

In this chapter we analyze simple electric circuits that contain devices such as batteries, resistors, and capacitors in various combinations. We begin by introducing **steady-state** electric circuits and the concept of a constant rate of flow of electric charges, known as **direct current (dc)**. We also introduce **Kirchhoff's two rules**, which are used to simplify and analyze more complicated circuits. Finally, we consider circuits containing resistors and capacitors, in which currents can *vary* with time.

24.1 Electric Current and Electric Current Density

Electric Current

When there is a net flow of charge across any area, we say there is an **electric current** (or simply **current**) across that area. To maintain a continuous current, we must maintain a net force on the mobile charge in some way. The net force may result, for example, from an electrostatic field. We assume that an electric field \vec{E} is maintained within a conductor such that the charged particle q is acted on by a force $\vec{F} = q\vec{E}$. We refer to this force as the particle's *driving force*.

To define the current, we consider positive charges moving perpendicularly onto a surface area A as shown in Fig. 24.1.

Spotlight

The current I across an area A is defined as the *net charge* flowing perpendicularly to that area per unit time.

Thus, if a net charge ΔQ flows across an area A in a time Δt , the **average current** I_{av} across the area is:

$$I_{av} = \frac{\Delta Q}{\Delta t} \tag{24.1}$$

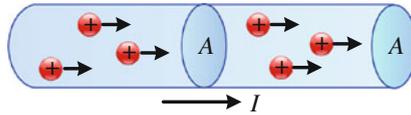


Fig. 24.1 Charged particles in motion perpendicular onto an area A . The current I represents the time rate of flow of charges and has by convention the direction of the motion of positive charges

When the rate of flow varies with time, we define the **instantaneous current** (or the **current**) I as:

$$I = \frac{dQ}{dt} \tag{24.2}$$

The SI unit of the current is **ampere** (abbreviated by A). That is:

$$1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}} \tag{24.3}$$

Thus, 1 A is equivalent to 1 C of charge passing through the surface area in 1 s. Small currents are more conveniently expressed in *milliamperes* ($1 \text{ mA} = 10^{-3} \text{ A}$) or *microamperes* ($1 \mu\text{A} = 10^{-6} \text{ A}$).

Currents can be due to positive charges, or negative charges, or both. In conductors, the current is due to the motion of only negatively charged free electrons (called *conduction electrons*). **By convention**, the direction of the current is the direction of the flow of positive charges. Therefore, the direction of the current is opposite to the direction of the flow of electrons, see Fig. 24.2b. A moving charge, positive or negative, is usually referred to as a mobile **charge carrier**.

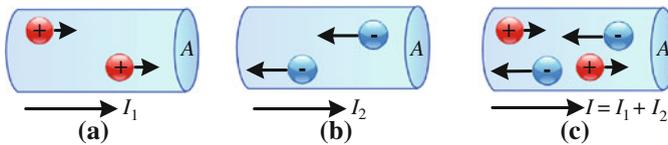


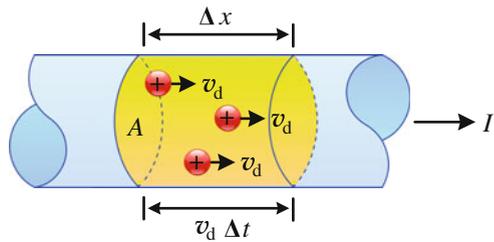
Fig. 24.2 Direction of current due to (a) positive charges, (b) negative charges, and (c) both positive and negative charges

Electric Current Density

The current across an area can be expressed in terms of the motion of the charge carriers. To achieve this we consider a portion of a cylindrical rod that has a cross-sectional area A , length Δx , and carries a *constant* current I , see Fig. 24.3. For convenience we consider positive charge carriers each having a charge q , and the number of carriers per unit volume in the rod is n . Therefore, in this portion, the number of carriers is $n A \Delta x$ and the total charge ΔQ is:

$$\Delta Q = (n A \Delta x) q \quad (24.4)$$

Fig. 24.3 A portion of a straight rod of uniform cross-sectional area A , carrying a constant current I . The mobile charge carriers are assumed to be positive and move with an average speed v_d



Suppose that all the carriers move with an average speed v_d (called the **drift speed**). Therefore, during a time interval Δt , all carriers must achieve a displacement $\Delta x = v_d \Delta t$ in the x direction. Now, let us choose Δt such that the carriers in the cylindrical portion move through a displacement whose magnitude is equal to the length of the cylinder, see Fig. 24.3. During such a time interval, all the charge carriers in this cylindrical portion must pass through the circular area A at the right end. Accordingly, we write the last relation as:

$$\Delta Q = (n A v_d \Delta t) q \quad (24.5)$$

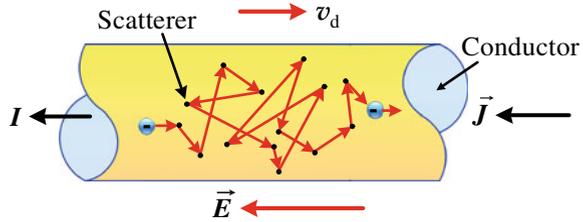
Therefore, the current $I = \Delta Q / \Delta t$ in the rod will be given by:

$$I = n q v_d A \quad (24.6)$$

The charge carriers in a solid conductor are all free electrons. If the conductor is isolated, these electrons move with speeds of the order of 10^6 m/s, and because of their collisions with the scatterers (atoms or molecules in the conductor), they move randomly in all directions. This results in a *zero drift velocity* and hence no *net*

charge transport, which means *zero current*. When an electric field \vec{E} is established across the conductor, this field exerts an electric force $\vec{F} = -e\vec{E}$ on each electron, producing a current. Of course, the electrons do not move in a straight line along the conductor, but their resultant motion is complicated and zigzagged, see Fig. 24.4. Regardless of the collisions of these electrons, they move slowly along the conductor in a direction opposite to \vec{E} with a drift velocity \vec{v}_d , see Fig. 24.4.

Fig. 24.4 A schematic representation of the random zigzag motion and the drift of a free electron with an average speed v_d in a conductor, due to the effect of an external electric field \vec{E}



The **current density** J is defined as the current per unit area, i.e.:

$$J = \frac{I}{A} \quad (24.7)$$

Using the relation $I = nq v_d A$, we get:

$$J = nq v_d \quad (24.8)$$

where the SI unit of the current density is A/m^2 . Equation 24.8 is valid only if J is uniform and the direction of I is perpendicular to the cross-sectional area A . Generally, the current density is a **vector** quantity that has the direction of $q\vec{v}_d$, for both signs of q ; that is:

$$\vec{J} = nq\vec{v}_d \quad (24.9)$$

The amount of current that passes through an element of area dA , can be written as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the vector area of the element. The current that passes throughout the entire area A is thus:

$$I = \int \vec{J} \cdot d\vec{A} \quad (24.10)$$

If the current density is uniform across the area and parallel to $d\vec{A}$, then this equation leads to Eq. 24.7.

Example 24.1

Estimate the drift speed of the conduction electrons in a copper wire that is 2 mm in diameter and carries a current of 1 A. Comment on your result. The density of copper is $8.92 \times 10^3 \text{ kg/m}^3$.

[Hint: Assume that each copper atom contributes one free conduction electron to the current.]

Solution: To get the drift speed v_d , we need to find the free-electron density n . To get n , we need to know the volume occupied by one kmol of copper. From the periodic table of elements, see Appendix C, the molar mass of copper is $M(\text{Cu}) = 63.546 \text{ kg/kmol}$. Recall that the mass of one kmol of $^{63.5}\text{Cu}$ contains Avogadro's number of atoms ($N_A = 6.022 \times 10^{26} \text{ atoms/kmol}$). Thus:

$$\text{Volume of 1 kmol} = \frac{\text{Mass of 1 kmol}}{\text{Density}} \Rightarrow V = \frac{M}{\rho}$$

$$\text{Number of copper atoms/m}^3 = \frac{\text{Avogadro's number}}{\text{Volume of 1 kmol}} \Rightarrow n = \frac{N_A}{V} = \frac{N_A \rho}{M}$$

Therefore:

$$\begin{aligned} n &= \frac{N_A \rho}{M} = \frac{(6.022 \times 10^{26} \text{ atoms/kmol})(8.92 \times 10^3 \text{ kg/m}^3)}{63.546 \text{ kg/kmol}} \\ &= 8.45 \times 10^{28} \text{ atoms/m}^3 \end{aligned}$$

Since the density of free-electrons is equal to the density of copper atoms, then we use Eq. 24.6 to find the drift speed as follows:

$$\begin{aligned} v_d &= \frac{I}{neA} = \frac{1 \text{ C/s}}{(8.45 \times 10^{28} \text{ electrons/m}^3)(1.6 \times 10^{-19} \text{ C})(\pi \times (10^{-3} \text{ m})^2)} \\ &= 2.35 \times 10^{-5} \text{ m/s} = 8.46 \text{ cm/h} \quad (\text{Very small speed}) \end{aligned}$$

You might ask why, even though v_d is so small, that regular light bulbs light up very quickly when one turns on its circuit switch? The answer is that the electric field travels along the connecting wires of the circuit at almost the speed of light, so electrons everywhere in the wires all begin to *drift at once* with a small drift speed.

Example 24.2

One end of the copper wire in example 1 is welded to one end of an aluminum wire with a 4 mm diameter. The composite wire carries a steady current equal to that of Example 24.1 (i.e. $I = 1$ A). (a) What is the current density in each wire? (b) What is the value of the drift speed v_d in the aluminum? [Aluminum has one free electron per atom and density 2.7×10^3 kg/m³]

Solution: (a) Except near the junction, the current density in a copper wire of radius $r_{\text{Cu}} = 1$ mm and aluminum wire of radius $r_{\text{Al}} = 2$ mm are:

$$J_{\text{Cu}} = \frac{I}{A_{\text{Cu}}} = \frac{I}{\pi r_{\text{Cu}}^2} = \frac{1 \text{ A}}{\pi \times (10^{-3} \text{ m})^2} = 3.18 \times 10^5 \text{ A/m}^2$$

$$J_{\text{Al}} = \frac{I}{A_{\text{Al}}} = \frac{I}{\pi r_{\text{Al}}^2} = \frac{1 \text{ A}}{\pi \times (2 \times 10^{-3} \text{ m})^2} = 7.96 \times 10^4 \text{ A/m}^2$$

(b) From the periodic table of elements, see Appendix C, the molar mass of aluminum is $M(\text{Al}) = 26.98$ kg/kmol. As in Example 24.1, we find:

$$n = \frac{N_A \rho}{M} = \frac{(6.022 \times 10^{26} \text{ atoms/kmol})(2.7 \times 10^3 \text{ kg/m}^3)}{26.98 \text{ kg/kmol}}$$

$$= 6.03 \times 10^{28} \text{ atoms/m}^3$$

$$v_d = \frac{I}{neA} = \frac{1 \text{ C/s}}{(6.03 \times 10^{28} \text{ electrons/m}^3)(1.6 \times 10^{-19} \text{ C})(\pi \times (2 \times 10^{-3} \text{ m})^2)}$$

$$= 8.25 \times 10^{-6} \text{ m/s} = 2.97 \text{ cm/h}$$

24.2 Ohm's Law and Electric Resistance

As a result of maintaining a potential difference ΔV across a conductor, an electric field \vec{E} and a current density \vec{J} are established in the conductor. For materials with electrical properties that are the same in all directions (isotropic materials), the electric field is found to be proportional to the current density. That is:

$$\vec{E} = \rho \vec{J} \quad (\text{Ohm's law}) \quad (24.11)$$

where the constant ρ^1 is called the **resistivity** of the conductor. Materials that obey this relation are said to obey **Ohm's law**:

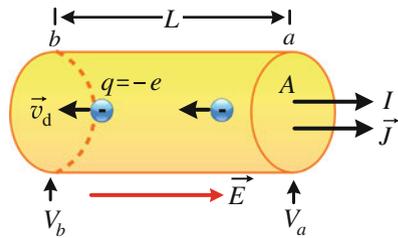
¹ Not to be confused with ρ referring to mass density or charge density.

Spotlight

For many materials and most metals, the ratio of the magnitude of the electric field to the magnitude of the current density is a constant and does not depend on the electric field producing the current.

Since it is difficult to measure \vec{E} and \vec{J} directly, we need to put Ohm's law into a more practical form. This can be obtained by considering a portion of a straight conductor that has a uniform cross-sectional area A and length L , as shown in Fig. 24.5. In addition, a potential difference $\Delta V = V_b - V_a$ between the ends of the conductor (denoted by a and b) will create a *straight* electric field and current, as also shown in Fig. 24.5. Since charge carriers in conductors are electrons, they will drift from face a to face b , against the field \vec{E} .

Fig. 24.5 A potential difference $\Delta V = V_b - V_a$ across a conductor of cross-sectional area A and length L sets up a field \vec{E} and current I



Recall that for uniform electric fields we have:

$$\Delta V = E L \quad (24.12)$$

Using this relation to eliminate E from the scalar form of Eq. 24.11, we get:

$$\frac{\Delta V}{L} = \rho J \quad (24.13)$$

Also, using $J = I/A$, the potential difference ΔV can be written as:

$$\Delta V = \left(\rho \frac{L}{A} \right) I \quad (24.14)$$

The quantity in brackets is called the **electrical resistance** (or simply **resistance**) of the conductor and is denoted by the symbol R ; that is:

$$R = \rho \frac{L}{A} \Rightarrow R \propto \rho \quad (24.15)$$

We can define the resistance R as a proportionality constant to the relation $\Delta V \propto I$ and write the equivalent Ohm's law as:

$$\Delta V = IR \quad \left\{ \begin{array}{l} \text{Equivalent form} \\ \text{of Ohm's law} \end{array} \right\} \quad (24.16)$$

The SI unit of resistance is **ohm** (abbreviated by Ω). That is:

$$1 \Omega = \frac{1 \text{ V}}{1 \text{ A}} \quad (24.17)$$

This means that if one applies a potential difference of 1 V across a conductor and this causes 1 A to flow, then the resistance of the conductor is 1 Ω . Note that according to Eq. 24.15, the SI unit of resistivity is ohm-meter ($\Omega \cdot \text{m}$). Also, since $\Delta V = V_b - V_a$, we note that the *direction of the current* is in the direction of *decreasing potential*.

The inverse of resistivity is called the conductivity σ , thus:

$$\sigma = \frac{1}{\rho} \quad (24.18)$$

where the SI unit of σ is $(\Omega \cdot \text{m})^{-1}$. The resistance of a conductor can also be written in terms of the conductivity as follows:

$$R = \frac{1}{\sigma} \frac{L}{A} \quad (24.19)$$

Equations 24.15 and 24.19 hold true only for isotropic conductors. Additionally, the resistance in Eq. 24.15 depends on the geometry of the resistor through the length L , area A , and resistivity ρ , which is a constant for a specific metallic conductor (assuming a constant temperature).

A material obeying Ohm's law is called an *ohmic* material or a *linear* material. If a material does not obey Ohm's law, the material is called a *non-ohmic* or a *nonlinear material*.

Variation of Resistance with Temperature

The variation of resistivity with temperature is mostly linear over a broad range. Since $R \propto \rho$, then for most engineering purposes a good empirical linear approximation for ρ and R can be written as:

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad \text{or} \quad R = R_0[1 + \alpha(T - T_0)] \quad (24.20)$$

where ρ is the resistivity at temperature T (in degrees Celsius), ρ_o is the resistivity at a reference temperature T_o (usually selected to be 20°C), and α is the **temperature coefficient of resistivity**. The same applies for the resistance. The coefficient α is selected such that Eq. 24.20 matches best with experimental measurements for the selected range of temperatures. From Eq. 24.20, we find that:

$$\alpha = \frac{1}{\rho_o} \frac{\Delta\rho}{\Delta T} = \frac{1}{R_o} \frac{\Delta R}{\Delta T} \quad \text{with} \quad \begin{cases} \Delta\rho = \rho - \rho_o \\ \Delta R = R - R_o \\ \Delta T = T - T_o \end{cases} \quad (24.21)$$

Table 24.1 lists the resistivity ρ and the temperature coefficient of resistivity α for some materials at 20°C .

Table 24.1 The resistivity and temperature coefficient of resistivity for various materials at 20°C

Material	Resistivity ρ ($\Omega\cdot\text{m}$)	Temperature coefficient of resistivity α [$(\text{C}^\circ)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.98×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome ^a	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	$10^{10}\text{--}10^{14}$	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Fused quartz	75×10^{16}	

^a A nickel–chromium alloy commonly used in heating elements

Most electric circuits use elements called **resistors** to control the current flowing through the circuit. Values of the resistance are normally indicated by color-coding as shown in Tables 24.2 and 24.3.

Table 24.2 Color-coding for resistors

Color	Number	Multiplier
Black	0	1
Brown	1	10 ¹
Red	2	10 ²
Orange	3	10 ³
Yellow	4	10 ⁴
Green	5	10 ⁵
Blue	6	10 ⁶
Violet	7	10 ⁷
Gray	8	10 ⁸
White	9	10 ⁹
Gold	–	10 ⁻¹
Silver	–	10 ⁻²

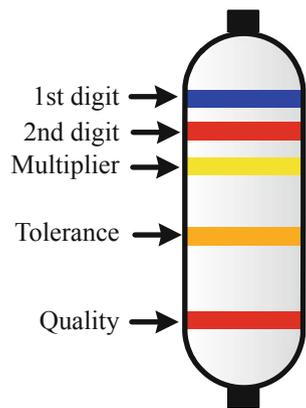
Table 24.3 Tolerance-coding

Color	Number	Multiplier	Tolerance
Gold	–	10 ⁻¹	5%
Silver	–	10 ⁻²	10%
Colorless	–	–	20%

How to Read the Color-coding

- First find the tolerance band; it will typically be gold (5%) or silver (10%), and sometimes colorless (20%), see the example shown in Fig. 24.6. In this example, the color is Gold, so 5% tolerance.

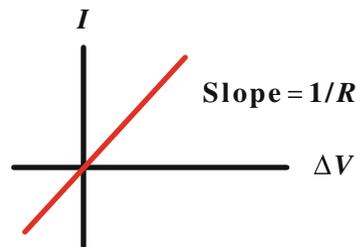
Fig. 24.6



- Starting from the other end, identify the first band, write down the number associated with that color; in this example Blue is '6'.
- Now read the next color, in this example it is Red, so write down a '2' next to the six (You should have '62' so far).
- Now read the third color, which indicates the multiplier exponent band, and write that down as the power of ten for the multiplier of the resistance value. In this example the multiplier is Yellow which represents 'four', so we get ' $62 \times 10^4 \Omega$ '.
- If the resistor has one extra band past the tolerance band, it is a quality band. Read the number as the % Failure rate per 1,000 h. In this example it is Red, so that we can expect a 2% failure rate per 1,000 h.

All Ohmic resistors have a linear-potential-difference relationship over a broad band of applied potential differences. The slope of the I versus ΔV curve in the linear region yields a value for $1/R$, see Fig. 24.7.

Fig. 24.7



Example 24.3

At 20°C , a copper wire has a diameter of 4 mm, a length of 10 m, a resistivity of $1.7 \times 10^{-8} \Omega\cdot\text{m}$, a temperature coefficient of resistivity of $3.9 \times 10^{-3} (\text{C}^\circ)^{-1}$, and carries a current of 1 A. (a) What is the current density in the wire? (b) What is the magnitude of the electric field applied to the wire? (c) What is the potential difference between the two ends of the wire? (d) What is the resistance of the wire? (e) When the wire is used in a thermometer for measuring the melting point of indium, the resistance calculated in part (d) increases to 0.0207Ω . Find the melting point temperature of indium.

Solution: (a) The current density in a copper wire of radius 2 mm is:

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1 \text{ A}}{\pi \times (2 \times 10^{-3} \text{ m})^2} = 7.96 \times 10^4 \text{ A/m}^2$$

(b) From Eq. 24.11, the electric field is given by:

$$E = \rho J = (1.7 \times 10^{-8} \Omega \cdot \text{m})(7.96 \times 10^4 \text{ A/m}^2) = 1.353 \times 10^{-3} \text{ V/m}$$

(c) Using Eq. 24.12, the potential difference will be given by:

$$\Delta V = EL = (1.353 \times 10^{-3} \text{ V/m})(10 \text{ m}) = 1.353 \times 10^{-2} \text{ V}$$

(d) From Eq. 24.15, the resistance of the wire is:

$$R = \rho \frac{L}{A} = 1.7 \times 10^{-8} \Omega \cdot \text{m} \frac{10 \text{ m}}{\pi \times (2 \times 10^{-3} \text{ m})^2} = 0.0135 \Omega$$

(e) Solving Eq. 24.21 for ΔT and then using: (1) the calculated resistance $R_o = 0.0135 \Omega$ at the reference temperature $T_o = 20^\circ \text{C}$, (2) the value of α , and (3) the final resistance $R = 0.0207 \Omega$, we obtain:

$$\Delta T = \frac{\Delta R}{\alpha R_o} = \frac{R - R_o}{\alpha R_o} = \frac{0.0207 - 0.0135 \Omega}{[3.9 \times 10^{-3} (\text{C}^\circ)^{-1}](0.0135 \Omega)} = 136.8 \text{ C}^\circ$$

Since $T_o = 20^\circ \text{C}$, we find that the melting point of indium is:

$$T = T_o + \Delta T = 20^\circ \text{C} + 136.8 \text{ C}^\circ = 156.8^\circ \text{C}$$

Example 24.4

A cylindrical shell of length $L = 20 \text{ cm}$ is made of aluminum and has an inner radius $a = 2 \text{ mm}$ and an outer radius $b = 4 \text{ mm}$, see Fig. 24.8. Assume that the shell has a uniform current density $J = 2 \times 10^5 \text{ A/m}^2$ in the direction of the wire's length. (a) What is the current through the shell? (b) What is the resistance of the shell and the potential difference ΔV ?

Solution: (a) Since the current density is uniform across any plane perpendicular to the length of the shell, we can use the relation $I = JA$ to find the current. First, we calculate the cross-sectional area of the shell as follows:

$$\begin{aligned} A &= \pi b^2 - \pi a^2 = \pi[b^2 - a^2] \\ &= \pi[(4 \times 10^{-3} \text{ m})^2 - (2 \times 10^{-3} \text{ m})^2] = 3.77 \times 10^{-5} \text{ m}^2 \end{aligned}$$

Then, we use this result to find I as follows:

$$I = JA = (2 \times 10^5 \text{ A/m}^2)(3.77 \times 10^{-5} \text{ m}^2) = 7.54 \text{ A}$$

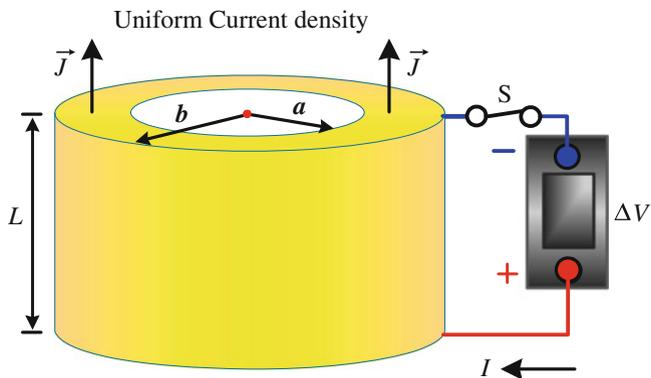


Fig. 24.8

(b) From Table 24.1, the resistivity of aluminum is $2.82 \times 10^{-8} \Omega \cdot \text{m}$. We use Eq. 24.15 to find the resistance of the shell, and then use Eq. 24.16 to find the potential difference ΔV as follows:

$$R = \rho \frac{L}{A} = 2.82 \times 10^{-8} \Omega \cdot \text{m} \frac{0.2 \text{ m}}{3.77 \times 10^{-5} \text{ m}^2} = 1.5 \times 10^{-4} \Omega$$

$$\Delta V = IR = (7.54 \text{ A})(1.5 \times 10^{-4} \Omega) = 1.13 \times 10^{-3} \text{ V}$$

Example 24.5

A conducting rod of radius $a = 2 \text{ mm}$ is concentric with a conducting cylindrical shell that has a radius $b = 4 \text{ mm}$ and length $L = 2.94 \text{ cm}$, see Fig. 24.9a. The space between the rod and the shell is tightly packed with silicon of resistivity $\rho = 640 \Omega \cdot \text{m}$. A battery of potential difference $\Delta V = 12 \text{ V}$ is connected in such a way that the current through the silicon flows in the radial direction. (a) Find the resistance of the silicon between the rod and the shell. (b) Find the radial current in the circuit. (c) Find the radial current density and electric field at the inner and outer surfaces of the silicon.

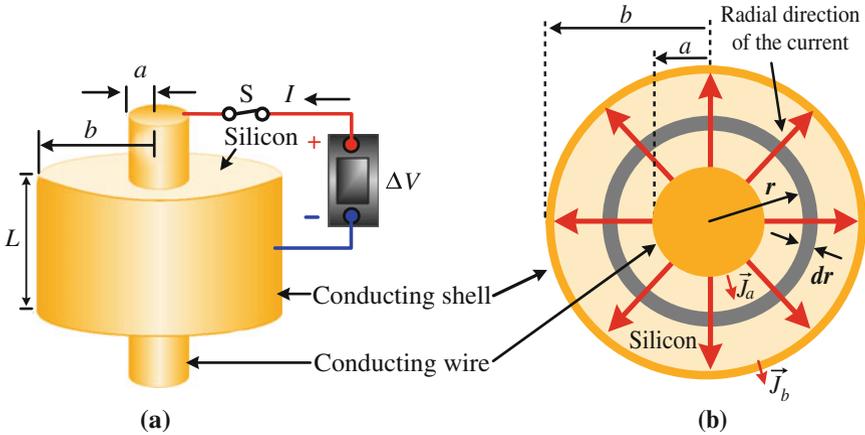


Fig. 24.9

Solution: (a) Cylindrical symmetry of the silicon suggests a radial flow of the current density \vec{J} . Equation 24.15 cannot be used directly because the cross section through which the charge travels varies from $2\pi aL$ (at the inner cylindrical face) to $2\pi bL$ (at the outer cylindrical face). Therefore, we consider a cylindrical silicon shell element of an inner radius r , height L , face area $A = 2\pi rL$, and thickness dr , see Fig. 24.9b. This shell element has a resistance dR . In this case, Eq. 24.15 will take the following form:

$$dR = \rho \frac{dr}{2\pi rL}$$

To find the total resistance across the entire silicon, we must integrate the previous expression from $r = a$ to $r = b$. Thus:

$$R = \int_a^b dR = \int_a^b \rho \frac{dr}{2\pi rL} = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$$

Now, substituting with the given values, we get:

$$R = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right) = \frac{640 \Omega \cdot \text{m}}{2\pi (2.94 \times 10^{-2} \text{ m})} \ln\left(\frac{4 \text{ mm}}{2 \text{ mm}}\right) = 2.4 \times 10^3 \Omega$$

(b) Knowing the resistance R and the potential difference ΔV , we use Ohm's law given by Eq. 24.16 to find the total current in the silicon (which is the current in the circuit) as follows:

$$I = \frac{\Delta V}{R} = \frac{12 \text{ V}}{2.4 \times 10^3 \Omega} = 5 \times 10^{-3} \text{ A} = 5 \text{ mA}$$

(c) At the inner and outer faces of the silicon, namely $2\pi aL$ and $2\pi bL$, respectively, we use Eq. 24.7 to find the corresponding current density as follows:

$$J_a = \frac{I}{A_a} = \frac{I}{2\pi aL} = \frac{5 \times 10^{-3} \text{ A}}{2\pi(2 \times 10^{-3} \text{ m})(2.94 \times 10^{-2} \text{ m})} = 13.53 \text{ A/m}^2$$

$$J_b = \frac{I}{A_b} = \frac{I}{2\pi bL} = \frac{5 \times 10^{-3} \text{ A}}{2\pi(4 \times 10^{-3} \text{ m})(2.94 \times 10^{-2} \text{ m})} = 6.77 \text{ A/m}^2$$

Finally, we use Ohm's law given by Eq. 24.11 to find the corresponding electric fields at the inner and outer faces of the silicon as follows:

$$E_a = \rho J_a = (640 \Omega \cdot \text{m})(13.53 \text{ A/m}^2) = 8.659 \times 10^3 \text{ V/m}$$

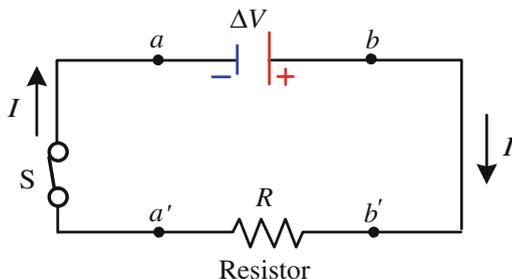
$$E_b = \rho J_b = (640 \Omega \cdot \text{m})(6.77 \text{ A/m}^2) = 4.333 \times 10^3 \text{ V/m}$$

24.3 Electric Power

When a battery is used to establish an electric current in a light bulb, the battery transforms its stored chemical energy to kinetic energy of the electrons. These electrons flow through the filament of the light bulb, and result in an increase in the temperature of the filament. It is important to calculate the rate of this energy transfer.

Figure 24.10 shows a battery of potential difference ΔV connected to a simple circuit (our system) containing a resistor of resistance R . The resistor is usually represented by the symbol $\text{---}\text{---}\text{---}$. Unless noted otherwise, we assume that the connecting wires have zero resistance.

Fig. 24.10 A simple circuit containing one battery and one resistor



Now, imagine a positive charge dQ flowing clockwise from point a through the battery and the resistor, and back to the same point a . In a time interval dt a quantity of charge dQ enters point a , and an equal quantity leaves point b . Thus, the electric potential energy of the system *increases* by the amount $dU = dQ \Delta V$, see Eq. 22.18, while the stored chemical potential energy of the battery *decreases* by the same amount. On the other hand, as the charge enters the resistor at b' and an equal quantity leaves a' (which is identical to a) over the same time dt , the system *loses* this energy through collisions with the molecules of the resistor. The net result is that some of the chemical energy of the battery has been delivered to the resistor as internal energy associated with molecular vibration (rise in temperature). This rise in temperature will ultimately transfer to the surroundings through thermal radiation.

The rate at which the system loses energy as the charges pass through the resistor is:

$$\frac{dU}{dt} = \frac{dQ \Delta V}{dt} = \frac{dQ}{dt} \Delta V = I \Delta V \quad (24.22)$$

where I is the current. This rate is equal to the rate at which the resistor *gains* internal energy, and is defined as the power P :

$$P = I \Delta V \quad (24.23)$$

Using the relation $\Delta V = IR$ for a resistor of resistance R , the electric power P delivered in the resistor can be written in the following form:

$$P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R} \quad (24.24)$$

Because $P = I \Delta V$, the same amount of power P can be transported either at high I and low ΔV , or at low I and high ΔV .

Example 24.6

A 220 V potential difference is maintained across an electric heater that is made from a nichrome wire of resistance 20Ω . (a) Find the current in the wire and the power rating of the heater. (b) At an estimated price of 0.35 LE (Egyptian pound) per kilowatt-hour of electricity, what is the cost of operating the heater for 2 h?

Solution: (a) Using $\Delta V = IR$, we get:

$$I = \frac{\Delta V}{R} = \frac{220 \text{ V}}{20 \Omega} = 11 \text{ A}$$

Using the power expression $P = I^2 R$, we find that:

$$P = I^2 R = (11 \text{ A})^2 (20 \Omega) = 2,420 \text{ W}$$

(b) The amount of energy transferred in time Δt is $P \Delta t$. Thus:

$$P \Delta t = (2,420 \text{ W})(2 \text{ h}) = 4,840 \text{ Wh} = 4.84 \text{ kWh}$$

If energy is purchased at 35 piaster per kilowatt-hour then the cost is:

$$\text{Cost} = (4.84 \text{ kWh})(0.35 \text{ LE/kWh}) = 1.69 \text{ LE}$$

24.4 Electromotive Force

We previously introduced the battery as a device that produces a potential difference and causes charges to move. In fact, it is a device that works as an *energy converter*. A battery is often called a *source of electromotive force* or, a *source of emf* (this unfortunate historical name describes a potential difference in volts, but not a force).

Spotlight

The emf \mathcal{E} of a battery is the maximum possible potential difference that the battery can provide between its terminals, usually the voltage at zero current.

Figure 24.11a shows a device (a battery) with an emf \mathcal{E} that is used in a simple circuit containing a resistor of resistance R . The battery keeps one terminal (labeled with the sign $+$) at a higher electric potential than the other (labeled with the sign $-$). Therefore, within the battery, the *conventional positive charge carriers* move from a region of low electric potential (at the negative terminal) to a region of higher electric potential (at the positive terminal).

Because a real battery is made of matter, there is a resistance against the flow of charge within the battery. This resistance is called the battery's **internal resistance** and is usually denoted by r . For an *ideal battery* with *zero internal resistance*, the

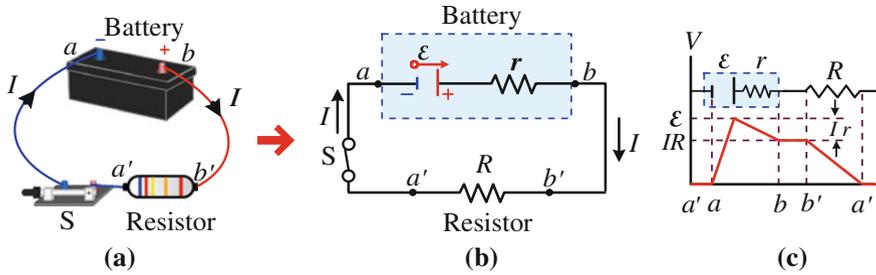


Fig. 24.11 (a) A simple circuit containing a resistor connected to a battery. (b) A circuit diagram of a source of emf \mathcal{E} (the battery) of internal resistance r , connected to a resistor of resistance R . (c) Graphical representation of the electric potential at different points

potential difference between its terminals is equal to its emf \mathcal{E} (directed from the $-$ terminal to the $+$ terminal). For *real batteries*, this is not the case.

We now consider the circuit diagram in Fig. 24.11b, which is the same as the *real* emf device of Fig. 24.11a, except we represent the battery with a dashed rectangular box containing an *ideal* emf \mathcal{E} in series with an internal resistance r . Let us start at point a (where the potential is V_a), and move *clockwise* to point b (where the potential is V_b), and measure the electric potential at different locations. When we move from the negative terminal to the positive terminal, the potential *increases* by the amount of the emf \mathcal{E} . However, as we move through the internal resistance r in the *direction of the current* I , the potential *drops* by an amount Ir . Thus, the potential difference between the terminals of the battery $\Delta V = V_b - V_a$ is:

$$\Delta V = \mathcal{E} - Ir \quad (\Delta V = \mathcal{E} \text{ for an open-circuit}) \tag{24.25}$$

We always assume that the wires in the circuit have no resistance, unless otherwise indicated. This means that the potentials of points a and a' are the same. The same applies to points b and b' . Thus:

$$V_b - V_a = V_{b'} - V_{a'} = \Delta V \tag{24.26}$$

But according to Ohm’s law, given by Eq. 24.16, $V_{b'} - V_{a'}$ must equal IR . Thus, $V_b - V_a = V_{b'} - V_{a'} = IR$. Combining this expression with Eq. 24.25, we find that:

$$\mathcal{E} = IR + Ir \tag{24.27}$$

Solving for the current, we get:

$$I = \frac{\mathcal{E}}{R + r} \quad (24.28)$$

Note that the current I depends on the resistance R of the external resistor (which is called the load) and the internal resistance r of the battery. Since $R \gg r$ in most circuits, we can usually neglect r .

Example 24.7

A device is connected to a battery that has an emf $\mathcal{E} = 9 \text{ V}$ and internal resistance $r = 0.02 \Omega$. Find the current in the circuit and the terminal voltage of the battery when the device is a: (a) light bulb that has a resistance $R = 4 \Omega$, see Fig. 24.12a. (b) conducting wire having zero resistance, i.e. the battery is short circuited by this conductor, see Fig. 24.12b.

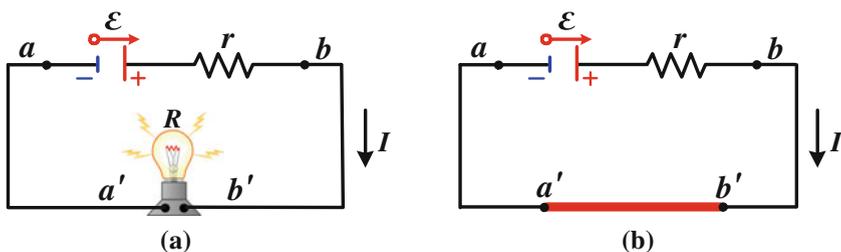


Fig. 24.12

Solution: (a) Equation 24.28 gives us the value of the current as:

$$I = \frac{\mathcal{E}}{R + r} = \frac{9 \text{ V}}{4 \Omega + 0.02 \Omega} = 2.24 \text{ A}$$

From Eq. 24.25, the terminal voltage of the battery will be given by:

$$\Delta V = \mathcal{E} - Ir = 9 \text{ V} - (2.24 \text{ A})(0.02 \Omega) = 8.96 \text{ V}$$

(b) When we use a conducting wire, it is as if we have a device of $R = 0$. This results in a current and terminal voltage of the battery as follows:

$$I = \frac{\mathcal{E}}{r} = \frac{9 \text{ V}}{0.02 \Omega} = 450 \text{ A}$$

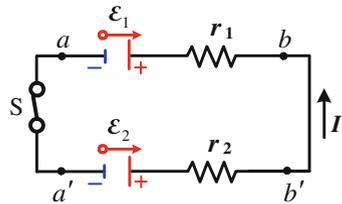
$$\Delta V = \mathcal{E} - Ir = 9 \text{ V} - (450 \text{ A})(0.02 \Omega) = 0$$

Such large values for the current I would result in a very quick depletion of the battery as all of its stored energy would be quickly transferred to the conducting wire in the form of heat energy. The term “short circuit” is applied to such cases, and they can cause fire or burns.

Example 24.8

A battery that has an emf $\mathcal{E}_1 = 9 \text{ V}$ and internal resistance $r_1 = 0.02 \Omega$ is connected to a second battery of $\mathcal{E}_2 = 12 \text{ V}$ and $r_2 = 0.04 \Omega$, such that their like terminals are connected, see Fig. 24.13. Find the current in the circuit and the terminal voltage across each battery.

Fig. 24.13



Solution: The two batteries are oppositely directed around the circuit. Since $\mathcal{E}_2 > \mathcal{E}_1$, then the net emf \mathcal{E}_{net} in this circuit will be in the counterclockwise direction, i.e.:

$$\mathcal{E}_{\text{net}} = \mathcal{E}_2 - \mathcal{E}_1 = 12 - 9 \text{ V} = 3 \text{ V} \quad (\text{Counterclockwise direction})$$

Consequently, the current I in this circuit will also be in the counterclockwise direction as indicated in Fig. 24.13. This current is opposite to the *discharging* current that the $\mathcal{E}_1 = 9 \text{ V}$ battery should produce when connected to circuits containing only resistors. Actually, this current will *charge* the $\mathcal{E}_1 = 9 \text{ V}$ battery.

The total resistance of this circuit is only due to the presence of the internal resistances r_1 and r_2 of the two batteries. Therefore, Eq. 24.28 gives us the value of the current as follows:

$$I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + r_2} = \frac{12 \text{ V} - 9 \text{ V}}{0.02 \Omega + 0.04 \Omega} = \frac{3 \text{ V}}{0.06 \Omega} = 50 \text{ A}$$

Depending on the direction of the current in each battery, the terminal voltages across the batteries are:

$$\begin{aligned} \Delta V &= V_b - V_a = \mathcal{E}_1 + Ir_1 = 9\text{ V} + (50\text{ A})(0.02\ \Omega) \\ &= 10\text{ V} \quad (\text{Gain from } a \text{ to } b) \end{aligned}$$

$$\begin{aligned} \Delta V &= V_{b'} - V_{a'} = \mathcal{E}_2 - Ir_2 = 12\text{ V} - (50\text{ A})(0.04\ \Omega) \\ &= 10\text{ V} \quad (\text{Drop from } a' \text{ to } b') \end{aligned}$$

24.5 Resistors in Series and Parallel

Resistors in a circuit may be used in different combinations, and we can sometimes replace a combination of resistors with one *equivalent resistor*. In this section, we introduce two basic combinations of resistors that allow such a replacement.

Resistors in a Series Combination

Figure 24.14a shows two resistors R_1 and R_2 that are connected in **series** with a battery B. Figure 24.14b shows a circuit diagram for this combination of resistors.

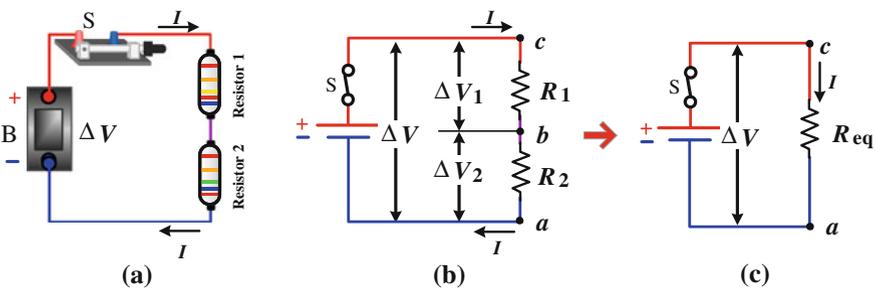


Fig. 24.14 (a) Two resistors are connected in series to a battery B that has a potential difference ΔV . (b) The circuit diagram for this series combination. (c) An equivalent resistance R_{eq} replacing the original resistors set up in a series combination

When the circuit is connected, the amount of charge that passes through R_1 must also pass through R_2 in the same time interval. Otherwise, charge will accumulate on the wire between resistors. Thus, for series combination of resistors, the current I is the same in both resistors. Figure 24.14c shows a single resistor R_{eq} that is equivalent to this combination and has the same effect on the circuit. This means that when the

potential difference ΔV is applied across the equivalent resistor, it must produce the same current I as in the series combination.

The potential difference ΔV is divided to ΔV_1 and ΔV_2 across the resistors R_1 and R_2 , respectively. Thus:

$$\Delta V = \Delta V_1 + \Delta V_2 \quad (24.29)$$

For the two resistors in Fig. 24.14b, we have:

$$\Delta V_1 = V_c - V_b = IR_1 \quad \text{and} \quad \Delta V_2 = V_b - V_a = IR_2 \quad (24.30)$$

Substituting in Eq. 24.29, we get:

$$\Delta V = IR_1 + IR_2 \quad (24.31)$$

The equivalent resistor R_{eq} has the same applied potential difference ΔV and the same circuit current I flowing through it; thus:

$$\Delta V = IR_{\text{eq}} = IR_1 + IR_2 \quad (24.32)$$

Canceling I , we arrive at the following relationship:

$$R_{\text{eq}} = R_1 + R_2 \quad (\text{Series combination}) \quad (24.33)$$

We can extend this treatment to n resistors connected in series as:

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n \quad (\text{Series combination}) \quad (24.34)$$

Thus, the equivalent resistor of a series combination of resistors is simply the algebraic sum of the individual resistances and will always be greater than any one of them.

Example 24.9

In Fig. 24.14, let $R_1 = 6 \Omega$, $R_2 = 3 \Omega$, and $\Delta V = 18 \text{ V}$. Find I in the circuit and the potential differences ΔV_1 and ΔV_2 .

Solution: The equivalent resistance of the series combination is:

$$R_{\text{eq}} = R_1 + R_2 = 6 \Omega + 3 \Omega = 9 \Omega$$

Using Ohm's law, given by Eq. 24.16, we find:

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{18 \text{ V}}{9 \Omega} = 2 \text{ A}$$

$$\Delta V_1 = IR_1 = (2 \text{ A})(6 \Omega) = 12 \text{ V}$$

$$\Delta V_2 = IR_2 = (2 \text{ A})(3 \Omega) = 6 \text{ V}$$

Resistors in a Parallel Combination

Figure 24.15a shows two resistors of resistances R_1 and R_2 that are connected in **parallel** with a battery B. Figure 24.15b shows a circuit diagram for this combination of resistors. The potential difference ΔV between the battery's terminals is the same as the potential difference across each resistor. Figure 24.15c shows a single resistance R_{eq} that is equivalent to this combination and has the same effect on the circuit.

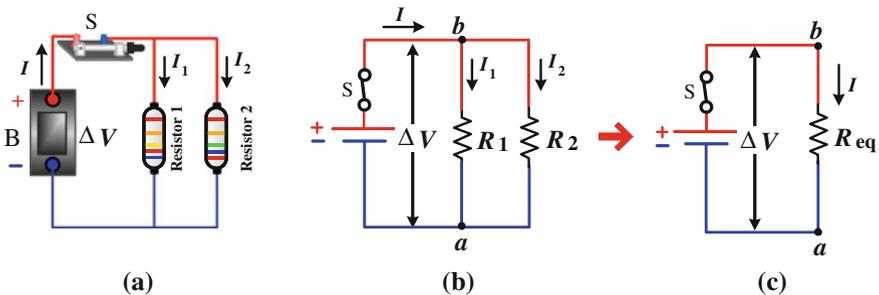


Fig. 24.15 (a) Two resistors of resistances R_1 and R_2 are connected in parallel to a battery B that has a potential difference ΔV . (b) The circuit diagram for this parallel combination. (c) The equivalent resistance R_{eq} replacing the parallel combination

When the current I reaches *junction b*, it will split into two parts, I_1 in R_1 and I_2 in R_2 . Because electric charge is conserved, the current I that enters *junction b* must equal the total current leaving that junction; that is:

$$I = I_1 + I_2 \quad (24.35)$$

Because the potential difference ΔV across the resistors is the same, then from Fig. 24.15b, we have:

$$\Delta V = I_1 R_1 \quad \text{and} \quad \Delta V = I_2 R_2 \quad (24.36)$$

Substituting into Eq. 24.35, we get:

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Delta V \quad (24.37)$$

An equivalent resistor with the same applied potential difference ΔV and total current I has a resistance R_{eq} given by $\Delta V = I R_{\text{eq}}$. Thus:

$$I = \frac{\Delta V}{R_{\text{eq}}} \quad (24.38)$$

Substituting in Eq. 24.37 and canceling ΔV , we arrive at the following relationship:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (\text{Parallel combination}) \quad (24.39)$$

We can extend this treatment to n resistors connected in parallel as:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \quad (\text{Parallel combination}) \quad (24.40)$$

Thus, the equivalent resistance of a parallel combination of resistors is simply the algebraic sum of the reciprocal of the individual resistances and is less than any one of them.

Example 24.10

In Fig. 24.15, let $R_1 = 6 \Omega$, $R_2 = 3 \Omega$, and $\Delta V = 18 \text{ V}$. Find the three currents I , I_1 , and I_2 in the circuit.

Solution: The equivalent resistance of the parallel combination is:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6 \Omega} + \frac{1}{3 \Omega} = \frac{1}{2 \Omega}$$

Then :

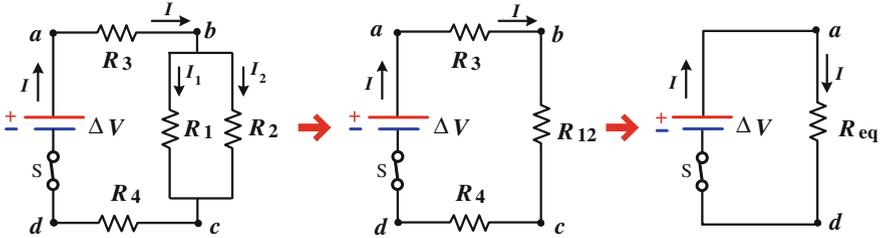
$$R_{\text{eq}} = 2 \Omega$$

Now we calculate the three currents in the circuit as follows:

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{18 \text{ V}}{2 \Omega} = 9 \text{ A} \quad I_1 = \frac{\Delta V}{R_1} = \frac{18 \text{ V}}{6 \Omega} = 3 \text{ A} \quad I_2 = \frac{\Delta V}{R_2} = \frac{18 \text{ V}}{3 \Omega} = 6 \text{ A}$$

Example 24.11

In Fig. 24.16, let $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 7\ \Omega$, and $\Delta V_{da} = V_a - V_d = 30\ \text{V}$. (a) What is the equivalent resistance between points a and d ? (b) Evaluate the current passing through each resistor.

**Fig. 24.16**

Solution: (a) We can simplify the circuit by the rule of adding resistances in series and in parallel in steps. The resistors R_1 and R_2 are in parallel and their equivalent resistance R_{12} between b and c is:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{1}{2\ \Omega}$$

Then : $R_{12} = 2\ \Omega$

Now R_3 , R_{12} , and R_4 are in series between points a and d . Hence, their equivalent resistance R_{eq} is:

$$R_{\text{eq}} = R_3 + R_{12} + R_4 = 1\ \Omega + 2\ \Omega + 7\ \Omega = 10\ \Omega$$

(b) The current I that passes through the equivalent resistor also passes through R_3 and R_4 . Thus, using Ohm's law, we find that:

$$I = \frac{\Delta V_{da}}{R_{\text{eq}}} = \frac{30\ \text{V}}{10\ \Omega} = 3\ \text{A} \quad (\text{Current through the battery, } R_3 \text{ and } R_4)$$

Since $\Delta V_{cb} = IR_{12} = I_1 R_1 = I_2 R_2$, then we find I_1 and I_2 as follows:

$$I_1 = \frac{IR_{12}}{R_1} = \frac{(3\ \text{A})(2\ \Omega)}{3\ \Omega} = 2\ \text{A} \quad \text{and} \quad I_2 = \frac{IR_{12}}{R_2} = \frac{(3\ \text{A})(2\ \Omega)}{6\ \Omega} = 1\ \text{A}$$

24.6 Kirchhoff's Rules

Not all circuits can be reduced to simple series and parallel combinations. A technique that is applied to loops in complicated circuits consists of two principles called Kirchhoff's Rules.

Kirchhoff's Rules:

1. Junction rule

At any junction in a circuit, the sum of the ingoing currents must equal the sum of the outgoing currents. That is:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \quad (24.41)$$

2. Loop rule

For any closed loop in a circuit, the sum of the potential differences across all elements must be zero. That is:

$$\sum_{\text{closed loop}} \Delta V = 0 \quad (24.42)$$

The first rule merely states that no charge can accumulate at a junction. This rule is based on the principle of conservation of charge within any system. The second rule follows from the law of conservation of energy but is expressed in terms of potential energy.

When we apply Kirchhoff's second rule to a loop, we should note the following sign conventions:

- (1) When a resistor is traversed in the direction of the current, the potential difference ΔV is $-IR$ (Fig. 24.17a).
- (2) When a resistor is traversed in the direction opposite the current, the potential difference ΔV is $+IR$ (Fig. 24.17b).
- (3) When a source of emf is traversed in the direction of its emf (from $-$ to $+$), the potential difference ΔV is $+\mathcal{E}$ (Fig. 24.17c).
- (4) When a source of emf is traversed in a direction opposite to its emf (from $+$ to $-$), the potential difference ΔV is $-\mathcal{E}$ (Fig. 24.17d).

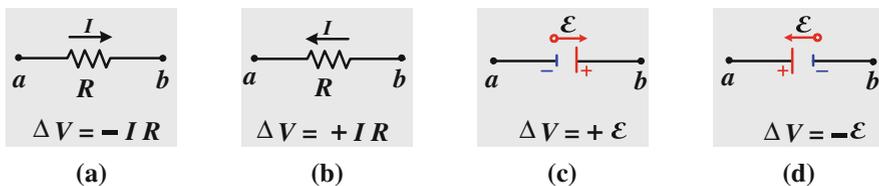


Fig. 24.17 The potential differences $\Delta V = V_b - V_a$ across a resistor of resistance R and a battery of emf \mathcal{E} (assumed to have zero internal resistance), when each element is traversed from a to b

Example 24.12

Apply Kirchhoff's loop on the circuit of Example 24.8 to find the current in the circuit, see Fig. 24.13.

Solution: Applying Kirchhoff's loop rule to the loop $abb'a'a$ of Fig. 24.13, and traversing the loop clockwise, we obtain the following expression:

$$\begin{aligned} \text{Loop } abb'a'a: \quad \mathcal{E}_1 + Ir_1 + Ir_2 - \mathcal{E}_2 &= 0 \\ 9\text{ V} + (0.02\ \Omega)I + (0.04\ \Omega)I - 12\text{ V} &= 0 \end{aligned}$$

Then:
$$I = 50\text{ A}$$

Of course, we do not need Kirchhoff's rules to solve this simple loop circuit. We are just using it to practice applying the loop rule.

Example 24.13

In Fig. 24.18, let $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 4\ \Omega$, $\mathcal{E}_1 = 10\text{ V}$, and $\mathcal{E}_2 = 14\text{ V}$. Find the currents I_1 , I_2 , and I_3 in the circuit.

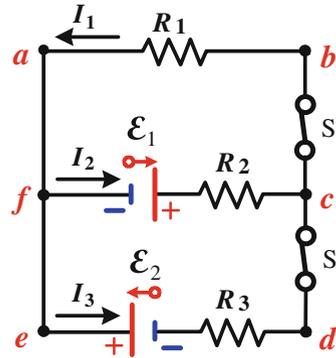
Solution: We cannot simplify the circuit by the rule of adding resistances in series and in parallel. Thus, we must use Kirchhoff's rules. By applying Kirchhoff's junction rule to the junction f , we get:

$$(1) \text{ Junction } f: \quad I_1 = I_2 + I_3$$

We have three loops in this circuit, but we need only two loop equations to determine the three unknown currents. Applying Kirchhoff's loop rule to the loops

$abcfa$ and $fcdef$ and traversing these loops clockwise, we obtain the following equations (after temporarily omitting the units, since they are all consistent SI units):

Fig. 24.18



$$(2) \text{ Loop } abcfa: \quad I_1 R_1 + I_2 R_2 - \mathcal{E}_1 = 0 \quad \Rightarrow \quad 2I_1 + 6I_2 - 10 = 0$$

$$(3) \text{ Loop } fcdef: \quad \mathcal{E}_1 - I_2 R_2 + I_3 R_3 + \mathcal{E}_2 = 0 \quad \Rightarrow \quad 24 - 6I_2 + 4I_3 = 0$$

Substituting Eqs. (1) into (3) gives:

$$24 - 10I_2 + 4I_1 = 0$$

Dividing this equation by 2 gives:

$$(4) \quad 12 - 5I_2 + 2I_1 = 0$$

Subtracting Eqs. (4) from (2) gives:

$$(2I_1 + 6I_2 - 10) - (12 - 5I_2 + 2I_1) = 0 \quad \Rightarrow \quad I_2 = 2 \text{ A}$$

Using this value of I_2 in Eq. (4) gives I_1 a value of:

$$12 - 5 \times 2 + 2I_1 = 0 \quad \Rightarrow \quad I_1 = -1 \text{ A}$$

Finally, from Eq. (1) we have:

$$I_3 = I_1 - I_2 = -1 \text{ A} - 2 \text{ A} = -3 \text{ A}$$

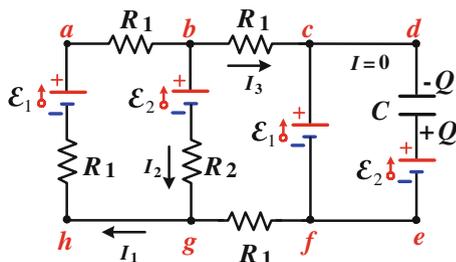
Thus $(I_1 = -1 \text{ A}, I_2 = 2 \text{ A}, I_3 = -3 \text{ A})$

We notice that I_1 and I_2 are both negative. This means that the currents are opposite to the direction we chose. However, the numerical values are correct.

Example 24.14

In Fig. 24.19, let $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $\mathcal{E}_1 = 6\ \text{V}$, $\mathcal{E}_2 = 3\ \text{V}$, and $C = 2\ \mu\text{F}$. Find the steady currents I_1 , I_2 , and I_3 and the charge Q .

Fig. 24.19



Solution: Applying Kirchoff's junction rule at point b , we get:

$$(1) \text{ Junction } b: \quad I_1 = I_2 + I_3$$

The application of the loop rule to the loops $abgha$ and $bcfgb$ gives:

$$\text{Loop } abgha: \quad -I_1 R_1 - \mathcal{E}_2 - I_2 R_2 - I_1 R_1 + \mathcal{E}_1 = 0$$

$$-2I_1 - 3 - 4I_2 - 2I_1 + 6 = 0$$

$$(2) \quad -4I_1 - 4I_2 + 3 = 0$$

$$\text{Loop } bcfgb: \quad -I_3 R_1 - \mathcal{E}_1 - I_3 R_1 + I_2 R_2 + \mathcal{E}_2 = 0$$

$$-2I_3 - 6 - 2I_3 + 4I_2 + 3 = 0$$

$$(3) \quad -4I_3 + 4I_2 - 3 = 0$$

Substituting Eqs. (1) into (3) gives:

$$(4) \quad -4I_1 + 8I_2 - 3 = 0$$

Subtracting Eqs. (4) from (2) gives:

$$(-4I_1 - 4I_2 + 3) - (-4I_1 + 8I_2 - 3) = 0 \Rightarrow I_2 = 0.5\ \text{A}$$

Using this value of I_2 in Eq. (4) gives I_1 a value of:

$$-4I_1 + 8(0.5) - 3 = 0 \Rightarrow I_1 = 0.25 \text{ A}$$

Finally, from Eq. (1) we have:

$$I_3 = I_1 - I_2 = 0.5 - 0.25 \text{ A} = 0.25 \text{ A}$$

Applying Kirchhoff's loop rule to the loop $cdefc$ gives:

$$\text{Loop } cdefc: \quad Q/C - \mathcal{E}_2 + \mathcal{E}_1 = 0 \Rightarrow Q = (\mathcal{E}_2 - \mathcal{E}_1)C = -6 \mu\text{C}$$

24.7 The RC Circuit

In all previous analyses, we considered steady-state situations where the current remains constant. However, when a circuit contains a resistor and a capacitor (called an RC circuit), the current in that circuit is found to vary with time until a steady current is reached.

Charging a Capacitor

Consider the simple RC series circuit shown in Fig. 24.20a. We assume that the capacitor is initially uncharged when the switch S is open, see Fig. 24.20a, b. Once the switch S is closed at time $t = 0$, charge begins to flow, setting up a current I in the circuit, until the capacitor is fully charged and the current becomes zero.

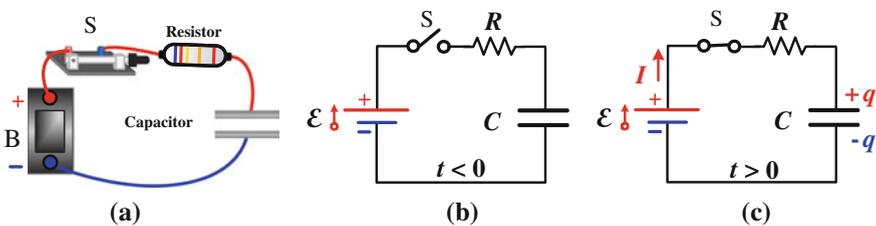


Fig. 24.20 (a) A capacitor in series with a resistor, switch, and battery. (b) The circuit diagram before the switch is closed ($t < 0$). (c) The circuit diagram at time $t > 0$ after the switch is closed at $t = 0$

Assume that the current in the circuit at time $t > 0$ is I and the magnitude of the charge on the capacitor is q (Fig. 24.20c). Applying Kirchhoff's loop rule and traversing the circuit clockwise, we get:

$$\mathcal{E} - IR - \frac{q}{C} = 0 \quad (\text{At time } t > 0) \quad (24.43)$$

Since the capacitor is uncharged at $t = 0$, then substituting $q = 0$, into this equation indicates that the current, denoted by I_o , is maximum:

$$I_o = \frac{\mathcal{E}}{R} \quad (\text{Current at time } t = 0) \quad (24.44)$$

At $t = 0$, the potential difference across the battery appears entirely on the resistor. At $t = \infty$, when the capacitor is fully charged to its maximum value Q , the current is zero and the potential difference across the battery appears entirely on the capacitor. Therefore, Eq. 24.43 gives:

$$Q = C \mathcal{E} \quad (\text{Maximum charge at } t = \infty) \quad (24.45)$$

To find the charge as a function of time, we substitute $I = dq/dt$ into Eq. 24.43 and rearrange the equation as follows:

$$\begin{aligned} \frac{dq}{dt} &= \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{q - C\mathcal{E}}{RC} \\ \frac{dq}{q - C\mathcal{E}} &= -\frac{dt}{RC} \end{aligned}$$

Substituting with $C\mathcal{E} = Q$ in this expression and integrating from the initial charge $q = 0$ at $t = 0$ to an arbitrary charge q at time t , we get:

$$\int_0^q \frac{dq}{q - Q} = -\int_0^t \frac{dt}{RC} \Rightarrow \ln \frac{q - Q}{-Q} = -\frac{t}{RC}$$

By using the definition of the natural logarithm, we can rewrite the last expression as:

$$q = Q(1 - e^{-t/RC}), \quad Q = C\mathcal{E} \quad (24.46)$$

This relation conforms with the facts that we already know, i.e. that $q = 0$ at $t = 0$ and that $q = Q = C\mathcal{E}$ at $t = \infty$. Using $I = dq/dt$, we differentiate the charge q in Eq. 24.46 to find the current I as a function of time as follows:

$$I = I_o e^{-t/RC}, \quad I_o = \frac{\mathcal{E}}{R} \quad (24.47)$$

This relation shows that $I = I_o = \mathcal{E}/R$ at $t = 0$ as obtained in Eq. 24.44 and $I = 0$ at $t = \infty$ as expected.

The quantity RC in the exponents of Eqs. 24.46 and 24.47 is called the **time constant** τ of the circuit. Therefore, the quantity $\tau = RC$ represents the time interval during which the charge on the capacitor increases to $Q(1 - e^{-1}) = 0.632Q$, i.e., $\sim 63\%$ increase. Similarly, after a time interval τ , the current decreases to $1/e$ of its initial value; that is, $I = e^{-1}I_0 = 0.368I_0$ ($\sim 37\%$ decrease).

Figure 24.21 shows the variation of the capacitor charge q and the circuit current I as a function of time.

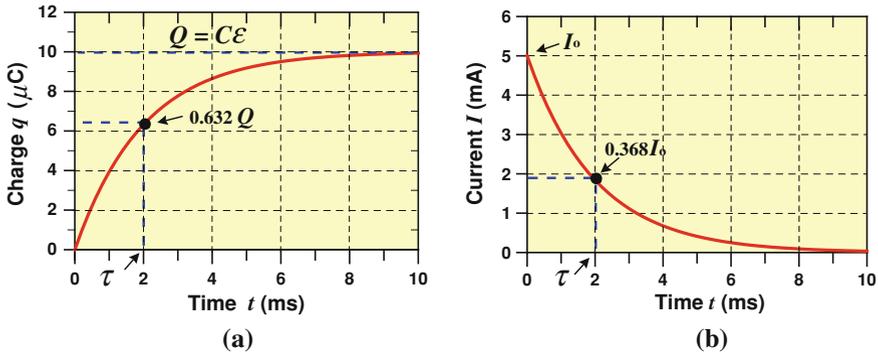


Fig. 24.21 (a) A plot of the charge q on the capacitor of Fig. 24.20 versus time t . (b) A plot of the current I in the same figure versus time t . The two curves are for $R = 2 \text{ k}\Omega$, $C = 1 \text{ }\mu\text{F}$, and $\mathcal{E} = 10 \text{ V}$

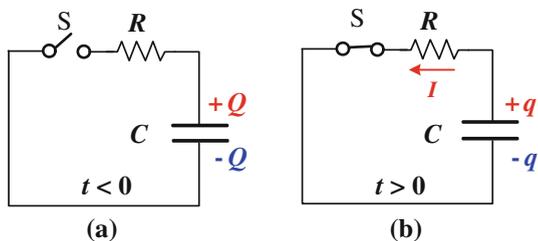
Discharging a Capacitor

Let us consider the circuit shown in Fig. 24.22a, in which we have a capacitor of capacitance C carrying an initial charge Q , a resistor of resistance R , and an open switch S . When the switch is closed at time $t = 0$, the capacitor begins to discharge through the resistor. If the current in the circuit at time $t > 0$ is I and the magnitude of the charge on the capacitor is q (Fig. 24.22b), then by applying Kirchhoff's loop rule and traversing the circuit clockwise, we get:

$$+IR - \frac{q}{C} = 0 \quad (\text{At time } t) \quad (24.48)$$

To find the charge as a function of time, we substitute with $I = -dq/dt$ (the rate of decrease of charge on the capacitor) into Eq. 24.48 and rearrange the equation as follows:

Fig. 24.22 (a) A capacitor with an initial charge Q is connected to a resistor and an open switch ($t < 0$). (b) A circuit diagram showing the charge and current at $t > 0$, after the switch is closed at time $t = 0$



$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

Integrating this expression from the initial charge $q = Q$ at $t = 0$ to an arbitrary charge q at time t , we get:

$$\int_Q^q \frac{dq}{q} = -\int_0^t \frac{dt}{RC} \Rightarrow \ln \frac{q}{Q} = -\frac{t}{RC}$$

By using the definition of the natural logarithm, we can rewrite the last expression as:

$$q = Qe^{-t/RC} = Qe^{-t/\tau} \quad (24.49)$$

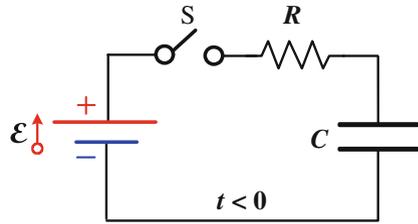
Using $I = -dq/dt$, we differentiate q in Eq. 24.49 to find the current I as a function of time as follows:

$$I = I_0 e^{-t/RC} = I_0 e^{-t/\tau}, \quad I_0 = \frac{Q}{RC} \quad (24.50)$$

We must note that the *discharging current* in Fig. 24.22 is opposite to the direction of the *charging current* in Eq. 24.20.

Example 24.15

In the circuit of Fig. 24.23a, let $R = 2 \text{ k}\Omega$, $\mathcal{E} = 10 \text{ V}$, and $C = 1 \text{ }\mu\text{F}$. The capacitor is uncharged before closing the switch S. (a) Find the time constant of the circuit. After closing S at $t = 0$, find the maximum current in the circuit and find the maximum charge on the capacitor at $t = \infty$. (b) Find the charge and current as a function of time.

Fig. 24.23

Solution: (a) The time constant of the circuit is:

$$\tau = RC = (2 \times 10^3 \Omega)(1 \times 10^{-6} \text{ F}) = 2 \times 10^{-3} \text{ s} = 2 \text{ ms}$$

The maximum current in the circuit (see Eq. 24.44) is:

$$I_o = \frac{\mathcal{E}}{R} = \frac{10 \text{ V}}{2 \times 10^3 \Omega} = 5 \times 10^{-3} \text{ A} = 5 \text{ mA}$$

At $t = \infty$, the magnitude of the maximum charge on the capacitor is:

$$Q = C\mathcal{E} = (1 \times 10^{-6} \text{ F})(10 \text{ V}) = 10^{-5} \text{ C} = 10 \mu\text{C}$$

(b) Substituting the obtained values of part (a) in Eqs. 24.46 and 24.47, we get for the charge the following relation:

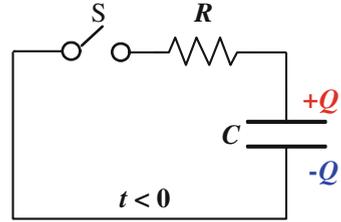
$$\begin{aligned} q &= Q(1 - e^{-t/RC}) = Q(1 - e^{-t/\tau}) \\ &= (10 \mu\text{C})(1 - e^{-t/(2 \times 10^{-3} \text{ s})}) \end{aligned}$$

and for the current the following relation:

$$\begin{aligned} I &= I_o e^{-t/RC} = I_o e^{-t/\tau} \\ &= (5 \times 10^{-3} \text{ A})(1 - e^{-t/(2 \times 10^{-3} \text{ s})}) \end{aligned}$$

Example 24.16

In Fig. 24.24, let $R = 2 \text{ k}\Omega$, $C = 5 \mu\text{F}$, and $Q = 50 \mu\text{C}$. (a) After how many time constants, $\tau = RC$, will the charge on the capacitor be half of its initial value when the switch is closed? (b) When the stored energy in the capacitor becomes half of its initial value?

Fig. 24.24

Solution: (a) The time constant of the circuit is:

$$\tau = RC = (2 \times 10^3 \Omega)(5 \times 10^{-6} \text{ F}) = 10^{-2} \text{ s} = 10 \text{ ms}$$

After closing the switch at $t = 0$, the charge on the capacitor is given by Eq. 24.49, $q = Qe^{-t/\tau}$. To find the time interval during which q drops to one-half its initial value, we substitute $q = Q/2$ into this equation and solve for the time t as follows:

$$\frac{Q}{2} = Qe^{-t/\tau} \Rightarrow \frac{1}{2} = e^{-t/\tau}$$

Taking the logarithm of both sides, we find:

$$-\ln 2 = -\frac{t}{\tau} \Rightarrow t = (\ln 2)\tau = 0.69\tau = 0.69 \times (10 \text{ ms}) = 6.9 \text{ ms}$$

(b) From Eq. 23.24, the initial stored energy in the capacitor is $U_o = Q^2/2C$. Using Eq. 24.49, the energy stored at time t is:

$$U = \frac{q^2}{2C} = \frac{Q^2}{2C}e^{-2t/\tau} = U_o e^{-2t/\tau}$$

As in part (a), we set $U = U_o/2$ and solve for t as follows:

$$\frac{U_o}{2} = U_o e^{-2t/\tau} \Rightarrow \frac{1}{2} = e^{-2t/\tau}$$

Again, taking the logarithm of both sides and solving for t , we find:

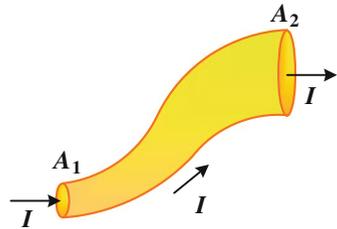
$$-\ln 2 = -2t/\tau \Rightarrow t = \frac{1}{2}(\ln 2)\tau = \frac{1}{2} \times 0.69\tau = \frac{1}{2} \times 0.69 \times (10 \text{ ms}) = 3.45 \text{ ms}$$

Note that the results of both parts are independent on the value of Q .

24.8 Exercises

Section 24.1 Electric Current and Electric Current Density

- (1) How many electrons per second would pass through a given cross section of a conductor carrying a current $I = 1.6 \text{ A}$?
- (2) A current of 10 A is maintained in a wire for 1 min . (a) How much charge flows through the wire in this period? (b) How many electrons flow through the wire in this period?
- (3) A 0.1 mol of electrons flows through a wire in 30 min . (a) What is the total charge that passes through the wire? (b) What is the value of the current in the wire?
- (4) A copper wire contains 2×10^{21} free electrons in 1 cm of its length. The electrons move with a drift speed of $2.5 \times 10^{-3} \text{ cm/s}$. (a) How many electrons pass through a given cross section of the wire each second? (b) How large is the current in the wire?
- (5) The current through a cross-sectional area of a wire is given by the relation $I = 2 + 3t^3$; where I is in amperes and t is in seconds. (a) Find the total charge that passes through this area between $t = 2 \text{ s}$ and $t = 8 \text{ s}$. (b) Find the average current needed to pass the same quantity of charge calculated in part (a) during the same time interval.
- (6) The charge that passes a cross-sectional area $A = 10^{-4} \text{ m}^2$ varies with time according to the relation $Q = 4 + 2t + t^2$, where Q is in coulombs and t is in seconds. (a) Find the relation that gives the instantaneous current at any time, and evaluate this current at time $t = 2 \text{ s}$. (b) Find the relation that gives the current density at any time, and evaluate this current density at time $t = 2 \text{ s}$.
- (7) A wire carrying a current of 3 A has a circular cross section everywhere with a non-uniform radius, see Fig. 24.25. The radius of the cross section A_1 is 2 cm . (a) Find the current density across A_1 . (b) Find the current density across A_2 if its radius is two times the radius at A_1 .
- (8) A copper wire with a 0.2 mm diameter and an iron wire with a 5 mm diameter are soldered together to form one wire in a circuit. A current of 8 A is found to pass through the copper wire. (a) What is the current and current density through the iron wire? (b) What is the current density through the copper wire?

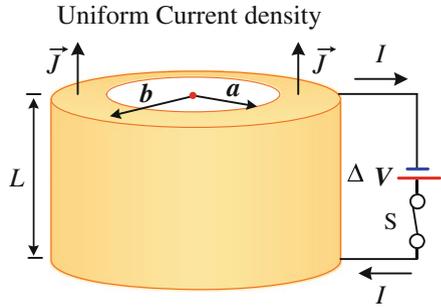
Fig. 24.25 See Exercise (7)

- (9) Given that the density of aluminum is $2.7 \times 10^3 \text{ kg/m}^3$, find the drift speed of the conduction electrons in an aluminum wire that has a cross-sectional area of 10^{-6} m^2 and carries a current of 10 A. Assume that each aluminum atom contributes one free conduction electron to the current.

Section 24.2 Ohm's Law and Electric Resistance

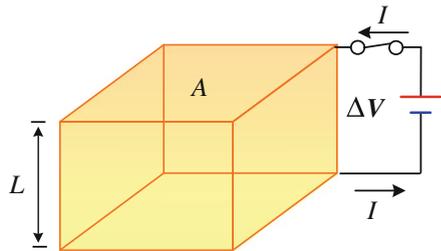
- (10) Use Table 24.1 to calculate the electric field that exists in a gold wire when the current density in the wire is $3 \times 10^7 \text{ A/m}^2$.
- (11) A metallic rod has a length $L = 1.5 \text{ m}$ and a diameter $D = 0.2 \text{ cm}$. The rod carries a current of 5 A when a potential difference of 75 V is applied between its ends. (a) Find the current density in the rod. (b) Calculate the magnitude of the electric field applied to the rod. (c) Calculate the resistivity and conductivity of the material of the rod.
- (12) Use Table 24.1 to calculate the resistance of a silver wire that has a length of 100 m and a cross section of 0.4 mm^2 .
- (13) At 20°C , a silver wire has a diameter of 2 mm, a length of 0.5 m, a resistivity of $1.6 \times 10^{-8} \Omega \cdot \text{m}$, a temperature coefficient of resistivity of $4 \times 10^{-3} (\text{C}^\circ)^{-1}$, and carries a current of 5 A. (a) What is the current density in the wire? (b) Find the magnitude of the electric field applied to the wire. (c) What is the potential difference between the ends of the wire? (d) What is the resistance of the wire? (e) Find the temperature of the wire when its resistance increases to $6.5 \times 10^{-4} \Omega$.
- (14) A cylindrical shell of length $L = 10 \text{ cm}$ is made of copper and has an inner radius $a = 2 \text{ mm}$ and an outer radius $b = 8 \text{ mm}$, see Fig. 24.26. Assume that the shell has a uniform current density $J = 10^5 \text{ A/m}^2$ directed upward as shown in the figure. (a) What is the current through the shell? (b) What are the values of the resistance of the shell and the potential difference ΔV ?

Fig. 24.26 See Exercise (14)



- (15) A cube of copper has a mass $m = 50$ g, see Fig. 24.27. The copper has a density of 8.92×10^3 kg/m³, a molar mass of 63.546 kg/kmol, a resistivity of 1.7×10^{-8} $\Omega \cdot m$, and contributes one conduction electron per atom. (a) What is the distance between opposite faces of the cube? (b) What is the resistance between opposite faces of the cube? (c) What is the current and the average drift speed of the conduction electrons when a potential difference ΔV of 10^{-4} V is applied between two opposite faces of the cube?

Fig. 24.27 See Exercise (15)



- (16) The temperature coefficient of resistivity of copper at 20°C is $3.9 \times 10^{-3} (\text{C}^\circ)^{-1}$. Calculate the percentage increase in its resistivity when its temperature increases to 220°C .
- (17) At a temperature of $1,800^\circ\text{C}$ the tungsten filament of a light bulb has a resistance of 250Ω . With the aid of Table 24.1, find its resistance at room temperature (assume it to be 20°C).
- (18) At 20°C a copper wire has a resistance of $4 \times 10^{-3} \Omega$ and a temperature coefficient of resistivity of $3.9 \times 10^{-3} (\text{C}^\circ)^{-1}$. What is its resistance at 100°C ?
- (19) At 70°C , an electric field $E = 0.2$ V/m is applied along a silver rod of length $L = 0.5$ m and radius $r = 0.05$ mm. Silver has a density of 10.5×10^3 kg/m³,

a molar mass of 107.868 kg/kmol, a coefficient α at 20 °C of $3.8 \times 10^{-3} (\text{C}^\circ)^{-1}$, and a resistivity of $1.59 \times 10^{-8} \Omega \cdot \text{m}$ at 20 °C. Assuming 1 free electron per atom, find: (a) the resistivity of the silver wire (b) the current density in the silver wire. (c) the current in the silver wire. (d) the resistance of the silver wire. (e) the drift speed of the conduction electrons. (f) the potential difference between the ends of the silver wire.

- (20) At 20 °C, a nichrome wire of resistance R_{on} and a carbon wire of resistance R_{oc} are attached end-to-end to form one wire of resistance R_o , where $R_o = R_{on} + R_{oc} = 9 \Omega$. What values of R_{on} and R_{oc} would give a combined resistance of R equal to R_o regardless of the temperature T ? [Hint: use Table 24.1.]

Section 25.3 Electric Power

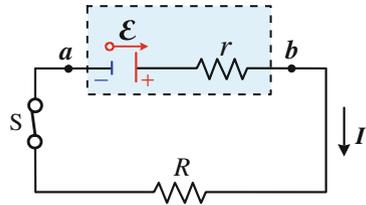
- (21) A light bulb rated 60 W at 240 V is operated from a 240 V source. (a) Find the current flowing through the bulb. (b) Find the resistance of the bulb. (c) Repeat (a) and (b) when the bulb is rated 100 W at 240 V.
- (22) A 550 W electric heater is designed to operate from a 220 V source. (a) What is the resistance of the heater? (b) What current does the heater draw from the source? (c) If the source voltage drops to 120 V, what power does the heater consume from the source?
- (23) A heating coil is made from a nichrome wire of radius 0.45 mm. The coil is designed to produce 240 W of thermal power when connected to a source that has a potential difference of 24 V. (a) What is the resistance of the coil? (b) What current does the heating coil draw from the source? (c) What is the length of the coil?
- (24) A 1 k Ω carbon resistor used in an electric circuit is rated 0.4 W. (a) Find the maximum allowable current that can pass through the resistor. (b) Find the maximum allowable potential difference that can be applied across the resistor.
- (25) Batteries are rated in terms of the quantity It , i.e. rated in ampere-hours (A.h). For instance, a battery that can produce a current of 4 A for 5 h is rated as a 20 A.h battery. (a) Find the total energy stored in a 12 V battery rated at 75 A.h. (Express your answer in kW.h, where 1 kW.h = 3.6×10^3 J). (b) At a price of 35 piaster per kilowatt-hour of electricity, what is the total cost of the electricity produced by this battery?

- (26) A beam of electrons in a TV set has a radius of 0.1 cm. The electrons move from the cathode to the screen with an electron current of 0.1 mA and a kinetic energy of 5 keV. (a) What is the current density in the beam? (b) How many electrons per second hit the screen? (c) How much power is dissipated at the screen? (d) What is the speed of each electron in the beam? (e) Find the number of electrons per unit volume in the beam.
- (27) A heating coil operating from a 220 V source increases the temperature of 2 kg of water from 20°C to 50°C in 20 min. Find the coil's resistance if water's specific heat is 4,186 J/kg.C°.

Section 24.4 Electromotive Force

- (28) In Fig. 24.28 the circuit contains a battery that has an emf $\mathcal{E} = 11$ V and internal resistance $r = 0.5$ Ω . The load in the circuit has a resistance $R = 5$ Ω . (a) Find the current in the circuit. (b) Find the potential difference between a and b .

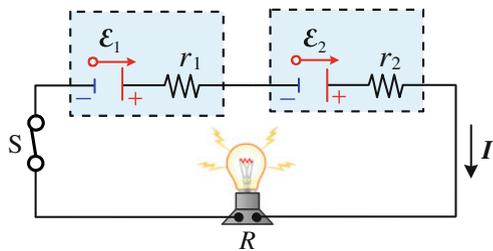
Fig. 24.28 See Exercise (28)



- (29) Assume a circuit similar to the one in Exercise 28 has an unknown emf \mathcal{E} and internal resistance r . It is found that when the current is 0.5 A, the load resistance is 16 Ω . Similarly, it is found that when the current is 1.5 A, the load resistance is 5 Ω . (a) Find the internal resistance of the battery. (b) Find the emf of the battery.
- (30) Two batteries, one old and the other new, each have an emf of 1.5 V. When each battery is short-circuited with a conducting wire of zero resistance, it is found that the new one establishes a 30 A current in the wire while the old one establishes a 10 A current. Find the internal resistance of the two batteries.
- (31) A battery has an emf $\mathcal{E}_1 = 9$ V and an internal resistance $r_1 = 0.4$ Ω . This battery is connected to a second battery of $\mathcal{E}_2 = 12$ V and $r_2 = 0.6$ Ω , and a light bulb of resistance R . If the batteries are connected with their positive

terminals in the same direction as shown in Fig. 24.29, a current of 0.7 A is established in the circuit. (a) Find the resistance of the light bulb. (b) What fraction of the transferred chemical energy is dissipated in the two batteries? (c) If we reverse the polarity of the $\mathcal{E}_1 = 9\text{ V}$ battery in the circuit, what is the value of the current in the circuit? Would the answer to part (b) change in this case?

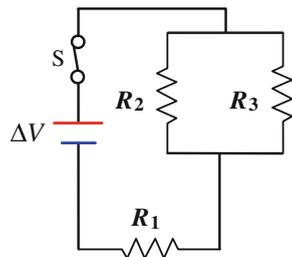
Fig. 24.29 See Exercise (31)



Section 24.5 Resistors in Series and Parallel

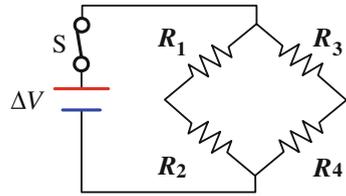
(32) When three resistors of resistances $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$, and $R_3 = 4\ \Omega$ are connected to a source of potential difference ΔV as shown in Fig. 24.30, the current in the circuit is found to be 5 A. (a) Find the equivalent resistance of the combination. (b) Determine the value of ΔV .

Fig. 24.30 See Exercise (32)



(33) For the circuit shown in Fig. 24.31, take $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 12\ \Omega$, $R_4 = 6\ \Omega$, and $\Delta V = 12\text{ V}$. (a) Find the equivalent resistance of the combination. (b) Find the current in the branch containing R_1 and R_2 . (c) Repeat (b) for the branch containing R_3 and R_4 . (d) Find the potential difference across each resistor.

Fig. 24.31 See Exercise (33)



(34) For each of the combinations shown in Fig. 24.32, find a formula that represents the equivalent resistance between the terminals A and B.

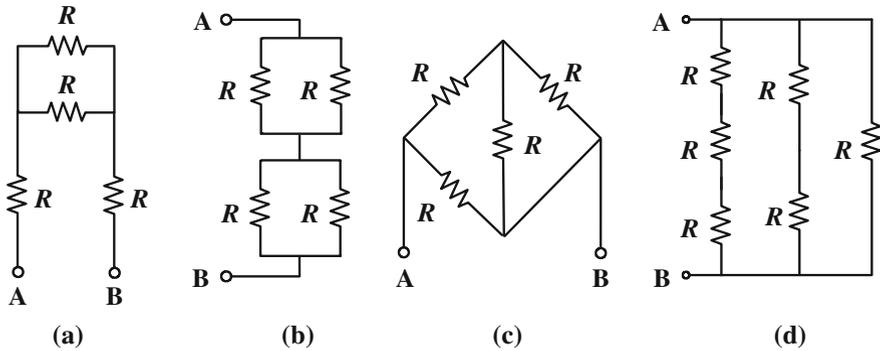


Fig. 24.32 See Exercise (34)

- (35) Assume that in exercise 34, $R = 2 \Omega$ and $\Delta V_{BA} = 12 \text{ V}$. For each combination, find the current in each branch of the circuit. Always start from the branch closest to A and move toward B.
- (36) It is recommended that the current through the human body not exceed $150 \mu\text{A}$. Assume a person stands barefoot on the ground, holding a wire connected through a resistor of high resistance R to a power source of potential difference $\Delta V = 220 \text{ V}$ as shown in Fig. 24.33. Assume that the circuit's wire makes a low-resistance contact with the person's hand. Also, assume that the resistance through the person's body is negligible compared to the resistance R . (a) Find R_{\min} , which is the safest resistance value of R . (b) While holding R_{\min} , the person decided to wear shoes of resistance R_S to reduce the current to $100 \mu\text{A}$. Find R_S .
- (37) A light bulb is rated 60 W at 240 V . The bulb is connected to a source of 240 V with two equal length wires, each having a resistance $R/2 = 120 \Omega$, see

Fig. 24.34. (a) What is the resistance R_b of the light bulb? (b) What is the value of the current I in the circuit? (c) What is the potential difference between the sockets of the light bulb? (d) What is the actual power delivered to the bulb in this circuit?

Fig. 24.33 See Exercise (36)

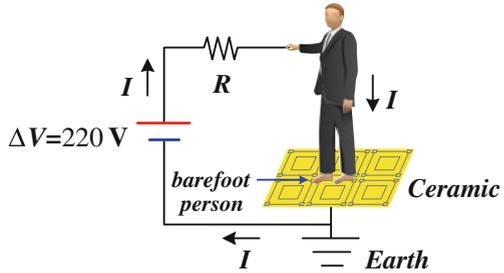
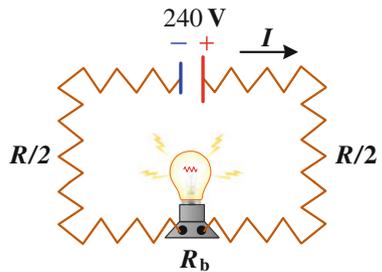
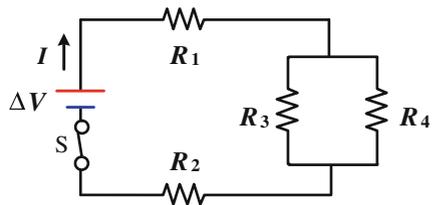


Fig. 24.34 See Exercise (37)



(38) The four resistances of Fig. 24.35 are $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, and $R_4 = 12\ \Omega$. The power source has a potential difference $\Delta V = 12\text{ V}$. (a) Find the equivalent resistance of the combination. (b) What is the value of the current I in the circuit? (c) Find the currents in R_3 and R_4 . (d) Calculate the power delivered to each resistor in the circuit.

Fig. 24.35 See Exercise (38)



Section 24.6 Kirchhoff's Rules

- (39) For the circuit shown in Fig. 24.36, let $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, $\mathcal{E}_1 = 10 \text{ V}$, and $\mathcal{E}_2 = 12 \text{ V}$. Find the values of the currents I_1 , I_2 , and I_3 in the circuit.

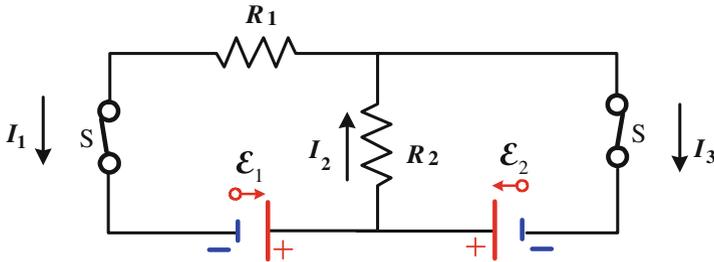
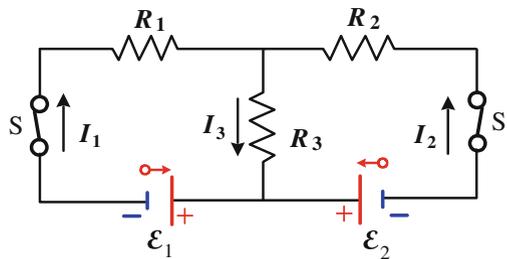


Fig. 24.36 See Exercise (39)

- (40) For the circuit shown in Fig. 24.37, let $R_1 = 1 \Omega$, $R_2 = 2 \Omega$, $R_3 = 3 \Omega$, $\mathcal{E}_1 = 10 \text{ V}$, and $\mathcal{E}_2 = 12 \text{ V}$. Find the values of the currents I_1 , I_2 , and I_3 in the circuit.

Fig. 24.37 See Exercise (40)



- (41) For the circuit shown in Fig. 24.38, let $R_1 = 5 \Omega$, $R_2 = R_3 = 15 \Omega$, $\mathcal{E}_1 = 60 \text{ V}$, $\mathcal{E}_2 = 80 \text{ V}$, and $\mathcal{E}_3 = 10 \text{ V}$. Find the values of the currents I_1 , I_2 , and I_3 in the circuit.
- (42) For the circuit shown in Fig. 24.39, let $R_1 = 2 \Omega$, $R_2 = R_3 = 4 \Omega$, $\mathcal{E}_2 = 20 \text{ V}$, and $\mathcal{E}_3 = 2 \text{ V}$. The ammeter, represented by the symbol $\text{---}\text{Ⓜ}\text{---}$, reads the current I_1 in the wire to be 0.5 A . Find the voltage of the unknown battery \mathcal{E}_1 and the values of the currents I_2 , and I_3 .

Fig. 24.38 See Exercise (41)

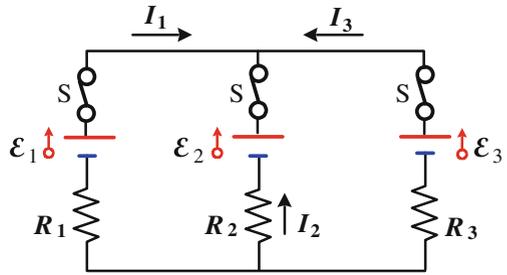
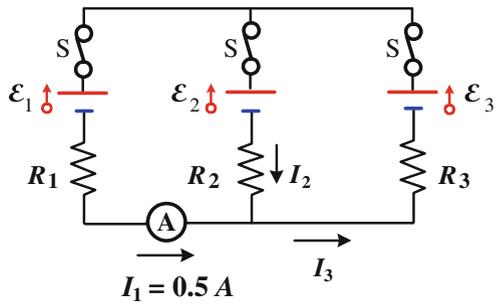


Fig. 24.39 See Exercise (42)



(43) For the circuit shown in Fig. 24.40, let $R_1 = 3 \Omega$, $R_2 = 6 \Omega$, $R_3 = 3 \Omega$, $R_4 = 6 \Omega$, and $\mathcal{E} = 7.5 \text{ V}$. Find the values of the currents I_1 , I_2 , I_3 , and I_4 in the circuit.

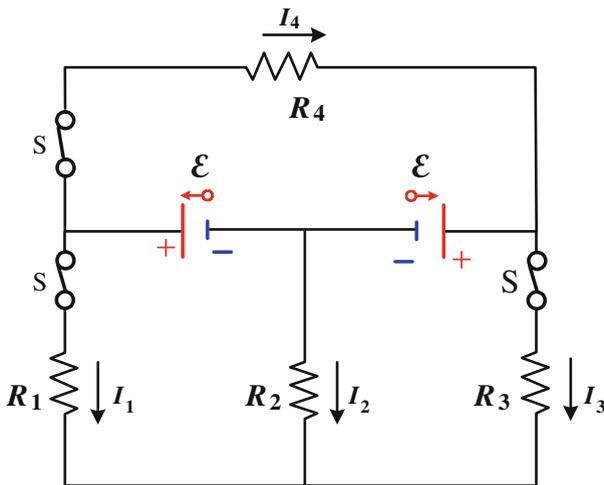


Fig. 24.40 See Exercise (43)

- (44) Each resistor in the different configurations of Fig. 24.41 has the same resistance R . Show that the equivalent resistance of the four parts of the figure are: (a) $7R/5$, (b) $2R/3$, (c) R , and (d) $3R/4$, respectively.

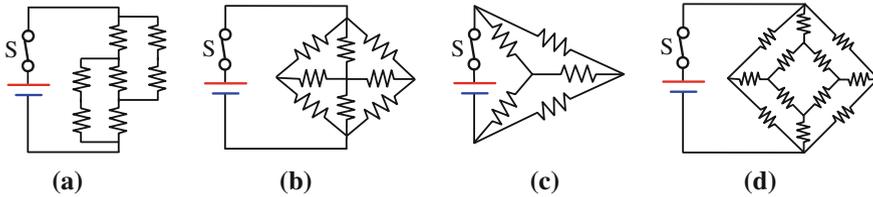
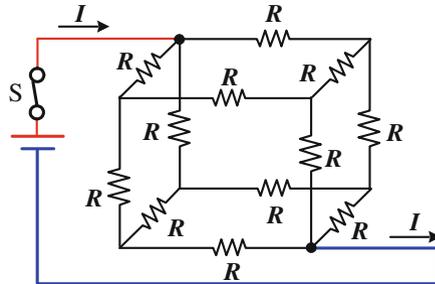


Fig. 24.41 See Exercise (44)

- (45) Apply symmetry arguments to the equal-valued resistors of Fig. 24.42 to show that: (a) the current passing through any resistor in the figure is either $I/3$ or $I/6$. (b) the equivalent resistance of the circuit is $5R/6$.

Fig. 24.42 See Exercise (45)



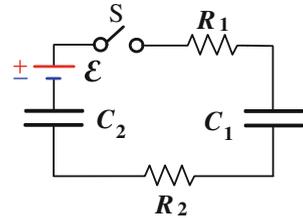
Section 24.7 The RC Circuit

- (46) In the process of charging a capacitor of capacitance C through a resistor of resistance R , about 63% of the maximum charge will accumulate on the capacitor in a time $t = RC$ (known as the time constant $\tau = RC$). In this time, what percentage of the maximum electrostatic energy is stored on the capacitor?
- (47) An uncharged capacitor has a capacitance of $2 \mu\text{F}$. A battery of 12 V charges this capacitor through a $1 \text{ M}\Omega$ resistor. (a) Find the time constant of the circuit, the maximum charge on the capacitor, and the maximum current in the circuit.

(b) How much time is required for the potential difference across the capacitor to reach 6 V?

- (48) Prove that when switch S in Fig. 24.43 is closed, the charge q at time t on any capacitor is $q = Q(1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2)(C_1 + C_2)$ and $Q = (C_1 + C_2) \mathcal{E}$.

Fig. 24.43 See Exercise (47)



- (49) A $5 \mu\text{F}$ capacitor is charged to 220 V. After disconnecting it from its source, a student holds its two lead wires with his bare hands. Assume that the resistance between the student's hands is $50 \text{ k}\Omega$. (a) What is the initial charge on the capacitor and the maximum current that passes through the student's body? (b) Find the charge that remains on the capacitor, and calculate the current that passes through the student's body after 0.5 s.
- (50) The switch in the circuit of Fig. 24.44 is left open for a long time, and then closed at $t = 0$. Let $R_1 = 50 \text{ k}\Omega$, $R_2 = 150 \text{ k}\Omega$, $C = 5 \mu\text{F}$, and $\mathcal{E} = 30 \text{ V}$. Find the time constant before and after the switch is closed. Then find the current in the switch as a function of time.

Fig. 24.44 See Exercise (50)

