Chapter 9
Particle Production in e^+e^- Collisions

So far, we have mainly discussed the light quarks, u and d, and those hadrons composed of these two quarks. The easiest way to produce hadrons with heavier quarks is in e^+e^- collisions. Free electrons and positrons may be produced rather easily. They can be accelerated, stored and made to collide in accelerators. In an electron-positron collision process, all particles which interact electromagnetically and weakly can be produced, as long as the energy of the beam particles is sufficiently high. In an electron-positron electromagnetic annihilation, a virtual photon is produced, which immediately decays into a pair of charged elementary particles Fig. 9.1 (left). In a weak interaction, the exchanged particle is the heavy vector boson Z^0 (cf. Fig. 9.1 (right) and see Chap. 12). The symbol f denotes an elementary fermion (quark or lepton) and $\bar{f}$ its antiparticle. The $f\bar{f}$ system must have the quantum numbers of the photon or the Z^0, respectively. In these reactions all fundamental, charged particle-antiparticle pairs can be produced; lepton-antilepton and quark-antiquark pairs. Neutrinos are electrically neutral; hence, neutrino-antineutrino pairs can only be produced by Z^0 exchange.

Colliding beams Which particle-antiparticle pairs can be produced only depends upon the energy of the electrons and positrons (Fig. 9.2). In a storage ring, electrons and positrons with beam energies E_1 and E_2 orbit in opposite directions and collide head-on. It is conventional to use the Lorentz-invariant energy variable s, the square of the centre-of-mass energy:

$$\begin{aligned} s &= (p_1c + p_2c)^2 \\ &= m_1^2c^4 + m_2^2c^4 + 2E_1E_2 - 2\boldsymbol{p}_1\boldsymbol{p}_2c^2 \,. \end{aligned} \tag{9.1}$$

In a storage ring with colliding particles of the same energy E,

$$s = 4E^2 \,. \tag{9.2}$$

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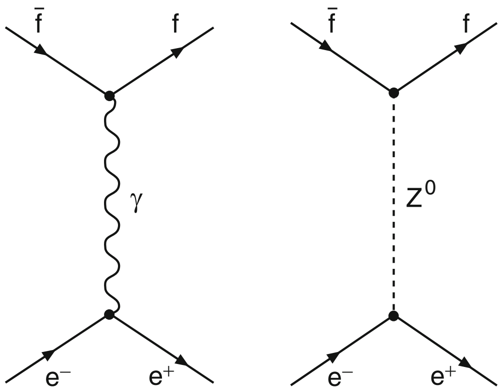

Fig. 9.1 Fermion-antifermion production in electron-positron collisions via the exchange of a virtual photon (left) and a Z^0-boson (right)

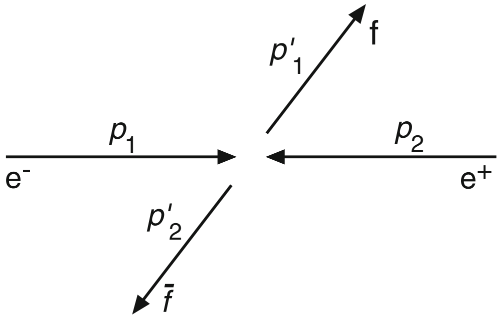

Fig. 9.2 For colliding particles of the same energy the fermion and anti fermion are produced back-to-back

Hence, particle-antiparticle pairs with masses of up to $2m = \sqrt{s}/c^2$ can be produced. To discover new particles, the storage ring energy must be raised. One then looks for an increase in the reaction rate, or for resonances in the cross-section.

The great advantage of colliding beam experiments is that the total beam energy is available in the centre-of-mass system. In a fixed target experiment, with m satisfying $mc^2 \ll E$, s is related to E by:

$$s \approx 2mc^2 \cdot E\,. \tag{9.3}$$

Here, the centre-of-mass energy only increases proportionally to the square root of the beam energy.

Particle detection To detect the particles produced back-to-back in e$^+$e$^-$ annihilation (Fig. 9.2) one requires a detector set up around the collision point which covers as much as possible of the total 4π solid angle. The detector should permit us to trace the tracks back to the interaction point and to identify the particles themselves. The basic form of such a detector is sketched in Fig. 9.3.

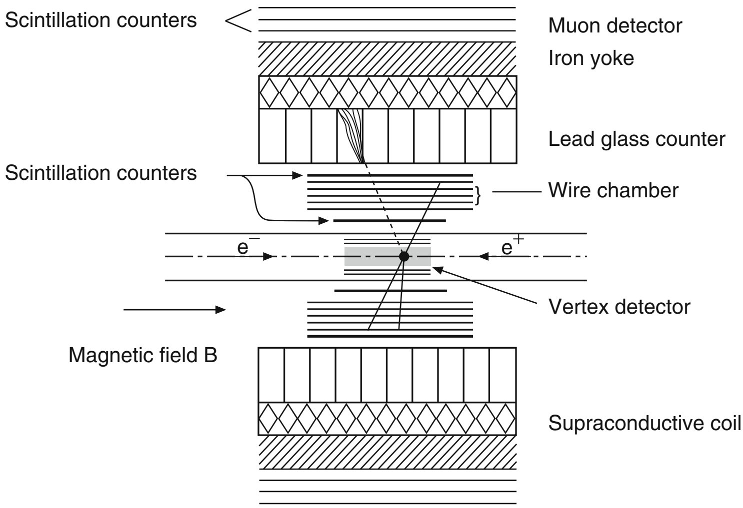

Fig. 9.3 Sketch of a 4π-detector, as used in e^+e^- collision experiments. The detector is inside the coil of a solenoid, which typically produces a magnetic field of around 1 T along the beam direction. Charged particles are detected in a vertex detector, mostly composed of silicon microstrip counters, and in wire chambers. The vertex detector is used to locate the interaction point. The curvature of the tracks in the magnetic field tell us the momenta. Photons and electrons are detected as shower formations in electromagnetic calorimeters (of, e.g., lead glass). Muons pass through the iron yoke with little energy loss. They are then seen in the exterior scintillation counters

9.1 Lepton Pair Production

Before we turn to the creation of heavy quarks, we want to initially consider the leptons. *Leptons* are elementary spin-1/2 particles which feel the weak and, if they are charged, the electromagnetic interaction – but not, however, the strong interaction.

Muons The lightest particles which can be produced in electron-positron collisions are muon pairs:

$$e^+ + e^- \rightarrow \mu^+ + \mu^- .$$

The muon μ^- and its antiparticle[1] the μ^+ both have a mass of only $105.7\,\text{MeV}/c^2$ and they are produced in all usual e^+e^- storage ring experiments. They penetrate matter very easily,[2] whereas electrons because of their small mass and hadrons because of the strong interaction have much smaller ranges. After that of the

[1] Antiparticles are generally symbolised by a bar (e.g., $\overline{\nu}_e$). This symbol is generally skipped over for charged leptons since knowledge of the charge alone tells us whether we have a particle or an antiparticle. We thus write e^+, μ^+, τ^+.

[2] Muons from cosmic radiation can still be detected in underground mines!

neutron, theirs is the longest lifetime (2 µs) of any unstable particle. This means that experimentally they may easily be identified. Therefore the process of muon pair production is often used as a reference point for other e^+e^- reactions.

Tau leptons If the centre-of-mass energy in an e^+e^- reaction suffices, a further lepton pair, the τ^- and τ^+, may be produced. Their lifetime, $3 \cdot 10^{-13}$ s, is much shorter. They may weakly decay into muons or electrons as will be discussed in Sect. 10.1f.

The tau was discovered at the SPEAR e^+e^- storage ring at SLAC when oppositely charged electron-muon pairs were observed whose energy was much smaller than the available centre-of-mass energy [16].

These events were interpreted as the creation and subsequent decay of a heavy lepton-antilepton pair:

$$\begin{array}{l} e^+ + e^- \longrightarrow \tau^+ + \tau^- \\ \qquad \tau^- \to \mu^- + \overline{\nu}_\mu + \nu_\tau \quad \text{or} \quad e^- + \overline{\nu}_e + \nu_\tau \\ \qquad \tau^+ \to e^+ + \nu_e + \overline{\nu}_\tau \quad \text{or} \quad \mu^+ + \nu_\mu + \overline{\nu}_\tau\,. \end{array}$$

The neutrinos which are created are not detected.

The threshold for $\tau^+\tau^-$-pair production, and hence the mass of the τ-lepton, may be read off from the increase of the cross-section of the e^+e^- reaction with the centre-of-mass energy. One should use as many leptonic and hadronic decay channels as possible to provide a good signature for τ-production (Fig. 9.4). The experimental threshold at $\sqrt{s} = 2m_\tau c^2$ implies that the tau mass is $1.777\,\text{GeV}/c^2$.

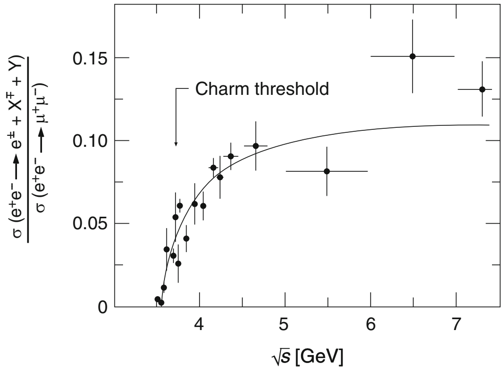

Fig. 9.4 Ratio of the cross-sections for the production of two particles with opposite charges in the reaction $e^+ + e^- \to e^\pm + X^\mp + Y$, to the cross-sections for the production of $\mu^+\mu^-$ pairs [5, 6]. Here $X^\mp$ denotes a charged lepton or meson and Y symbolises the unobserved, neutral particles. The sharp increase at $\sqrt{s} \approx 3.55\,\text{GeV}$ is a result of τ-pair production, which here becomes energetically possible. The threshold for the creation of mesons containing a charmed quark (*arrow*) is only a little above that for τ-lepton production. Both particles have similar decay modes which makes it more difficult to detect τ-leptons

Cross-section The creation of charged lepton pairs may, to a good approximation, be viewed as a purely electromagnetic process (γ exchange). The exchange of Z^0 bosons, and interference between photon and Z^0 exchange, may be neglected if the energy is small compared to the mass of the Z^0. The cross-section may then be found relatively easily. The most complicated case is the elastic process $e^+e^- \to e^+e^-$, *Bhabha scattering*. Here two processes must be taken into account: the annihilation of the electron and positron into a virtual photon with subsequent e^+e^--pair creation (Fig. 9.5 (left)) and secondly the scattering of the electron and positron off each other (Fig. 9.5 (right)). These processes lead to the same final state and so their amplitudes must be added in order to obtain the cross-section.

Muon pair creation is more easily calculated. Other e^+e^- reactions are therefore usually normalised with respect to it. The differential cross-section for this reaction is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(\hbar c)^2 \cdot \left(1 + \cos^2\theta\right) . \tag{9.4}$$

Integrating over the solid angle Ω yields the total cross-section:

$$\sigma = \frac{4\pi\alpha^2}{3s}(\hbar c)^2 , \tag{9.5}$$

and one finds

$$\sigma(e^+e^- \to \mu^+\mu^-) = 21.7 \, \frac{\text{nbarn}}{(E^2/\text{GeV}^2)} . \tag{9.6}$$

The formal derivation of (9.4) may be found in many standard text books [10, 14, 15], we will merely try to make it plausible: The photon couples to two elementary charges. Hence the matrix element contains two powers of e and the cross-section, which is proportional to the square of the matrix element, is proportional to e^4 or α^2.

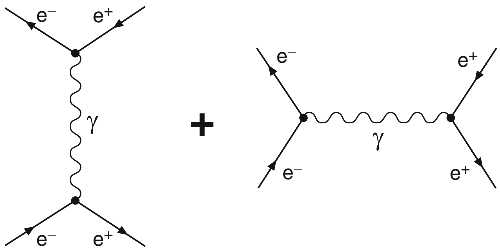

Fig. 9.5 The two processes contributing to Bhabha scattering

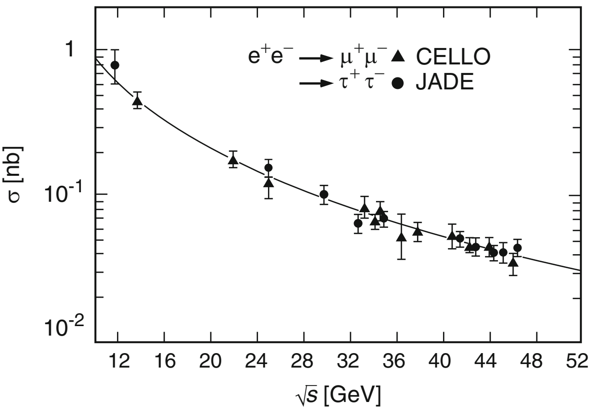

Fig. 9.6 Cross-sections of the reactions $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ as functions of the centre-of-mass energy $\sqrt{s}$ (From [7] and [8]). The *solid line* shows the cross-section (9.6) predicted by quantum electrodynamics

The length scale is proportional to $\hbar c$, which enters twice over since cross-sections have the dimension of area. We must further divide by a quantity with dimensions of [energy^2]. Since the masses of the electron and the muon are very small compared to s, this last is the only reasonable choice. The cross-section then falls off with the square of the storage ring's energy. The $(1 + \cos^2\theta)$ angular dependence is typical for the production of two spin-1/2 particles such as muons. Note that (9.4) is, up to this angular dependence, completely analogous to the equation for Mott scattering (5.39) once we recognise that $Q^2c^2 = s = 4E^2 = 4E'^2$ holds here.

Figure 9.6 shows the cross-section for $e^+e^- \to \mu^+\mu^-$ and the prediction of quantum electrodynamics. One sees an excellent agreement between theory and experiment. The cross-section for $e^+e^- \to \tau^+\tau^-$ is also shown in the figure. If the centre-of-mass energy $\sqrt{s}$ is large enough that the difference in the μ and τ rest masses can be neglected, then the cross-sections for $\mu^+\mu^-$ and $\tau^+\tau^-$ production are identical. One speaks of *lepton universality*, which means that the electron, the muon and the tau behave, apart from their masses and associated effects, identically in all reactions. The muon and the tau may to a certain extent be viewed as being heavier copies of the electron.

Since (9.6) describes the experimental cross-section so well, the form factors of the μ and τ are unity – which according to Table 5.1 means they are point-like particles. No spatial extension of the leptons has yet been seen. The upper limit for the electron is 10^{-18} m. Since the hunt for excited leptons so far has also been unsuccessful, it is currently believed that leptons are indeed elementary, point-like particles.

9.2 Resonances

If the cross-sections for the production of muon pairs and hadrons in e^+e^- scattering are plotted as a function of the centre-of-mass energy $\sqrt{s}$, one finds in both cases the $1/s$-dependence of (9.5). In the hadronic final state channels this trend is broken by various strong peaks which are sketched in Fig. 9.7. These so-called *resonances* are short lived states which have a fixed mass and well-defined quantum numbers such as angular momentum. It is therefore reasonable to call them particles.

Breit-Wigner formula The energy dependence of the cross-section of a reaction between two particles a and b close to a resonance energy E_0 is generally described by the *Breit-Wigner formula* (see, e.g., [15]). In the case of elastic scattering, it is approximately given by:

$$\sigma(E) = \frac{\pi \lambda\!\!\!^{-2}\,(2J+1)}{(2s_a+1)(2s_b+1)} \cdot \frac{\Gamma^2}{(E-E_0)^2+\Gamma^2/4}\,. \tag{9.7}$$

Here $\lambda\!\!\!^{-}$ is the reduced wavelength in the centre-of-mass system, s_a and s_b are the spins of the reacting particles and Γ is the *width* (half width) of the resonance. The lifetime of such a resonance is $\tau = \hbar/\Gamma$. This formula is similar to that for the resonance of a forced oscillator with large damping. The energy E corresponds to the excitation frequency ω, E_0 to the resonance frequency ω_0 and the width Γ to the damping.

For an inelastic reaction like the case at hand, the cross-section depends upon the *partial widths* Γ_i and Γ_f in the initial and final channels and on the total width Γ_{tot}. The latter is the sum of the partial widths of all possible final channels. The result

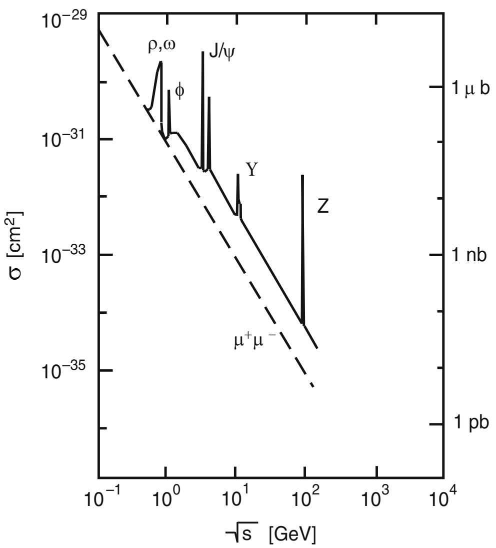

Fig. 9.7 Cross-section of the reaction $e^+e^- \rightarrow$ *hadrons* as a function of the centre-of-mass energy $\sqrt{s}$ (sketch) [11]. The cross-section for direct muon pair production (9.5) is denoted by a *dashed curve*

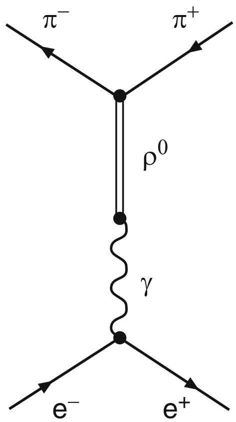

Fig. 9.8 Production of a ϱ^0 vector meson in e^+e^- annihilation with its subsequent decay into a charged pion pair

for an individual decay channel f is

$$\sigma_f(E) = \frac{3\pi \lambda\!\!\!^{-2}}{4} \cdot \frac{\Gamma_i \Gamma_f}{(E-E_0)^2 + \Gamma_{\text{tot}}^2/4}\,, \tag{9.8}$$

where we have replaced s_a and s_b by the spins of the electrons (1/2) and J by the spin of the photon (1).

The resonances ϱ, ω, and ϕ First, we discuss resonances at low energies. The width Γ of these states varies between 4 and 150 MeV, corresponding to lifetimes from about 10^{-22} to 10^{-24} s. These values are typical of the strong interaction. These resonances are therefore interpreted as quark-antiquark bound states whose masses are just equal to the total centre-of-mass energy of the reaction. The quark-antiquark states must have the same quantum numbers as the virtual photon; in particular, they must have total angular momentum $J = 1$ and negative parity. Such quark-antiquark states are called *vector mesons*; they decay into lighter mesons. Figure 9.8 depicts schematically the production and the decay of the ϱ^0 resonance.

The analysis of the peak at 770–780 MeV reveals that it is caused by the interference of two resonances, the $\varrho^0 -$ *meson* ($m_{\varrho^0} = 776\,\text{MeV}/c^2$) and the ω-meson ($m_\omega = 782\,\text{MeV}/c^2$). These resonances are produced via the creation of $u\bar{u}$ and $d\bar{d}$ pairs. Since u-quarks and d-quarks have nearly identical masses, the $u\bar{u}$- and $d\bar{d}$-states are approximately degenerate. The ϱ^0 and ω are mixed states of $u\bar{u}$ and $d\bar{d}$.

These two mesons undergo different decays and may be experimentally identified by them (cf. Sect. 15.3):

$$\varrho^0 \to \pi^+\pi^-\,,$$
$$\omega \to \pi^+\pi^0\pi^-\,.$$

At an energy of 1,019 MeV, the ϕ-resonance is produced. It has a width of only $\Gamma = 4.3$ MeV, and hence a relatively long lifetime compared to other hadrons. The main decay modes (≈85 %) of the ϕ are into two kaons, which have masses of 494 MeV/c^2 ($K^\pm$) and 498 MeV/c^2 (K^0):

$$\phi \to K^+ + K^- ,$$
$$\phi \to K^0 + \overline{K}^0 .$$

Kaons are examples of the so-called *strange particles.* This name reflects the unusual fact that they are produced by the strong interaction, but only decay by the weak interaction; this despite the fact that their decay products include hadrons, i.e., strongly interacting particles.

This behaviour is explained by the fact that kaons are quark-antiquark combinations containing an s or "strange" quark:

$$|K^+\rangle = |u\bar{s}\rangle \qquad |K^0\rangle = |d\bar{s}\rangle$$
$$|K^-\rangle = |\bar{u}s\rangle \qquad |\overline{K}^0\rangle = |\bar{d}s\rangle .$$

The constituent mass attributed to the s-quark is 450 MeV/c^2. In a kaon decay, the s-quark must turn into a light quark which can only happen in weak interaction processes. Kaons and other "strange particles" can be produced in the strong interaction, as long as equal numbers of s-quarks and $\bar{s}$-antiquarks are produced. At least two "strange particles" must therefore be produced simultaneously. We introduce the quantum number S (the *strangeness*), to indicate the number of $\bar{s}$-antiquarks minus the number of s-quarks. This quantum number is conserved in the strong and electromagnetic interactions, but it can be changed in weak interactions.

The ϕ meson decays mainly into two kaons because it is an $s\bar{s}$ system When it decays a $u\bar{u}$ pair or a $d\bar{d}$ pair are produced in the colour field of the strong interaction. The kaons are produced by combining these with the $s\bar{s}$ quarks, as shown in Fig. 9.9.

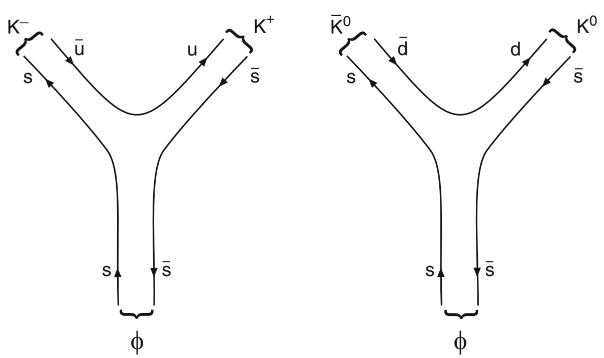

Fig. 9.9 The decay of the ϕ meson into two Kaons with continuous s- and $\bar{s}$-quark lines

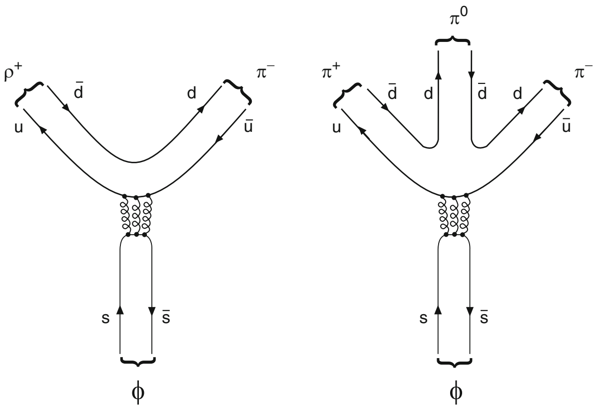

Fig. 9.10 The decay of the ϕ meson into light mesons requires the annihilation of the $s\bar{s}$ pair and a virtual state with three gluons

Because of the small mass difference $m_\phi - 2m_K$, the phase space available to this decay is very small. This accounts for the narrow width of the ϕ resonance.

One could ask: why does the ϕ not decay mainly into light mesons? The decay into pions is very rare (2.5 %), although the phase space available is much larger. Such a decay is only possible if the s and $\bar{s}$ first annihilate, producing two or three quark-antiquark pairs (Fig. 9.10). According to QCD, this proceeds through a virtual intermediate state with at least three gluons. Hence, this process is suppressed with respect to the decay into two kaons which can proceed through the exchange of one gluon. The enhancement of processes with continuous quark lines is called the *Zweig rule*.

The resonances J/ψ and Υ Although the s-quarks were known from hadron spectroscopy, it was a surprise when in 1974 an extremely narrow resonance whose width was only 93 keV was discovered at a centre-of-mass energy of 3,097 MeV. It was named J/ψ.[3] The resonance was attributed to the production of a new heavy quark. There were already theoretical suggestions that such a c-quark ("charmed" quark) exists. The long lifetime of the J/ψ is explained by its $c\bar{c}$ structure. The decay into two mesons each containing a c- (or $\bar{c}$)-quark plus a light quark (in analogy to the decay $\phi \rightarrow \mathrm{K} + \overline{\mathrm{K}}$) would be favoured by the Zweig rule, but is impossible due to energy conservation. This is because the mass of any pair of D mesons ($c\bar{u}$, $c\bar{d}$ etc.), which were observed in later experiments, is larger than the mass of the J/ψ. More resonances were found at centre-of-mass energies some 100 MeV higher. They were called ψ', ψ'' etc., and were interpreted as excited states of the $c\bar{c}$ system. The J/ψ is the lowest $c\bar{c}$ state with the quantum numbers of the photon $J^P = 1^-$.

[3]This particle was discovered nearly simultaneously in two differently conceived experiments (pp collision and e^+e^- annihilation). One collaboration called it J [3], the other ψ [4].

Table 9.1 Charges and masses of the quarks

Quark	Colour	Electr. charge	Mass (MeV/c^2) Bare quark	Const. quark
Down	b, g, r	$-1/3$	4.5–5.5	≈300
Up	b, g, r	$+2/3$	1.8–3.0	≈300
Strange	b, g, r	$-1/3$	90–100	≈450
Charm	b, g, r	$+2/3$	1,250–1,300	
Bottom	b, g, r	$-1/3$	4,150–4,210	
Top	b, g, r	$+2/3$	$172.5 \cdot 10^3$–$174.5 \cdot 10^3$	

A $c\bar{c}$ state, the η_c, exists at a somewhat lower energy, it has quantum numbers 0^- (cf. Sect. 14.2 ff) and cannot be produced directly in e^+e^- annihilation.

A similar behaviour in the cross-section was found at about 10 GeV. Here the series of Upsilon (Υ) resonances was discovered [12, 13]. These $b\bar{b}$ states are due to the even heavier b-quark ("bottom" quark). The lowest-lying state at 9.46 GeV also has an extremely narrow width (only 54 keV) and hence a long lifetime.

The t-quark ("top" quark) was found in 1995 by the two experiments D0 and CDF at the Tevatron (FNAL) in $p\bar{p}$ collisions [1, 2]. From these experiments and more recently also from the LHC experiments a t-quark mass of 173.5 ± 1.0 GeV/c^2 [17] has been derived. The e^+e^--storage ring LEP could only attain centre-of-mass energies of up to 209.2 GeV, which is not enough for $t\bar{t}$ pair production. An actual review of the experimental results and the properties of the t-quark can be found, e.g., in [18].

Table 9.1 shows a compilation of the colour charges, the electric charges and the masses of the quarks; b, g, r denote the colours blue, green and red. Listed are the masses of "bare" quarks (current quarks) which would be measured in the limit $Q^2 \to \infty$ [17] as well as the masses of constituent quarks, i.e., the effective masses of quarks bound in hadrons. The masses of the quarks, in particular those of the current quarks, are strongly model dependent. For heavy quarks, the relative difference between the two masses is small.

The Z^0 resonance At $\sqrt{s} = 91.2$ GeV, an additional resonance is observed with a width of 2,495 MeV. It decays into lepton and quark pairs. The properties of this resonance are such that it is thought to be a real Z^0, the vector boson of the weak interaction. In Sect. 12.2, we will describe what we can learn from this resonance.

9.3 Non-resonant Hadron Production

Up to now we have solely considered resonances in the cross-sections of electron-positron annihilation. Quark-antiquark pairs can, naturally, also be produced among the resonances. Further quark-antiquark pairs are then produced and form hadrons, around the primarily produced quark (or antiquark). This process is called *hadro-*

nisation. Of course only those quarks can be produced whose masses are less than half the centre-of-mass energy available.

In hadron production, a quark-antiquark pair is initially produced. Hence the cross-section is given by the sum of the individual cross-sections of quark-antiquark pair production. The production of the primary quark-antiquark pair by an electromagnetic interaction can be calculated analogously to muon pair production. Unlike muons, quarks do not carry a full elementary charge of $1 \cdot e$; but rather a charge $z_f \cdot e$ which is $-1/3\,e$ or $+2/3\,e$, depending on the quark flavour f. Hence the transition matrix element is proportional to $z_f\, e^2$, and the cross-section is proportional to $z_f^2\,\alpha^2$. Since quarks (antiquarks) carry colour (anticolour), a quark-antiquark pair can be produced in three different colour states. Therefore there is an additional factor of 3 in the cross-section formula. The cross-section is given by:

$$\sigma(\mathrm{e^+e^-} \to \mathrm{q}_f\overline{\mathrm{q}}_f) = 3 \cdot z_f^2 \cdot \sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)\,, \tag{9.9}$$

and the ratio of the cross-sections by

$$R := \frac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)} = \frac{\sum_f \sigma(\mathrm{e^+e^-} \to \mathrm{q}_f\overline{\mathrm{q}}_f)}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)} = 3 \cdot \sum_f z_f^2\,. \tag{9.10}$$

Here only those quark types f which can be produced at the centre-of-mass energy of the reaction contribute to the sum over the quarks.

Figure 9.11 shows schematically the ratio R as a function of the centre-of-mass energy $\sqrt{s}$. Many experiments had to be carried out at different particle accelerators, each covering a specific region of energy, to obtain such a picture. In the non-resonant regions R increases step by step with increasing energy $\sqrt{s}$. This becomes plausible if we consider the contributions of the individual quark flavours. Below the threshold for J/ψ production, only $\mathrm{u\overline{u}}$, $\mathrm{d\overline{d}}$, and $\mathrm{s\overline{s}}$ pairs can be produced. Above it, $\mathrm{c\overline{c}}$ pairs can also be produced; and at even higher energies, $\mathrm{b\overline{b}}$ pairs are produced. The sum in (9.10) thus contains at higher energies ever more terms. As a corollary, the increase in R tells us about the charges of the quarks involved. Depending on the energy region, i.e., depending upon the number of quark flavours involved, one expects:

$$R = 3 \cdot \sum_f z_f^2 = \underbrace{\underbrace{\underbrace{3 \cdot \Big\{ \underset{\mathrm{u}}{(\tfrac{2}{3})^2} + \underset{\mathrm{d}}{(-\tfrac{1}{3})^2} + \underset{\mathrm{s}}{(-\tfrac{1}{3})^2}}_{3 \cdot 6/9} + \underset{\mathrm{c}}{(\tfrac{2}{3})^2}}_{3 \cdot 10/9} + \underset{\mathrm{b}}{(-\tfrac{1}{3})^2} \Big\}}_{3 \cdot 11/9}\,. \tag{9.11}$$

These predictions are in good agreement with the experimental results. The measurement of R represents an additional way to determine the quark charges

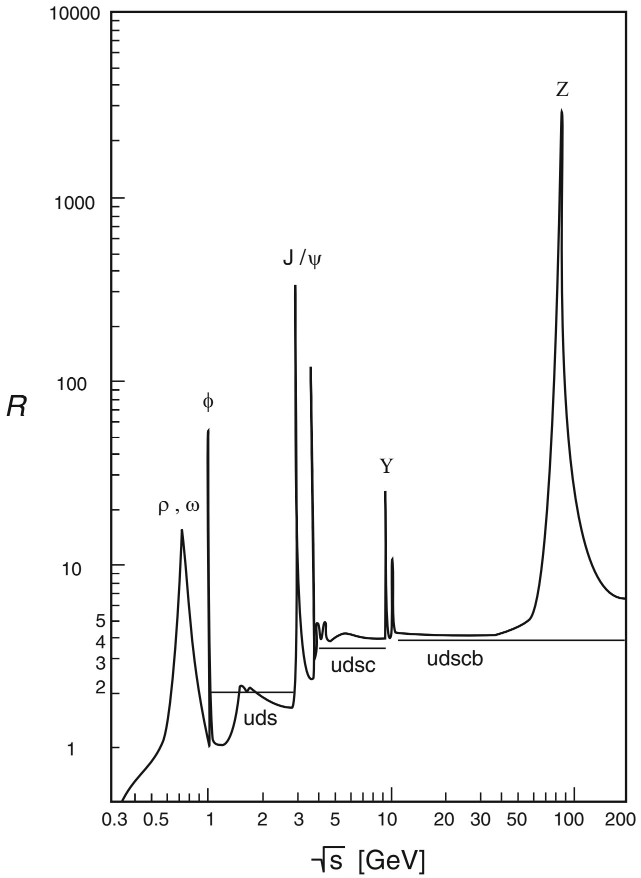

Fig. 9.11 Cross-section of the reaction $e^+e^- \to$ *hadrons*, normalised to $e^+e^- \to \mu^+\mu^-$, as a function of the centre of mass energy $\sqrt{s}$ (sketch). The *horizontal lines* correspond to $R = 6/3$, $R = 10/3$ and $R = 11/3$, the values we expect from (9.10), depending upon the number of quarks involved. The value $R = 15/3$ which is expected if the t-quark participates lies outside the plotted energy range (Courtesy of G. Myatt, Oxford)

and is simultaneously an impressive confirmation of the existence of exactly three colours.

9.4 Gluon Emission

Using e^+e^- scattering it has proven possible to experimentally establish the existence of gluons and to measure the value of α_s, the strong coupling constant.

The first indications for the existence of gluons were provided by deep-inelastic scattering of leptons off the "average nucleon". The integral of the structure function

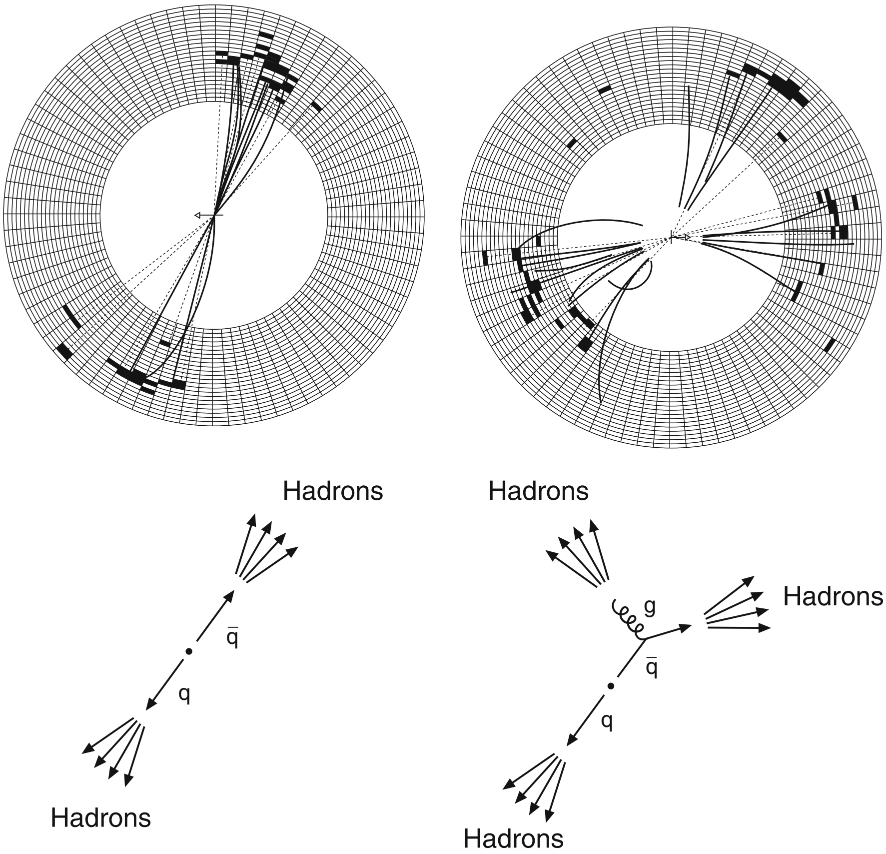

Fig. 9.12 Typical 2-jet and 3-jet events, measured with the JADE detector at the PETRA e^+e^- storage ring. The figures show a projection perpendicular to the beam axis, which is at the centre of the cylindrical detector. The tracks of charged particles (*solid lines*) and of neutral particles (*dotted lines*) are shown. They were reconstructed from the signals in the central wire chamber and in the lead glass calorimeter surrounding the wire chamber. In this projection, the concentration of the produced hadrons in two or three particle jets is clearly visible (Courtesy of DESY)

F_2 was only half the expected value (cf. Sect. 7.5). The missing half of the nucleon momentum was apparently carried by electrically neutral particles which were also not involved in weak interactions. They were identified with the gluons. The coupling constant α_s was determined from the scaling violation of the structure function F_2 (Sect. 8.3).

A direct measurement of these quantities is possible by analysing "jets". At high energies, hadrons are typically produced in two jets, emitted in opposite directions. These jets are produced in the hadronisation of the primary quarks and antiquarks (left side of Fig. 9.12).

In addition to simple $q\overline{q}$ production, higher-order processes can occur. For example, a high-energy ("hard") gluon can be emitted, which can then manifest itself as a third jet of hadrons. This corresponds to the emission of a photon in electromagnetic bremsstrahlung. Emission of a hard photon, however, is a relatively rare process, as the electromagnetic coupling constant α is rather small. By contrast, the probability of gluon bremsstrahlung is given by the coupling constant α_s. Such 3-jet events are indeed detected. Figure 9.12 (right) shows a particularly nice example. The coupling constant α_s may be deduced directly from a comparison of the 3- and 2-jet event rates. Measurements at different centre-of-mass energies also demonstrate that α_s decreases with increasing $Q^2 = s/c^2$ as (8.1) predicts. The experimental determination of the Q^2 dependence of the strong coupling constant α_s has been reviewed in detail, e.g., in Ref. [9].

Problems

1. **Electron-positron collisions**

   (a) Electrons and positrons each with a beam energy E of 4 GeV collide head on in a storage ring. What production rate of $\mu^+\mu^-$-pairs would you expect at a luminosity of $10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$? What production rate for events with hadronic final states would you expect?

   (b) It is planned to construct two linear accelerators aimed at each other (a linear collider) from whose ends electrons and positrons will collide head on with a centre-of-mass energy of 500 GeV. How big must the luminosity be if one wants to measure the hadronic cross-section within two hours with a 10 % statistical error?

2. **Υ resonance**

   Detailed measurements of the $\Upsilon(1S)$ resonance, whose mass is roughly 9,460 MeV, are performed at the CESR electron-positron storage ring.

   (a) Calculate the uncertainty in the beam energy E and the centre-of-mass energy W if the radius of curvature of the storage ring is $R = 100$ m. We have:

$$\delta E = \left(\frac{55}{32\sqrt{3}} \frac{\hbar c\, m_e c^2}{2R} \gamma^4 \right)^{1/2}$$

   What does this uncertainty in the energy tell us about the experimental measurement of the Υ (Use the information given in Part b)?

   (b) Integrate the Breit-Wigner formula across the region of energy where the $\Upsilon(1S)$ resonance is found. The experimentally observed value of this integral for hadronic final states is $\int \sigma(e^+e^- \to \Upsilon \to \text{hadrons})\,dW \approx 300\,\text{nb MeV}$.

The decay probabilities for $\Upsilon \to \ell^+\ell^-$ ($\ell = \mathrm{e},\ \mu,\ \tau$) are each around 2.5 %. How large is the total natural decay width of the Υ? What cross-section would one expect at the resonance peak if there was no uncertainty in the beam energy (and the resonance was not broadened by radiative corrections)?

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