Capacitors and Capacitance 23

In this chapter we introduce capacitors, which are one of the simplest *circuit elements*. Capacitors are charge-storing devices that can store energy in the form of an electric potential energy, and are commonly used in a variety of electric circuits.

Apart from being energy-storing devices, capacitors can be used to accumulate charges relatively slowly during the charging process, or to minimize voltage variations in electronic power supplies, or to detect electromagnetic waves, such as when tuning a radio receiver.

We shall first study the properties of capacitors and dielectrics, and follow that by studying capacitors in combination, and finally studying capacitors as electric charge-storing devices.

23.1 Capacitor and Capacitance

We can use a device called **capacitor** to store energy in the form of an electric potential. Beyond serving as storehouses for electric potential energy, capacitors have many uses in our electronic and microelectronic age.

Figure 23.1a shows the basic elements of an air-filled capacitor. It consists of two isolated conductors of any arbitrary shape, each of which carries an equal but opposite charge of magnitude Q.

Figure 23.1b shows a more convenient and practical arrangement of an air-filled capacitor, called a *parallel-plate capacitor*, consisting of two parallel conducting plates of area A separated by a distance d of air. We represent a capacitor of any geometry by the symbol ($\dashv\vdash$), which is based on the structure of a parallel plate capacitor.

H. A. Radi and J. O. Rasmussen, *Principles of Physics*,
Undergraduate Lecture Notes in Physics, DOI: 10.1007/978-3-642-23026-4_23,

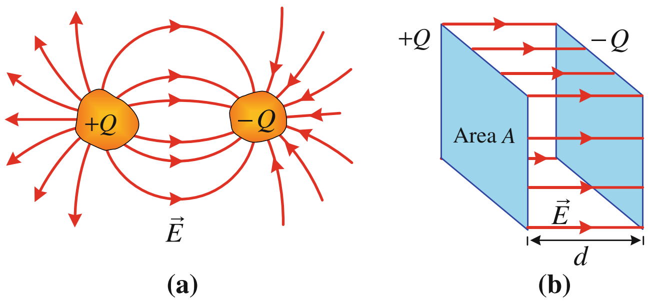


Fig. 23.1 (a) A capacitor made up of two conductors carrying an equal but opposite charge of magnitude Q. (b) A parallel-plate capacitor made up of two plates of area A separated by a distance d. Each plate carries an equal but opposite charge of magnitude Q

Experiments show that the magnitude of the charge on a capacitor is directly proportional to the potential difference between its conductors; i.e. $Q \propto \Delta V$; which can be written as $Q = C\,\Delta V$. Thus:

$$C = \frac{Q}{\Delta V} \tag{23.1}$$

The proportionality constant C is called the **capacitance** of the capacitor and depends on the shape and separation of the conductors. Furthermore, the charge Q and the potential difference ΔV are always expressed in Eq. 23.1 as positive quantities to produce a positive ratio $C = Q/\Delta V$. Hence:

> Spotlight
> The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors.

The SI unit of the capacitance is coulomb per volt, or **farad** (abbreviated by F). That is:

$$1\,\text{F} = 1\,\text{C/V} \tag{23.2}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads ($1\ \mu\text{F} = 10^{-6}\,\text{F}$), nanofarads ($1\ \text{nF} = 10^{-9}\,\text{F}$), to picofarads ($1\ \text{pF} = 10^{-12}\,\text{F}$).

23.2 Calculating Capacitance

For a capacitor with a charge of magnitude Q, we can calculate the potential difference ΔV using the technique described in the preceding chapter. Then we can use the expression $C = Q/\Delta V$ to calculate the capacitance for the capacitor under consideration.

A Parallel-Plate Capacitor

Figure 23.2a shows an uncharged parallel-plate capacitor of equal area A separated by a distance d. The capacitor is connected in a circuit containing a battery B that has a potential difference ΔV and an open switch S. When the switch is closed, the battery establishes an electric field in the wires and consequently charges flow in the circuit to charge the capacitor with a charge of magnitude Q, see Fig. 23.2b. Therefore, some of the stored chemical energy in the battery is transformed to the capacitor in the form of an electric field $\vec{E}$. Figure 23.2c shows the circuit schematic diagram, where we use the symbol ⁺|⊢⁻ to represent the battery, the symbol ⊣ ⊢ to represent the capacitor C, and the symbol –o–o– to represent the closed switch S. An open switch is represented by the symbol –o⁄ o–.

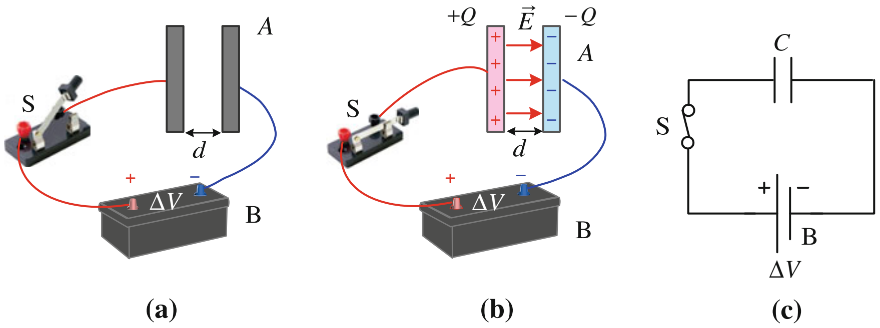


Fig. 23.2 (a) A parallel-plate capacitor is connected to a battery B and an open switch **S**. (b) When **S** is closed, each capacitor plate will carry equal but opposite charges of magnitude Q. (c) A schematic diagram of the circuit with symbols representing the elements used

To find the relation between the capacitance and the geometry of this parallel-plate capacitor, we first note that the magnitude of the surface charge density on either plate is $\sigma = Q/A$. Then according to Example 21.6, the magnitude of the electric field between the plates (assuming it uniform) is:

$$E = \frac{\sigma}{\epsilon_\circ} = \frac{Q}{\epsilon_\circ A} \tag{23.3}$$

Since the positive potential difference ΔV across the battery and the plates are identical, then according to Eq. 22.17 we have:

$$\Delta V = Ed = \frac{Qd}{\epsilon_\circ A} \tag{23.4}$$

Substituting this result into Eq. 23.1, we get:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_\circ A}$$

Thus, the capacitance of the parallel-plate capacitor is:

$$C = \frac{\epsilon_\circ A}{d} \quad \text{(Parallel-plate capacitor)} \tag{23.5}$$

A Cylindrical Capacitor

Figure 23.3a shows a cylindrical capacitor of length ℓ composed of a solid cylindrical conductor of radius a having a charge Q and a coaxial cylindrical conducting shell of radius b having a charge $-Q$. Thus, the magnitude of the linear charge density on either the cylinders is $\lambda = Q/\ell$. We assume that $\ell \gg b$ and hence neglect the fringing (non-uniformity) of the electric field at the cylinders' ends.

Figure 23.3b shows a cross-sectional view of the cylindrical capacitor. The electric field in the region between the cylinders is radial and perpendicular to the axis of the cylinders. In Chap. 21, we showed using Gauss's law that the electric field of a cylindrical charge distribution having a linear charge density λ is radial and is given by:

$$E_r = 2k\frac{\lambda}{r} \qquad (k = 1/4\pi\epsilon_\circ)$$

The same formula applies here since the charge on the outer shell does not contribute to any cylindrical Gaussian surface having $a < r < b$.

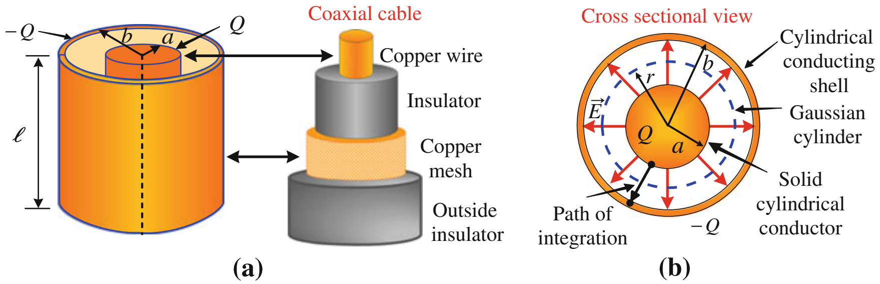


Fig. 23.3 (a) A cylindrical capacitor in the form of a cylindrical solid conductor surrounded by a coaxial shell. (b) A cross-sectional view of the capacitor showing a Gaussian cylinder of radius $a < r < b$

The potential difference $V_b - V_a$ between the cylinders is given by:

$$V_b - V_a = -\int_a^b \vec{E} \bullet d\,\vec{s} = -\int_a^b E_r\, d\,r = -2\,k\,\lambda \int_a^b \frac{d\,r}{r} = -2\,k\lambda \ln\left(\frac{b}{a}\right) \quad (23.6)$$

Therefore, the magnitude of the potential difference between the cylinders is $\Delta V = |V_b - V_a| = 2k\lambda \ln(b/a)$. Substituting this result into Eq. 23.1 and using the fact that $\lambda = Q/\ell$, we get:

$$C = \frac{Q}{\Delta V} = \frac{Q}{2k(Q/\ell)\ln(b/a)}$$

Thus, the capacitance of a cylindrical capacitor of length ℓ is:

$$C = \frac{\ell}{2k\ln(b/a)} = 2\pi\epsilon_\circ \frac{\ell}{\ln(b/a)} \quad \text{(Cylindrical capacitor)} \quad (23.7)$$

In addition, the capacitance per unit length of this configuration is:

$$\frac{C}{\ell} = \frac{1}{2k\ln(b/a)} = 2\pi\epsilon_\circ \frac{1}{\ln(b/a)} \quad \text{(Cylindrical capacitor)} \quad (23.8)$$

A Spherical Capacitor

Figure 23.4a shows a three-dimensional spherical capacitor consisting of a solid spherical conductor of radius a having a charge Q and a concentric spherical shell of radius b having a charge $-Q$.

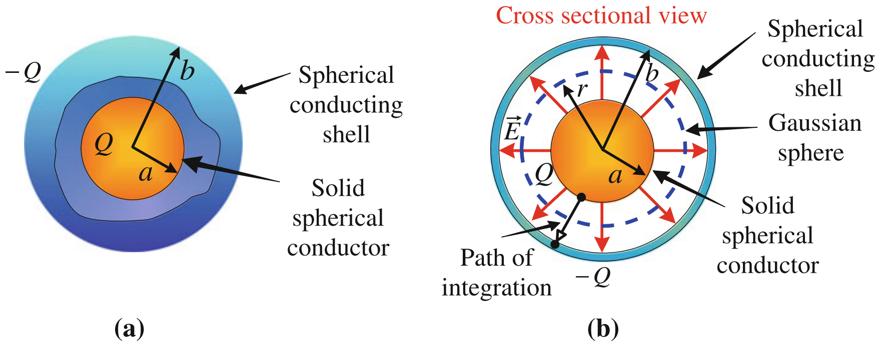


Fig. 23.4 (a) A spherical capacitor consists of a spherical solid conductor surrounded by a concentric spherical shell. (b) A cross-sectional view across the center of the spheres showing a Gaussian sphere of radius $a < r < b$

Figure 23.4b shows a cross-sectional view of the spherical capacitor. As shown in Chap. 21, the electric field outside a spherically symmetric charge distribution is radial and is given by:

$$E_r = k\frac{Q}{r^2}$$

This result applies only to the field between the spheres since the charge on the outer spherical shell does not contribute to any spherical Gaussian surface having $a < r < b$, see Fig. 23.4b.

The potential difference $V_b - V_a$ between the spheres is given by:

$$\begin{aligned} V_b - V_a &= -\int_a^b \vec{E} \bullet d\,\vec{s} = -\int_a^b E_r\,dr = -k\,Q\int_a^b \frac{dr}{r^2} \\ &= k\,Q\left|\frac{1}{r}\right|_a^b = k\,Q\left(\frac{1}{b} - \frac{1}{a}\right) \end{aligned} \tag{23.9}$$

Therefore, the magnitude of the potential difference between the spheres is $\Delta V = |V_b - V_a| = kQ\,(b-a)/ab$. Substituting this result into Eq. 23.1, we obtain:

$$C = \frac{Q}{\Delta V} = \frac{Q}{kQ\,(b-a)/ab}$$

Thus, the capacitance of the spherical capacitor is:

$$C = \frac{ab}{k\,(b-a)} = 4\pi\epsilon_\circ\frac{ab}{(b-a)} \qquad \text{(Spherical capacitor)} \tag{23.10}$$

An Isolated Sphere

The capacitance of a *single* isolated spherical conductor of radius R can be obtained by assuming that the missing second conducting sphere has an infinite radius. The electric field lines that leave or enter the isolated spherical conductor must therefore end at infinity. For practical purposes, the walls of the room in which the spherical conductor is housed can serve as our missing sphere of infinite radius. This proves that *any single conductor has a capacitance*.

To find the capacitance of the isolated spherical conductor, we rearrange Eq. 23.10 to be as follows:

$$C = \frac{a}{k\,(1 - a/b)}$$

Then we let $b \to \infty$ and replace a by R in this formula to find the following relation:

$$C = \frac{R}{k} = 4\pi\epsilon_\circ R \quad \text{(Isolated sphere)} \tag{23.11}$$

Note that all the formulas derived so far for the capacitance [Eqs. 23.5, 23.7, 23.10, and 23.11] involve the constants $1/k$ or $\epsilon_\circ$ multiplied by a quantity that has the dimension of a length. Thus, the units of k and $\epsilon_\circ$ may be expressed as m/F and F/m, respectively.

Example 23.1

The plates of a parallel-plate capacitor are separated in air by a distance $d = 1$ mm. (a) Find the capacitance of this capacitor if its area is $A = 1\ \text{cm}^2$. (b) What must be the plate area if its capacitance is to be 1 F?

Solution: (a) From Eq. 23.5, we have:

$$C = \frac{\epsilon_\circ A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(1 \times 10^{-4}\ \text{m}^2)}{(1 \times 10^{-3}\ \text{m})} = 8.85 \times 10^{-13}\ \text{F} = 0.885\ \text{pF}$$

(b) From Eq. 23.5, we have:

$$A = \frac{Cd}{\epsilon_\circ} = \frac{(1\ \text{F})(1 \times 10^{-3}\ \text{m})}{(8.85 \times 10^{-12}\ \text{F/m})} = 1.13 \times 10^8\ \text{m}^2$$

This is an area of a square that has a side of more than 10.6 km. Therefore, the farad is indeed a large unit. However, modern technology has permitted the construction of a 1 F capacitor of a very modest size. This capacitor is used as a backup power supply (up to many months) for computer memory chips in case of a power failure.

Example 23.2

Show that the capacitance of the cylindrical capacitor shown in Fig. 23.3a approaches the capacitance of a parallel-plate capacitor if the separation d between the two cylinders is very small.

Solution: When $d = b - a$ is very small, then d/a must also be very small. If we use the approximation $\ln(1 + x) \approx x$ for $x \ll 1$, in the natural logarithm of the denominator of Eq. 23.7, we find that:

$$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{a+d}{a}\right) = \ln\left(1 + \frac{d}{a}\right) \approx \frac{d}{a} \quad \text{(When } d/a \ll 1\text{)}$$

Then, using the surface area of the inner cylinder $A = 2\pi a\ell$, we find that Eq. 23.7 approaches Eq. 23.5 as follows:

$$C = 2\pi\epsilon_\circ \frac{\ell}{\ln(b/a)} \approx 2\pi\epsilon_\circ \frac{\ell}{d/a} = \epsilon_\circ \frac{2\pi a\ell}{d} = \frac{\epsilon_\circ A}{d}$$

Example 23.3 **(Spherical Capacitor)**

(a) How much charge is stored in a spherical capacitor consisting of two concentric spheres of radii $a = 20$ cm and $b = 21$ cm if the potential difference between them is 200 V? (b) Show that if the separation d between the two spheres is small compared to their radii, then the capacitance is given by the parallel-plate capacitance formula $\epsilon_\circ A/d$. (c) Does the answer to part (b) apply to part (a)? (d) Find the capacitance of the inner sphere of part (a) if it is isolated.

Solution: (a) For concentric spheres, Eq. 23.10 is used to calculate the capacitance as follows:

$$C = \frac{ab}{k(b-a)} = \frac{(0.2\,\text{m})(0.21\,\text{m})}{(9 \times 10^9\,\text{m/F})(0.21\,\text{m} - 0.2\,\text{m})} = 4.67 \times 10^{-10}\,\text{F} = 0.467\,\text{nF}$$

Then, by using Eq. 23.1, the magnitude of the charge on each sphere will be:

$$Q = C\Delta V = (4.67 \times 10^{-10}\ \text{F})(200\ \text{V}) = 93.4\ \text{nC}$$

(b) When the separation $d = b - a$ is small, we can write the surface area of each sphere as $A \approx 4\pi a^2 \approx 4\pi b^2 \approx 4\pi ab$. Then, we have:

$$C = 4\pi\epsilon_\circ \frac{ab}{(b-a)} = \epsilon_\circ \frac{4\pi ab}{d} \approx \frac{\epsilon_\circ A}{d}$$

(c) Since the separation d in part (a) is very small compared to the radii of the spheres, then according to part (b) the capacitance is:

$$C \approx \frac{\epsilon_\circ A}{d} = \frac{4\pi a^2 \epsilon_\circ}{d} = \frac{4\pi (0.2\,\text{m})^2 (8.85 \times 10^{-12}\,\text{F/m})}{(1 \times 10^{-2}\,\text{m})} = 4.45 \times 10^{-10}\ \text{F}$$

This is very close to the answer 4.67×10^{-10} F obtained in part (a).

(d) Substituting with $R = a = 20$ cm in Eq. 23.11, we find that:

$$C = 4\pi\epsilon_\circ R = 4\pi (8.85 \times 10^{-12}\,\text{F/m})(0.2\,\text{m}) = 2.22 \times 10^{-11}\ \text{F}$$

23.3 Capacitors with Dielectrics

An Electrical Description of Dielectrics

Capacitance was found to increase when a non-conducting material (such as oil, rubber, plastic, glass, or waxed paper) is inserted between the capacitor's plates. These non-conducting materials are called **dielectrics**. If the dielectric completely fills the space between the plates, the capacitance is found to increase by a dimensionless factor κ (the Greek alphabet Kappa), called the **dielectric constant**.

Fixed Charge

Consider a parallel-plate capacitor without a dielectric to have a capacitance $C_\circ$, a charge $Q_\circ$, and potential difference $\Delta V_\circ$, i.e. $C_\circ = Q_\circ / \Delta V_\circ$, see Fig. 23.5a. When a dielectric is inserted between the plates, see Fig. 23.5b, the potential difference between the plates is found to decrease to a value ΔV related to $\Delta V_\circ$ by the relation:

$$\Delta V = \frac{\Delta V_\circ}{\kappa} \tag{23.12}$$

Note that, $\kappa > 1$ because $\Delta V < \Delta V_\circ$.

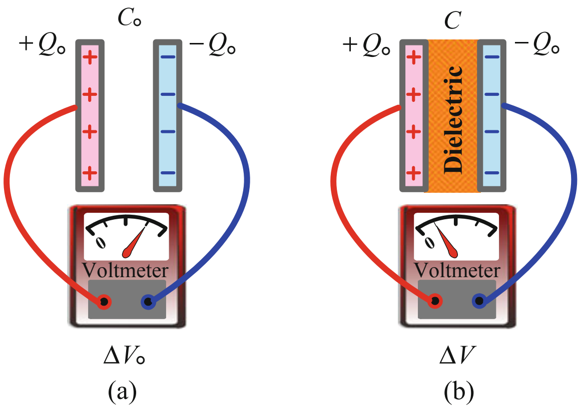


Fig. 23.5 (a) A capacitor with capacitance $C_\circ$ has a charge $Q_\circ$ when the potential difference between the plates is $\Delta V_\circ$. (b) When the capacitor's charge is maintained, inserting a dielectric reduces the potential difference to ΔV, where $\Delta V < \Delta V_\circ$

After inserting the dielectric, the capacitance C of the capacitor can be obtained from Eq. 23.1 as follows:

$$C = \frac{Q_\circ}{\Delta V} = \frac{Q_\circ}{\Delta V_\circ/\kappa} = \kappa \frac{Q_\circ}{\Delta V_\circ} \tag{23.13}$$

Using $C_\circ = Q_\circ/\Delta V_\circ$, we find that:

$$\boxed{C = \kappa\, C_\circ} \tag{23.14}$$

This indicates that the capacitance increases by a factor κ when the dielectric completely fills the space between the plates of the capacitor. Using Eq. 23.5, $C_\circ = \epsilon_\circ A/d$, the capacitance becomes:

$$\boxed{C = \frac{\kappa\, \epsilon_\circ A}{d} = \frac{\epsilon A}{d}} \tag{23.15}$$

where $\epsilon = \kappa\, \epsilon_\circ$ and is known as the *permittivity of the dielectric*.

On the other hand, if $\vec{E}_\circ$ is the electric field without the dielectric, then a reduction of the potential difference from $\Delta V_\circ$ to $\Delta V = \Delta V_\circ/\kappa$ means that the electric field decreases from $\vec{E}_\circ$ to $\vec{E} = \vec{E}_\circ/\kappa$. That is:

$$\boxed{\vec{E} = \frac{\vec{E}_\circ}{\kappa}} \tag{23.16}$$

Fixed Potential Difference

Now, consider a parallel-plate capacitor without a dielectric, having a capacitance $C_\circ$, a charge $Q_\circ$, and connected to a battery that has a potential difference $\Delta V_\circ$, i.e. $C_\circ = Q_\circ/\Delta V_\circ$, see Fig. 23.6a. If the dielectric is inserted between the plates while the potential difference is held constant by keeping the capacitor connected to the battery, see Fig. 23.6b, then the capacitance has to increase as before by the relation $C = \kappa C_\circ$. Consequently, the magnitude of the charge on the capacitor has to increase by a factor κ according to the relation:

$$Q = \kappa Q_\circ \tag{23.17}$$

The extra charge comes from the battery attached to the capacitor.

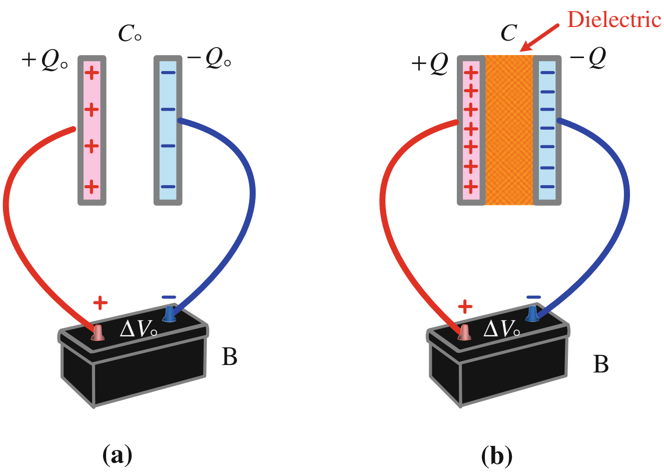


Fig. 23.6 (a) A capacitor with capacitance $C_\circ$ has a charge $Q_\circ$ when connected to a battery that has a potential difference $\Delta V_\circ$. (b) When the potential difference is maintained by the battery, inserting a dielectric increases the charge to Q, where $Q = \kappa Q_\circ$

An Atomic Description of Dielectrics

The molecules of some dielectrics have randomly oriented *permanent electric dipole moments* as shown in Fig. 23.7a. The presence of an external electric field $\vec{E}_\circ$ in such materials (called *polar dielectrics*), will exert a torque on the dipoles, causing them to partially align with the field, as shown in Fig. 23.7b. We can now describe the dielectric as being polarized, and the degree of alignment depends generally on the strength of $\vec{E}_\circ$.

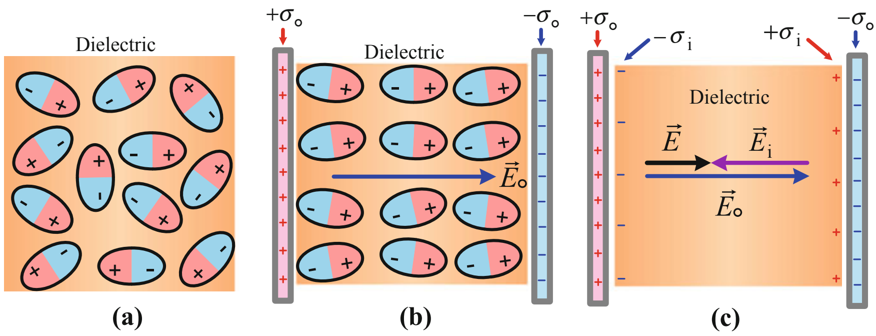


Fig. 23.7 (a) A dielectric that has randomly oriented molecules. (b) The partial alignment of molecules in the presence of an external electric field $\vec{E}_\circ$ due to a charged parallel plate capacitor with a surface charge density of magnitude $\sigma_\circ$. (c) The formation of an induced charge density $+\sigma_i$ and $-\sigma_i$ on either sides of the capacitor sets up an induced electric field $\vec{E}_i$. The resultant electric field $\vec{E}$ inside the dielectric has the same direction as $\vec{E}_\circ$ but is less in magnitude

Even when the dielectric material is *non-polar*, the applied external electric field $\vec{E}_\circ$ tends to separate the centers of the positive and negative charges of the molecules, producing *induced electric dipole moments*. Therefore, the induced electric dipole moments tend to align with the external electric field, and the dielectric is polarized.

The net effect on the dielectric is the formation of an *induced* positive and negative charge density $+\sigma_i$ and $-\sigma_i$ on the right and left faces of the dielectric, respectively, see Fig. 23.7c. Therefore, an *induced electric field* $\vec{E}_i$ will be established in a direction opposite to the external electric field $\vec{E}_\circ$. Accordingly, the net electric field $\vec{E}$ in the dielectric will have a magnitude given by:

$$E = E_\circ - E_i \tag{23.18}$$

In the case of the parallel-plate capacitor shown in Fig. 23.7c, we use the relations $E_\circ = \sigma_\circ/\epsilon_\circ$, $E_i = \sigma_i/\epsilon_\circ$, and $E = E_\circ/\kappa = \sigma_\circ/\epsilon$, to get:

$$\frac{\sigma_\circ}{\kappa\epsilon_\circ} = \frac{\sigma_\circ}{\epsilon_\circ} - \frac{\sigma_i}{\epsilon_\circ} \tag{23.19}$$

or

$$\sigma_i = \frac{\kappa - 1}{\kappa}\sigma_\circ \quad \text{(Parallel-plate capacitor)} \tag{23.20}$$

where $\sigma_i < \sigma_o$ because $\kappa > 1$. When the dielectric is replaced by a conductor, for which $E = 0$, then $E_i = E_o$ and hence $\sigma_i = \sigma_o$. This means that the induced charge on the conductor is equal in magnitude but opposite in sign to that on the plates of the parallel-plate capacitor.

Equation 23.15 indicates that the capacitance C increases drastically when d diminishes. However, d is limited by the electric discharge that could occur through the dielectric medium. Every dielectric material has a specific *dielectric strength* E_{max}, which is the maximum value of the electric field that the dielectric can withstand without *electrical breakdown*. Above this value the dielectric breaks down and forms a conducting path between the capacitor's plates. The largest potential difference ΔV_{max} that can be applied to a dielectric without exceeding the dielectric strength is called the *breakdown potential difference*. In fact, insulating materials have $\kappa > 1$ and their E_{max} is greater than that of air.

Table 23.1 displays approximate dielectric constants κ and dielectric strengths E_{max} of some materials at room temperature.

Table 23.1 Approximate values of the dielectric constants and dielectric strengths of some materials at room temperature

Material	κ	E_{max} (10^6 V/m $\equiv$ kV/mm)
Vacuum	1.00000	–
Air (1 atm)	1.00059	3
Teflon	2.1	60
Silicon oil	2.5	15
Mylar	3.2	7
Nylon	3.4	14
Paraffin-impregnated paper	3.5	11
Paper	3.7	16
Pyrex glass	5.6	14
Distilled Water	80	–

Types of Capacitors

Low-voltage capacitors are usually made of metallic foil interlaced with thin sheets of a dielectric material, made of either paraffin-impregnated paper or Mylar. The metallic foil and dielectric are rolled into a cylinder to form a small package, see Fig. 23.8a.

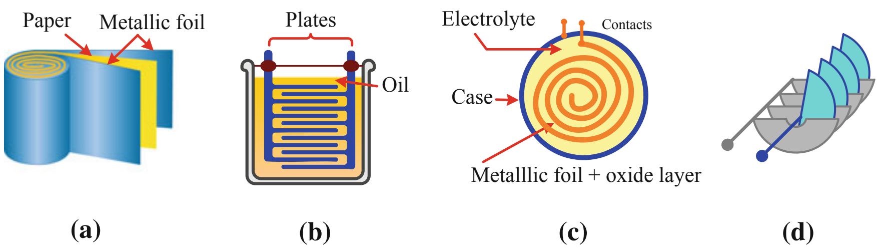


Fig. 23.8 (a) A low-voltage capacitor whose plates are separated by paper as a dielectric. (b) A high-voltage capacitor consisting of a number of plates separated by insulating oil as a dielectric. (c) An electrolytic capacitor used to store a large amount of charge. (d) A variable air capacitor

High-voltage capacitors are usually made of a number of interwoven metallic plates immersed in silicon oil, see Fig. 23.8b.

Large-charge storage capacitors consist of a metallic foil in contact with an electrolyte. When a voltage is applied between the foil and the electrolyte, a very thin layer of metal oxide is formed on the foil, and that layer serves as a dielectric, see Fig. 23.8c. Because the dielectric layer is very thin, the capacitance obtained with this type is very large. Such capacitors are assigned a polarity, which is indicated by positive and negative signs. If the polarity of the applied voltage is reversed, the oxide layer is removed, and the capacitor starts conducting electricity instead of storing charge.

Variable capacitors whose capacitance may vary are widely used in tuning circuits of radio receivers. They are constructed from a set of fixed parallel-plates connected together to form one plate of the capacitor, while the second set of movable plates are connected together to form the other plate. The plates are separated by air as a dielectric, see Fig. 23.8d.

Example 23.4

The parallel plates in Fig. 23.9a have an area $A = 0.2\ \text{m}^2$ and separation distance $d = 0.01$ m. The original potential difference between them is $\Delta V_\circ = 300$ V which decreases to $\Delta V = 100$ V when a dielectric sheet fills the space between the plates, see Fig. 23.9b. (a) Calculate the capacitance $C_\circ$, the magnitude of the charge $Q_\circ$, and the magnitude of the electric field $E_\circ$. (b) Calculate the final capacitance C and the dielectric constant κ. (c) Find the magnitudes of the induced charge density σ_i, the induced electric field E_i, and the final electric field E.

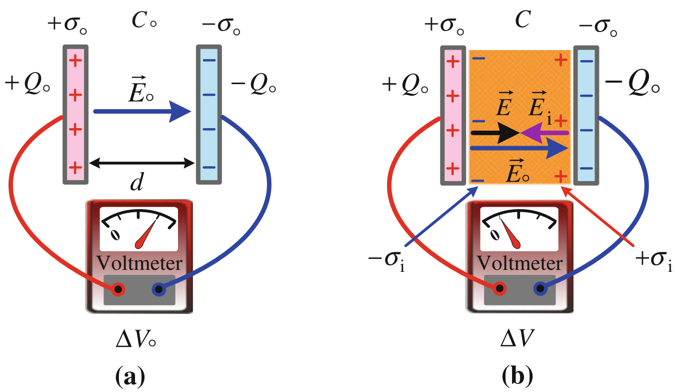


Fig. 23.9

Solution: (a) Using the parallel-plate capacitor Eq. 23.5, we get:

$$C_\circ = \frac{\epsilon_\circ A}{d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})(0.2\,\text{m}^2)}{0.01\,\text{m}} = 1.77 \times 10^{-10}\ \text{F} = 177\ \text{pF}$$

Then, when using Eq. 23.1, the magnitude of the charge on each plate will be the following:

$$Q_\circ = C_\circ\,\Delta V_\circ = (1.77 \times 10^{-10}\,\text{F})(300\,\text{V}) = 5.31 \times 10^{-8}\,\text{C} = 53.1\,\text{nC}$$

Finally, we use Eq. 22.17 to find the magnitude of the uniform electric field $E_\circ$ as follows:

$$E_\circ = \frac{\Delta V_\circ}{d} = \frac{300\ \text{V}}{0.01\ \text{m}} = 3 \times 10^4\ \text{V/m}$$

Alternatively, we can use the relation $E_\circ = \sigma_\circ/\epsilon_\circ$ to find $E_\circ$. First, we calculate $\sigma_\circ$ as follows:

$$\sigma_\circ = \frac{Q_\circ}{A} = \frac{5.31 \times 10^{-8}\,\text{C}}{0.2\,\text{m}^2} = 2.655 \times 10^{-7}\ \text{C/m}^2$$

Then we find the value of $E_\circ$ as follows:

$$E_\circ = \frac{\sigma_\circ}{\epsilon_\circ} = \frac{2.655 \times 10^{-7}\ \text{C/m}^2}{8.85 \times 10^{-12}\,\text{F/m}} = 3 \times 10^4\ \text{C/F.m} = 3 \times 10^4\ \text{V/m}$$

(b) We first use Eq. 23.1 to find C as follows:

$$C = \frac{Q_\circ}{\Delta V} = \frac{5.31 \times 10^{-8}\ \text{C}}{100\ \text{V}} = 5.31 \times 10^{-10}\,\text{F} = 531\,\text{pF}$$

Then, by using equation $C=\kappa C_\circ$, we find that:

$$\kappa=\frac{C}{C_\circ}=\frac{5.31\times10^{-10}\ \text{F}}{1.77\times10^{-10}\ \text{F}}=3$$

(c) The induced charge density σ_i can be obtained from Eq. 23.20 as follows:

$$\sigma_\text{i}=\frac{\kappa-1}{\kappa}\sigma_\circ=\frac{(3-1)(2.655\times10^{-7}\ \text{C/m}^2)}{(3)}=1.77\times10^{-7}\ \text{C/m}^2$$

The magnitude of the induced electric field is therefore:

$$E_\text{i}=\frac{\sigma_\text{i}}{\epsilon_\circ}=\frac{1.77\times10^{-7}\ \text{C}}{8.85\times10^{-12}\ \text{F/m}}=2\times10^4\ \text{V/m}$$

The magnitude of the final electric field can be obtained from Eq. 23.16 as follows:

$$E=\frac{E_\circ}{\kappa}=\frac{3\times10^4\ \text{V/m}}{3}=10^4\ \text{V/m}$$

Alternatively, we can find E from Eq. 23.18 as follows:

$$E=E_\circ-E_\text{i}=3\times10^4\ \text{V/m}-2\times10^4\ \text{V/m}=10^4\ \text{V/m}$$

Example 23.5

Assume that the parallel-plate capacitor of Fig. 23.10a has a plate area $A=0.2\ \text{m}^2$, separation distance $d=10^{-2}$ m, and original potential difference $\Delta V_\circ=300$ V. A dielectric slab of thickness $a=5\times10^{-3}$ m and dielectric constant $\kappa=2.5$ is inserted between the plates as shown in Fig. 23.10b. (a) Find the magnitudes of the final electric field E in the slab, the final potential difference ΔV between the plates, and the final capacitance C with the dielectric slab in place. (b) Find an expression for C in terms of $C_\circ$, a, d, and κ.

Solution: (a) From Example 23.4, we have $E_\circ=3\times10^4$ V/m. Therefore, the magnitude of the final electric field in the slab can be obtained from Eq. 23.16 as follows:

$$E=\frac{E_\circ}{\kappa}=\frac{3\times10^4\ \text{V/m}}{2.5}=1.2\times10^4\ \text{V/m}$$

By applying Eq. 22.6, we can find ΔV by integrating against the electric field along a straight line from the negative plate $(-)$ to the positive plate $(+)$. Within the dielectric, we must note that $\vec{E} \bullet d\vec{s} = -E\,ds$, the path length is a, and the magnitude of the field is E. But within the right and left gaps, the total path length is $d - a$ and the magnitude of the field is $E_\circ$. Thus, Eq. 22.6 yields:

$$\begin{aligned}\Delta V &= V_+ - V_- = -\int_-^+ \vec{E} \bullet d\vec{s} = \int_-^+ E\,ds = E_\circ\,(d-a) + Ea\\ &= (3\times 10^4\ \text{V/m})(10^{-2}\ \text{m} - 5\times 10^{-3}\ \text{m}) + (1.2\times 10^4\ \text{V/m})(5\times 10^{-3}\ \text{m})\\ &= 210\ \text{V}\end{aligned}$$

From Example 23.4, we found that $Q_\circ = 5.31\times 10^{-8}$ C and from Eq. 23.1 we can find the value of C as follows:

$$C = \frac{Q_\circ}{\Delta V} = \frac{5.31\times 10^{-8}\,\text{C}}{210\ \text{V}} = 2.53\times 10^{-10}\,\text{F} = 0.253\,\text{nF}$$

Note that we cannot use the relation $C = \kappa\, C_\circ$, because it is true only if the dielectric material *fills* the space between the capacitor's plates.

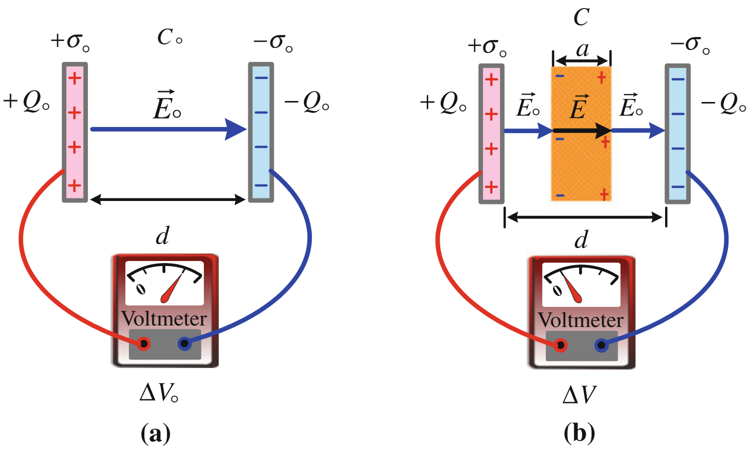


Fig. 23.10

(b) We start with the proven formula of part (a); that is:

$$\Delta V = E_\circ(d-a) + Ea$$

Then, using $\Delta V = Q_\circ/C$, $E_\circ = \sigma_\circ/\epsilon_\circ = Q_\circ/\epsilon_\circ A$, $C_\circ = \epsilon_\circ A/d$, and $E = E_\circ/\kappa = \sigma_\circ/\epsilon$, we can find an expression for C by performing the following steps:

$$\frac{Q_\circ}{C} = \frac{Q_\circ}{\epsilon_\circ A}(d-a) + \frac{Q_\circ}{\kappa\epsilon_\circ A}a$$

$$\frac{1}{C} = \frac{d-a}{\epsilon_\circ A} + \frac{a}{\kappa\epsilon_\circ A}$$

$$C = \frac{\epsilon_\circ A}{\left[(d-a)+\dfrac{a}{\kappa}\right]} \quad \Rightarrow \quad C = \frac{d}{\left[(d-a)+\dfrac{a}{\kappa}\right]}\frac{\epsilon_\circ A}{d}$$

$$C = \frac{d}{\left[(d-a)+\dfrac{a}{\kappa}\right]}C_\circ$$

In the second step, $(d-a)/\epsilon_\circ A$ is the inverse of the capacitance of an air capacitor of separation $d-a$, and $a/\kappa\epsilon_\circ A$ is the inverse of the capacitance of a capacitor of separation a but filled with a dielectric.

23.4 Capacitors in Parallel and Series

Capacitors in a circuit may be used in different combinations, and we can sometimes replace a combination of capacitors with one *equivalent capacitor*. In this section, we introduce two basic combinations of capacitors that allow such a replacement.

Capacitors in a Parallel Combination

Figure 23.11a shows two capacitors of capacitances C_1 and C_2, that are connected in **parallel** with a battery B. Figure 23.11b shows a circuit diagram for this combination of capacitors. The potential difference ΔV between the battery's terminals is the same as the potential difference across each capacitor. Figure 23.11c shows a single capacitance C_{eq} that is equivalent to this combination and has the same effect on the circuit. This means that when the potential difference ΔV is applied across the equivalent capacitor, it will store the same magnitude of the maximum total charge Q as stored in the combination being replaced.

When the circuit is first connected, electrons are transferred between the wires and the plates. This transfer leaves the top plates of the two capacitors positively charged, and the bottom plates negatively charged. If the magnitude of the maximum charges stored on the two capacitors are Q_1 and Q_2, then we must have:

$$Q = Q_1 + Q_2 \tag{23.21}$$

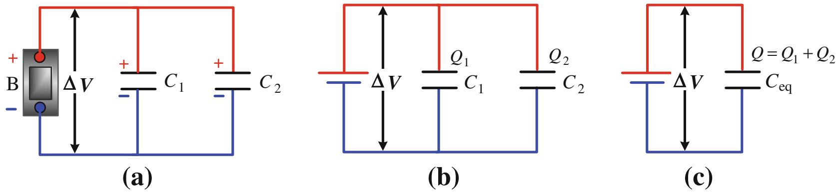


Fig. 23.11 (a) Two capacitors of capacitances C_1 and C_2 are connected in parallel to a battery B that has a potential difference ΔV. (b) The circuit diagram for this parallel combination. (c) The equivalent capacitance C_{eq} replaces the parallel combination

For the two capacitors in Fig. 23.11b, we have:

$$Q_1 = C_1 \Delta V \quad \text{and} \quad Q_2 = C_2 \Delta V \tag{23.22}$$

Substituting in Eq. 23.21, we get:

$$Q = (C_1 + C_2)\Delta V \tag{23.23}$$

The equivalent capacitor with the same total charge Q and applied potential difference ΔV has a capacitance C_{eq} given by:

$$C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2 \quad \text{(Parallel combination)} \tag{23.24}$$

We can extend this treatment to n capacitors connected in parallel as:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n \quad \text{(Parallel combination)} \tag{23.25}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is simply the algebraic sum of the individual capacitances and is greater than any one of them.

Example 23.6

In Fig. 23.11, let $C_1 = 6\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$, and $\Delta V = 18\,\text{V}$. Find the equivalent capacitance as well as the charges on C_1 and C_2.

Solution: The equivalent capacitance of the parallel combination is:

$$C_{eq} = C_1 + C_2 = 6\,\mu\text{F} + 3\,\mu\text{F} = 9\,\mu\text{F}$$

The magnitudes of the charges Q_1 and Q_2 on the two capacitors are:

$$Q_1 = C_1 \Delta V = (6\,\mu\,\text{F})(18\text{ V}) = 108\ \mu\text{C}$$
$$Q_2 = C_2 \Delta V = (3\,\mu\text{F})(18\text{ V}) = 54\ \mu\text{C}$$

Capacitors in a Series Combination

Figure 23.12a shows two capacitors of capacitances C_1 and C_2 that are connected in **series** with a battery B. Figure 23.12b shows a circuit diagram for this combination of capacitors.

When the circuit is first connected, the electrons are transferred out of the upper plate of C_1 (leaving it with an excess of positive charge) into the lower plate of C_2. As this negative charge accumulates on the lower plate of C_2, an exact amount of negative charge is forced off the upper plate of C_2 (leaving it with an excess positive charge) into the lower plate of C_1. As a result, all the upper plates acquire a positive charge $+Q$, and the lower plates acquire a negative charge $-Q$. Figure 23.11c shows a single capacitance C_{eq} that is equivalent to this combination and has the same effect on the circuit. This means that when the potential difference ΔV is applied across the equivalent capacitor, it must have a positive charge $+Q$ on its upper plate and a negative charge $-Q$ on its lower plate.

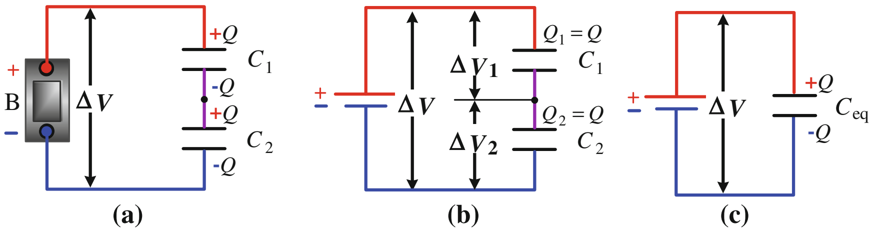


Fig. 23.12 (a) Two capacitors are connected in series to a battery B that has a potential difference ΔV. (b) The circuit diagram for this series combination. (c) An equivalent capacitance C_{eq} replacing the original capacitors set up in a series combination

The potential difference ΔV is divided to ΔV_1 and ΔV_2 across the capacitors C_1 and C_2, respectively. Thus:

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{23.26}$$

For the two capacitors in Fig. 23.12b, we have:

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1} \quad \text{and} \quad \Delta V_2 = \frac{Q_2}{C_2} = \frac{Q}{C_2} \tag{23.27}$$

Substituting in Eq. 23.26, we get:

$$\Delta V = \frac{Q}{C_1} + \frac{Q}{C_2} \tag{23.28}$$

The equivalent capacitor C_{eq} has the same charge Q and applied potential difference ΔV; thus:

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \tag{23.29}$$

Canceling Q, we arrive at the following relationship:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(Series combination)} \tag{23.30}$$

We can extend this treatment to n capacitors connected in series as:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \quad \text{(Series combination)} \tag{23.31}$$

Thus, the equivalent capacitance of a series combination of capacitors is simply the algebraic sum of the reciprocals of the individual capacitances and will always be less than any one of them.

Example 23.7

In Fig. 23.12, let $C_1 = 6\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$, and $\Delta V = 18$ V. Find C_{eq}, Q, ΔV_1, and ΔV_2.

Solution: The equivalent capacitance of the series combination is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6\,\mu\text{F}} + \frac{1}{3\,\mu\text{F}} = \frac{1}{2\,\mu\text{F}} \quad \Rightarrow \quad C_{eq} = 2\,\mu\text{F}$$

Consequently: $Q = C_{eq}\,\Delta V = (2\,\mu\text{F})(18\text{ V}) = 36\,\mu\text{C}$

$$\Delta V_1 = \frac{Q}{C_1} = \frac{36\,\mu\text{V}}{6\,\mu\text{F}} = 6\text{ V} \quad \text{and} \quad \Delta V_2 = \frac{Q}{C_2} = \frac{36\,\mu\text{V}}{3\,\mu\text{F}} = 12\text{ V}$$

Example 23.8

For the combination of capacitors shown in Fig. 23.13a, assume that $C_1 = 2\ \mu\text{F}$, $C_2 = 4\ \mu\text{F}$, and $C_3 = 3\ \mu\text{F}$, and $\Delta V = 12\ \text{V}$. (a) Find the equivalent capacitance of the combination. (b) What is the charge on C_1?

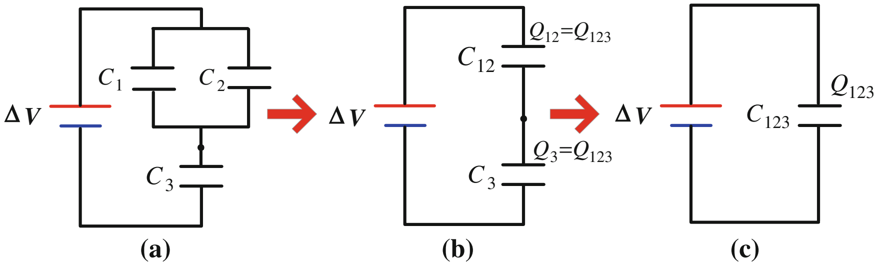


Fig. 23.13

Solution: (a) Capacitors C_1 and C_2 in Fig. 23.13a are in parallel and their equivalent capacitance C_{12} is:

$$C_{12} = C_1 + C_2 = 2\,\mu\text{F} + 4\,\mu\text{F} = 6\,\mu\text{F}$$

From Fig. 23.13b, we find that C_{12} and C_3 form a series combination and their equivalent capacitance C_{123} is given by:

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{6\,\mu\text{F}} + \frac{1}{3\,\mu\text{F}} = \frac{1}{2\,\mu\text{F}} \quad \Rightarrow \quad C_{123} = 2\,\mu\text{F}$$

(b) We first find the charge Q_{123} on C_{123} in Fig. 23.13c as follows:

$$Q_{123} = C_{123}\Delta V = (2\,\mu\text{F})(12\ \text{V}) = 24\ \mu\text{C}$$

This same charge exists on each capacitor in the series combination of Fig. 23.13b. Therefore, if Q_{12} represents the charge on C_{12}, then $Q_{12} = Q_{123} = 24\ \mu\text{C}$. Accordingly, the potential difference across C_{12} is:

$$\Delta V_{12} = \frac{Q_{12}}{C_{12}} = \frac{24\,\mu\text{C}}{6\,\mu\text{F}} = 4\ \text{V}$$

This same potential difference exists across C_1, i.e. $\Delta V_1 = \Delta V_{12}$. Thus:

$$Q_1 = C_1 \Delta V_1 = (2\,\mu\text{F})(4\ \text{V}) = 8\ \mu\text{C}$$

23.5 Energy Stored in a Charged Capacitor

When the switch S of Fig. 23.14a is closed, the process of charging the capacitor starts by transferring electrons from the left plate (leaving it with an excess of positive charge) to the right plate. In the process of charging this capacitor, the battery must do work at the expense of its stored chemical energy.

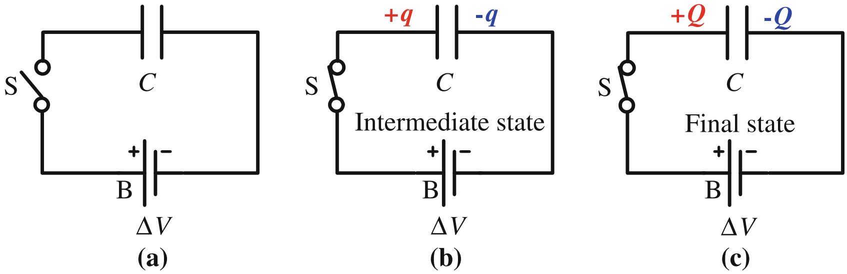


Fig. 23.14 (a) A circuit consisting of a battery B, a switch S, and a capacitor C. (b) An intermediate state when the magnitude of the charge on the capacitor is q. (c) A final state when $q = Q$.

In principle, the charging process occurs *as if* positive charges were pulled off from the right plate and transferred directly to the left plate. Suppose that, at a given instant during the charging process, as shown in Fig. 23.14b, the charge on the capacitor is q, i.e. $q = C\Delta V$. Moreover, according to Eq. 22.11, the differential applied work necessary to transfer a differential charge dq from the plate having charge $-q$ to the plate having a charge $+q$ is given by:

$$dW(\text{app}) = dq\ \Delta V = \frac{q}{C} dq \tag{23.32}$$

The total work required to charge the capacitor from a charge $q = 0$ to a final charge $q = Q$, see Fig. 23.14c, is thus:

$$W(\text{app}) = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q\ dq = \frac{Q^2}{2\,C} \tag{23.33}$$

According to Eqs. 22.6 and 22.10, this work done by the battery is stored as electrostatic potential energy U in the capacitor. Thus:

$$U = \frac{Q^2}{2C} \quad \text{(Electric potential energy)} \tag{23.34}$$

From Eq. 23.1, we can write this stored electric potential energy in the following forms:

$$U = \tfrac{1}{2}C(\Delta V)^2 \quad \text{(Electric potential energy)} \tag{23.35}$$

or

$$U = \tfrac{1}{2}Q\Delta V \quad \text{(Electric potential energy)} \tag{23.36}$$

It is important to note that Eqs. 23.34 to 23.36 hold for any capacitor, regardless of its shape.

When we neglect the fringing effect (nonuniform $\vec{E}$) in a parallel-plate capacitor filled with a dielectric, we know that the electric field has the same value at any point between the plates. Thus, the potential energy per unit volume between the plates, known as the **energy density** u_E, should also be uniform. Then we can find u_E by dividing the electric potential energy U by the volume Ad between the plates:

$$u_E = \frac{U}{Ad} = \frac{C(\Delta V)^2}{2Ad} \tag{23.37}$$

Using $C = \kappa\,\epsilon_\circ A/d$ and $\Delta V = Ed$ for parallel-plate capacitors, we get:

$$u_E = \tfrac{1}{2}\kappa\epsilon_\circ E^2 \quad \text{(Electric energy density)} \tag{23.38}$$

Although this equation is derived for a parallel-plate capacitor, it holds true for any source of electric field. When the electric field $\vec{E}$ exists at any point in a dielectric material of dielectric constant κ, the potential energy per unit volume at this point is given by Eq. 23.38. When $\kappa = 1$, this relation reduces to $u_E = \frac{1}{2}\epsilon_\circ E^2$.

Example 23.9

A capacitor $C_1 = 4\,\mu\text{F}$ is charged by an initial potential difference $\Delta V_i = 12\,\text{V}$, see Fig. 23.15a. The charging battery is then removed, as shown in Fig. 23.15b, and the capacitor is connected to the uncharged capacitor $C_2 = 2\,\mu\text{F}$, as shown in Fig. 23.15c. (a) Find the final potential difference ΔV_f as well as Q_{1f} and Q_{2f}. (b) Find the stored energy before and after the switch is closed.

Solution: (a) The original charge is now shared by C_1 and C_2, so:

$$Q_{1i} = Q_{1f} + Q_{2f}$$

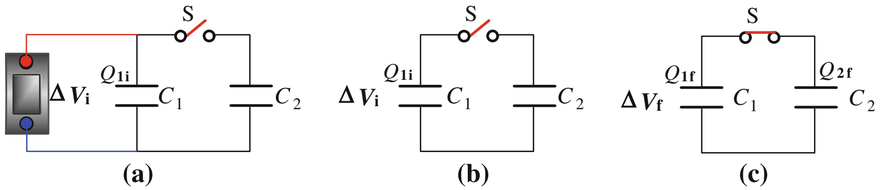


Fig. 23.15

Using of the relation $Q = C\Delta V$ in each term of this equation, we get:

$$C_1 \Delta V_i = C_1 \Delta V_f + C_2 \Delta V_f$$

Thus:

$$\Delta V_f = \frac{C_1}{C_1 + C_2} \Delta V_i = \frac{(4\,\mu F)}{4\,\mu F + 2\,\mu F}(12\,V) = 8\,V$$

and:

$$Q_{1f} = C_1 \Delta V_f = (4\,\mu F)(8\ V) = 32\ \mu C$$
$$Q_{2f} = C_2 \Delta V_f = (2\,\mu F)(8\ V) = 16\ \mu C$$

(b) The initial potential energy is:

$$U_i = \tfrac{1}{2} C_1 (\Delta V_i)^2 = \tfrac{1}{2}(4\,\mu F)(12\ V)^2 = 288\ \mu J$$

The final potential energy is:

$$U_f = \tfrac{1}{2} C_1 (\Delta V_f)^2 + \tfrac{1}{2} C_2 (\Delta V_f)^2 = \tfrac{1}{2}(4\,\mu F + 2\,\mu F)(8\ V)^2 = 192\ \mu J$$

Although $U_i > U_f$, this is not a violation of the conservation of energy principle. The missing energy is transferred as thermal energy into the connecting wires and as radiated electromagnetic waves.

23.6 Exercises

Section 23.1 Capacitor and Capacitance

(1) A capacitor has a capacitance of 15 μF. How much charge must be removed to lower the potential difference between its conductors to 10 V?

(2) Two identical coins carry equal but opposite charges of magnitude 1.6 μC. The capacitance of this combination is 20 pF. What is the potential difference between the coins?

(3) A capacitor with a charge of magnitude 10^{-4} C has a potential difference of 50 V. What charge value is needed to produce a potential difference of 15 V?

Section 23.2 Calculating Capacitance

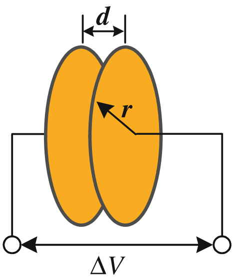
(4) A computer memory chip contains a large number of capacitors, each of which has a plate area $A = 20 \times 10^{-12}\,\text{m}^2$ and a capacitance of 50 f F (50 femtofarads). Assuming a parallel-plate configuration, find the order of magnitude of the separation distance d between the plates of such a capacitor.

(5) A parallel-plate capacitor has a plate area $A = 0.04\,\text{m}^2$ and a vacuum separation $d = 2 \times 10^{-3}$ m. A potential difference of 20 V is applied between the plates of the capacitor. (a) Find the capacitance of the capacitor. (b) Find the magnitude of the charge and charge density on the plates of the capacitor. (c) Find the magnitude of the electric field between the plates.

(6) An electric spark occurs if the electric field in air exceeds the value 3×10^6 V/m. Find the maximum magnitude of the charge on the plates of an air-filled parallel-plate capacitor of area $A = 30\,\text{cm}^2$ such that a spark is avoided.

(7) A parallel-plate capacitor has circular plates, each with a radius $r = 5$ cm. Assume a vacuum separation $d = 1$ mm exists between the plates, see Fig. 23.16. How much charge is stored on each plate of the capacitor when their potential difference has the value $\Delta V = 50$ V.

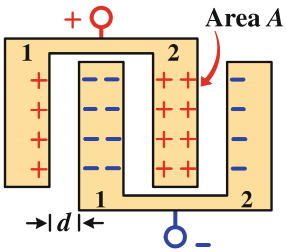
Fig. 23.16 See Exercise (7)



(8) Figure 23.17 shows a set of two parallel sheets of a conductor connected together to form one plate of a capacitor, while the second set is connected

together to form the other plate of the capacitor. Assume that the effective area of adjacent sheets is A and that the air separation is d. From the figure, confirm that the number of adjoining sheets of positive and negative charges is 3 and the capacitor has a capacitance $C = 3\epsilon_\circ A/d$.

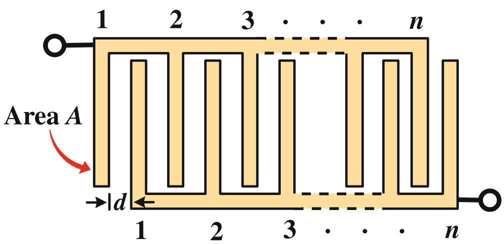
Fig. 23.17 See Exercise (8)



(9) If each set in Exercise 8 consists of n plates, see Fig. 23.18, then show that the capacitance of the capacitor will be given by:

$$C = \frac{(2n-1)\epsilon_\circ A}{d}$$

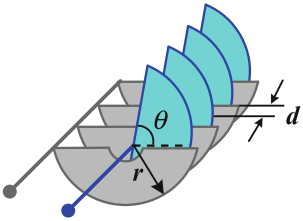
Fig. 23.18 See Exercise (9)



(10) A variable air capacitor used in radio tuning consists of a set of n fixed semi-circular plates, each of radius r, and located a distance d from a neighboring plate of an identical yet rotatable set, see Fig. 23.19. Show that when one set is rotated by an angle θ, the capacitance is:

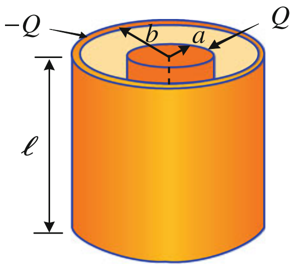
$$C = \frac{(2n-1)\epsilon_\circ(\pi - \theta)\, r^2}{2d}$$

Fig. 23.19 See Exercise (10)



(11) A coaxial cable of length $\ell = 5$ m consists of a solid cylindrical conductor surrounded by a cylindrical conducting shell. The inner conductor has a radius $a = 2.5$ mm and carries a charge Q, while the surrounding shell has a radius $b = 8.5$ mm and carries a charge $-Q$, see Fig. 23.20. Assume that $Q = +8 \times 10^{-8}$ C and that air fills the gap between the conductors. (a) What is the capacitance of this cable? (b) What is the magnitude of the potential difference between the two cylinders?

Fig. 23.20 See Exercise (11)



(12) An isolated spherical conductor carries a charge $Q = 4$ nC, see Fig. 23.21. The potential difference between the sphere and its surroundings is $\Delta V = 100$ V. What is the capacitance formed from the sphere and its surroundings?

Fig. 23.21 See Exercise (12)

(13) A capacitor consists of two concentric spheres of radii $a = 30$ cm and $b = 36$ cm, see Fig. 23.22. Assume the gap between the conductors is filled with air. (a) What is the capacitance of this capacitor? (b) How much charge is stored in the capacitor if the potential difference between the two spheres is $\Delta V = 50$ V?

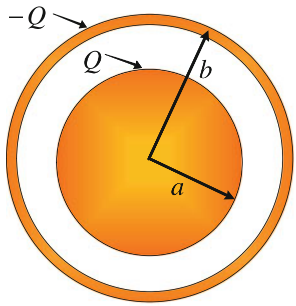


Fig. 23.22 See Exercise (13)

(14) Find the capacitance of Earth by assuming that the "missing second conducting sphere" has an infinite radius. The radius of Earth is $R = 6.37 \times 10^6$ m.

(15) A spherical drop of mercury has a capacitance of 2.78 fF. If two such drops combine into one, what would its capacitance be?

Section 23.3 Capacitors with Dielectrics

(16) Two parallel plates of area $A = 0.01$ m^2 are separated by a distance $d = 5 \times 10^{-3}$ m. The region between these plates is filled with a dielectric material of $\kappa = 3$, and the plates are given equal but opposite charges of 2 μC. (a) What is the capacitance of this capacitor? (b) Find the potential difference between the plates.

(17) An air-filled parallel-plate capacitor of 15 μF is connected to a 50 V battery; then the battery is removed. (a) Find the charge on the capacitor. (b) If the air is replaced with oil having $\kappa = 2.2$, find the new values of the capacitance and the potential difference between the plates.

(18) A parallel-plate capacitor has an area $A = 4$ cm^2. (a) Find the maximum stored charge on the capacitor if air fills the space between the plates. (b) Redo part (a) when paper is used instead of the air (use the dielectric strengths given in Table 23.1).

(19) The charged air capacitor shown in Fig. 23.23 is first placed at a pressure of 1 atm and found to have a potential difference $\Delta V = 10{,}376$ V. Then, the capacitor is placed in a vacuum chamber and the air is removed. The potential difference is found to rise to $\Delta V_\circ = 10{,}382$ V. Determine the dielectric constant of the air.

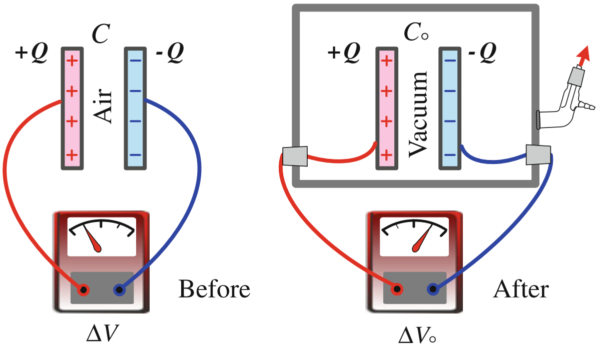


Fig. 23.23 See Exercise (19)

(20) A parallel-plate capacitor having an area $A = 0.2\,\text{m}^2$, and a plate separation $d = 1$ mm filled with air as an insulator, is connected to a battery that has a potential difference $\Delta V_\circ = 12$ V, see Fig. 23.24. While the battery is still connected to the capacitor, a sheet of glass ($\kappa = 4.5$) is inserted to fill the space between the plates, see the figure. (a) Determine both the initial capacitance ($C_\circ$) and the initial charge ($Q_\circ$), then find C and Q after inserting the glass. (b) If σ_i is the magnitude of the induced surface charge density on the glass and $\sigma_\circ$ is the magnitude of the charge density of the plates before the insertion of the glass, then show that:

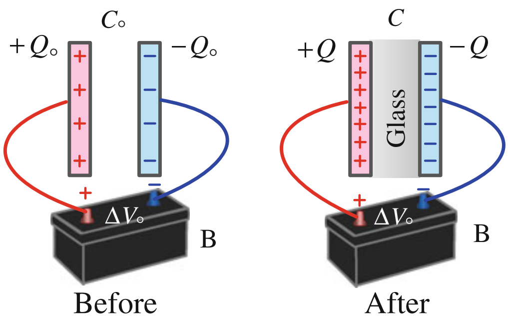
$$\sigma_i = (\kappa - 1)\sigma_\circ$$

(c) Find the values of σ_i and the induced electric field E_i.

Section 23.4 Capacitors in Parallel and Series

(21) Two capacitors, $C_1 = 2\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$, are connected in parallel to a battery that has a potential difference $\Delta V = 9$ V. (a) Find the equivalent capacitance of the combination. (b) Find the charge on each capacitor. (c) Find the potential difference across each capacitor.

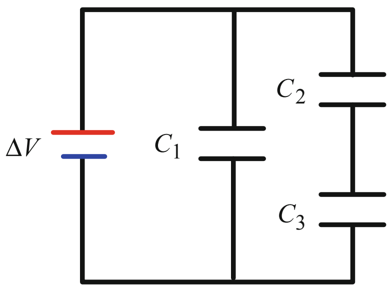
Fig. 23.24 See Exercise (20)



(22) The two capacitors of exercise 21 are now connected in series to the same battery (i.e. with a potential difference $\Delta V = 9$ V). (a) Find the equivalent capacitance of the combination. (b) Find the charge on each capacitor. (c) Find the potential difference across each capacitor.

(23) For the combination of capacitors shown in Fig. 23.25, assume that $C_1 = 1\,\mu$F, $C_2 = 2\,\mu$F, $C_3 = 3\,\mu$F, and $\Delta V = 6$ V. (a) Find the equivalent capacitance of the combination. (b) Find the charge on each capacitor. (c) Find the potential difference across each capacitor.

Fig. 23.25 See Exercise (23)



(24) Three capacitors, $C_1 = 6\,\mu$F, $C_2 = 4\,\mu$F, and $C_3 = 12\,\mu$F, are connected in four different ways, as shown in Fig. 23.26. In all configurations, the potential difference is 22 V. How many coulombs of charge pass from the battery to each combination?

(25) When the three capacitors $C_1 = 2\,\mu$F, $C_2 = 1\,\mu$F, and $C_3 = 4\,\mu$F are connected to a source of a potential difference ΔV, as shown Fig. 23.27, the charge Q_2 on C_2 is found to be 10 μC. (a) Find the values of the charges on the two capacitors C_1 and C_3. (b) Determine the value of ΔV.

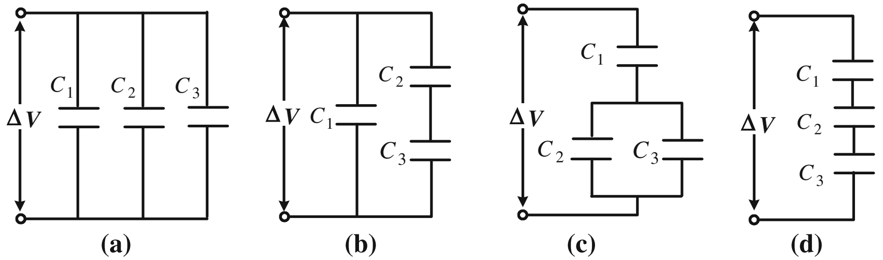
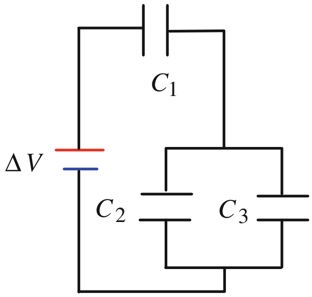


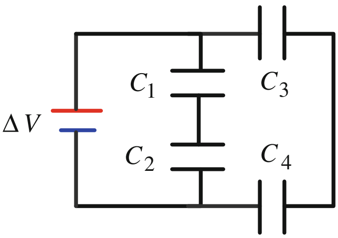
Fig. 23.26 See Exercise (24)

Fig. 23.27 See Exercise (25)



(26) For the circuit shown in Fig. 23.28, $C_1 = 3\,\mu$F, $C_2 = 6\,\mu$F, $C_3 = 6\,\mu$F, $C_4 = 12\,\mu$F, and $\Delta V = 12$ V. (a) Find the equivalent capacitance of the combination. (b) Find the potential difference across each capacitor.

Fig. 23.28 See Exercise (26)



(27) For each of the combinations shown in Fig. 23.29, find a formula that represents the equivalent capacitance between the terminals A and B.

(28) Assume that in Exercise 27, $C = 12\,\mu$F and $\Delta V_{BA} = 12$ V. For each combination, find the magnitude of the total charge that the source between A and B will distribute on the capacitors.

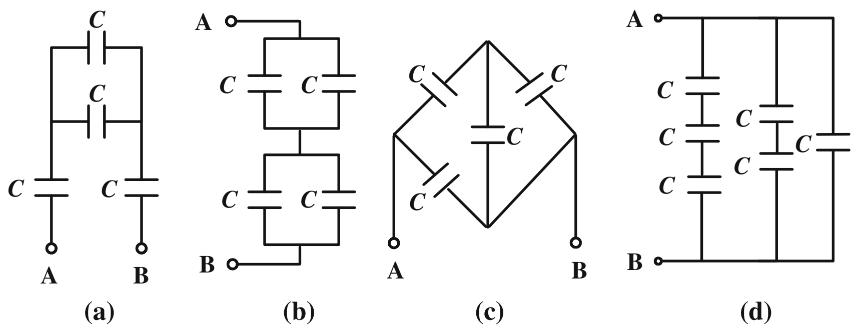


Fig. 23.29 See Exercise (27)

(29) Two capacitors, $C_1 = 25\ \mu\text{F}$ and $C_2 = 40\ \mu\text{F}$, are charged by being connected to batteries that have a potential difference $\Delta V = 50\ \text{V}$, see part (a) of Fig. 23.30. They are then disconnected from their batteries and connected to each other, with each positive plate connected to the other's negative plate; see part (b) of Fig. 23.30. (a) Find the equivalent capacitance between A and B. (b) What is the charge Q on the equivalent capacitor? (c) What is the potential difference ΔV_{BA} between A and B? (d) Find the final charge on each capacitor.

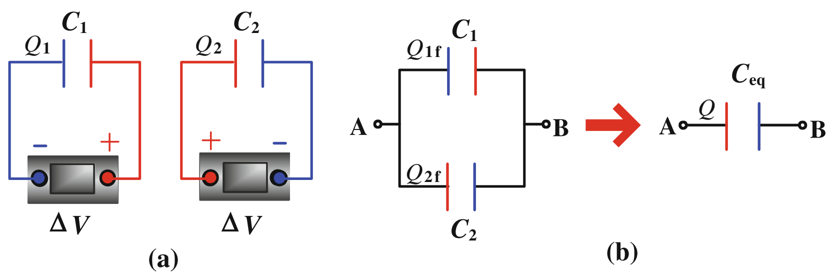
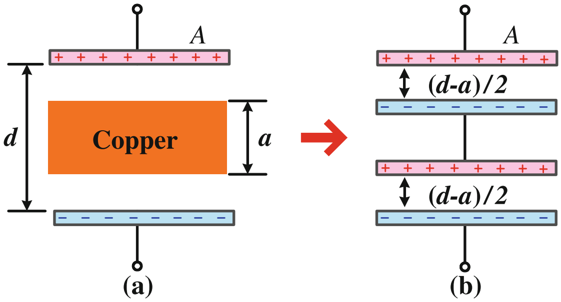


Fig. 23.30 See Exercise (29)

(30) A parallel-plate capacitor has an area A and separation d. A slab of copper of thickness a is inserted midway between the plates, see part (a) of Fig. 23.31. Show that the capacitor is equivalent to two capacitors in series, each having a plate separation $(d - a)/2$, as shown in part (b) of the figure, and show that the capacitance after inserting the slab is given by:

$$C = \frac{\epsilon_\circ A}{d - a}$$

Fig. 23.31 See Exercise (30)



(31) Show that the capacitance of the capacitor in Fig. 23.10b can be obtained by finding the equivalent capacitance of two capacitors in series, one capacitor with a dielectric of thickness a and the second an air-filled capacitor of thickness $d - a$.

(32) A parallel-plate capacitor of plate area A and separation d is filled in two different ways with two dielectrics κ_1 and κ_2 as shown in parts (a) and (b) of Fig. 23.32. Show that the capacitances of the two capacitors of parts (a) and (b) are:

$$C = \frac{\epsilon_\circ A}{d}\,\frac{\kappa_1 + \kappa_2}{2} \quad \text{and} \quad C = \frac{2\epsilon_\circ A}{d}\,\frac{\kappa_1\,\kappa_2}{\kappa_1 + \kappa_2} \quad \text{respectively,}$$

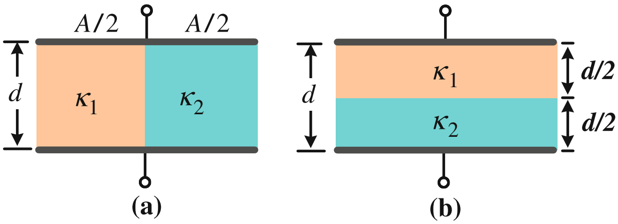


Fig. 23.32 See Exercise (32)

Section 23.6 Energy Stored in a charged Capacitor

(33) How much energy is stored in one cubic meter of air due to an electric field of magnitude 100 V/m?

(34) The two capacitors shown in Fig. 23.33 are uncharged when the switch S is open. Assume that $C_1 = 4\,\mu\text{F}$, $C_2 = 6\,\mu\text{F}$, and $\Delta V = 10\,\text{V}$. The two capacitors become fully charged when the switch S is closed. (a) Find the energy stored

in these two capacitors. (b) Does the stored potential energy in the equivalent capacitor equal the total stored energy in the two capacitors?

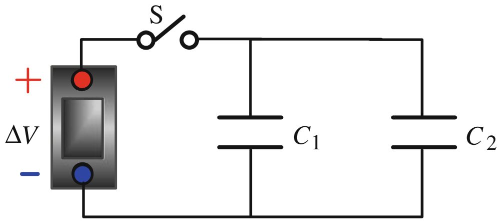


Fig. 23.33 See Exercise (34)

(35) Redo Example 23.9 when $C_1 = C_2 = 5\,\mu\text{F}$, and $\Delta V_i = 10\,\text{V}$. Does the initial and final stored potential energy remains the same?

(36) A capacitor is charged to a potential difference ΔV. How much should you increase ΔV so that the stored potential energy is increased by 20%?

(37) Calculate the electric field, the energy density, and the stored potential energy in the parallel-plate capacitor of Exercise 7.

(38) A parallel-plate capacitor has a capacitance of $4\,\mu\text{F}$ when a mica sheet with dielectric constant $\kappa = 5$ fills the space between the plates. The capacitor is charged by a battery that has a potential difference 50 V, and is later disconnected. How much work must be done to slowly pull the dielectric from the capacitor?

(39) For the circuit shown in Fig. 23.34, $C_1 = 2\,\mu\text{F}$, $C_2 = 3\,\mu\text{F}$, $C_3 = 6\,\mu\text{F}$, $C_4 = 1\,\mu\text{F}$, and $C_5 = 2\,\mu\text{F}$. (a) Find the potential difference between A and B needed to give C_3 a charge of $20\,\mu\text{C}$. (b) Under these considerations, what is the electric potential energy stored in the combination?

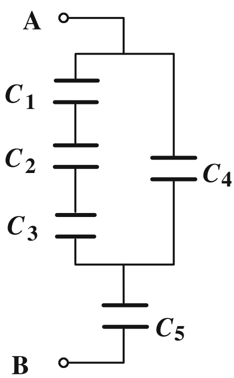


Fig. 23.34 See Exercise (39)

(40) Confirm the relationships shown in Fig. 23.35, where $\Delta V_\circ$ is shortened by $V_\circ$ and ΔV is shortened by V.

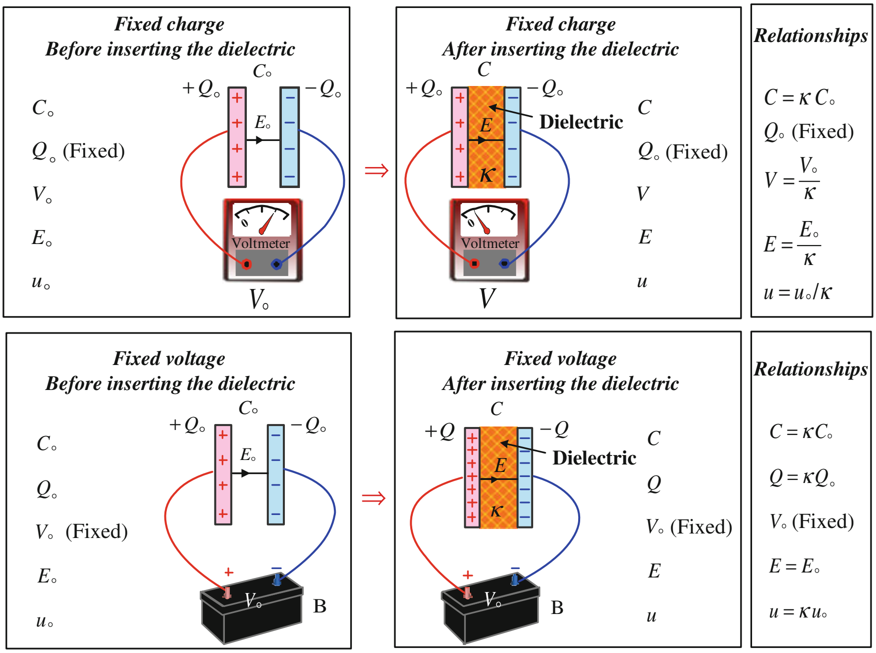


Fig. 23.35 See Exercise (40)