Chapter 14
Quarkonia

Analogy is perhaps the physicist's most powerful conceptual tool for understanding new phenomena or opening new areas of investigation. Early in this century, for example, Ernest Rutherford and Niels Bohr conceived the atom as a miniature solar system in which electrons circle the nucleus as planets circle the Sun.

V. L. Telegdi [9]

In the second part of this book we are going to consider hadronic bound-states. We will at first discuss the properties of mesons and baryons and subsequently details of the structure of atomic nuclei. The simplest example are heavy quark-antiquark ($c\bar{c}$ and $b\bar{b}$) pairs, which are known as *quarkonia*. Due to the large quark masses they may be approximately treated in a non-relativistic manner. The *hydrogen atom* and *positronium* will serve as electromagnetic analogues.

14.1 The Hydrogen Atom and Positronium Analogues

The simplest atomic bound-state is the hydrogen atom, which is composed of a proton and an electron. To a first approximation the bound-states and energy levels may be calculated from the non-relativistic Schrödinger equation. The static Coulomb potential $V_C \propto 1/r$ is then incorporated into the Hamiltonian

$$\left(-\frac{\hbar^2}{2m}\triangle - \frac{\alpha\hbar c}{r}\right)\psi(\boldsymbol{r}) = E\,\psi(\boldsymbol{r})\,. \tag{14.1}$$

The eigenstates are characterised by the number of nodes N in the radial wave functions and the orbital angular momentum ℓ. For the particular case of the Coulomb potential, states with identical $n = N + \ell + 1$ are degenerate and n is therefore called the *principal quantum number*. The allowed energy levels E_n are found to be

$$E_n = -\frac{\alpha^2 mc^2}{2n^2}\,, \tag{14.2}$$

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where α is the electromagnetic coupling constant and m is the reduced mass of the system:

$$m = \frac{M_p m_e}{M_p + m_e} \approx m_e = 0.511\,\mathrm{MeV}/c^2\,. \tag{14.3}$$

The binding energy of the hydrogen ground state ($n = 1$) is $E_1 = -13.6\,\mathrm{eV}$. The Bohr radius r_b is given by

$$r_b = \frac{\hbar \cdot c}{\alpha \cdot mc^2} \approx \frac{197\,\mathrm{MeV \cdot fm}}{137^{-1} \cdot 0.511\,\mathrm{MeV}} = 0.53 \cdot 10^5\,\mathrm{fm}\,. \tag{14.4}$$

The spin-orbit interaction ("fine structure") and the spin-spin-interaction ("hyperfine structure") split the degeneracy of the principal energy levels as is shown in Fig. 14.1. These corrections to the general $1/n^2$ behaviour of the energy levels are, however, very small. The fine structure correction is of order α^2 while that of the hyperfine structure is of order $\alpha^2 \cdot \mu_p/\mu_e$. The ratio of the hyperfine splitting of the $1s_{1/2}$ level to the gap between the $n = 1$ and $n = 2$ principal energy levels is therefore merely $E_{HFS}/E_n \approx 5 \cdot 10^{-7}$. Here we employ the notation $n\ell_j$ for states when fine structure effects are taken into account. The orbital angular momenta quantum numbers $\ell = 0, 1, 2, 3$ are then denoted by the letters s, p, d, f. The quantum number j is the total angular momentum of the electron, $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$. A fourth quantum number f is used to describe the hyperfine effects (see Fig. 14.1 left). This describes the total angular momentum of the atom, $\boldsymbol{f} = \boldsymbol{j} + \boldsymbol{i}$, with the proton's spin $\boldsymbol{i}$ included.

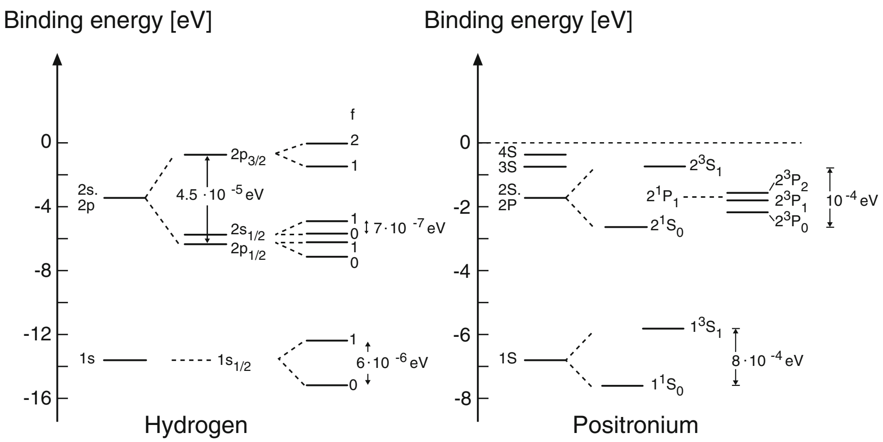

Fig. 14.1 The energy levels of the hydrogen atom and of positronium. The ground states ($n = 1$) and the first excited states ($n = 2$) are shown together with their fine and hyperfine splitting. The shown splitting is not to scale

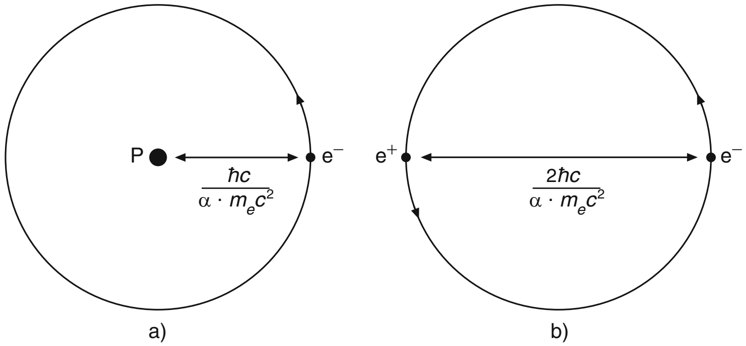

Fig. 14.2 The first Bohr orbits of the hydrogen atom (**a**) and positronium (**b**) (From [6]). The Bohr radius describes the average separation of the two bound particles

The energy states of positronium, the bound e^+e^- system, can be found in an analogous way to the above. The main differences are that the reduced mass ($m = m_e/2$) is only half the value of the hydrogen case and the spin-spin coupling is much larger than before, since the electron magnetic moment is roughly 650 times larger than that of the proton. The smaller reduced mass means that the binding energies of the bound states are only half the size of those of the hydrogen atom while the Bohr radius is twice its previous value (Fig. 14.2). The stronger spin-spin coupling now means that the positronium spectrum does not display the clear hierarchy of fine and hyperfine structure effects that we know from the hydrogen atom. The spin-orbit and spin-spin forces are of a similar size (Fig. 14.1).

Thus for positronium the total spin S and the total angular momentum J as well as the principal quantum number n and the orbital angular momentum L are the useful quantum numbers. S can take on the values 0 (singlet) and 1 (triplet), and J obeys the triangle inequality, $|L - S| \leq J \leq L + S$. The notation $n^{2S+1}L_J$ is commonly employed, where the orbital angular momentum L is represented by the capital letters (S, P, D, F). Thus $2\,{}^3\mathrm{P}_1$ signifies a positronium state with $n = 2$ and $S = L = J = 1$.

Since electrons and positrons annihilate, positronium has a finite lifetime. It primarily decays into two or three photons, depending upon whether the total spin is 0 or 1. The decay width for the two-photon decay of the $1^1\mathrm{S}_0$ state is found to be [6]

$$\Gamma(1^1\mathrm{S}_0 \to 2\gamma) = \frac{4\pi\alpha^2\hbar^3}{m_e^2 c}|\psi(0)|^2 \,. \tag{14.5}$$

Note that $|\psi(0)|^2$ is the square of the wave function at the origin, i.e. the probability that e^+ and e^- meet at a point. Equation (14.5) yields a lifetime of $\approx 10^{-10}$ s.

The potential and the coupling constant of the electromagnetic interaction are very well known, and electromagnetic transitions in positronium as well as its lifetime can be calculated to high precision and excellent agreement with experiment is found. Quarkonia, i.e., systems built up of strongly interacting heavy quark-antiquark pairs, can be investigated in an analogous manner. The effective potential and the coupling strength of the strong interaction can thus be determined from the experimental spectrum and transition strengths between the various states.

14.2 Charmonium

Bound states of c- and $\bar{c}$-quarks are, in analogy to positronium, called *charmonium*. For historical reasons a somewhat different nomenclature is employed for charmonium states than is used for positronium. The first number is $n_{q\bar{q}} = N + 1$, where N is the number of nodes in the radial wave function, while for positronium the atomic convention, according to which the principal quantum number is defined as $n_{\text{atom}} = N + \ell + 1$, is used.

$c\bar{c}$ pairs are most easily produced in the decay of virtual photons generated in e^+e^- collisions (Fig. 14.3) with a centre-of-mass energy of around 3–4.5 GeV

$$e^+ + e^- \to \gamma \to c\bar{c}\,.$$

Various resonances may be detected by varying the beam energy and looking for peaks in the cross-section. These are then ascribed to the various charmonium states (Fig. 14.4). Because of the intermediate virtual photon, only $c\bar{c}$ states with the quantum numbers of a photon, ($J^P = 1^-$), can be created in this way. The lowest state with such quantum numbers is the 1^3S_1, which is called the J/ψ (see p. 132)

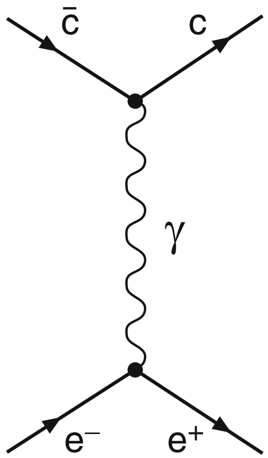

Fig. 14.3 Production of $c\bar{c}$ pairs in e^+e^- collisions

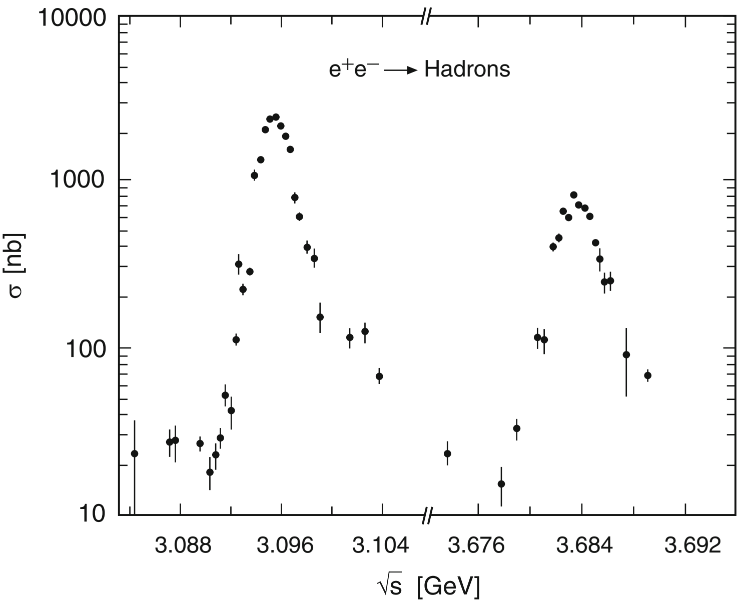

Fig. 14.4 The cross-section of the reaction $e^+e^- \rightarrow$ *hadrons*, plotted against the centre-of-mass energy in two different intervals each of 25 MeV. The two peaks which are both 100 times larger than the continuum represent the lowest charmonium states with $J^P = 1^-$ (the J/ψ (1^3S_1) and the ψ (2^3S_1)). That the experimental width of these resonances is a few MeV is a consequence of the detector's resolution: widths of 87 and 286 keV respectively may be extracted from the lifetimes of the resonances. The results shown are early data from the e^+e^- ring SPEAR at Stanford [1]

and has a mass of 3.097 GeV/c^2. Higher resonances with masses up to 4.4 GeV/c^2 have been detected.

Charmonium states only have a finite lifetime. They predominantly decay via the strong interaction into hadrons. Excited states can, however, by the emission of a photon, decay into lower energy states, just as in atomic physics or for positronium. The emitted photons may be measured with a detector that covers the entire solid angle around the e^+e^- interaction zone (4π detectors). *Crystal balls,* which are composed of spherically arranged scintillators (NaI crystals) are particularly well suited to this task (Fig. 14.5).

If one generates, say, the excited charmonium ψ (2^3S_1) state one then may measure the photon spectrum shown in Fig. 14.6, in which various sharp lines are clearly visible. The photon energy is between 100 and 700 MeV. The stronger lines are electric dipole transitions which obey the selection rules, $\Delta L = 1$ and $\Delta S = 0$. Intermediate states with total angular momentum 0, 1 or 2 and positive parity must therefore be created in such decays. The parity of the spatial wave function is just $(-1)^L$, where L is the orbital angular momentum. Furthermore from the Dirac theory fermions and antifermions have opposite intrinsic parity. Thus the parity of $q\bar{q}$ states is generally $(-1)^{L+1}$. Armed with this information we can reconstruct the diagram

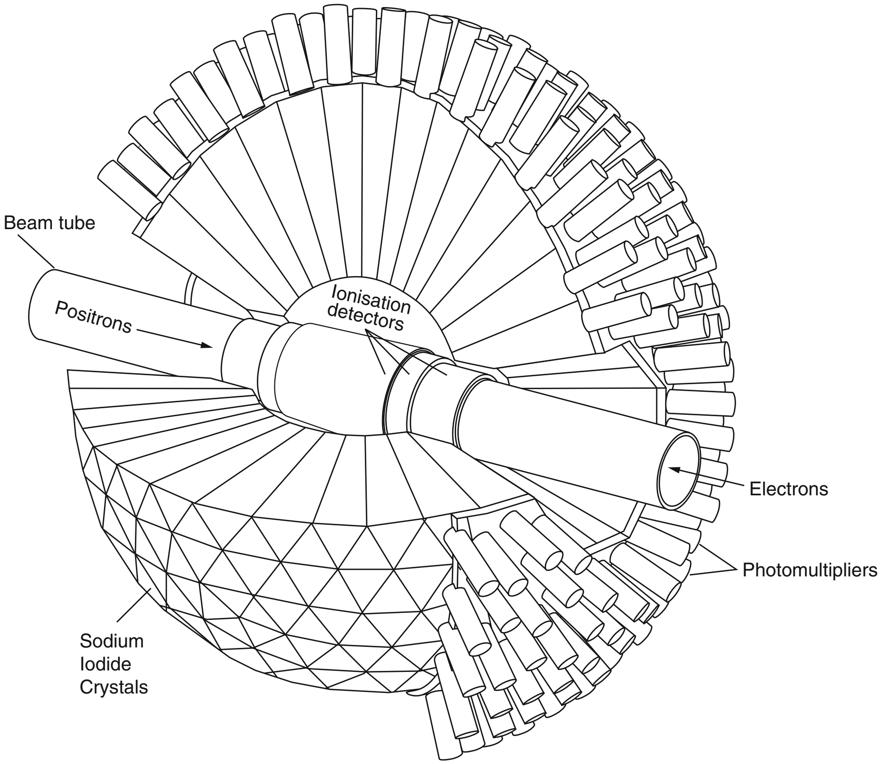

Fig. 14.5 A (crystal ball) detector built out of spherically arranged NaI crystals. High energy photons from electromagnetic $c\bar{c}$ transitions are absorbed by the crystals. This creates a shower of electron-positron pairs which generate many low energy, visible photons. These are then detected by photomultipliers attached to the rear of the crystals. The current measured from the photomultipliers is proportional to the energy of the initial photon (From [3])

in Fig. 14.6. We see that after the $\psi\,(2^3S_1)$ state is generated it primarily decays into the 1^3P_J charmonium triplet system which is known as χ_c. These χ_c states then decay into J/ψ's. The spin-0 charmonium states (n^1S_0), which are called η_c, and cannot be produced in e^+e^- collisions, are only produced in magnetic dipole transitions from J/ψ or $\psi\,(2^3S_1)$. These obey the selection rules $\Delta L = 0$ and $\Delta S = 1$ and thus connect states with the same parity. They correspond to a spin flip of one of the c-quarks. Magnetic dipole transitions are weaker than electric dipole transitions. They are, however, observed in charmonium, since the spin-spin interaction for $c\bar{c}$ states is significantly stronger than in atomic systems. This is due to the much smaller separation between the partners compared to atomic systems.

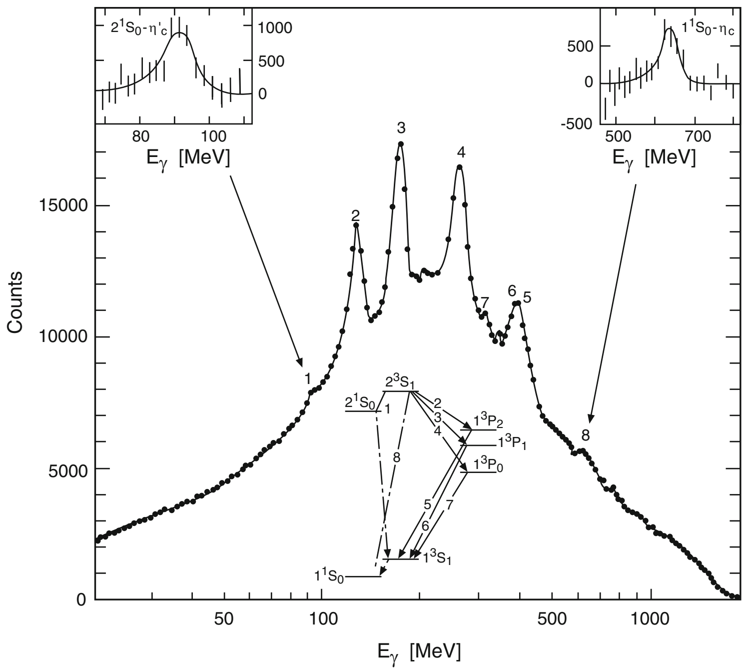

Fig. 14.6 The photon spectrum in the decay of ψ (2^3S_1), as measured in a crystal ball, and a sketch of the so extracted charmonium energy levels. The strong peaks in the photon spectrum represent the so numbered transitions in the sketch. The *continuous lines* in the sketch represent parity changing electric dipole transitions and the *dashed lines* denote magnetic dipole transitions which do not change parity [3]

14.3 Quark-Antiquark Potential

If we compare the spectra of charmonium and positronium, we find that the states with $n = 1$ and $n = 2$ are very similarly arranged once an overall increase in the positronium scale of about 10^8 is taken into account (Fig. 14.7). The higher charmonium states do not, on the other hand, display the $1/n^2$ behaviour we see in positronium.

What can we learn from this about the potential and the coupling constant of the strong interaction? Since the potential determines the relative positions of the energy levels, it is clear that the potential of the strong interaction must, similarly to the electromagnetic one, be of a Coulomb type (at least at very short distances, i.e., for $n = 1, 2$). This observation is supported by quantum chromodynamics which describes the force between the quarks via gluon exchange and predicts a r^{-1} potential at short distances. The absence, in comparison to positronium, of any degeneracy between the 2^3S and 1^3P states suggests that the potential is not of a pure Coulomb form even at fairly small quark-antiquark separations. Since free quarks have not been experimentally observed, it is plausible to postulate a potential which

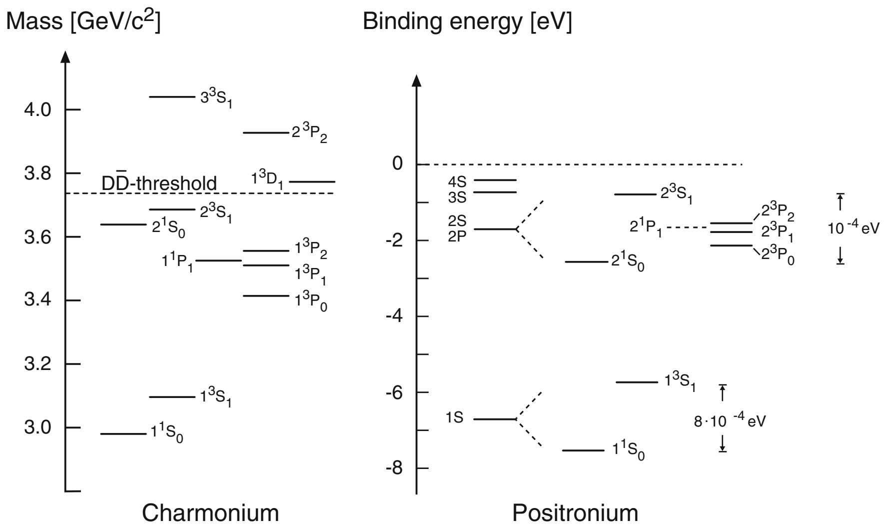

Fig. 14.7 Comparison of the energy levels of positronium and charmonium. The energy scales were chosen such that the 1S and 2S states of the two systems coincide horizontally. As a result of the differences in nomenclature for the first quantum number, the 2P states in positronium actually correspond to the 1P levels in charmonium. The splitting of the positronium states has been magnified. Dashed states have been calculated but not yet experimentally detected. Note that the $n = 1$ and $n = 2$ level patterns are very similar, while the 2S-3S separations are distinctly different. The *dashed*, *horizontal line* marks the threshold where positronium breaks up and charmonium decays into two D mesons (see Sect. 14.6)

is of a Coulomb type at short distances and grows linearly at greater separations, thus leading to the confinement of quarks in hadrons.

An ansatz for the potential is therefore

$$V = -\frac{4}{3}\frac{\alpha_s(r)\hbar c}{r} + k \cdot r \,, \tag{14.6}$$

which displays the asymptotic behaviour $V(r \to 0) \propto 1/r$ and $V(r \to \infty) \to \infty$. The factor of $4/3$ is a theoretical consequence of quarks coming in three different colours. The strong coupling constant α_s is actually not a constant at all, but depends upon the separation r of the quarks (8.1), becoming smaller as the separation increases. This is a direct consequence of QCD and results in the so-called *asymptotic freedom* property of the strong force. This behaviour allows us to view quarks as quasi-free particles at short distances as we have already discussed for deep-inelastic scattering.

While a Coulomb potential corresponds to a dipole field, where the field lines are spread out in space (Fig. 14.8a), the kr term leads to a so-called flux tube. The lines of force between the quarks are "stretched" (Fig. 14.8b) and the field energy increases linearly with the separation of the quarks. The constant k in the second

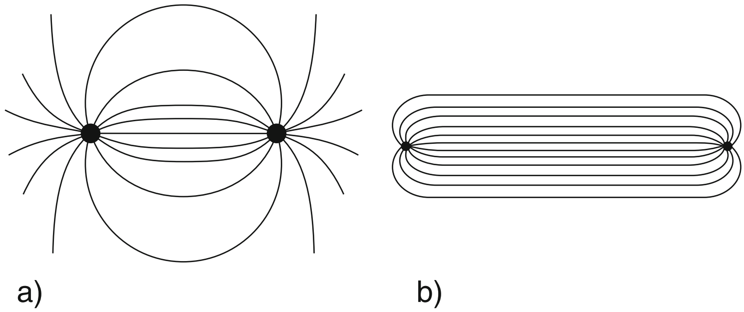

Fig. 14.8 Field lines for (**a**) a dipole field ($V \propto 1/r$) between two electric charges, (**b**) a potential $V \propto r$ between two widely separated quarks

term of the potential determines the field energy per unit length and is called the "string tension".

The charmonium energy levels depend not only upon the potential but also upon the kinetic terms in the Hamiltonian, which contain the a priori unknown c-quark mass m_c. The three unknown quantities α_s, k and m_c may be roughly determined by fitting the principal energy levels of the $c\bar{c}$ states from the non-relativistic Schrödinger equation with the potential (14.6). Typical results are: $\alpha_s \approx 0.15$–0.25, $k \approx 1\,\text{GeV/fm}$ and $m_c \approx 1.5\,\text{GeV}/c^2$. Note that m_c is the constituent mass of the c-quark. The strong coupling constant in the charmonium system is about 20–30 times larger than the electromagnetic coupling, $\alpha = 1/137$. Figure 14.9 shows a potential, based upon (14.6), where the calculated radii of the charmonium states are given. The J/ψ (1^3S_1) has, for example, a radius[1] of approximately $r \approx 0.4\,\text{fm}$, which is five orders of magnitude smaller than that of positronium.

To fully describe the energy levels of Fig. 14.7 one must incorporate further terms into the potential. Similarly to the case of atomic physics, one can describe the splitting of the P states very well through a spin-orbit interaction. The splitting of the S states of charmonium and the related spin-spin interaction will be treated in the next section.

The Coulomb potential describes forces that decrease with distance. The integral of this force is the ionisation energy. The strong interaction potential, (14.6), on the other hand, describes a force between quarks which remains constant at large separations. To remove a coloured particle such as a quark from a hadron would require an infinitely high energy. Thus, since the isolation of coloured objects is impossible, we find only colourless objects in nature. This does not, however, mean that quarks cannot be detached from one another.

Quarks are not liberated in such circumstances, rather fresh hadrons are produced if the energy in the flux tube crosses a specific threshold. The now detached quarks become constituents of these new hadrons. If, for example, a quark is knocked out

[1] By this we mean the average separation between the quark and the antiquark (see Fig. 14.2).

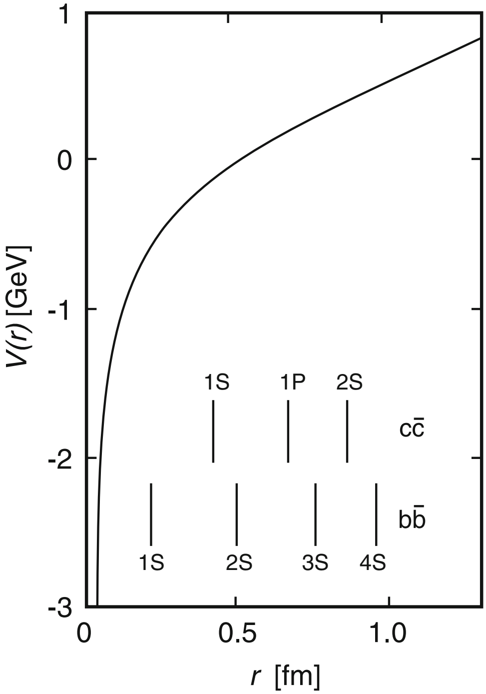

Fig. 14.9 Strong interaction potential versus the separation r of two quarks. This potential is roughly described by (14.6). The *vertical lines* mark the radii of the $c\bar{c}$ and $b\bar{b}$ states as calculated from such a potential (From [2])

of a hadron in deep-inelastic scattering, the flux tube between this quark and the remainder of the original hadron breaks when the tube reaches a length of about 1–2 fm. The field energy is converted into a quark and an antiquark. These then separately attach themselves to the two ends of the flux tube and thus produce two colour neutral hadrons. This is the previously mentioned *hadronisation* process.

14.4 The Chromomagnetic Interaction

The similarity between the potential of the strong force and that of the electromagnetic interaction is due to the short distance r^{-1} Coulombic term. This part corresponds to 1-gluon (1-photon) exchange. Charmonium displays a strong splitting of the S states, as does positronium, and this is due to a spin-spin interaction. This force is only large at small distances and thus 1-gluon exchange should essentially account for it in quarkonium. The spin-spin interaction splitting, and hence the force itself, is, however, roughly 1,000 times larger for charmonium than in positronium.

The spin-spin interaction for positronium takes the form

$$V_{\mathrm{ss}}(\mathrm{e}^+\mathrm{e}^-) = \frac{-2\mu_0}{3}\,\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2\,\delta(\boldsymbol{x})\,, \tag{14.7}$$

where μ_0 is the vacuum permeability. This equation describes the point interaction of the magnetic moments $\boldsymbol{\mu}_{1,2}$ of e^+ and e^-. The magnetic moment of the electron (positron) is just

$$\boldsymbol{\mu}_i = \frac{z_i e\hbar}{2m_i}\boldsymbol{\sigma}_i\,, \qquad \text{where} \quad z_i = Q_i/e = \pm 1\,, \tag{14.8}$$

and the components of the vector $\boldsymbol{\sigma}$ are the Pauli matrices; $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \mathbb{1}$. The potential $V_{\rm ss}(e^+e^-)$ may then be expressed as

$$V_{\rm ss}(e^+e^-) = \frac{-\hbar^2\,\mu_0}{6}\frac{z_1 z_2 e^2}{m_1 m_2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,\delta(\boldsymbol{x}) = \frac{2\pi\hbar^3}{3\,c}\,\alpha\,\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{m_{\rm e}^2}\,\delta(\boldsymbol{x})\,. \tag{14.9}$$

The quark colour charges lead to a spin-spin interaction called the *chromomagnetic* or *colour magnetic interaction*. To generalise the electromagnetic spin-spin force to describe the chromomagnetic spin-spin interaction we have to replace the electromagnetic coupling constant α by $\alpha_{\rm s}$ and alter the factor to take the three colour charges into account. We thus obtain for the quark-antiquark spin-spin interaction

$$V_{\rm ss}(q\overline{q}) = \frac{8\pi\hbar^3}{9\,c}\alpha_{\rm s}\frac{\boldsymbol{\sigma}_{\rm q}\cdot\boldsymbol{\sigma}_{\overline{\rm q}}}{m_{\rm q}m_{\overline{\rm q}}}\,\delta(\boldsymbol{x})\,. \tag{14.10}$$

The chromomagnetic energy thus depends upon the relative spin orientations of the quark and the antiquark. The expectation value of $\boldsymbol{\sigma}_{\rm q}\cdot\boldsymbol{\sigma}_{\overline{\rm q}}$ is found to be

$$\begin{aligned}\boldsymbol{\sigma}_{\rm q}\cdot\boldsymbol{\sigma}_{\overline{\rm q}} = 4\boldsymbol{s}_{\rm q}\cdot\boldsymbol{s}_{\overline{\rm q}}/\hbar^2 &= 2\cdot\left[S(S+1) - s_{\rm q}(s_{\rm q}+1) - s_{\overline{\rm q}}(s_{\overline{\rm q}}+1)\right]\\ &= \begin{cases} -3 \text{ for } S=0\,,\\ +1 \text{ for } S=1\,,\end{cases}\end{aligned} \tag{14.11}$$

where $\boldsymbol{S}$ is the total spin of the charmonium state and we have used the identity $\boldsymbol{S}^2 = (\boldsymbol{s}_{\rm q} + \boldsymbol{s}_{\overline{\rm q}})^2$. One thus obtains an energy splitting from this chromomagnetic interaction of the form

$$\Delta E_{\rm ss} = \langle\psi|V_{\rm ss}|\psi\rangle = 4\cdot\frac{8\pi\hbar^3}{9\,c}\frac{\alpha_{\rm s}}{m_{\rm q}m_{\overline{\rm q}}}\,|\psi(0)|^2\,. \tag{14.12}$$

This splitting is only important for S states, since only then the wave function at the origin $\psi(0)$ is non-vanishing.

The observed charmonium transition from the state 1^3S_1 to 1^1S_0 (i.e., $J/\psi \to \eta_c$) is a magnetic transition, which corresponds to one of the quarks flipping its spin. The measured photon energy, and hence the gap between the states, is approximately

120 MeV. The colour magnetic force (14.12) should account for this splitting. Although an exact calculation of the wave function is not possible, we can use the values of α_s and m_c from the last section to see that our ansatz for the chromomagnetic interaction is consistent with the observed splitting of the states. We will see in Chap. 15 that the spin-spin force also plays a role for light mesons and indeed describes their mass spectrum very well.

The c-quark's mass The c-quark mass which we obtained from our study of the charmonium spectrum is its constituent-quark mass, i.e., the effective quark mass in the bound state. This constituent mass has two parts: the intrinsic (or "bare") quark mass and a "dynamical" part which comes from the cloud of sea quarks and gluons that surrounds the quark. The fact that charmed hadrons are 4–10 times heavier than light hadrons implies that the constituent mass of the c-quark is predominantly intrinsic since the dynamical masses themselves should be more or less similar for all hadrons. We should not forget that even if the dynamical masses are small compared to the heavy quark constituent mass, the potential we have used is a phenomenological one which merely describes the interaction between *constituent* quarks.

14.5 Bottonium and Toponium

A further group of narrow resonances are found in e^+e^- scattering at centre-of-mass energies of around 10 GeV. These are understood as $b\bar{b}$ bound states and are called *bottonium*. The lowest $b\bar{b}$ state which can be obtained from e^+e^- annihilation is called the Υ and has a mass of 9.46 GeV/c^2. Higher $b\bar{b}$ excitations have been found with masses up to 11 GeV/c^2.

Various electromagnetic transitions between the various bottonium states are also observed. As well as a 1^3P_J state, a 2^3P_J state has been observed. The spectrum of these states closely parallels that of charmonium (Fig. 14.10). This indicates that the quark-antiquark potential is independent of quark flavour. The b-quark mass is about 3 times as large as that of the c-quark. The radius of the quarkonium ground state is from (14.4) inversely proportional both to the quark mass and to the strong coupling constant α_s. The 1S $b\bar{b}$ state thus has a radius of roughly 0.2 fm (cf. Fig. 14.9), i.e., about half that of the equivalent $c\bar{c}$ state. Furthermore the non-relativistic treatment of bottonium is better justified than was the case for charmonium. The approximately equal mass difference between the 1S and 2S states in both systems is, however, astounding. A purely Coulombic potential would cause the levels to be proportional to the reduced mass of the system, (14.2). It is thus clear that the long distance part of the potential kr cancels the mass dependence of the energy levels at the c- and b-quark mass scales.

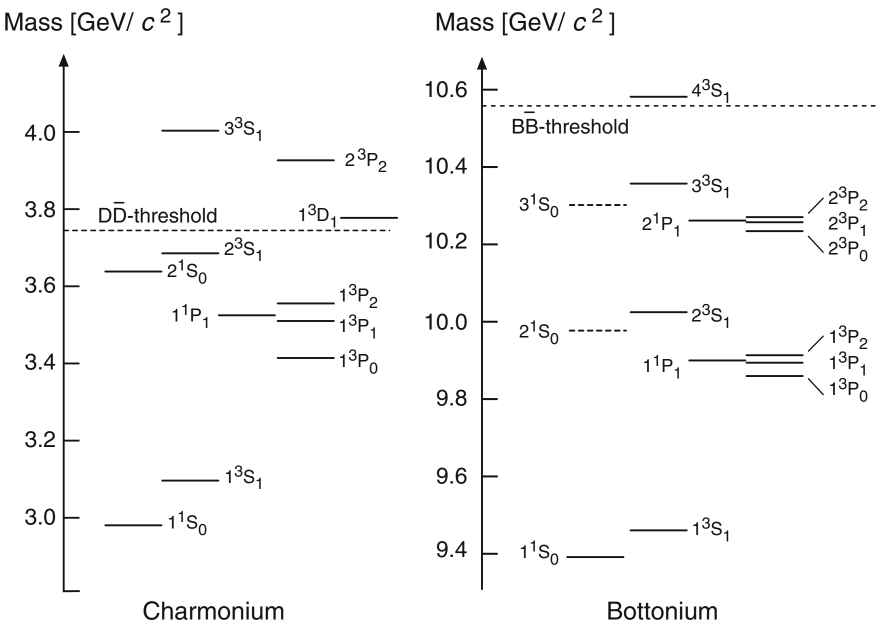

Fig. 14.10 Energy levels of charmonium and bottonium. Dashed levels are theoretically predicted, but not yet experimentally observed. The spectra display a very similar structure. The *dashed line* shows the threshold beyond which charmonium (bottonium) decays into hadrons containing the initial quarks, i.e., D (B) mesons. Below the threshold electromagnetic transitions from ^{3}S states into ^{3}P and ^{1}S states are observed. For bottonium the first and second excitations ($n = 2, 3$) lie below this threshold, for charmonium only the first does

The t-quark has, due to its large mass, only a fleeting lifetime. Thus no pronounced $\mathrm{t\bar{t}}$ states *(toponium)* are expected.

14.6 The Decay Channels of Heavy Quarkonia

Up to now we have essentially dealt with the electromagnetic transitions between various levels of quarkonia. But actually it is astonishing that electromagnetic decays occur at all at an observable rate. One would naively expect a strongly interacting object to decay “strongly”. The decays of heavy quarkonia have been in fact investigated very thoroughly [4] so as to obtain the most accurate possible picture of the quark-antiquark interaction. There are in principle four different ways in which quarkonia can change its state or decay. They are:

(a) A change of excitation level via photon emission (electromagnetic), e.g.,

$$\chi_{c1}\,(1^3\mathrm{P}_1) \;\rightarrow\; \mathrm{J}/\psi\,(1^3\mathrm{S}_1) + \gamma\,.$$

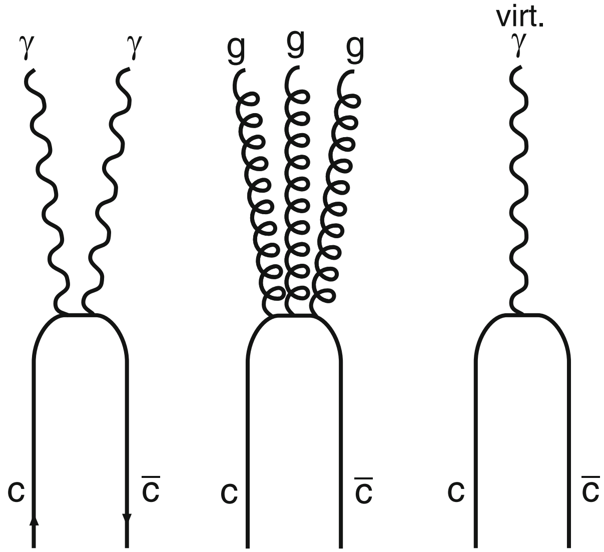

Fig. 14.11 Various channels of $c\bar{c}$ annihilation

(b) Quark-antiquark annihilation (Fig. 14.11) into real or virtual photons or gluons (electromagnetic or strong), e.g.,

$$\begin{aligned}
\eta_c\ (1^1S_0) &\to 2\gamma \\
J/\psi\ (1^3S_1) &\to ggg \ \to \ \text{hadrons} \\
J/\psi\ (1^3S_1) &\to \text{virt. } \gamma \ \to \ \text{hadrons} \\
J/\psi\ (1^3S_1) &\to \text{virt. } \gamma \ \to \ \text{leptons} .
\end{aligned}$$

The J/ψ decays about 30 % of the time electromagnetically into hadrons or charged leptons and about 70 % of the time strongly. The electromagnetic route can, despite the smallness of α, compete with the strong one, since in the strong case three gluons must be exchanged to conserve colour and parity. A factor of α_s^3 thus lowers this decay probability (compared to α^2 in the electromagnetic case). States such as η_c, which have $J = 0$, can decay into two gluons or two real photons. The decay of the J/ψ ($J = 1$) is mediated by three gluons or a single virtual photon.

(c) Creation of one or more light $q\bar{q}$ pairs from the vacuum to form light mesons via the strong interaction (Fig. 14.12 (left)).

(d) Weak decay of one or both heavy quarks (Fig. 14.12 (right)).

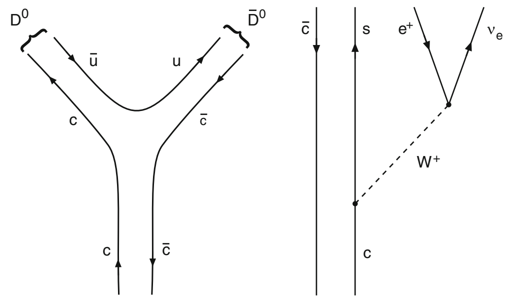

Fig. 14.12 Strong decay $\psi(3778) \rightarrow \mathrm{D}^{\circ} + \bar{\mathrm{D}}^{\circ}$ (*left*); weak decay $\mathrm{J}/\psi \rightarrow \mathrm{D}_s^- + \mathrm{e}^+ + \nu_e$ (*right*)

In practice the weak decay (d) is unimportant since the strong and electromagnetic decays proceed much more quickly. The strong decay (c) is, in principle, the most likely, but this can only take place above a certain threshold since the light $\mathrm{q\bar{q}}$ pairs need to be created from the quarkonia binding energy. Hence only options (a) and (b) are available to quarkonia below this threshold.

Electromagnetic processes like deexcitation via photon emission are relatively slow. Furthermore, although hadronisation via the annihilation (b) into gluons is a strong process such decays are, according to the *Zweig rule* (cf. Sect. 9.2) suppressed relative to those decays (c) where the initial quarks still exist in the final state. For these reasons the width of those quarkonium levels below the mesonic threshold is very small (e.g., $\Gamma = 93\,\mathrm{keV}$ for the J/ψ).

The first charmonium state beyond this threshold is the $\psi\ (1^3\mathrm{D}_1)$ which has a mass of 3,778 MeV/c^2. It has, compared to the J/ψ, rather a large width, $\Gamma \approx 27\,\mathrm{MeV}$. For the more strongly bound $\mathrm{b\bar{b}}$ system the decay channel into mesons with b-quarks is first open to the third excitation, the $\Upsilon\ (4^3\mathrm{S}_1)$ (10,579 MeV/c^2) (cf. Fig. 14.10).

The lightest quarks are the u- and d-quarks and their pair production opens the mesonic decay channels (cf. Fig. 14.12 (left)). Charmonium, say, decays into

$$\mathrm{c\bar{c}} \rightarrow \mathrm{c\bar{u}} + \bar{c}u\,,$$
$$\mathrm{c\bar{c}} \rightarrow \mathrm{c\bar{d}} + \mathrm{\bar{c}d}\,,$$

where $\mathrm{c\bar{u}}$ is called the D^0 meson, $\mathrm{\bar{c}u}$ the $\overline{\mathrm{D}}^0$, $\mathrm{c\bar{d}}$ the D^+ and $\mathrm{\bar{c}d}$ the D^-. The masses of these mesons are 1,864.9 MeV/c^2 (D^0) and 1,869.6 MeV/c^2 ($\mathrm{D}^{\pm}$). The preferred decays of bottonium are analogously

$$\mathrm{b\bar{b}} \rightarrow \mathrm{b\bar{u}} + \bar{b}u\,,$$
$$\mathrm{b\bar{b}} \rightarrow \mathrm{b\bar{d}} + \bar{b}d\,.$$

These mesons are called[2] B^- and B^+ ($m = 5{,}279.3\,\mathrm{MeV}/c^2$), as well as $\overline{B}^0$ and B^0 ($m = 5,279.6\,\mathrm{MeV}/c^2$). For higher excitations decays into mesons with s-quarks are also possible:

$$\begin{aligned} c\bar{c} &\rightarrow c\bar{s} + \bar{c}s \qquad (D_s^+ \text{ and } D_s^-)\,, \\ b\bar{b} &\rightarrow b\bar{s} + \bar{b}s \qquad (\overline{B}_s^0 \text{ and } B_s^0)\,. \end{aligned}$$

Such mesons are accordingly heavier. The mass of $D_s^\pm$ meson is, for example, $1{,}968.5\,\mathrm{MeV}/c^2$. All of these mesons eventually decay weakly into lighter mesons such as pions.

14.7 Decay Widths as a Test of QCD

The decays and decay rates of quarkonia can provide us with information about the strong coupling constant α_s. Let us consider the 1^1S_0 charmonium state (η_c) which can decay into either two photons or two gluons. (In the latter case we will experimentally only observe the end products of hadronisation.) Measurements of the ratio of these two decay widths can determine α_s, in principle, in a very elegant way.

The formula for the decay width into two *real* photons is essentially just the same as for positronium (14.5), one needs only to recall that the c-quarks have fractional electric charge $z_c = 2/3$ and come in three flavours.

$$\Gamma(1^1S_0 \rightarrow 2\gamma) = \frac{3 \cdot 4\pi z_c^4 \alpha^2 \hbar^3}{m_c^2 c} |\psi(0)|^2 (1 + \varepsilon')\,. \tag{14.13}$$

The ε' term signifies higher order QCD corrections which can be approximately calculated.

To consider the two gluon decay, one must replace α by α_s. In contrast to photons, gluons do not exist as real particles but rather have to hadronise. For this process we set the strong coupling constant to one. The different colour-anticolour combinations also mean we must use a different overall colour factor which takes the various gluon combinations into account:

$$\Gamma(1^1S_0 \rightarrow 2g \rightarrow \text{hadrons}) = \frac{8\pi}{3} \frac{\alpha_s^2 \hbar^3}{m_c^2 c} |\psi(0)|^2 (1 + \varepsilon'')\,, \tag{14.14}$$

[2]The standard nomenclature for mesons containing heavy quarks is such that the neutral meson with a b-quark is called a $\overline{B}^0$ and the meson with a $\bar{b}$ is known as a B^0. An electrically neutral $\overline{qq'}$ state is marked with a bar, if the heavier quark/antiquark is negatively charged [8].

where ε'' signifies QCD corrections once again. The ratio of these decay widths is

$$\frac{\Gamma(2\gamma)}{\Gamma(2\mathrm{g})} = \frac{8}{9}\frac{\alpha^2}{\alpha_\mathrm{s}^2}(1+\varepsilon)\,. \tag{14.15}$$

The correction factor ε itself depends upon α_s and is about $\varepsilon \approx -0.5$. From the experimentally determined ratio $\Gamma(2\gamma)/\Gamma(2\mathrm{g}) \approx (3.0 \pm 1.2)\cdot 10^{-4}$ [8] one finds the value $\alpha_\mathrm{s}(m^2_{\mathrm{J}/\psi}\,c^2) \approx 0.25 \pm 0.05$. This is consistent with the value from the charmonium spectrum. From (8.1) we see that α_s always depends upon a distance or, equivalently, energy or mass scale. In this case the scale is fixed by the constituent mass of the c-quark or by the J/ψ mass.

The above result, despite the simplicity of the original idea, suffers from both experimental and theoretical uncertainties. As well as QCD corrections, there are further corrections from the relativistic motion of the quarks. For a better determination of α_s from charmonium physics one can investigate other decay channels. The comparison, for instance, of the decay rates

$$\frac{\Gamma(\mathrm{J}/\psi \to 3\mathrm{g} \to \text{hadrons})}{\Gamma(\mathrm{J}/\psi \to \gamma \to 2\text{ leptons})} \propto \frac{\alpha_\mathrm{s}^3}{\alpha^2}\,, \tag{14.16}$$

is simpler from an experimental viewpoint. Both here and in studies of other channels one finds $\alpha_\mathrm{s}(m^2_{\mathrm{J}/\psi}c^2) \approx 0.2\ldots0.3$ [5].

The comparison of various bottonium decays yields the coupling strength α_s in a more accurate way since both QCD corrections and relativistic effects are smaller. From QCD one expects α_s to be smaller, the coupling is supposed to decrease with the separation. This is indeed the case. One finds from the ratio

$$\frac{\Gamma(\Upsilon \to \gamma\mathrm{gg} \to \gamma + \text{hadrons})}{\Gamma(\Upsilon \to \mathrm{ggg} \to \text{hadrons})} \propto \frac{\alpha}{\alpha_\mathrm{s}}\,, \tag{14.17}$$

which is (2.75 ± 0.04) %, that $\alpha_\mathrm{s}(m^2_\Upsilon c^2) = 0.163\pm0.016$ [7]. The error is dominated by uncertainties in the theoretical corrections.

These examples demonstrate that the annihilation of a $\mathrm{q}\overline{\mathrm{q}}$ pair in both the electromagnetic and strong interactions may formally be described in the same manner. The only essential difference is the coupling constant. This comparison can be understood as a test of the applicability of QCD at short distances, which, after all, is where the $\mathrm{q}\overline{\mathrm{q}}$ annihilation takes place. In this region QCD and QED possess the same structure since both interactions are well described by the exchange of a single vector boson (a gluon or a photon).

Problems

1. **Weak charge**
Bound states are known to exist for the strong interaction (hadrons, nuclei), electromagnetism (atoms, solids) and gravity (the solar system, stars) but we do not have such states for the weak force. Estimate, in analogy to positronium, how heavy two particles would have to be if the Bohr radius of their bound state would be roughly equal to the range of the weak interaction.
2. **Muonic and hadronic atoms**
Negatively charged particles that live long enough (μ^-, π^-, K^-, $\overline{p}$, Σ^-, Ξ^-, Ω^-), can be captured by the field of an atomic nucleus. Calculate the energy of atomic ($2p \to 1s$) transitions in hydrogen-type "atoms" where the electron is replaced by the above particles. Use the formulae of Chap. 14. The lifetime of the 2p state in the H atom is $\tau_H = 1.76 \cdot 10^9$ s. What is the lifetime, as determined from electromagnetic transitions, of the 2p state in a $p\overline{p}$ system (protonium)? Remember to take the scaling of the matrix element and of phase space into account.
3. **Hyperfine structure**
In a two-fermion system the hyperfine structure splitting between the levels 1^3S_1 and 1^1S_0 is proportional to the product of the magnetic moments of the fermions, $\Delta E \propto |\psi(0)|^2 \mu_1 \mu_2$, where $\mu_i = g_i \frac{e_i}{2m_i}$. The g-factor of the proton is $g_p = 5.5858$ and those of the electron and the muon are $g_e \approx g_\mu \approx 2.0023$. In positronium an additional factor of 7/4 arises in the formula for ΔE, which takes the level shifts of the triplet state by pair annihilation graphs into account.
In the hydrogen atom, the level splitting corresponds to a transition frequency $f_H = 1{,}420$ MHz. Estimate the values for positronium and muonium ($\mu^+ e^-$). (Hint: $\psi(0) \propto r_b^{-3/2}$; use the reduced mass in the expression for $|\psi(0)|^2$.) Compare your result with the measured values of the transition frequencies, 203.4 GHz for positronium and 4.463 GHz for muonium. How can the (tiny) difference be explained?

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