Chapter 20
Nuclear Thermodynamics

So far we have concerned ourselves with the properties of nuclei in the ground state or the lower lying excited states. We have seen that the observed phenomena are characterised, on the one hand, by the properties of a degenerate fermion system and, on the other, by the limited number of the constituents. The nuclear force generates, to a good approximation, an overall mean field in which the nucleons move like free particles. In the shell model the finite size of nuclei is taken into account and the states of the individual nucleons are classified according to radial excitations and angular momenta. Thermodynamically speaking, we assign such systems zero temperature.

In the first part of this chapter we want to concern ourselves with highly excited nuclei. At high excitation energies the mean free path of the nucleon inside the nucleus is reduced; it is only about 1 fm. The nucleus is then no longer a degenerate fermionic system, but rather resembles, ever more closely for increasing excitations, the state of a normal liquid. It is natural to use statistical methods in the description of such systems. A clear description may be gained by employing thermodynamical quantities. The excitation of the nucleus is characterised by the temperature. We should not forget that strictly speaking one can only associate a temperature to large systems in thermal equilibrium and even heavy nuclei do not quite correspond to such a system. As well as this, excited nuclei are not in thermal equilibrium, but rather rapidly cool down via the emission of nucleons and photons. In any thermodynamical interpretation of experimental results we must take these deficiencies into account. In connection with nuclear thermodynamics one prefers to speak about *nuclear matter* rather than nuclei, which implies that many experimental results from nuclear physics may be extrapolated to large systems of nucleons. As an example of this we showed, when we considered the nuclear binding energy, that by taking the surface and Coulomb energies into account one can calculate the binding energy of a nucleon in nuclear matter. This is just the volume term of the mass formula (2.8).

B. Povh et al., *Particles and Nuclei*, Graduate Texts in Physics,
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Heavy ion reactions have proven themselves especially useful in the investigation of the thermodynamical properties of nuclear matter. In nucleus-nucleus collisions the nuclei melt together to form for a brief time a nuclear-matter system with increased density and temperature. We will try below to describe the phase diagram of nuclear matter using experimental and theoretical results about these reactions.

The results of nuclear thermodynamics are also of great importance for cosmology and astrophysics. According to our current understanding, the universe in the early stages of its existence went through phases where its temperature and density were many orders of magnitude higher than in the universe of today. These conditions cannot be reconstructed in the laboratory. Many events in the history of the universe have, however, left lasting traces. With the help of this circumstantial evidence one can try to draw up a model of the development of the universe.

20.1 Thermodynamical Description of Nuclei

We have already in Sect. 3.4 (Fig. 3.10) distinguished between three sorts of excitations in nuclei:

- The ground state and the low-lying states can be described in terms of single-particle excitations or via collective motion. This was treated in Chaps. 18 and 19.
- Far above the particle threshold there are no discrete states but only a continuum.
- In the transition region below and barely above the particle threshold there are lots of narrow resonances. These states do not, however, contain any information about the structure of the nucleus. The phenomena in this energy range in nuclei are widely referred to as *quantum chaos*.

In the following we shall concern ourselves with the last two of these domains. Their description involves statistical methods and so we will initially turn our attention to the concept of *nuclear temperature*.

Temperature We want to introduce the idea of temperature in nuclear physics through the example of the spontaneous fission of ^{252}Cf. The half-life of ^{252}Cf is 2.6 years and it has a 3.1 % probability of decaying via spontaneous fission. There is some friction in the separation of the fission fragments. Therefore, not all of the available energy from the fission process is converted into kinetic energy for the fragments. Rather the internal energy of the fragments is increased: the two fragments heat up.

The cooling-down process undergone by the fission fragments is shown schematically in Fig. 20.1. Initially cooling down takes place via the emission of slow neutrons. Typically 4 neutrons are emitted, each of them carrying off, on average, 2.1 MeV. Once the fragments have cooled below the threshold for neutron emission, they can only cool further by photon emission.

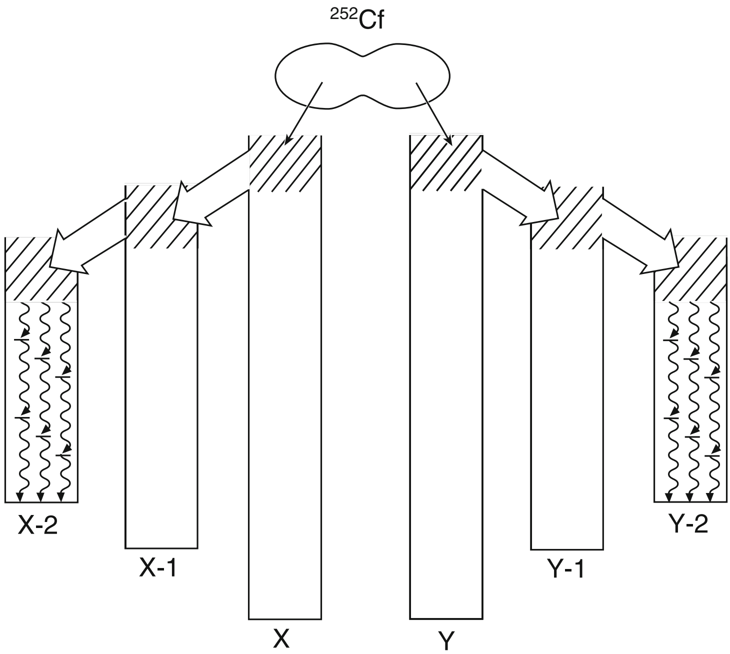

Fig. 20.1 Cooling of fission fragments (schematic). A ^{252}Cf nucleus splits into two parts with mass numbers X and Y which then cool down by emitting first neutrons and later photons

The energy spectrum of the emitted neutrons has the form of an evaporation spectrum. It may be described by a Maxwell distribution:

$$N_\mathrm{n}(E_\mathrm{n}) \sim \sqrt{E_\mathrm{n}} \cdot \mathrm{e}^{-E_\mathrm{n}/kT} . \tag{20.1}$$

Figure 20.2 shows the experimental spectrum normalised by a factor of $\sqrt{E_\mathrm{n}}$. The exponential fall-off is characterised by the temperature T of the system, in this case $kT = 1.41$ MeV. Fission fragments from different nuclei are found to have different temperatures. One finds, e.g., a smaller value in the fission of ^{236}U, namely $kT = 1.29$ MeV.

Figure 20.3 displays the energy spectrum of the photons emitted in the de-excitation of the produced daughter nuclei. On average about 20 photons are set free for each spontaneous fission, and 80 % of these photons have energies of less than 1 MeV. This spectrum also closely resembles an evaporation spectrum. The stronger fall-off of the photon spectrum compared to the neutron spectrum signals that the temperature in the photon emission phase, which takes place for lower nuclear excitations, is significantly lower.

Our successful statistical interpretation of these neutron and photon spectra leads to the important conclusion that the states in the neighbourhood of the particle threshold, which may be understood as a reflection of the corresponding transitions,

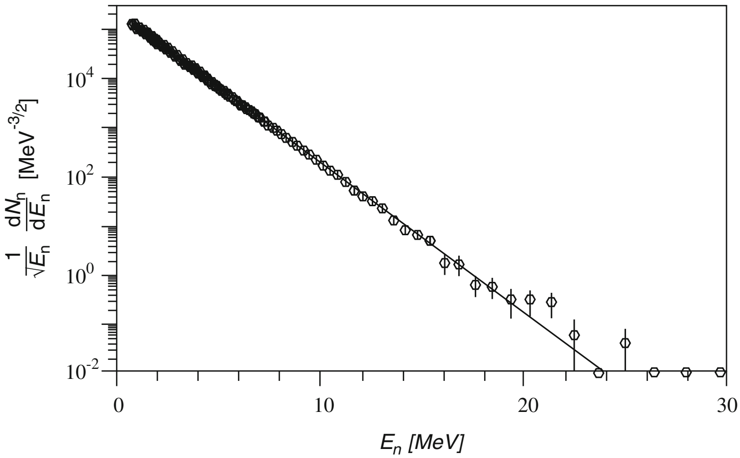

Fig. 20.2 Energy spectrum of neutrons emitted in the spontaneous fission of ^{252}Cf (From [6]). The distribution is divided by $\sqrt{E_n}$ and then fitted to the exponential behaviour of a Maxwell distribution (*solid line*)

can also be described with statistical methods. Indeed the observed form of the spectrum may be formally derived from a statistical study of the density of states of a degenerate Fermi gas.

20.2 Compound Nuclei and Quantum Chaos

Many narrow resonances may be found in the transition region below and just above the particle threshold of a heavy nucleus. The states below the particle threshold are discrete and each one of these states possesses definite quantum numbers. The same is true for the states immediately above the threshold. Decays into these states are only described statistically through the density of these states. These states therefore do not contain any specific information about the structure of the nucleus.

Compound nuclei In neutron capture by heavy nuclei a multitude of resonances is observed. An example of such a measurement is seen in Fig. 20.4 where the cross-section for neutron scattering off thorium displays very many resonances. One should note that the energy scale is in eV, the separation of these resonances is thus six orders of magnitude smaller than the gaps in energy separating lower lying states. This observation was already explained in the thirties by Niels Bohr in the so-called compound-nucleus model. Neutrons in the nucleus have a very short free path due to the strong interaction and they very rapidly distribute their energy among the nucleons in the nucleus. The probability that all the energy supplied is held by one single nucleon is small. Therefore, the nucleons cannot escape from the nucleus

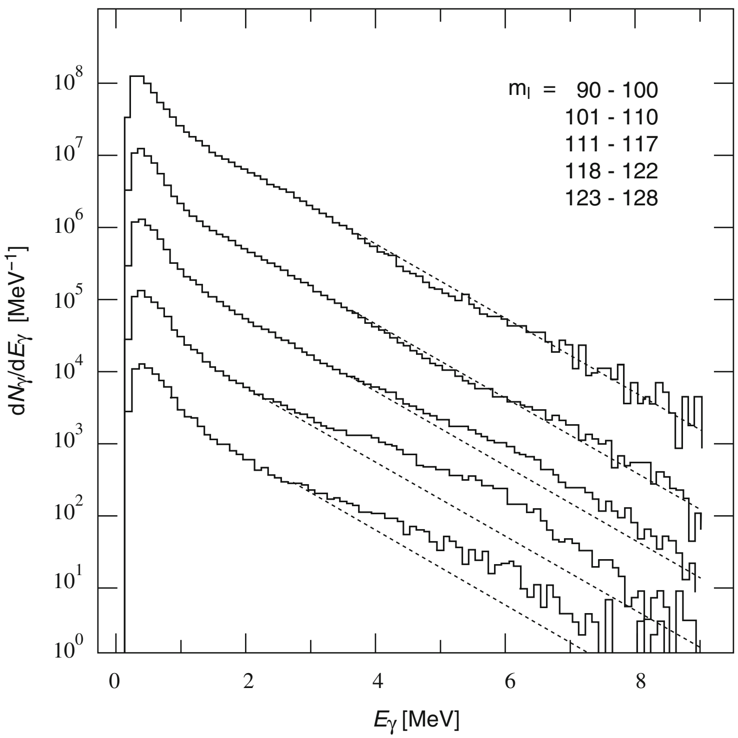

Fig. 20.3 Photon emission energy spectra in the spontaneous fission of ^{252}Cf. The various spectra correspond to different mass numbers, m_1, of the lighter fission products (*from top to bottom*). The *dotted line* is a common fit of an exponential function (From [11])

and this leads to a long lifetime for the compound-nucleus states. This lifetime is mirrored in the narrow widths of the resonances.

This picture has been greatly refined in the intervening decades. Thus the compound-nucleus state is not reached immediately, but rather the system, via successive collisions, passes through a series of intermediate states. The compound-nucleus state is the limiting case in which the nucleons are in thermal equilibrium.

Quantum chaos in nuclei In the theory of classical deterministic systems we distinguish between regular and chaotic orbits. Regular orbits are stable orbits which are not greatly affected by small external perturbations. The particles undergo periodic motion and the entire configuration of the system thus repeats itself. Chaotic orbits are very different. They are not periodic and infinitesimally small perturbations lead to big changes. While predictions for the development of regular systems may be made to an arbitrary accuracy, the uncertainties associated with predicting chaotic systems increase exponentially.

In quantum mechanics regular orbits correspond to states whose wave functions may be calculated with the help of the Schrödinger equation in some model, e.g.,

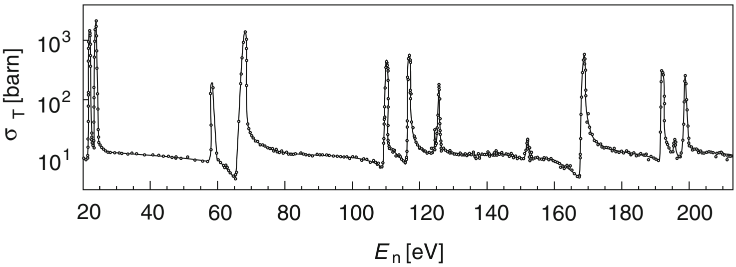

Fig. 20.4 Total cross-section for the reaction ^{232}Th + n as a function of the neutron energy. The sharp peaks correspond to resonances with orbital angular momentum $\ell = 0$ (From [5])

for nuclei the shell model. The quantum mechanical equivalent of classical chaotic motion are states which are stochastically made up of single-particle wave functions. In both the classical and quantum mechanical cases a system in a chaotic state does not contain any information about the interactions between the particles.

The stochastic composition of chaotic states can be experimentally demonstrated by measuring the energy separations between these states. For this one considers resonance spectra such as that of Fig. 20.4. In the excitation region of the compound nucleus the states are very dense, so a statistical approach is justified.

It is apparent here that states with the same spin and parity (in Fig. 20.4 all the sharp resonances) attempt to keep as far apart as possible. The most likely separation of these states is significantly greater than the most likely separation of the energy levels of states if they were, for the same state density, distributed in a statistical fashion, according to a Poisson distribution independently of each other. This behaviour of the chaotic states is just what one expects if they are made up from a mixture of single-particle states with the same quantum numbers. Such quantum mechanical mixed states attempt to repel each other, i.e., their energy levels arrange themselves as far apart from each other as possible.

The existence of collective states, such as, e.g., the giant dipole resonance, for excitations above the particle threshold, i.e., in the region where the behaviour of the states is chaotic, is a very pretty example of the coexistence of regular and chaotic nuclear dynamics. Excitation of the collective state of the giant resonance takes place through photon absorption. The collective state couples to the many chaotic states via the nucleon-nucleon interaction. These partially destroy the coherence and thus reduce the lifetime of the collective state.

The continuum The continuum is by no means flat, rather strong fluctuations are seen in the cross-section. The reason for this is that, on the one hand, at higher energies the widths of the resonances increase because more decay channels stand open to them, but on the other hand the density of states also increases. Resonances with the same quantum numbers thus interfere with each other which leads to fluctuations in the total cross-section. These fluctuations do not correspond to single resonances but to the interference of many resonances. The size of the fluctuations and their average separation can be quantitatively calculated from the known state density [9].

20.3 The Phases of Nuclear Matter

The liquid-gas phase transition Peripheral heavy ion reactions have proven themselves most useful as a way to heat up nuclei in a controlled way. In a glancing collision of two nuclei (Fig. 20.5) two main fragments are produced which are heated up by friction during the reaction. In such reactions one can measure rather well both the temperature of the fragments and also the energy supplied to the system. The temperature of the fragments is found from the Maxwell distribution of the decay products, while the total energy supplied to the system is determined

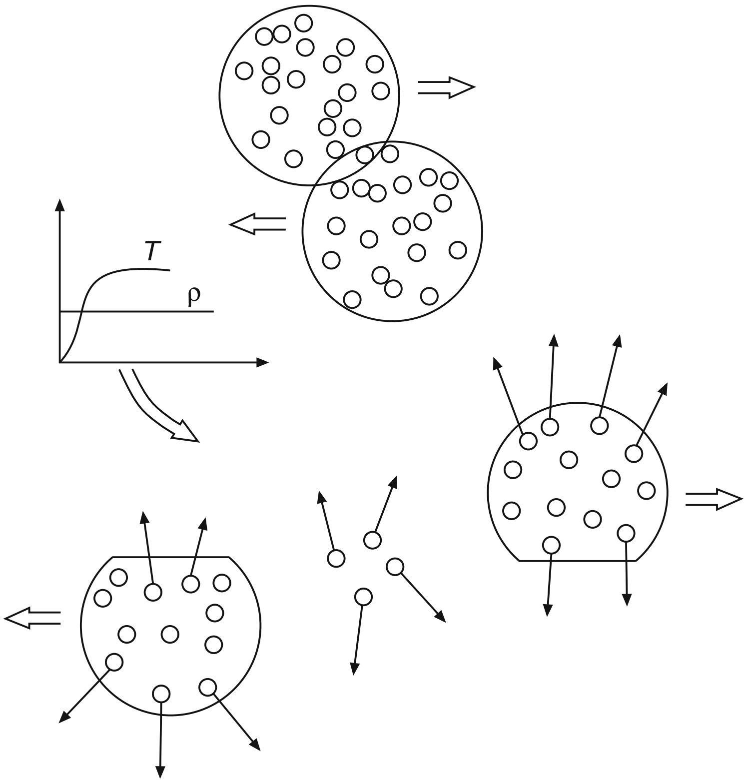

Fig. 20.5 A peripheral nuclear collision. The large fragments are heated up by friction. As well as this, individual nucleons and smaller nuclear fragments are also produced in the collision. The diagram describes the time evolution of the density ϱ and temperature T of the fragments during the collision

by detecting all of the particles produced in the final state. Since the fragment which came from the projectile moves off in the direction of the projectile, its decay products will also move in that direction and may be thus kinematically distinguished both from the decay products of the target fragments and also from the frictionally induced evaporative nucleons. The contributions from the energy supplied to the fragments and from the energy lost to friction during the glancing collision may thus be separated from one another.

Let us take as an example an experiment where gold nuclei with an energy of 600 MeV/nucleon were fired at a gold target. The reaction products were then tracked down using a detector which spanned almost the entire solid angle (a 4π detector).

The dependence of the fragments' temperature on the energy supplied to the system is shown in Fig. 20.6. For excitation energies E/A up to about 4 MeV/nucleon one observes that the temperature sharply increases. In the region 4 MeV $< E/A <$ 10 MeV the temperature hardly varies at all, while at higher energies it again grows rapidly. This behaviour is reminiscent of the process of water evaporation where, around the boiling point, at the phase transition from liquid into steam, the temperature remains constant, even though energy is added to the system, until the entire liquid has been converted into a gaseous state. It is therefore natural to

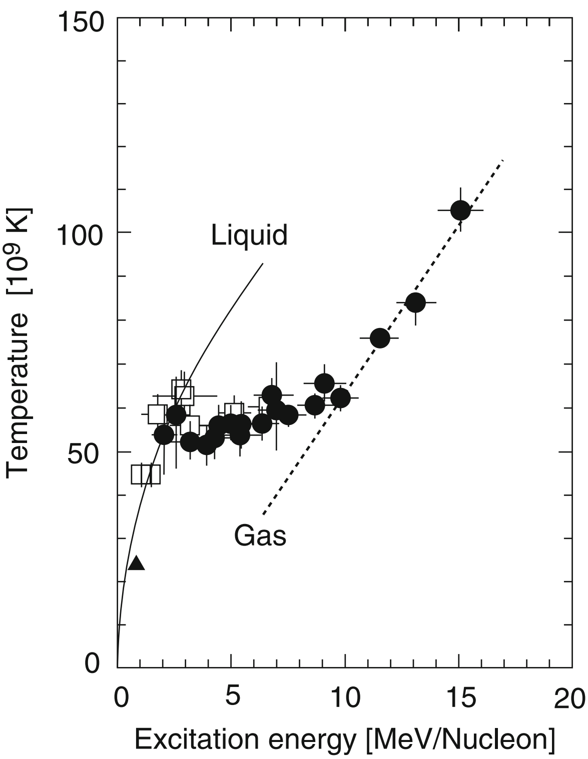

Fig. 20.6 Temperature of the fragments in a peripheral collision of two ^{197}Au nuclei as a function of the excitation energy per nucleon (From [17]). The behaviour of the temperature can be understood as a phase transition in nuclear matter

interpret the temperature dependence described above as a nuclear matter phase transition from a liquid to a gas-like state.

The terms which we have used come from equilibrium thermodynamics. For such conditions a logical interpretation of the phase transition would be the following: at a temperature of about $kT \sim 4\,\mathrm{MeV}$ a layer of nucleons in a gaseous phase forms around the nucleus. This does not evaporate away but remains in equilibrium with the liquid nucleus and exchanges nucleons with it. The nucleon gas can only be further heated up after the whole of the nucleon liquid has evaporated.

Hadronic matter If we wish to investigate central, and not peripheral, collisions in gold-gold collisions, we have to select in the experiment those events in which many charged and neutral pions are emitted (Fig. 20.7). To keep the discussion simple, we will choose projectile energies of 10 GeV/nucleon or more for which a large number of pions is created.

At such energies the nucleonic excitation $\mathrm{N} + \mathrm{N} \rightarrow \Delta + \mathrm{N}$ has a cross-section of $\sigma = 40\,\mathrm{mb}$. The corresponding path length $\lambda \approx 1/\sigma\varrho_{\mathrm{N}}$ in the nucleus is of the order of 1 fm. This means that multiple collisions take place in heavy ion collisions and that for sufficiently high energies every nucleon will on average be excited once or more into a Δ baryon. In the language of thermodynamics this excitation corresponds to the opening up of a new degree of freedom.

The Δ baryons decay rapidly but they are continually being reformed through the inverse reaction $\pi N \rightarrow \Delta$. Creation and decay via $\pi N \leftrightarrow \Delta$ thus stand in a dynamical equilibrium. This mix of nucleons, Δ baryons, pions and, in significantly smaller amounts, other mesons is called *hadronic matter*.

Pions, since they are much lighter than the other hadrons, are primarily responsible for energy exchange inside hadronic matter. The energy density and temperature of hadronic matter produced in a collision of two atomic nuclei can be experimentally determined with the help of these pions. The temperature is found from the energy distribution of those pions which are emitted orthogonally to the beam direction. Their energy spectrum has the exponential behaviour expected of a Boltzmann distribution:

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{kin}}} \propto \mathrm{e}^{-E_{\mathrm{kin}}/kT}\,, \tag{20.2}$$

where E_{kin} is the kinetic energy of the pion. One finds experimentally that the temperature of the pionic radiation is never greater than $kT \approx 150\,\mathrm{MeV}$, no matter how high the energies of the colliding nuclei are. This may be understood as follows: hot nuclear matter expands and in doing so cools down. Below a temperature $kT \approx 150\,\mathrm{MeV}$, the hadronic interaction probability of the pions, and thus energy exchange between them and other particles, decreases sharply. This process is referred to as the pions *freezing out*.[1]

[1] A similar process takes place in stars: the electromagnetic radiation in the interior of the Sun is at many millions of K. On its way out it cools down via interactions with matter. What we observe is

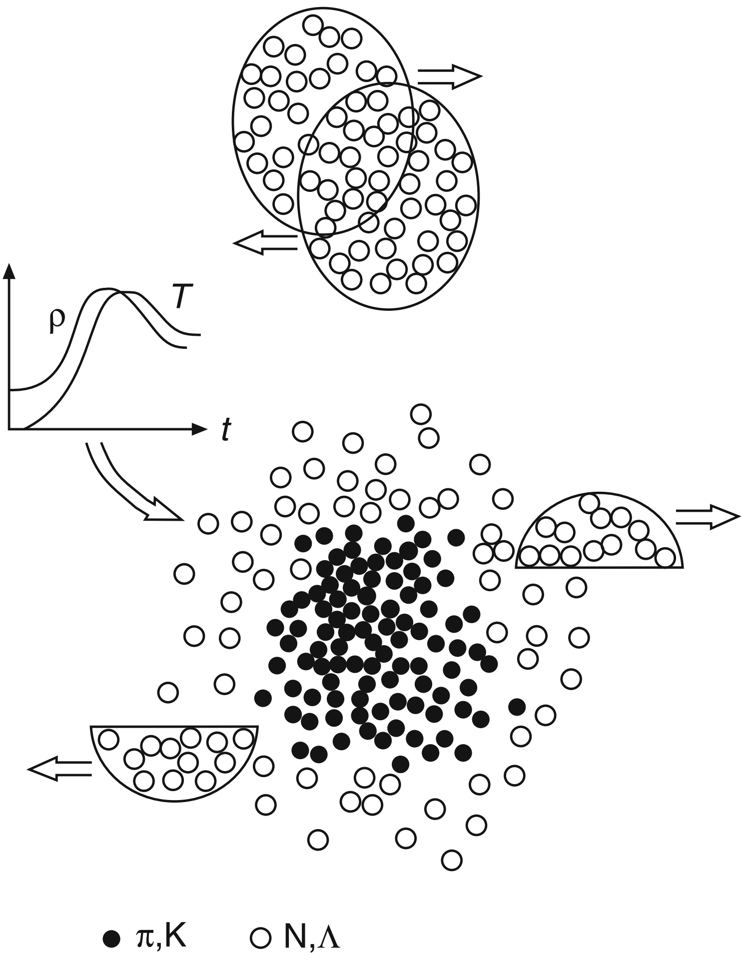

Fig. 20.7 Central collision of two heavy nuclei at high energies. A large number of pions is produced here. The curves show the increase of density, ϱ, and temperature, T, in the central region of the collision

Phase diagram for nuclear matter The various phases of nuclear matter are summarised in Fig. 20.8. We want to clarify this phase diagram by comparing nuclear matter with usual matter (that composed of atoms or molecules). Cold nuclei have density ϱ_N and temperature $kT = 0$. A neutron star corresponds to a state with $kT = 0$, however, its density is about 3–10 times as big as that of nuclei.

white light whose spectrum corresponds to the temperature of the solar surface. In contrast to hot nuclear matter, the Sun is of course in equilibrium and is not expanding.

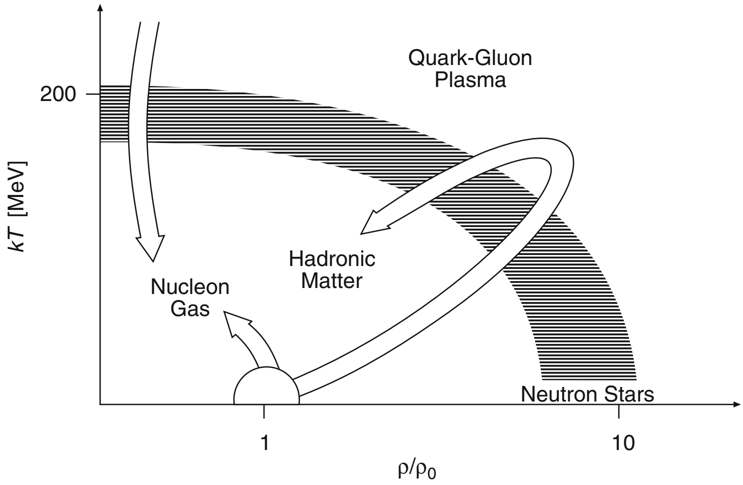

Fig. 20.8 Phase diagram for nuclear matter. Normal nuclei have $\varrho = \varrho_0$ ($= \varrho_N$) and temperature $T = 0$. The *arrows* show the paths followed by nuclei in various heavy ion reactions. The *short arrow* symbolises the heating up of nuclei in peripheral collisions; the *long arrow* corresponds to relativistic heavy ion collisions, in which nuclear matter possibly crosses the quark-gluon plasma phase. The cooling of the universe at a time of $\approx 1\,\mu$s after the big bang is represented by the *downwards pointing arrow*

If one supplies energy to a normal nucleus, it heats up and emits nucleons or small nuclei, mainly α-clusters, just as a liquid droplet evaporates atoms or molecules. If, however, one confines the material, increasing the energy supplied leads to the excitation of internal degrees of freedom. In a molecular gas these are rotational and vibrational excitations. In nuclei nucleons can be excited into $\Delta(1{,}232)$ resonances or to still higher nucleon states. We have called the mish-mash of nucleons and pions, which are then created by decays, *hadronic matter*.

Quark-gluon plasma The complete dissociation of atoms into electrons and atomic nuclei (a plasma) has its equivalent in the disintegration of nucleons and pions into quarks and gluons. Qualitatively the positions of the phase boundary in the temperature-density diagram (Fig. 20.8) may be understood as follows: at normal nuclear densities each nucleon occupies a volume of about $6\,\text{fm}^3$, whereas the actual volume of a nucleon itself is only about a third of this. If one then were to compress a cold nucleus ($T = 0$) to many times its usual density, the individual nucleons would overlap and cease to exist as individual particles. Quarks and gluons would then be able to move "freely" in the entire nuclear volume. If on the other hand one were to follow a path along the temperature axis, i.e., increase the temperature without thereby altering the nucleon density in the nucleus, then at a temperature of $kT \approx 200\,\text{MeV}$ enough energy would be available to the individual nucleon-nucleon interactions to increase, via pion production, the hadronic density and the frequency of the collisions between them so much that it would be impossible to assign a quark or gluon to any particular hadron.

This state is referred to as a *quark-gluon plasma*. As we have already mentioned, this state, where the hadrons are dissolved, cannot be observed through the study of emitted hadrons. There are attempts to detect a quark-gluon plasma state via electromagnetic radiation. The coupling of photons to quarks is about two orders of magnitude smaller than that of strongly interacting matter is. Thus any electromagnetic radiation produced in any potential creation of a quark-gluon plasma, e.g., in relativistic heavy ion collisions, could be directly observed. It would not be cooled down in the expansion of the system.[2]

There is a great deal of interest in detecting a quark-gluon plasma because it would mean an experimental confirmation of our ideas of the structure of strongly interacting matter. Therefore, it is searched for intensively in collisions of ultra-relativistic heavy ions, e.g., in experiments at RHIC (*Relativistic Heavy Ion Collider*) at Brookhaven/USA or the experiment ALICE (*A Large Ion Collider Experiment*) at the LHC. If the assignment of quarks and gluons to individual hadrons were removed, the constituent quarks would lose their masses and turn into partonic quarks; one would be able to simulate the state of the universe at a very early stage in its history.

20.4 Particle Physics and Thermodynamics in the Early Universe

In all societies men have constructed myths about the origins of the universe and of man. The aim of these myths is to define man's place in nature, and thus give him a sense of purpose and value.

John Maynard Smith [20]

The interplay between cosmology and particle physics during the last few decades has lead to surprising insights for both areas. In what follows we want to depict current ideas about the evolution of the universe and show what consequences this evolution has had for our modern picture of particle physics. We will here make use of the standard cosmological model, the big bang model, according to which the universe began as an infinitely hot and dense state. This fireball then expanded explosively and its temperature and density have continued to decrease till the present day. This expansion of an initially hot plasma of elementary particles was the origin of all nowadays known macroscopic and microscopic forms of matter:

[2]The above analogy from astrophysics is also applicable here: the neutrinos which are created in fusion reactions in the solar interior are almost unhindered in their escape from the Sun. Their energy spectrum thus corresponds to the temperature at their production point and not to that of the surface.

stars and galaxies; leptons, quarks, nucleons and nuclei. This model for the time development of the universe was motivated and then confirmed by two important experimental observations: the continuous expansion of the universe and the cosmic background radiation.

The expanding universe The greatest part of the mass of the universe is located in galaxies. These spatially concentrated star systems are held together by the force of gravity and, depending upon their size, have masses of between 10^7 and 10^{13} solar masses. It is believed that there are about 10^{23} stars in the universe – a number comparable to the number of molecules in a mole.

With the help of large telescopes it is possible to measure the distance to and the velocities of galaxies which are very far away from the Earth. The velocity of a galaxy relative to the Earth can be determined from the Doppler shift of atomic spectral lines, which are known from laboratory measurements. One so finds a shift of the observed lines into the red, i.e., the longer wavelength region. This corresponds to a motion of the galaxies away from us. This observation holds no matter what direction in the heavenly sphere the galaxy under observation is in. A determination of the distance to the galaxy is carried out by measuring its light intensity and estimating its luminosity; these quantities are related by the well-known $1/r^2$ law. Such distance estimates are particularly imprecise for very distant galaxies.

The measured velocities v of the observed galaxies are roughly linearly proportional to their separation d from the Earth

$$v = H_0 \cdot d\,, \tag{20.3}$$

where H_0 is called the *Hubble constant* after the discoverer of this relationship. The measurements of H_0 have been improved appreciably in recent years. Its present value from a combination of data of various experiments is [13]:

$$H_0 = 69.32 \pm 0.88\ \text{km}\,\text{s}^{-1}/\text{MPc} \qquad (1\ \text{Pc} = 3.1 \cdot 10^{13}\ \text{km} = 3.3\ \text{light years})\,.$$

These observations taken together are interpreted as implying an isotropic expansion of the universe.

According to the big bang theory, the initial hot plasma filled the universe with extremely short wavelength electromagnetic radiation, which, though, increased its wavelength as the universe expanded and cooled. The observation, by Penzias and Wilson [16], of this radiation in the microwave-length region, which we now call the *cosmic background radiation*, was therefore a very important confirmation of the big bang model. This microwave radiation corresponds to black body radiation at a temperature

$$T = (2.7255 \pm 0.0006)\ \text{K}\,.$$

It was measured, e.g., by the COBE satellite [10], in every direction of the universe as being extraordinarily isotropic. This isotropy is, however, not exact. Tiny temperature fluctuations with an amplitude of $\Delta T/T \approx 10^{-5}$ have been observed. They are supposed to have been generated from small density perturbations in the early universe giving rise to seeds of galaxies and clusters. The anisotropy in the cosmic microwave background contains, therefore, important information about cosmological parameters. Presently such fluctuations are being searched for and studied with high precision by satellite experiments like WMAP [13] or Planck [1].

A relation between the age and the size of the universe can be derived with the help of general relativity theory and the observed expansion of the universe. In the simplest model, the *Friedman model* of the expanding universe, one distinguishes between three cases which depend upon the average mass density of the universe: if the average density is greater than a critical density, then the mutual attraction of the galaxies will slow the expansion of the universe down and eventually produce a contraction. The universe will then collapse into a point (*closed universe*). If the average density is smaller than the critical density, gravitation cannot reverse the expansion. In such a case the universe will expand forever (*open universe*). If the average and critical densities are approximately the same, the universe would asymptotically approach a limiting radius and not expand further (*flat universe*). Present observations point to a flat universe with critical density.

The consequences for the future expansion of the universe that we discussed so far are only valid when we neglect the mysterious *dark energy*. This is a contribution to the critical density aggregating to 70 %, but its origin is currently not understood at all. For instance, in a flat universe it would cause an accelerated expansion as opposed to an eventual standstill. Roughly speaking, the effect of dark energy on the evolution of the universe is inverse to the one of gravity. The latter tends to slow down the expansion, which is easy to understand, as it causes mutual attracts of all matter. A large number of cosmological observations can only be explained if dark energy is present.

Furthermore, the density measured with optical methods (about 4 % of the critical density) is smaller than the density obtained from observations based on gravitational effects. This means that there exists at least one type of *dark matter*, contributing with 23 % to the critical density. Obvious candidates would be massive neutrinos, which would, even though their mass is small, contribute strongly to the density, because of their huge number in the universe. However, particles with small masses such as neutrinos (at most 2 eV/c^2, see Sect. 18.6 for experiments determining neutrino mass) are unfortunately no suitable dark matter candidates. Namely, observations of the distribution of galaxies are compatible only with heavy neutral dark matter particles (10–1,000 GeV/c^2) whose interactions are of the order of the weak interaction. Such *WIMPs* (*Weakly Interacting Massive Particles*) show up typically in so-called supersymmetric theories, which are the most popular extension of the Standard Model. The contribution of dark matter to the critical density corresponds for a WIMP with mass 100 GeV/c^2 about 3,000 particles per cubic metre. Direct detection of dark matter can take place in very sensitive experiments that search for the elastic scattering of dark matter particles with nuclei. The recoil of the nucleus results, depending on the material, in phonons, scintillation light or an ionisation signal. One also hopes to produce dark matter particles in collider experiments at the LHC. A modern overview about properties and candidates for dark matter is [3]. A general introduction to cosmology can be found in [2, 19].

Since the universe is still in an early stage of its expansion the previous history of our universe would be similar in all three cases. The age of a universe with a sub-critical density is given by the inverse Hubble constant

$$t_0 = \frac{1}{H_0}, \tag{20.4}$$

and is about 14 billion years.

The first three minutes of the universe In the initial phase of the universe all the (anti)particles and the gauge bosons were in thermodynamical equilibrium, i.e., there was so much thermal, and thus kinematical, energy available that all the (anti)particles could transform into each other at will. There was therefore no difference between quarks and leptons, which means that the strength of all the interactions was the same.

After about 10^{-35} s the temperature had decreased so much due to the expansion that a phase transition took place and the strong interaction decoupled from the electroweak interaction, i.e., the strongly interacting quarks barely interacted with the leptons any more. At this stage the ratio between the numbers of quarks and photons was fixed at about 10^{-9}.

After about 10^{-11} s, at a temperature $kT \approx 100\,\text{GeV}$, a further phase transition took place in which the weak interaction decoupled from the electromagnetic interaction. We will discuss this process below.

When, after about 10^{-6} s, the continuous expansion of the universe had lowered its temperature down to $kT \approx 100\,\text{MeV}$, which is the typical energy scale for hadronic excitations, the quarks formed bound states in the shape of baryons and mesons. The protons and neutrons so-produced were in thermal equilibrium due to weak processes.

After about 1 s and at a temperature $kT \approx 1\,\text{MeV}$, the difference between the neutron and proton masses, the neutrinos had too little energy to maintain the state of equilibrium between the protons and neutrons. They decoupled from matter, i.e., they henceforth essentially no longer interacted at all and propagated freely through the universe. Meanwhile the ratio of protons to neutrons increased up to a value of 7.

After about 3 min of expansion the temperature had fallen to $kT \approx 100\,\text{keV}$. From this moment the thermal equilibrium between nucleons and photons was broken, since the photon energies were no longer sufficient to break up the light nuclei, through photofission processes, into their constituents at the same pace as they were produced by nucleon fusion. In this phase the big bang nucleosynthesis of deuterium, helium and lithium nuclei took place.

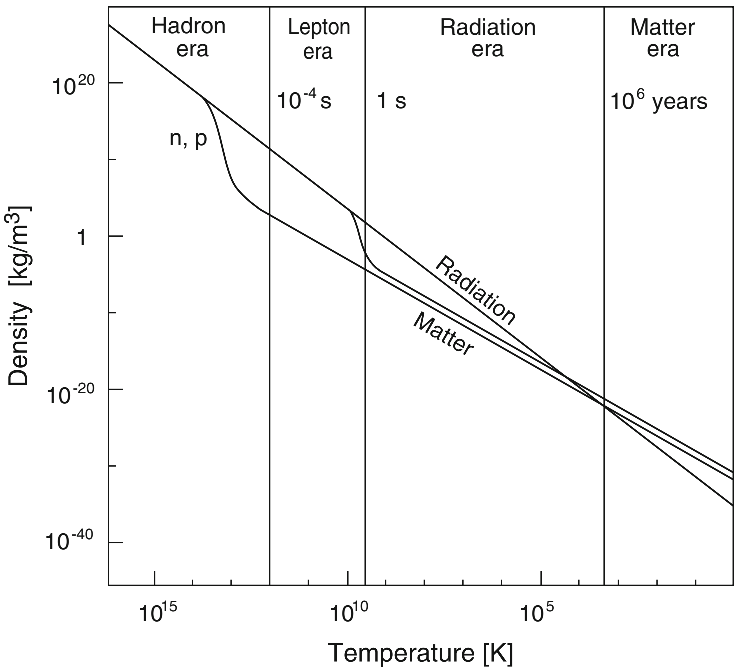

Fig. 20.9 The evolution of the energy density of the universe, as a function of temperature, after the electroweak phase transition ($T \approx 10^{15}$ K). In the early development of the universe radiation was in thermal equilibrium with matter and antimatter. Over a period of time matter decoupled from radiation and the matter and radiation energy densities developed different temperature dependencies, so that the universe finally became matter dominated

Figure 20.9 schematically shows the early history of the universe from the electroweak phase transition on. The curves represent the time (or temperature) dependent evolution of the energy density of radiation and matter. One can see the sharp drop in the energy density caused by the expansion of the universe. At temperatures of 10^{13} K the hadrons, and later the leptons, decouple from the radiation. At $T \approx 10^4$ K a matter dominated universe takes over from a previously radiation dominated universe. The current temperature of the universe is 2.73 K, the temperature of the cosmic background radiation.

Below we want to delve further into some important events from this early history of the universe.

Matter-antimatter asymmetry All observations show that the present universe is made up solely of matter and there is no evidence for some parts of the universe being composed of antimatter. Since according to our ideas all (anti)particles at a very early stage of the universe were in thermal equilibrium, i.e., fermion-antifermion creation from gauge bosons was just as frequent as fermion-antifermion annihilation into gauge bosons, then if this symmetry had survived the development of the universe, there ought to be just as many fermions as antifermions or, more especially, as many quarks as antiquarks (which means as many baryons and antibaryons) in the universe. Furthermore there ought to be free photons which were produced in fermion-antifermion annihilation, but which due to the expansion and cooling of the universe could not go through the reverse reaction. One finds today that the ratio of baryons to photons is $6 \cdot 10^{-10}$. If all of these photons came from quark-antiquark annihilation, then a quark-antiquark asymmetry in the hot plasma of the early universe of

$$\Delta \mathrm{N_q} = \frac{\mathrm{N_q} - \mathrm{N_{\overline{q}}}}{\mathrm{N_q} + \mathrm{N_{\overline{q}}}} = 6 \cdot 10^{-10} \tag{20.5}$$

would be sufficient to explain the current observed matter-antimatter asymmetry. The question is how did this small but decisive surplus of quarks arise in the early universe?

To generate a matter-antimatter asymmetry we have to fulfil three conditions: CP violation, baryon number violation and thermal non-equilibrium. In the framework of *grand unified theories,* GUTs, one can imagine that all of these conditions could be fulfilled.[3] Consider the situation of the universe at time $t < 10^{-35}$ s. At this moment all (anti)fermions were equivalent, so they could be transformed into each other which could in certain reactions lead to a violation of baryon number. A hypothetical exchange particle, which mediates such a transition, is the X boson whose mass would be about 10^{14} GeV/c^2. These X bosons could be produced as real particles at sufficiently high energies and would decay into a quark and an electron, similarly the $\overline{\mathrm{X}}$ boson decays into an antiquark and a positron. CP violation in the decay of the X boson would mean that the decay rates of the X and $\overline{\mathrm{X}}$ bosons would not be exactly equal. In thermal equilibrium, i.e., at temperatures or energies above the mass of the X boson, the effect of CP violation on the baryon number would be eliminated since the creation and decay of the X and $\overline{\mathrm{X}}$ bosons would be in equilibrium. This equilibrium would first be destroyed by the cooling of the universe and the asymmetry of the CP violating decay of the X boson would lead to a quark surplus, which eventually would be responsible for the matter-antimatter asymmetry we observe in the universe around us. The creation of a baryon excess from an initially baryon-antibaryon symmetric situation is called *baryogenesis*.

There are searches in progress for evidence of the existence of systems with CP violation and baryon number violation in the present universe. As discussed in Sects. 15.4 and 15.5, CP violation has been detected in K^0 and B^0 decays, but the observed effect is not sufficient to explain the matter-antimatter asymmetry. Experiments looking for proton decay have so far not yielded any evidence for baryon number violation.

[3]In principle the standard model of particle physics fulfils the three conditions, but predicts a matter-antimatter asymmetry that is smaller than the observed one (20.5) by ten orders of magnitude.

The discovery of small but non-zero neutrino mass receives in this context an important significance: in Sect. 11.4 we have outlined the seesaw mechanism as an explanation for the smallness of neutrino mass. The heavy Majorana neutrinos that were introduced, having masses of typically 10^{15}–10^{9} GeV/c^2 can be produced in the hot early universe. If their interactions, which lead in particular to decay into leptons, violate the CP symmetry, then due to their Majorana nature more leptons than antileptons are generated. The necessary departure from thermal equilibrium would be caused by the cooling of the universe. Non-perturbative standard model processes, which we will not discuss in detail here, transform this lepton asymmetry into a baryon asymmetry. The generation of a baryon excess via a lepton excess is called *leptogenesis*. Therefore, the search for CP violation in the lepton sector, and the proof that lepton number conservation is indeed violated, is conceptually of great importance (see Sect. 11.4).

Electroweak phase transition Let us now consider the universe at the age of just 10^{-11} s when it had a temperature of $kT \approx 100\,\text{GeV}$. It is believed that one can reconstruct the development of the universe from what is now known of elementary particle physics back to this stage. Extrapolations further back into the past may be based on plausible assumptions but they are in no way proven.

It is believed that the *electroweak phase transition* took place at this stage. Only after this phase transition did the now known properties of the elementary particles establish themselves. A loss of symmetry and an increase in order is characteristic of a phase transition of this type; just as in the phase transition from the paramagnetic to the ferromagnetic phase in iron when it drops below the Curie temperature. For temperatures equivalent to energies >100 GeV, in other words before the phase transition, the photon, W and Z gauge bosons had similar properties and the distinction between the electromagnetic and weak forces was removed. In this state there was also no significant difference between electrons and neutrinos. Below the critical temperature this symmetry was, however, destroyed. This phenomenon, known in the standard model of elementary particle physics as *spontaneous symmetry breaking*, caused the W and Z bosons to acquire their large masses from so-called Higgs fields and the elementary particles took on the properties that we are now familiar with (cf. Sect. 12.4).

Although today elementary particles may be accelerated up to energies of a few TeV and the W and Z bosons have been experimentally produced and detected, it will not be possible to reproduce in the laboratory the high energy-densities of 10^8 times the nuclear density which existed at the electroweak phase transition. We can therefore only try to reproduce and to demonstrate the traces left by the phase transition, i.e., the W, Z and Higgs bosons, so as to use them as witnesses of what went on in the initial stages of the universe.

Hadron formation An additional phase transition took place when the universe was about 1 μs old. At this stage the universe had an equilibrium temperature $kT \approx 100\,\text{MeV}$. The hadrons constituted themselves in this phase from the previously free quarks and gluons (*quark-gluon plasma*). Mostly nucleons were formed in this way.

Since the masses of the u- and d-quarks are very similar, they first formed roughly the same numbers of protons and neutrons, which initially existed as free nucleons since the temperature was too high to permit the formation of nuclei. These protons and neutrons were in thermal equilibrium until the temperature of the universe had sunk so much that the reaction rates for neutron creation processes (e.g., $\bar{\nu}_e\,p \rightarrow e^+n$) were, as a consequence of the greater mass of the neutron, significantly less than that of the inverse processes of proton formation (e.g., $\bar{\nu}_e\,p \leftarrow e^+n$). Thenceforth the numerical ratio of neutrons to protons decreased.

There are currently attempts to simulate this transition from a quark-gluon plasma to a hadronic phase in heavy ion reactions. In these reactions one tries to first create a quark-gluon plasma through highly energetic collisions of ions, in which the matter density is briefly increased to a multiple of the usual nuclear density. In such a state the quarks should only feel the short-range and not the long-range part of the strong potential, since this last should be screened by their tightly packed neighbours. In such a situation the quarks may be viewed as quasi-free and form a quark-gluon plasma. Such a quark-gluon plasma has, however, not yet been indubitably generated and a study of the transition to the hadronic phase is thus only possible in a rather limited fashion.

In the universe the transition from a quark-gluon plasma to the hadronic phase took place via the equilibrium temperature dropping at low matter densities. In the laboratory it is attempted to fleetingly create this transition by varying the matter density at high temperature (cf. Fig. 20.8 and Sect. 20.3).

Primordial synthesis of the elements At $t = 200\,\mathrm{s}$ in the cosmological calendar, the composition of baryonic matter was 88 % protons and 12 % neutrons. The creation of deuterium nuclei by the fusion of neutrons and protons was, until this stage, in equilibrium with the inverse reaction, the photodisintegration of the deuteron into a proton and a neutron, and the lifetime of the deuterons was extremely short. But now the temperature dropped below the level where the energy of the electromagnetic radiation sufficed to maintain the photodisintegration of the deuterons. Now long-lived deuterons were created by the reaction

$$\mathrm{n} + \mathrm{p} \rightarrow \mathrm{d} + \gamma + 2.22\,\mathrm{MeV}\,.$$

The lifetime of these deuterons was now limited by its fusion with protons and neutrons

$$\mathrm{p} + \mathrm{d} \rightarrow {}^3\mathrm{He} + \gamma + 5.49\,\mathrm{MeV}$$
$$\mathrm{n} + \mathrm{d} \rightarrow {}^3\mathrm{H} + \gamma + 6.26\,\mathrm{MeV}\,.$$

Finally the particularly stable ^{4}He nucleus was created in reactions like $^3\mathrm{H} + \mathrm{p}$, $^3\mathrm{He}+\mathrm{n}$, $^3\mathrm{He}+\mathrm{d}$ and $\mathrm{d}+\mathrm{d}$. The Li nuclei created by $^4\mathrm{He}+{}^3\mathrm{H} \rightarrow {}^7\mathrm{Li}+\gamma+2.47\,\mathrm{MeV}$

were on the other hand immediately destroyed again by the highly exothermic reaction

$$^{7}\mathrm{Li} + \mathrm{p} \rightarrow 2\,^{4}\mathrm{He} + 17.35\,\mathrm{MeV}\,.$$

Essentially all of the neutrons ended the primordial nucleosynthesis phase inside ^{4}He, which thus constitutes about 24 % of the mass of the universe.

Only traces of deuterium, ^{3}He and ^{7}Li are still present, so at that moment the greatest part of the baryonic mass must have been in the form of protons. Since there are no stable nuclei with masses $A = 5$ and $A = 8$ it was not possible at that stage of the universe's development to build up nuclei heavier than ^{7}Li through fusion processes. Such nuclei could only be produced much later in stellar interiors.

The primordial element-synthesis phase ended after about 10 min when the temperature had dropped so far that the Coulomb barrier prevented further fusion processes. The much later synthesis of heavy nuclei inside stars has not altered the composition of baryonic matter significantly. The ratio of hydrogen to helium which is observed in the present universe (cf. Fig. 2.2) is in excellent agreement with the theoretically calculated value. This is a strong argument in favour of the big bang model.

Cosmic microwave background The expanding universe, the helium to hydrogen ratio as the signature of the primordial synthesis of the elements and the cosmic microwave background (CMB) are the three most important experimental observations supporting the big bang model of the universe.

After the "first ten minutes", the universe was composed of a plasma of fully ionised hydrogen and helium and about 10^{10} times as many photons. The energy density in the universe was radiation dominated. The main mechanism for energy transport in this period was Compton scattering. The photon mean free path was small at the cosmic scale and the universe opaque.

One would expect that the decoupling of radiation from matter started when the temperature became too low to keep the thermal equilibrium via the reaction

$$p + e \leftrightarrow H + \gamma\,. \tag{20.6}$$

If this process took place under equilibrium conditions the decoupling temperature would be $kT_{\mathrm{dec}} = 0.32\,\mathrm{eV}$ ($T_{\mathrm{dec}} \approx 3{,}700\,\mathrm{K}$).

However, the recombination of hydrogen actually started later, at somewhat lower temperatures than $kT = 0.32\,\mathrm{eV}$. The reason is as follows. Hydrogen can be ionised by multiple absorption of low-energy photons from $2S$ or $2P$ excited states. Later recombination by a cascade passing through the $2P$ state can produce a photon of the correct energy (Lyman-α line), which in turn can excite another atom to the same state which then can be ionised by abundant low-energy photons. As photons from the $2P \rightarrow 1S$ transitions are confined in the universe, recombination is not possible via a direct cascade through the $2P$ level. The only leakage of the Lyman-α photons passes through the two-photon decay of the $2S$ state. The lifetime of this state is ≈ 0.1 s; therefore, hydrogen recombination is a non-equilibrium process. The transition from an opaque to a transparent universe took place at $T \approx 3{,}000$ K. Although the mean free path of photons increased dramatically at this temperature, photons still interacted with free electrons via Thomson scattering to a significant extent. Therefore, the photon background that we observe comes from the so-called last scattering surface, where Thomson scattering did not play a role any longer. This was the case about 4×10^5 years after the big bang. At present the decoupled radiation is a perfect black body spectrum, with temperature $T \approx 2.7$ K.

The cosmic background radiation is a rich source of information about the universe before the decoupling of the photons. The temperature fluctuations in order of 10^{-5} have been interpreted as quantum fluctuations in the early universe. They were the seeds from which the galaxies later developed. The fact that the fluctuations have not been smeared out is seen as a proof for the universe to be flat, i.e., its density has the critical value [12].

20.5 Stellar Evolution and Element Synthesis

The close weave linking nuclear physics and astrophysics stretches back to the thirties when Bethe, Weizsäcker and others tried to draw a quantitative balance between the energy emitted by the Sun and the energy that could be released by the known nuclear reactions. It was, though, Eddington who in 1920 had recognised that nuclear fusion is the source of energy production in stars.

The basis for modern astrophysics was, however, laid by Fred Hoyle [14, 15] at the end of the forties. The research programme he proposed required a consistent treatment of astronomical observations, study of the plasma dynamics of stellar interiors and calculations of the sources of energy using the cross-sections for nuclear reactions measured in laboratories. Stellar evolution and the creation of the elements had to be treated together. The observed abundance of the elements around us had to be explicable from element synthesis in the early stages of the universe and from nuclear reactions in stars and this would thus be a decisive test of the consistency of stellar evolution models. The results of this programme were presented by E. Burbidge, G. Burbidge, Fowler and Hoyle [7].

Stars are produced by the contraction of interstellar gas and dust. This matter is almost solely composed of primordial hydrogen and helium. The contraction heats up the centre of the star. When the temperature and pressure are sufficiently large to render nuclear fusion possible, radiation is produced whose pressure prevents a further contraction of the star. The virial theorem for the gravitational force law implies a fall-off in the temperature of stars from their centres to their exteriors. This means that at any separation from the centre of a star the average kinetic energy of an atom is half the size of its potential energy. The energy produced in nuclear reactions is primarily transported by radiation to the surface. The matter in the star is not greatly mixed up in the process. During the life of the star its chemical composition changes in the regions where the nuclear reactions take place, in other words most of all in the core of the star.

Fusion reactions A star in equilibrium produces as much energy through nuclear reactions as it radiates. The equilibrium state is thus highly dependent upon the rate of the fusion reactions. Energy may be released by fusing light nuclei together. It is especially effective to fuse hydrogen isotopes together to form ^{4}He, since the difference between its binding energy per nucleon, 7.07 MeV, and that of its neighbours is especially large (cf. Fig. 2.4). We will treat this reaction in more detail below. Fusion processes require a sufficiently high temperature, or energy, for the reaction partners to surmount the Coulomb barrier. It is not necessary that the energy of the nuclei involved is actually above the barrier, rather what really matters, in analogy to α-decay, is the probability, e^{-2G}, that the Coulomb barrier may be tunnelled through. The Gamow factor, G, depends upon the relative velocities and the charge numbers of the reaction partners. It is given by (see Sect. 3.15)

$$G = \frac{\pi \alpha Z_1 Z_2}{v/c} \propto \frac{1}{\sqrt{E}} . \tag{20.7}$$

Fusion reactions in stars normally take place below the Coulomb barrier and through the tunnel effect.

The reaction rate per unit volume is according to (4.3) and (4.4) given by

$$\dot{N} = n_1 n_2 \langle \sigma v \rangle , \tag{20.8}$$

where n_1 and n_2 are the particle densities of the two fusion partners. We have written the average value $\langle \sigma v \rangle$ since the velocity distribution in a hot stellar plasma is given by a Maxwell-Boltzmann distribution

$$n(v) \propto e^{-mv^2/2kT} = e^{-E/kT} \tag{20.9}$$

and the cross-section σ of the fusion reaction depends strongly, through the Gamow factor, upon the relative velocity of the reaction partners. This average value must be calculated by integration over v. Figure 20.10 schematically shows the convolution of the Gamow factor with a Maxwell distribution. The overlap of the distributions fixes the reaction rate and the energy range for which fusion reactions are possible. This depends upon the plasma temperature and the charges of the fusion partners. The higher the charge numbers, the higher the necessary temperatures at which fusion reactions become possible.

In this way the lightest nuclide in the star's interior, hydrogen, is burnt up, i.e., fused together. When this is used up, the temperature has to increase drastically for helium and, later, other heavier elements to be able to fuse together. The length of the various burn-phases depends upon the mass of the star in question. For heavier stars the pressure and thus the density of the plasma at the centre is higher and so

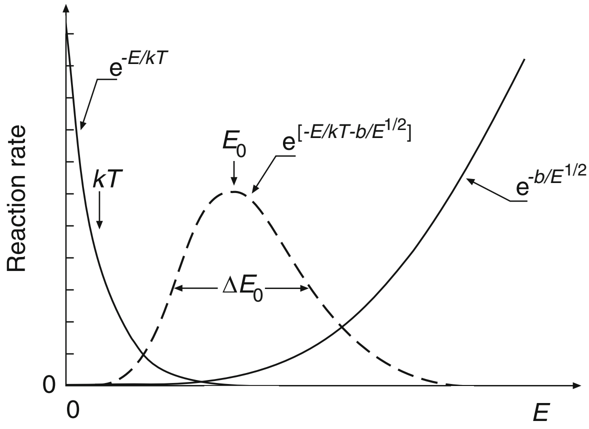

Fig. 20.10 Schematic representation of the convolution of a Maxwell distribution $\exp\{-E/kT\}$ with a Gamow factor $\exp\{-b/E^{1/2}\}$ as used to calculate the rate of fusion reactions. The product of the curves is proportional to the fusion probability (*dashed curve*). Fusion essentially takes place in a very narrow energy interval with width ΔE_0. The integral over this curve is proportional to the total reaction rate

the reaction rate is higher compared to lighter stars. Thus heavier stars are shorter lived than heavy ones.

Hydrogen burning In the formation phase of stars with masses greater than about one tenth of a solar mass, the temperatures inside the stars reach values of $T > 10^7$ K, and thus the first nuclear fusion processes are possible. In the early part of their lives stars gain their energy by burning hydrogen into helium in the *proton-proton cycle:*

$$
\begin{aligned}
p + p &\to d + e^+ + \nu_e + 0.420\,\text{MeV} \\
p + d &\to {}^3\text{He} + \gamma + 5.493\,\text{MeV} \\
{}^3\text{He} + {}^3\text{He} &\to p + p + \alpha + 12.859\,\text{MeV} \\
e^+ + e^- &\to 2\gamma + 1.022\,\text{MeV}.
\end{aligned}
$$

All in all, in the net reaction $4p \to \alpha + 2e^+ + 2\nu_e$, 24.69 MeV of energy is released. The total released energy is 26.73 MeV when also the contribution from e^+e^- pair annihilation is taken into account. Of this ≈0.3 MeV is on average taken by neutrinos and thus lost to the star. The first reaction is the slowest in the cycle since it requires not only the fusion of two protons but also the simultaneous transformation of a proton into a neutron via a weak interaction process. This reaction thus determines the lifetime of the star in the first stage of its evolution. There are various possible branches to the proton-proton cycle, but they are of little importance for energy production in stars.

As long as the supplies of hydrogen are adequate the star remains stable. For our Sun this period will last about 10^{10} years, of which about half are already gone. Larger stars with higher central densities and temperatures burn faster. If in such stars ^{12}C is already present, then the *carbon cycle* can take place:

$$^{12}_{6}\mathrm{C} \xrightarrow{\mathrm{p}} {}^{13}_{7}\mathrm{N} \xrightarrow{\beta^+} {}^{13}_{6}\mathrm{C} \xrightarrow{\mathrm{p}} {}^{14}_{7}\mathrm{N} \xrightarrow{\mathrm{p}} {}^{15}_{6}\mathrm{O} \xrightarrow{\beta^+} {}^{15}_{7}\mathrm{N} \xrightarrow{\mathrm{p}} {}^{12}_{6}\mathrm{C} + \alpha\,. \qquad (20.10)$$

The amount of carbon which was transformed at the beginning of the cycle is again available for further use at the end and thus it acts as a catalyst. The net reaction is as in the proton-proton cycle, $4\mathrm{p} \to \alpha + 2\mathrm{e}^+ + 2\nu_\mathrm{e}$, and the amount of energy released is also 24.69 or 26.73 MeV, respectively. The carbon cycle can take place much faster than the proton-proton cycle. But this new cycle only starts at higher temperatures due to the greater Coulomb barrier. In the Sun this cycle contributes by about 1.6 % to the energy production.

Helium burning Once the hydrogen supplies have dried up, the core of the star, which is now composed of helium, cannot withstand the pressure and collapses. For stars much smaller than the Sun the gravitational pressure is not great enough to ignite further fusion reactions. Without the radiative pressure, the star collapses under its own gravity to a planet-sized sphere. Fermi pressure is the first thing to stop the collapse and the star becomes a white dwarf.

Heavier stars heat up until they reach a temperature of about 10^8 K and a density of 10^8 kg/m^3. Helium burning then starts up. There is still some hydrogen in the outermost regions of the star, which is heated up by the helium burning in the hot central region until in this layer hydrogen burning commences. The outer mantle swells up through the radiation pressure. Since the surface area increases the surface temperature drops, even though the energy production is increasing in this stage. The colour of the star turns red and it becomes a red giant.

A synthesis of nuclei heavier than ^{4}He appears to be impossible because there are no stable nuclei with $A = 5$ and $A = 8$. ^{8}Be has a lifetime of only 10^{-16} s, and ^{5}He and ^{5}Li are still less stable. But in 1952 E. Salpeter showed how heavy nuclei could be produced by helium fusion [18].

At high temperatures around 10^8 K, which are present in stellar interiors, the unstable ^{8}Be nucleus can be formed from helium-helium fusion and equilibrium for the reaction $^4\mathrm{He} + {}^4\mathrm{He} \leftrightarrow {}^8\mathrm{Be}$ is created. This reaction is only possible in sufficient amounts at such high temperatures, since as well as the Coulomb barrier an energy level difference of 92 keV must be overcome (Fig. 20.11). At a density of 10^8 kg/m^3 in the interior of the star an equilibrium concentration of one ^{8}Be nucleus for 10^9 ^{4}He nuclei is produced. This minuscule proportion would be enough to produce sizable amounts of carbon via $^4\mathrm{He} + {}^8\mathrm{Be} \to {}^{12}\mathrm{C}^*$ if there were a 0^+ state in ^{12}C a little above the production threshold over which a resonant reaction can take place. Shortly after this suggestion was made such a state at an excitation energy of 7.654 MeV was indeed found [8]. This state decays with a probability of $4 \cdot 10^{-4}$ into the ^{12}C ground state (Fig. 20.11). Although this state is 287 keV above the $^8\mathrm{Be} + \alpha$ threshold, it can indeed be populated by reaction partners from the high-energy tail

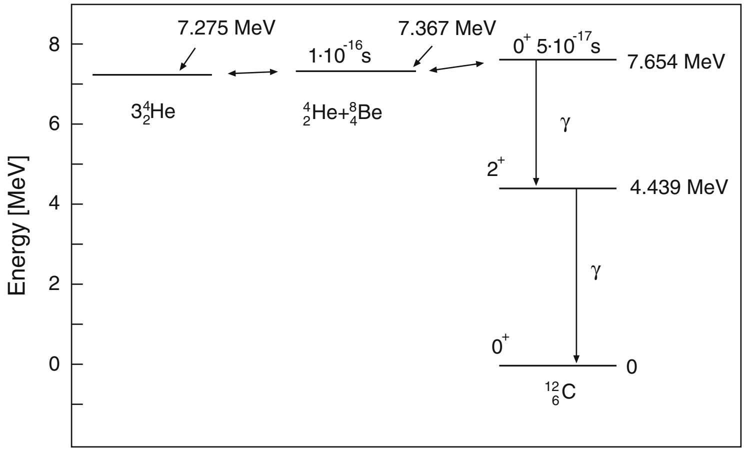

Fig. 20.11 Energy levels of the system: $3\,\alpha$, $\alpha + {}^8$Be and ^{12}C. Just above the ground states of the $3\,\alpha$ system and of the $\alpha + {}^8$Be system there is a 0^+ state in the ^{12}C nucleus, which can be created through resonant fusion of ^{4}He nuclei. This excited state decays with a 0.04 % probability into the ^{12}C ground state

of the Maxwell velocity distribution. The net reaction of helium fusion into carbon is thus

$$3\,{}^4\mathrm{He} \to \ {}^{12}\mathrm{C} + 2\gamma + 7.367\,\mathrm{MeV}\,.$$

This so-called triple-α reaction plays a key role in building up the heavier elements of the universe. Approximately 1 % of all the nuclei in the universe are heavier than helium and they were practically all created in the triple-α process.

Burning into iron When the helium supplies have been used up and the star is primarily made up of ^{12}C, then stars with masses of the order of the solar mass turn into white dwarfs.

More massive stars go through further phases of development. According to the temperature α-particles can fuse with ^{12}C, ^{16}O, ^{20}Ne etc., or carbon, oxygen, neon and silicon can simply fuse with each other.

As an example let us mention the reactions

$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\to {}^{20}_{10}\mathrm{Ne} + \alpha + 4.62\,\mathrm{MeV} \\ &\to {}^{23}_{11}\mathrm{Na} + \mathrm{p} + 2.24\,\mathrm{MeV} \\ &\to {}^{23}_{12}\mathrm{Mg} + \mathrm{n} - 2.61\,\mathrm{MeV} \\ &\to {}^{16}_{8}\mathrm{O} + 2\alpha - 0.11\,\mathrm{MeV}\,. \end{aligned}$$

Other reactions follow the same pattern and populate all the elements between carbon and iron.

The heavier the fusing nuclei are, the greater is the Coulomb repulsion and so the temperature must then be higher for fusion to take place. Since the temperature is greatest at the centre and falls off towards the surface, an onion-like stellar structure is formed. At the centre of the star iron is synthesised, towards the edges ever lighter elements are made. In the outermost layers the remnants of hydrogen and helium are burnt off.

The burning of the heavier nuclei takes place at ever shorter time scales, since the centre of the star needs to be ever hotter, but simultaneously the energy gained per nucleon-fusion decreases as the mass number increases (Fig. 2.4). The final phase, the fusion of silicon to form iron, lasts for only a matter of days [4]. The process of nuclear fusion in stars concludes with the formation of iron since iron has the largest binding energy per nucleon.

When the centre of the star is made of iron, there is no further source of energy available. There is neither radiative pressure nor thermal motion to withstand gravity. The star collapses. The outer material of the star collapses as if in free fall to the centre. Through this implosion the nuclear matter at the centre reaches a tremendous density and temperature which leads to an enormous explosion. The star emits at a stroke more energy, typically about 10^{47} J, than it has previously created in its entire life. This is called a supernova. The greater part of the stellar matter is then flung out into interstellar space and can later be used as building material for new stars. If the mass of the remaining stellar core is smaller than the mass of the Sun, the star ends its life as a white dwarf. If it is between one and two solar masses a neutron star is born. The matter from still heavier remnants ends up as a black hole. It is interesting to note that such an explosion of a star as supernova is a neutrino source of extremely high intensity. These neutrinos are produced, e.g., via the reaction $\mathrm{p} + \mathrm{e} \rightarrow \mathrm{n} + \nu_{\mathrm{e}}$ at the beginning of the collapse and carry about 99 % of the vast amount of energy released in a supernova explosion.

Synthesis of heavier nuclei Nuclei heavier than iron are synthesised by neutron accumulation. We distinguish between two processes.

The slow process (s-process). In the burning phase of the star neutrons are produced in nuclear reactions such as, e.g.,

$$ {}^{22}_{10}\mathrm{Ne} + \alpha \rightarrow {}^{25}_{12}\mathrm{Mg} + \mathrm{n} - 0.48\,\mathrm{MeV} \tag{20.11} $$

or

$$ {}^{13}_{6}\mathrm{C} + \alpha \rightarrow {}^{16}_{8}\mathrm{O} + \mathrm{n} - 0.91\,\mathrm{MeV}\,. \tag{20.12} $$

Through repeated neutron captures, neutron-rich isotopes are produced. If the isotopes are unstable under β-decay, they decay into their most stable isobar (Figs. 3.2 and 3.3). Thus the synthesis of heavier and heavier elements can proceed along a stability valley (Fig. 3.1). A limit is, however, reached at lead. Nuclei above lead are α-unstable. Isotopes built up by the slow process then decay again into α-particles and lead.

The rapid process (r-process). This process takes place during a supernova explosion when neutron fluxes of $10^{32}\,\mathrm{m^{-2}\,s^{-1}}$ can be reached and the successive accumulation of many neutrons is much quicker than β- or α-decay processes. Elements heavier than lead can be produced in this process. The upper limit for the creation of transuranic elements is determined by spontaneous fission.

All the elements (apart from hydrogen and helium) which make up the Earth and ourselves came originally from the interior of stars and were (probably several times in fact) released through supernova explosions. Even the absolute amounts as well as the distribution of the elements which are heavier than helium may be calculated from the age of the universe and from cross-sections measured in laboratories. The results are in excellent agreement with the measured values of the abundance of the elements (Fig. 2.2). This is definitely one of the great triumphs of the joint efforts of nuclear and astrophysicists.

Problems

1. **Sun**

   The solar mass is $M_\odot \approx 2 \cdot 10^{30}$ kg ($3.3 \cdot 10^5$ times the mass of the Earth). The chemical composition of the solar surface is 71 % hydrogen, 27 % helium and 2 % heavier elements (expressed as parts by mass). The luminosity of the Sun is $4 \cdot 10^{26}$ W.

   (a) How much hydrogen is converted into helium every second?
   (b) How much mass does the Sun lose in the same period?
   (c) What fraction of the original hydrogen content has been converted into helium since the creation of the Sun ($5 \cdot 10^9$ years)?
   (d) How large was the loss of mass in the same period?
   (e) Model calculations indicate that the Sun will burn hydrogen at a similar rate for a further $5 \cdot 10^9$ years. A shortage of hydrogen will then force it into a red giant state. Motivate this time scale.

2. **Supernova**

   A neutron star with mass, $M = 1.5\,M_\odot$ ($\approx 3.0 \cdot 10^{30}$ kg), and radius $R \approx 10$ km is the remnant of a supernova. The stellar material originates from the iron core ($R \gg 10$ km) of the supernova.

   (a) How much energy was released during the lifetime of the original star by converting hydrogen into iron? (The binding energy of ^{56}Fe is $B = 8.79$ MeV/nucleon.) *NB:* Since after the implosion only a part of the original iron core remains in the neutron star, the calculation should be performed only for this mass.
   (b) How much energy was released during the implosion of the iron core into a neutron star?
   (c) In what form was the energy radiated off?

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