238}U can therefore only lead to immediate nuclear fission if the neutron possesses a kinetic energy at least as large as this difference ("fast neutrons"). On top of this the reaction probability is proportional to v^{-1}, where v is the velocity of the neutron (4.21), and so it is very small. By contrast neutron capture in ^{235}U releases 6.4 MeV and the fission barrier of ^{236}U is just 5.5 MeV. Thus fission may be induced in ^{235}U with the help of low-energy (thermal) neutrons. This is exploited in nuclear reactors and nuclear weapons. Similarly both ^{233}Th and ^{239}Pu are suitable fission materials.

3.4 Decay of Excited Nuclear States

Nuclei usually have many excited states. Most of the lowest-lying states are understood theoretically, at least in a qualitative way as will be discussed in more detail in Chaps. 18 and 19.

Figure 3.10 schematically shows the energy levels of an even-even nucleus with $A \approx 100$. Above the ground state, individual discrete levels with specific J^P quantum numbers can be seen. The excitation of even-even nuclei generally corresponds to the break-up of nucleon pairs, which requires about 1–2 MeV. Even-even nuclei with $A \gtrsim 40$, therefore, rarely possess excitations below 2 MeV.[2] In odd-even and odd-odd nuclei, the number of low-energy states (with excitation energies of a few 100 keV) is considerably larger.

Electromagnetic decays Low lying excited nuclear states usually decay by emitting electromagnetic radiation. This can be described in a series expansion as a superposition of different multipolarities each with its characteristic angular distribution. Electric dipole, quadrupole, octupole radiation etc. are denoted by E1, E2, E3, etc. Similarly, the corresponding magnetic multipoles are denoted by M1, M2, M3 etc. Conservation of angular momentum and parity determine which multipolarities are possible in a transition. A photon of multipolarity Eℓ has angular momentum ℓ and parity $(-1)^{\ell}$, a photon of multipolarity Mℓ has angular momentum ℓ and parity $(-1)^{(\ell+1)}$ (Table 3.1). In a transition $J_i \rightarrow J_f$, conservation of angular momentum means that the triangle inequality $|J_i - J_f| \leq \ell \leq J_i + J_f$ must be satisfied.

Table 3.1 Selection rules for some electromagnetic transitions

Multi-	Electric			Magnetic		
polarity	Eℓ	$\|\Delta \boldsymbol{J}\|$	ΔP	Mℓ	$\|\Delta \boldsymbol{J}\|$	ΔP
Dipole	E1	1	−	M1	1	+
Quadrupole	E2	2	+	M2	2	−
Octupole	E3	3	−	M3	3	+

[2]Collective states in deformed nuclei are an exception to this: they cannot be understood as single particle excitations (Chap. 19).

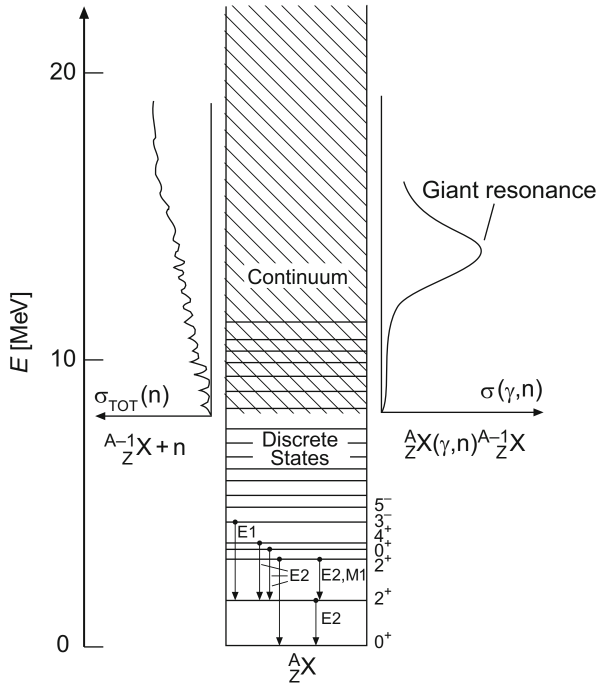

Fig. 3.10 Sketch of typical nuclear energy levels. The example shows an even-even nucleus whose ground state has the quantum numbers 0^+. To the left the total cross-section for the reaction of the nucleus ${}^{A-1}_{Z}\mathrm{X}$ with neutrons (elastic scattering, inelastic scattering, capture) is shown; to the right the total cross-section for γ-induced neutron emission ${}^{A}_{Z}\mathrm{X} + \gamma \to {}^{A-1}_{Z}\mathrm{X} + \mathrm{n}$

The lifetime of a state strongly depends upon the multipolarity of the γ-transitions by which it can decay. The lower the multipolarity, the larger the transition probability. A magnetic transition Mℓ has approximately the same probability as an electric E$(\ell + 1)$ transition. A transition $3^+ \to 1^+$, for example, is in principle a mixture of E2, M3, and E4, but will be easily dominated by the E2 contribution. A $3^+ \to 2^+$ transition will usually consist of an M1/E2 mixture, even though M3, E4, and M5 transitions are also possible. In a series of excited states 0^+, 2^+, 4^+, the most probable decay is by a cascade of E2-transitions $4^+ \to 2^+ \to 0^+$, and not by a single $4^+ \to 0^+$ E4-transition. The lifetime of a state and the angular distribution of the electromagnetic radiation which it emits are signatures for the multipolarity of the transitions, which in turn betray the spin and parity of the nuclear levels. The decay probability also strongly depends upon the energy. For radiation of multipolarity ℓ it is proportional to $E_\gamma^{2\ell+1}$ (cf. Sect. 19.1).

The excitation energy of a nucleus may also be transferred to an electron in the atomic shell. This process is called *internal conversion*. It is most important in transitions for which γ-emission is suppressed (high multipolarity, low energy) and the nucleus is heavy (high probability of the electron being inside the nucleus).

$0^+ \rightarrow 0^+$ transitions cannot proceed through photon emission. If a nucleus is in an excited 0^+-state, and all its lower lying levels also have 0^+ quantum numbers (e.g. in ^{16}O or ^{40}Ca, cf. Fig. 19.6), then this state can only decay in a different way: by internal conversion, by emission of 2 photons or by the emission of an e^+e^--pair, if this last is energetically possible. Parity conservation does not permit internal conversion transitions between two levels with $J = 0$ and opposite parity.

The lifetime of excited nuclear states typically varies between 10^{-9} and 10^{-15} s, which corresponds to a state width of less than 1 eV. States which can only decay by low energy and high multipolarity transitions have considerably longer lifetimes. They are called *isomers* and are designated by an "m" superscript on the symbol of the element. An extreme example is the second excited state of ^{110}Ag, whose quantum numbers are $J^P = 6^+$ and excitation energy is 117.7 keV. It relaxes via an M4-transition into the first excited state (1.3 keV; 2^-) since a decay directly into the ground state (1^+) is even more improbable. The half-life of ^{110}Ag^m is extremely long ($t_{1/2} = 235$ days) [2].

Continuum states Most nuclei have a binding energy per nucleon of about 8 MeV (Fig. 2.4). This is approximately the energy required to separate a single nucleon from the nucleus *(separation energy)*. States with excitation energies above this value can therefore emit single nucleons. The emitted nucleons are primarily neutrons since they are not hindered by the Coulomb threshold. Such a strong interaction process is clearly preferred to γ-emission.

The excitation spectrum above the threshold for particle emission is called the *continuum*, just as in atomic physics. Within this continuum there are also discrete, quasi-bound states. States below this threshold decay only by (relatively slow) γ-emission and are, therefore, very narrow. But for excitation energies above the particle threshold, the lifetimes of the states decrease dramatically, and their widths increase. The density of states increases approximately exponentially with the excitation energy. At higher excitation energies, the states therefore start to overlap, and states with the same quantum numbers can begin to mix.

The continuum can be especially effectively investigated by measuring the cross-sections of neutron capture and neutron scattering. Even at high excitation energies, some narrow states can be identified. These are states with exotic quantum numbers (high spin) which therefore cannot mix with neighbouring states.

Figure 3.10 shows schematically the cross-sections for neutron capture and γ-induced neutron emission *(nuclear photoelectric effect)*. A broad resonance is observed, the *giant dipole resonance,* which will be interpreted in Sect. 19.2.

Problems

1. **Alpha decay**
The α-decay of a ^{238}Pu ($\tau = 127$ years) nuclide into a long-lived ^{234}U ($\tau = 3.5 \cdot 10^5$ years) daughter nucleus releases 5.49 MeV kinetic energy. The heat so

produced can be converted into useful electricity by radio-thermal generators (RTG's). The *Voyager 2* space probe, which was launched on the 20.8.1977, flew past four planets, including Saturn which it reached on the 26.8.1981. Saturn's separation from the Sun is 9.5 AU; 1 AU = separation of the Earth from the Sun.

(a) How much plutonium would an RTG on Voyager 2 with 5.5 % efficiency have to carry so as to deliver at least 395 W electric power when the probe flies past Saturn?
(b) How much electric power would then be available at Neptune (24.8.1989; 30.1 AU separation)?
(c) To compare: the largest ever "solar paddles" used in space were those of the space laboratory *Skylab* which would have produced 10.5 kW from an area of 730 m^2 if they had not been damaged at launch. What area of solar cells would *Voyager 2* have needed?

2. **Radioactivity**
Naturally occurring uranium is a mixture of the ^{238}U (99.28 %) and ^{235}U (0.72 %) isotopes.

(a) How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? How do you interpret this result? The lifetime of ^{235}U is $\tau = 1.015 \cdot 10^9$ years. For the lifetime of ^{238}U use the data in Fig. 3.7.
(b) How much of the ^{238}U has decayed since the formation of the Earth's crust $2.5 \cdot 10^9$ years ago?
(c) How much energy per uranium nucleus is set free in the decay chain $^{238}\mathrm{U} \rightarrow {}^{206}\mathrm{Pb}$? A small proportion of ^{238}U spontaneously splits into, e.g., $^{142}_{54}$Xe and $^{96}_{38}$Sr.

3. **Radon activity**
After a lecture theatre whose walls, floor and ceiling are made of concrete ($10 \times 10 \times 4\,\mathrm{m}^3$) has not been aired for several days, a specific activity A from ^{222}Rn of $100\,\mathrm{Bq/m^3}$ is measured.

(a) Calculate the activity of ^{222}Rn as a function of the lifetimes of the parent and daughter nuclei.
(b) How high is the concentration of ^{238}U in the concrete if the effective thickness from which the ^{222}Rn decay product can diffuse is 1.5 cm?

4. **Mass formula**
   Isaac Asimov in his novel *The Gods Themselves* describes a universe where the stablest nuclide with $A = 186$ is not $^{186}_{74}$W but rather $^{186}_{94}$Pu. This is claimed to be a consequence of the ratio of the strengths of the strong and electromagnetic interactions being different to that in our universe. Assume that only the electromagnetic coupling constant α differs and that both the strong interaction and the nucleon masses are unchanged. How large must α be in order that $^{186}_{82}$Pb, $^{186}_{88}$Ra and $^{186}_{94}$Pu are stable?
5. **Alpha decay**
   The binding energy of an α particle is 28.3 MeV. Estimate, using the mass formula (2.8), from which mass number A onwards α-decay is energetically allowed for all nuclei.
6. **Quantum numbers**
   An even-even nucleus in the ground state decays by α-emission. Which J^P states are available to the daughter nucleus?

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