Force and Motion

5

The kind of interaction that accelerates an object is called a **force**, which could be a push or pull. From now on, we shall use the capital letter $\vec{F}$ (with an arrow over it) to represent a general force vector. In addition, we shall use the symbol $\Sigma\vec{F}$ for the vector sum of several forces, which we call the **resultant force** or the **net force**.

5.1 The Cause of Acceleration and Newton's Laws

The relationship between forces and the produced acceleration is an aspect of **mechanics** called **dynamics**. Isaac Newton (1642–1727) first formulated this relationship in terms of laws known by his name.

Newton's First Law of Motion

Newton's original first law reads:

> Newton's First Law
> An object will remain at rest, or in motion with constant velocity, unless it experiences a net external force.

$$\text{If}\ \ \Sigma\vec{F} = 0, \quad \text{then} \begin{cases} \vec{v} = 0 \\ \quad \text{or} \\ \vec{v} = \text{constant} \end{cases} \qquad \text{(Newton's first law)} \tag{5.1}$$

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Newton's first law is sometimes called the *law of inertia*, and the set of coordinates that are used to describe the object are called the *inertial reference frames* or alternatively *inertial frames*.

Inertial Frames
An inertial frame is one in which an object experiences zero net force.

Consequently, Newton's first law declares that if the net force on an object is zero, it must stay at rest or move with constant velocity with respect to any inertial frame.

Newton's Second Law of Motion

All observations reveal that the acceleration of an object is directly proportional to the net acting force. These observations are expressed in Newton's second law.

Newton's Second Law
The acceleration of an object, $\vec{a}$, is related to its mass, m, and the resultant force acting on it, $\Sigma\vec{F}$, by the relation:

$$\Sigma\vec{F} = m\vec{a} \quad \text{(Newton's second law)} \tag{5.2}$$

This equation is valid only when the speed of the object is much less than the speed of light. In SI units, we define the unit of force that accelerates a standard 1 kg by 1 m/s^2 as 1 newton (abbreviated to 1 N). Thus, according to Eq. 5.2, we have:

$$1\,\text{N} = (1\,\text{kg})(1\,\text{m/s}^2) = 1\,\text{kg.m/s}^2, \tag{5.3}$$

Although we shall use SI units only from now on, other systems like the CGS (centimeter-gram-second) system and the British system are still in use. Table 5.1 compares lists of all systems currently in use.

Table 5.1 Units in Newton's second law

System	Force[a]	Mass	Acceleration
SI	Newton (N)	Kilogram (kg)	m/s^2
CGS	dyne[b]	gram (g)	cm/s^2
British	Pound (lb)[c]	slug	ft/s^2

[a] 1 N = 10^5 dyne = 0.255 lb. [b] 1 dyne = 1 g.cm/s^2. [c] 1 lb = 1 slug.ft/s^2

Newton's Third Law of Motion

Forces come in pairs. For example, if you lean against a wall with a certain force, the wall reacts and pushes back on you with a force of equal magnitude. Another example of two interacting bodies is shown in Fig. 5.1, where body 1 exerts an action force $\vec{F}_{21}$ (a pull) on body 2. ($\vec{F}_{21}$ is read: force exerted on body 2 by body 1). Experiments show that body 2 would also exert a reaction force $\vec{F}_{12}$ on body 1. These two forces are equal in magnitude and opposite in direction. That is:

$$\vec{F}_{12} = -\vec{F}_{21} \quad \text{(Newton's third law)} \tag{5.4}$$

Equation 5.4 implies that $F_{12} = F_{21}$. Moreover, this equation holds true regardless of whether the two bodies move or remain stationary.

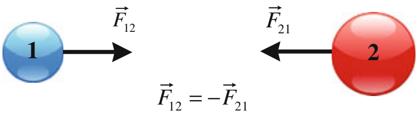


Fig. 5.1 The force exerted by body 1 on body 2 is equal in magnitude but opposite to the force exerted by body 2 on body 1

All observations similar to the previous two examples are summarized in Newton's third law, which states that:

> Newton's Third Law
> To every action there must be a reaction equal in magnitude and opposite in direction.

Forces of an action-reaction pair act on different bodies, i.e. they do not combine to give a net force. In Fig. 5.2, we display an orbiting satellite, where the only force that acts on it is $\vec{F}_{SE}$ (the gravitational force). The corresponding reaction force is $\vec{F}_{ES}$. This force causes the Earth to attain a very small yet undetectable acceleration.

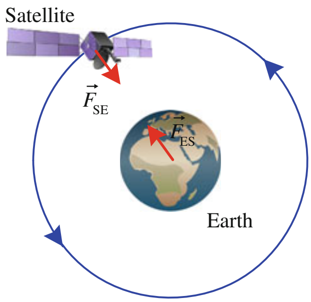


Fig. 5.2 Forces on a satellite and the Earth as action-reaction pair

5.2 Some Particular Forces

Weight ($\vec{W}$)

The **weight** $\vec{W}$ of a body is the force exerted by the Earth on the body. This force is directed toward the center of the Earth and is primarily due to an attraction (called a **gravitational attraction**) between the body and the Earth.

Since a freely falling body experiences an acceleration $\vec{g}$ acting toward the center of the Earth, then applying Newton's second law to a body of mass m, with $\vec{a} = \vec{g}$ and $\Sigma\vec{F} = \vec{W}$, gives the following:

$$\vec{W} = m\vec{g} \tag{5.5}$$

The magnitude of $\vec{W}$ in SI units is in newtons. We can weigh a body with a spring scale (see Fig. 5.3a). The body stretches the spring, moving its pointer along a scale that has been calibrated and marked in either mass or weight units. Alternatively, we can weigh a body by placing it on one pan of an equal-arm balance (Fig. 5.3b) and then adding reference masses on the other pan until we achieve a balance.

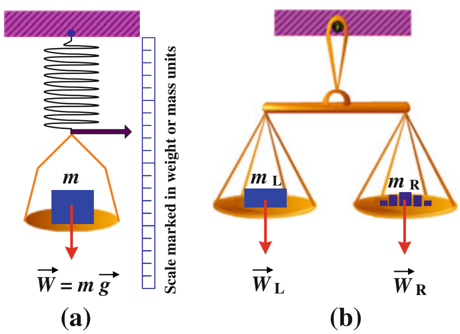


Fig. 5.3 (a) A spring scale. The reading gives the weight if marked in weight units. (b) An equal-arm balance. When balance is achieved, the masses on the *left* (L) and *right* (R) pans are equal

Normal Force ($\vec{N}$)

The reaction of a block of weight $\vec{W}$ is the force exerted on the Earth $\vec{W}'$, see Fig. 5.4a. When this block rests on a table, the table exerts an upward action force, $\vec{N}$, called the **normal force**; the name comes from the mathematical term *normal*, meaning "perpendicular", see Fig. 5.4b. The normal force is the force that prevents the block from falling through the table, and can have any value up to the point of breaking the table. The reaction to $\vec{N}$ is the force that the block exerts on the table, $\vec{N}'$, see Fig. 5.4c. Therefore, we conclude that:

$$\vec{W} = -\vec{W}', \quad \vec{N} = -\vec{N}' \tag{5.6}$$

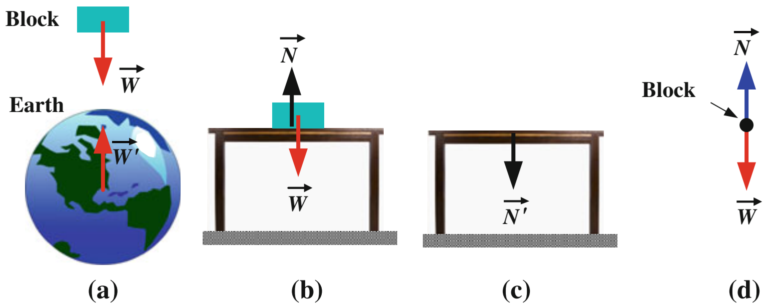


Fig. 5.4 (a) The reaction of a block of weight $\vec{W}$ is the force $\vec{W}'$. (b) A block resting on a table experiences a normal force $\vec{N}$ perpendicular to the table. (c) The reaction force $\vec{N}'$ exerted on the table. (d) The free-body diagram used to solve the block problem

The forces acting on the block are only $\vec{W} = m\vec{g}$ and $\vec{N}$, as seen in Fig. 5.4b. So, the normal force balances the weight of the block and provides equilibrium ($\vec{a} = 0$). To solve problems with Newton's laws, we often draw a **free-body diagram**, representing the body by a dot (or a sketch of the body) and each external force by a vector with its tail on the dot, see Fig. 5.4d. With this figure $\Sigma\vec{F} = m\vec{a}$ becomes:

$$\Sigma\vec{F} = 0 \quad \Rightarrow \quad \vec{N} + \vec{W} = 0 \quad \Rightarrow \quad N - W = 0$$

Thus:

$$N = W = mg \tag{5.7}$$

Foces of Friction ($\vec{f}$)

When we attempt to slide a block over a surface, the intended motion is resisted by a bonding between the block and the surface. We represent this resistance by a force $\vec{f}$ called the **force of friction** or simply **friction**. This force is directed along the surface, opposite to the intended motion. Sometimes, we simplify situations by neglecting friction, and the surface is said to be *frictionless*.

Consider a block resting on a horizontal table, as in Fig. 5.5a, where its weight $\vec{W}$ is balanced by an equal but opposite normal force $\vec{N}$. In Fig. 5.5b, we apply a force $\vec{F}$ on the block, attempting to pull it to the right. The block will remain stationary if $\vec{F}$ is not large enough. The frictional force $\vec{f}$ acts to the left and keeps the block stationary, i.e. $F = f$. We call this frictional force the **force of static friction** $\vec{f}_s$. If we increase F, the static frictional force $\vec{f}_s$ increases, while the block remains at rest. When the applied force F reaches a certain value, the block will be on the verge of slipping and the frictional force will be maximum and denoted by $f_{s,max}$, see Fig. 5.5c. When F exceeds $f_{s,max}$, the block moves to the right, see Fig. 5.5d. When the block is in motion, the frictional force becomes less than $\vec{f}_{s,max}$ and is called the **force of kinetic friction** $\vec{f}_k$, see Fig. 5.5e. The horizontal net force $F - f_k$ accelerates the block to the right.

Experimentally, one finds that both f_s and f_k are proportional to the magnitude of the normal force N acting on the block through a *dimensionless constant* μ. These observations can be summarized as:

1. If the block is not moving, the force of static friction is opposite to the applied force and can have values given by:

$$f_s \leq \mu_s N \tag{5.8}$$

   where the constant μ_s is called the **coefficient of static friction**.

2. When the block is on the **verge of slipping**, we have:

$$f_{s,max} = \mu_s N \tag{5.9}$$

3. If the block begins to move along the surface, the magnitude of the frictional force rapidly decreases to the value f_k given by:

$$f_k = \mu_k N, \quad \mu_k < \mu_s \tag{5.10}$$

where the constant μ_k is called the **coefficient of kinetic friction**.

(a)

(b)

(c)

(e)

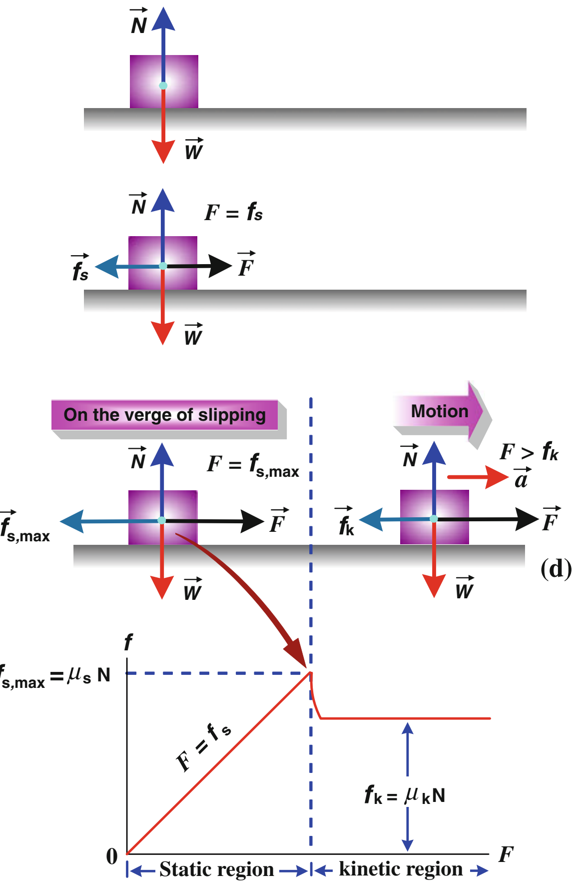


Fig. 5.5 (a) A block at rest on a horizontal table. The static frictions $f_{\rm s}$ and $f_{\rm s,max}$ are shown in parts (b) and (c). When the block moves, the kinetic friction $f_{\rm k}$ becomes less than $f_{\rm s,max}$ as in parts (d) and (e)

The values of the dimensionless coefficients $\mu_{\rm s}$ and $\mu_{\rm k}$ depend on the nature of the surfaces, not on their areas. Regardless, $\mu_{\rm k} < \mu_{\rm s}$ since $f_{\rm k} < f_{\rm s,max}$ as in Fig. 5.5e. Typical values of the coefficients lie in the range $0.05 \le \mu \le 1.5$. Table 5.2 lists some reported values.

A highly polished surface is far from being perfectly flat on the atomic scale. Moreover, the surfaces of everyday objects have layers of oxides and other contaminants. When two such surfaces are placed together, only high points touch each other, see Fig. 5.6. In addition, many contact points weld together, which is called *cold-welding*.

Table 5.2 Some approximate coefficients of friction

Material surfaces	μ_s	μ_k
Ice on ice	0.1	0.03
Wood on ice	0.08	0.06
Metal on metal (lubricated)	0.15	0.06
Wood on wood	0.25–0.5	0.2
Copper on steel	0.53	0.36
Glass on glass	0.94	0.4
Aluminum on steel	0.61	0.47
Steel on steel	0.74	0.57
Rubber on concrete	$\sim$ 0.9	$\sim$ 0.7

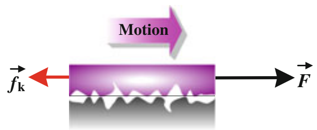


Fig. 5.6 A highly magnified cross section showing some welding at high spots. Force is required to break these welds to maintain motion

When two surfaces are pulled across each other, there is first a tearing of formed welds and then a continuous tearing of reforming welds as additional contacts are made. The kinetic friction $\vec{f}_k$ is the vector sum of all forces (against motion) at those many contacts.

Foces of Tension ($\vec{T}$)

When a rope (or a cord, cable, etc) is attached to a body and pulled, the rope is said to be in **tension**. The rope's function is to transfer force between two bodies. The tension in the rope is defined as the force that the rope exerts on the body. This force is denoted usually by the symbol $\vec{T}$, see Fig. 5.7a and b.

A rope is considered to be *massless* (i.e., its mass is negligible compared to the body's mass) and *non-stretchable*. It pulls on both bodies with a force of the magnitude T, even if the two bodies are accelerating, or the rope is run around a pulley as in Fig. 5.7c and d. Such a pulley is *massless* (has a negligible mass compared to the bodies) and *frictionless* (has negligible friction on its rotational axel).

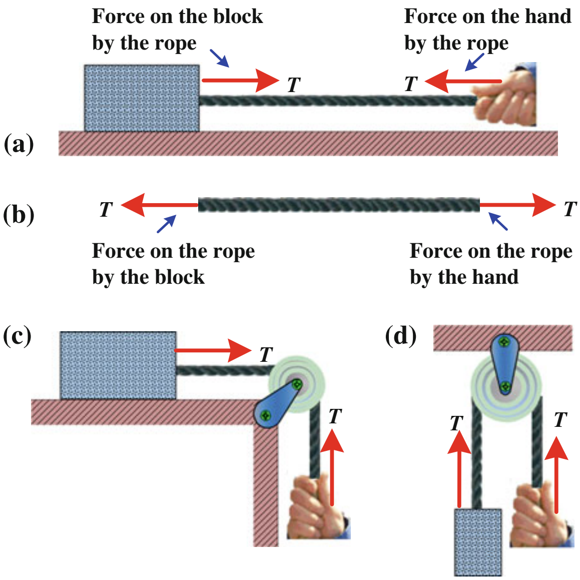


Fig. 5.7 When a rope is under tension, it pulls the block and the hand of parts (a), (c), and (d) with a force of magnitude T. According to Newton's third law, the block and the hand both exert a force on the rope of magnitude T, as shown in part (b) only

Drag Forces ($\vec{F}_D$)—Small Objects

When a small object moves at a low speed v through a viscous medium, it experiences a resistive **drag force** $\vec{F}_D$ that opposes its motion. In such situations, the force has a magnitude given by:

$$F_D = bv \tag{5.11}$$

where b is a proportionality constant that depends on the properties of the medium and on the shape of the object, and b has the units kg/s.

If we assume that a sphere of mass m and weight $W = mg$ is released from rest in a fluid as in Fig. 5.8, then application of Newton's second law $\Sigma \vec{F} = m\vec{a}$ in the vertical direction will give:

$$mg - bv = ma \quad \Rightarrow \quad mg - bv = m\frac{dv}{dt} \tag{5.12}$$

Note that when the initial speed $v = 0$, the resistive force is zero and the acceleration $a = dv/dt$ is g. As the time t increases, the speed increases and the resistive force increases, while the acceleration decreases. Finally, the acceleration becomes zero when the resistive force equals the weight mg. At this stage, the speed has reached its *terminal speed* v_t. The terminal speed can be obtained from Eq. 5.12 by setting $v = v_t$ and $a = dv/dt = 0$. Thus:

$$v_t = \frac{mg}{b} \tag{5.13}$$

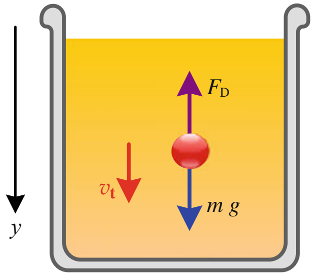


Fig. 5.8 A small sphere falling through a viscous fluid with a *low* speed. The resistive drag force $\vec{F}_D$ opposes the motion of the sphere

Drag Forces ($\vec{F}_D$)—Large Objects

When a large object (such as a baseball, skydiver, or an airplane) moves at a high speed v in a medium (gas or liquid) of density ρ (mass per unit volume), it experiences a **drag force** $\vec{F}_D$ that opposes the motion. From experiments, it was found that in these situations the magnitude F_D will be given by:

$$F_D = \tfrac{1}{2}\, C \rho A v^2 \tag{5.14}$$

where C is a dimensionless proportionality constant called the **drag coefficient**, and A is the effective cross sectional area of the object, taken to be perpendicular to its velocity $\vec{v}$. If v varies significantly, C can vary as well, but we ignore such complications.

If we assume a body of mass m and weight $W = mg$ falls from rest in air, as shown in Fig. 5.9, then application of Newton's second law $\Sigma\vec{F} = m\vec{a}$ in the vertical direction as in part (c) of the figure gives:

$$mg - \tfrac{1}{2}\, C \rho A v^2 = ma \quad \Rightarrow \quad mg - \tfrac{1}{2}\, C \rho A v^2 = m\frac{dv}{dt} \tag{5.15}$$

By setting $a = 0$ in this equation, the terminal speed is given by:

$$v_t = \sqrt{\frac{2mg}{C\rho A}} \tag{5.16}$$

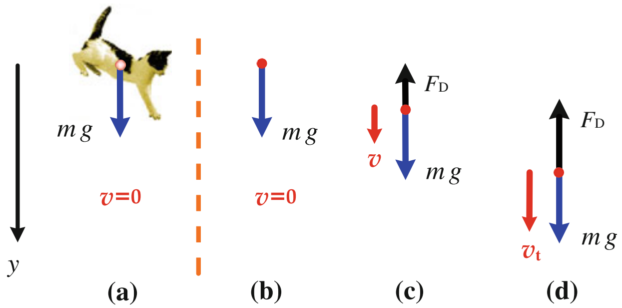


Fig. 5.9 Part (a) shows a body (cat) when it has just begun to fall through air and part (b) shows its corresponding free-body diagram. (c) Later, the drag force F_D has developed. (d) F_D has increased until it balances mg and the body falls with constant terminal speed v_t

5.3 Applications to Newton's Laws

This section is devoted to applications related to Newton's three laws of motion. The idea behind the examples is to let you know how to tackle a problem and how to translate a sketch of a situation to a free-body diagram with appropriate axes.

Example 5.1

(a) How much force is needed to give a 20,000 kg heavy loaded truck on a leveled track an acceleration of 1.5 m/s^2, and what is the force exerted by the track on the truck? (b) If the truck starts from rest, find its speed and position after 2 s.

Solution: Part (a) of Fig. 5.10 depicts the truck's travel. In part (b) we choose the coordinate axes and show the truck's free-body diagram. In this part we show the truck's weight $\vec{W}$ (acting downwards), the normal force $\vec{N}$ (acting perpendicularly to the track), and the driving force $\vec{F}$ (acting to the right).

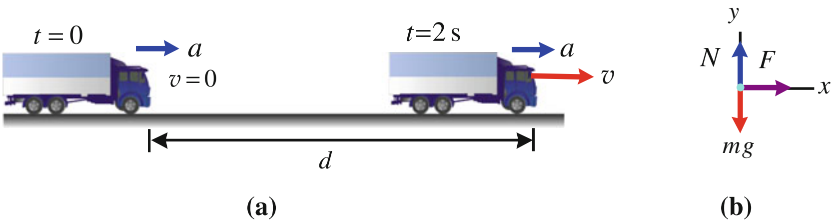


Fig. 5.10

(a) Applying Newton's second law in the component form, we find:

$$\Sigma F_x = F = ma \quad \text{and} \quad \Sigma F_y = N - mg = 0$$

From the first and second equations we find F and N as follows:

$$F = ma = (20{,}000 \text{ kg})(1.5 \text{ m/s}^2) = 30{,}000 \text{ N}$$

$$N = mg = (20{,}000 \text{ kg})(9.8 \text{ m/s}^2) = 196{,}000 \text{ N}$$

(b) Since a is constant, we can use $v = v_\circ + at$ and $x = v_\circ t + \frac{1}{2}at^2$ to find the speed v and the distance d after 2 s as follows:

$$v = v_\circ + at = 0 + (1.5 \text{ m/s}^2)(2 \text{ s}) = 3 \text{ m/s} = 10.8 \text{ km/h}$$

$$d = v_\circ t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(1.5 \text{ m/s}^2)(2 \text{ s})^2 = 3 \text{ m}$$

Example 5.2

A block of mass $m = 21$ kg hangs from three cords as shown in part (a) of Fig. 5.11. Taking $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\sin\phi = 5/13$, and $\cos\phi = 12/13$, find the tensions in the three cords.

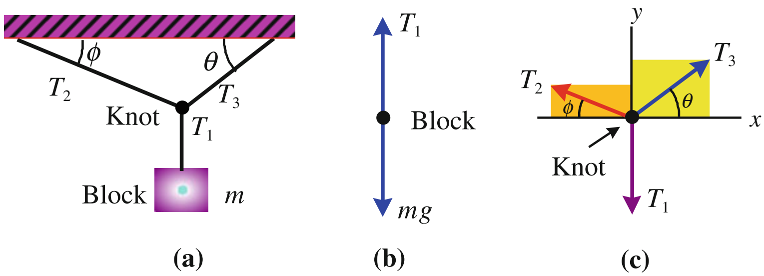


Fig. 5.11

Solution: We construct a free-body diagram for the block as shown in part (b) of Fig. 5.11. The tension in the vertical cord balances the weight of the block. Thus, by taking $g = 10$ m/s^2, we get:

$$T_1 = mg = (21 \text{ kg})(10 \text{ m/s}^2) = 210 \text{ N}$$

In part (c) of Fig. 5.11, we first construct a free-body diagram of the *stationary* knot that holds the three cords together, and then we choose the coordinate axes. By applying Newton's second law in the x and y directions of part (c) of the figure, we find that:

$$\boxed{\Sigma F_x = T_3 \cos\theta - T_2 \cos\phi = 0} \quad \text{and} \quad \boxed{\Sigma F_y = T_3 \sin\theta + T_2 \sin\phi - T_1 = 0}$$

From the x-component equation we get the following relation:

$$T_3 = \frac{\cos\phi}{\cos\theta}\, T_2 = \frac{12/13}{3/5}\, T_2 = \frac{20}{13}\, T_2$$

When we substitute the result of T_3 into the y component equation, after putting $T_1 = mg = 210$ N, we get:

$$\frac{20}{13}\,\frac{4}{5}\, T_2 + \frac{5}{13} T_2 - 210 = 0 \quad \Rightarrow \quad \left(\frac{16}{13} + \frac{5}{13}\right) T_2 = 210 \quad \Rightarrow \quad T_2 = 130 \text{ N}$$

Consequently, one can find the value of the third tension to be:

$$T_3 = \frac{20}{13}\, T_2 = \frac{20}{13} \times 130\,\text{N} = 200 \text{ N}$$

Example 5.3

Two masses m_1 and m_2 ($m_2 > m_1$), are connected by a light cord that passes over a massless, frictionless pulley as shown in part (a) of Fig. 5.12. This arrangement is called Atwood's machine and sometimes is used to measure the acceleration due to gravity. Find the magnitude of acceleration of the two masses and the tension in the cord (consider $m_1 = 4$ kg and $m_2 = 6$ kg).

Solution: We construct a free-body diagram for the two masses as shown in parts (b) and (c) of Fig. 5.12. When Newton's second law is applied to m_1 in part (b) of the figure, we find:

$$\Sigma F_y = T - m_1 g = m_1 a$$

Also, we do the same for m_2 of part (c) of the figure, to get:

$$\Sigma F_y = m_2 g - T = m_2 a$$

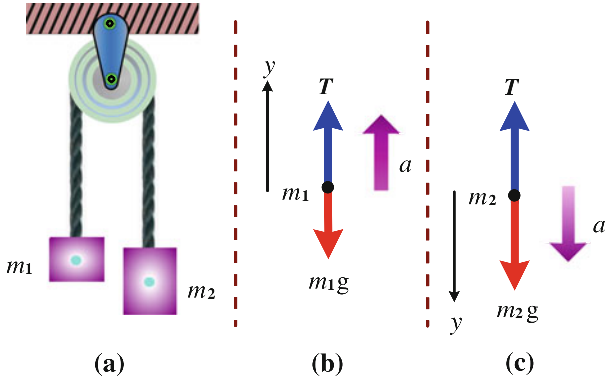


Fig. 5.12

When we add the last two equations, T will cancel out, and we get:

$$m_2 g - m_1 g = m_2 a + m_1 a$$

Thus: $$\boxed{a = \frac{m_2 - m_1}{m_1 + m_2} g} \quad \Rightarrow \quad a = \frac{6\text{ kg} - 4\text{ kg}}{6\text{ kg} + 4\text{ kg}} \times 9.8\text{ m/s}^2 = 1.96\text{ m/s}^2$$

If we substitute with a into the first equation we get:

$$\boxed{T = \frac{2m_1 m_2}{m_1 + m_2} g} \quad \Rightarrow \quad T = \frac{2(4\text{ kg})(6\text{ kg})}{6\text{ kg} + 4\text{ kg}} \times 9.8\text{ m/s}^2 = 47\text{ N}$$

Example 5.4

A car moving with an initial speed $v_\circ = 30$ m/s suddenly brakes, locking its wheels (i.e. it starts to skid). The car travels on the road a distance $d = 75$ m before it comes to a complete stop. Find the coefficient of kinetic friction between the tires of the car and the road.

Solution: Part (a) of Fig. 5.13 depicts the car's travel. In part (b) we choose the coordinate axes and show the car's free-body diagram during its skid. In this part we show the car's weight $\vec{W}$ (acting downwards), the normal force $\vec{N}$ (acting perpendicularly to the road), and kinetic frictional force $\vec{f}_k$ (acting to the left).

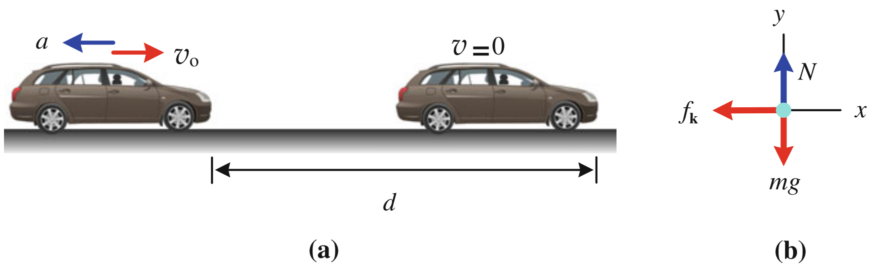


Fig. 5.13

Applying Newton's second law in the component form, we find that:

$$\Sigma F_x = -f_k = ma$$

$$\Sigma F_y = N - mg = 0$$

From the last equation we get $N = mg$. Since $f_k = \mu_k N = \mu_k mg$, then the first equation gives:

$$-\mu_k mg = ma$$

Thus: $$a = -\mu_k g$$

The negative sign means that the acceleration is to the left. Since a is constant, we can use $v^2 = v_\circ^2 + 2ax$, with $v = 0$ and $x = d$. This gives:

$$0 = v_\circ^2 + 2ad = v_\circ^2 - 2\mu_k\, gd$$

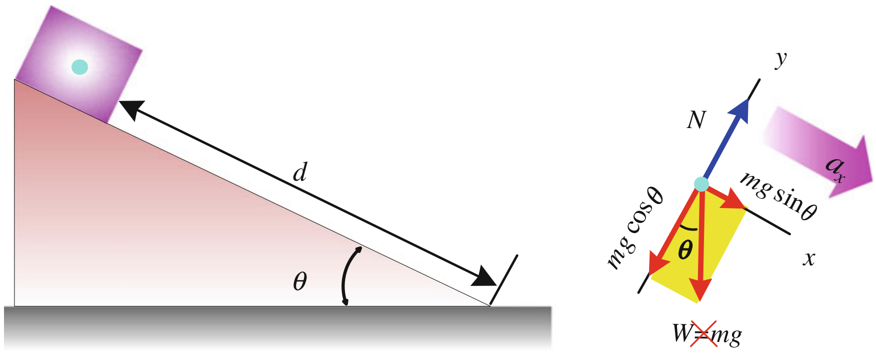
Thus: $$\mu_k = \frac{v_\circ^2}{2gd} = \frac{(30\text{ m/s})^2}{2(9.8\text{ m/s}^2)(75\text{m})} = 0.61$$

Example 5.5

A block of mass $m = 2$ kg is placed on an inclined plane of angle $\theta = 30°$, as shown in Fig. 5.14. The block is released from rest at the top of the plane, where the distance from the bottom is $d = 10$ m. (a) Find the magnitude of the acceleration of the block and the normal force exerted on the block. (b) How long does it take the block to reach the bottom, and what is its speed just as it gets there?

Solution: (a) We construct a free-body diagram to this example as shown in part (b) of Fig. 5.14. The only forces on the block are the weight $\vec{W}$ (acting downward) and the normal force $\vec{N}$ (acting perpendicular to the inclined plane). We choose a coordinate system with x axis parallel to the incline and y-axis perpendicular to it. With this choice, the angle between the weight vector and the negative direction of the y-axis equals the angle θ of the inclined plane. After that, we decompose the weight to a component of magnitude $mg\cos\theta$ along the negative y-axis and a component of magnitude $mg\sin\theta$ along the x axis. The block will slide along the inclined plane with acceleration a_x and will never leave the plane; i.e. $a_y = 0$. Applying Newton's second law to the x and y components gives:

$$\Sigma F_x = mg\sin\theta = m\,a_x$$
$$\Sigma F_y = N - mg\cos\theta = 0$$



(a) **(b)**

Fig. 5.14

From the x-component form, we see that the acceleration along the incline is provided by the component of the weight down the incline. By taking $g = 10$ m/s^2, we get:

$$a_x = g\sin\theta = (10\ \text{m/s}^2)(\sin 30°) = 5\ \text{m/s}^2$$

Notice that when $\theta = 0$ (i.e. when the plane is horizontal) we have $a_x = 0$ (the minimum acceleration value). Also, we see that when $\theta = 90°$ (i.e. when the plane is vertical) the case resembles a free fall scenario, resulting in $a_x = g$ (the maximum acceleration value).

From the y component of Newton's second law we find N to be:

$$N = mg\cos\theta = (2\text{ kg})(10\text{ m/s}^2)(\cos 30°) = 17.32\text{ N}$$

Also, notice that when $\theta = 0$, we have $N = mg = 20$ N (its maximum value), and when $\theta = 90°$, we have $N = 0$ (its minimum value).

(b) Since $a_x =$ constant, we apply the equation $x = v_{x\circ}\, t + \frac{1}{2} a_x\, t^2$ to the block with $v_{x\circ} = 0$ and $x = d = 10$ m to get the following relation:

$$d = \frac{1}{2} a_x\, t^2$$

Then, Solving for t and taking the positive root yields:

$$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2 \times (10\text{ m})}{5\text{ m/s}^2}} = 2\text{ s}$$

Also, we can apply the kinematics equation $v_x^2 = v_{x\circ}^2 + 2a_x\, x$ with $v_{x\circ} = 0$ and $x = d = 10$ m to get the following relation:

$$v_x^2 = 2a_x\, d$$

Then, solving for v_x and taking the positive root yields:

$$v_x = \sqrt{2a_x\, d} = \sqrt{2(5\text{ m/s}^2)(10\text{ m})} = 10\text{ m/s}$$

Example 5.6

A block of mass $m_1 = 4$ kg lying on a rough horizontal surface is connected to a second block of mass $m_2 = 6$ kg by a light non-stretchable cord over a massless, frictionless pulley as shown in part (a) of Fig. 5.15. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$. (a) Find the magnitudes of the acceleration of the system and the tension in the cord. (b) Find the relation between m_1 and m_2 in the case when the system is on the verge of slipping.

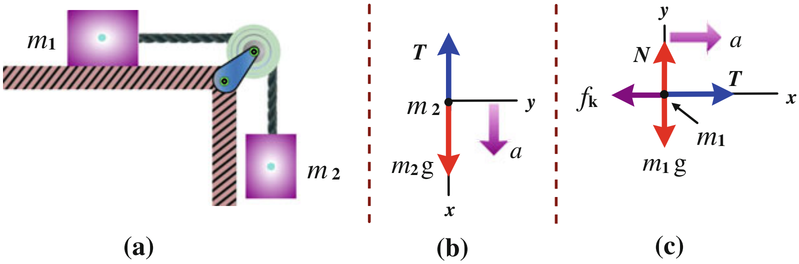


Fig. 5.15

Solution: (a) Since the cord is *non-stretchable*, the two masses have the same magnitude of acceleration. Consequently, we construct a free-body diagram for the two masses as shown in parts (b) and (c) of Fig. 5.15, where we take the x axis always along any of the body's motion. In this case a cannot take negative values. When Newton's second law is applied to m_2 in part (b) of the figure, we find:

$$(1) \qquad \Sigma F_x = m_2 g - T = m_2 a$$
$$\Sigma F_y = 0$$

From (1), we can find the magnitude of the tension in terms of g and a. That is:

$$(2) \qquad T = m_2 g - m_2 a$$

Doing the same for m_1 (see part (c) of Fig. 5.15) we get:

$$(3) \qquad \Sigma F_x = T - f_k = m_1 a$$
$$(4) \qquad \Sigma F_y = N - m_1 g = 0$$

Since $f_k = \mu_k N$, and from (4) we have $N = m_1 g$, then:

$$(5) \qquad f_k = \mu_k m_1 g$$

When this result is substituted into (3), we get:

$$(6) \qquad T = \mu_k m_1 g + m_1 a$$

Equating the magnitude of the tension in (2) and (6), we get:

$$\mu_k m_1 g + m_1 a = m_2 (g - a)$$

Solving for a we get:

$$\boxed{a = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g}$$

Note that, when $m_2 > \mu_k m_1$ we have accelerated motion, and when $m_2 = \mu_k m_1$, we have motion with zero acceleration, i.e. the speed is constant. The value of a can then be evaluated as follows:

$$a = \frac{6 \text{ kg} - 0.5(4 \text{ kg})}{6 \text{ kg} + 4 \text{ kg}} \times 9.8 \text{ m/s}^2 = 3.92 \text{ m/s}^2$$

We can find T by substituting the expression of a into (6), to get:

$$\boxed{T = \frac{(\mu_k + 1) m_1 m_2}{m_1 + m_2} g}$$

Thus: $$T = \frac{(0.5 + 1)(4 \text{ kg})(6 \text{ kg})}{6 \text{ kg} + 4 \text{ kg}} \times 9.8 \text{ m/s}^2 = 35.28 \text{ N}$$

(b) When the system is on the verge of slipping, the magnitude of the force T that acts on mass m_1 must equal the maximum static friction $f_{s,max} = \mu_s N$, i.e. $T = \mu_s N = \mu_s m_1 g$. Also, the weight of the mass m_2 must equal the magnitude of the tension, i.e. $T = m_2 g$. Thus:

$$m_2 g = \mu_s m_1 g$$

Finally:

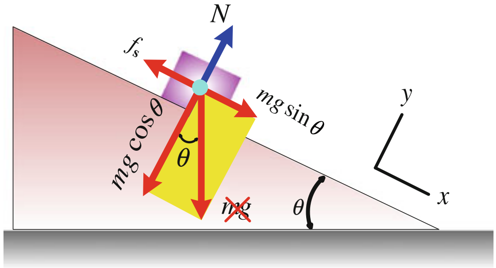
$$\boxed{m_2 = \mu_s m_1} \quad \text{(On the verge of slipping)}$$

Example 5.7

A block is at rest on a rough inclined plane of angle θ, as shown in Fig. 5.16. (a) Find the static frictional force f_s in terms of N and θ. (b) When the angle is increased until the block is on the verge of slipping at $\theta = \theta_c = 38.7°$, find the value of the coefficient of static friction μ_s. (c) After we increase θ further to allow the block to accelerate and then decrease θ again to the value $\theta = \theta' = 26.6°$

to allow the block to move with constant speed, find the coefficient of kinetic friction μ_k.

Fig. 5.16



Solution: (a) The block is balanced under its weight mg, the normal force N, and the static frictional force f_s. Taking x parallel to the plane and y perpendicular to it, then Newton's second law will give:

$$\Sigma F_x = mg \sin\theta - f_s = 0$$
$$\Sigma F_y = N - mg \cos\theta = 0$$

From the last equation we find $mg = N/\cos\theta$. Therefore, we can eliminate mg from the first equation to get:

$$f_s = mg \sin\theta = \frac{N}{\cos\theta} \sin\theta = N \tan\theta$$

(b) When the inclined plane is at the critical angle θ_c, the block is on the verge of slipping and $f_s = f_{s,max} = \mu_s N$. So, at this angle the last equation becomes $\mu_s N = N \tan\theta_c$.

Thus: $$\boxed{\mu_s = \tan\theta_c} \xrightarrow[\text{when } \theta_c = 38.7^\circ]{} \mu_s = \tan 38.7^\circ = 0.8$$

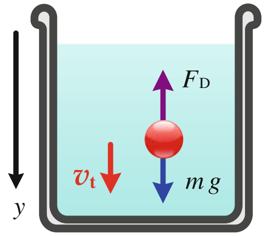
(c) When the block moves with constant speed at $\theta' = 26.6^\circ$, the kinetic friction $f_k = \mu_k N$ equals the weight component $mg \sin\theta'$.

Thus: $$\boxed{\mu_k = \tan\theta'} \xrightarrow[\text{when } \theta' = 26.6^\circ]{} \mu_k = \tan 26.6^\circ = 0.5$$

Example 5.8

A small sphere of mass $m = 1.5$ g is released from rest in a large vessel filled with liquid, see Fig. 5.17. The sphere reaches a terminal speed of $v_t = 2.45$ cm/s. Assume that the resistive drag force is given by Eq. 5.11.* (a) Solve Eq. 5.12 to find the speed of the sphere as a function of time. (b) Find the time t it takes the sphere to reach a speed of 0.9 v_t.

Fig. 5.17



Solution: * (a) To solve Eq. 5.12, we set $\tau = m/b$, which is called the **time constant**, and perform the following steps:

$$mg - bv = m\frac{dv}{dt} \quad \Rightarrow \quad \int_0^v \frac{dv}{v_t - v} = \tau^{-1}\int_0^t dt$$

The previous integration can be performed to get:

$$\boxed{v = v_t(1 - e^{-t/\tau})}$$

One can find from this result that the time $\tau = m/b$ is the time it takes the sphere to reach 63% of its terminal speed.

(b) Let us first determine the coefficient b in Eq. 5.11. Since the terminal speed is given by the relation $F_D = bv_t = mg$, i.e. $v_t = mg/b$, then the value of b will be given by:

$$b = \frac{mg}{v_t} = \frac{(1.5 \times 10^{-3}\,\text{kg})(9.8\ \text{m/s}^2)}{2.45 \times 10^{-2}\ \text{m/s}} = 0.6\ \text{kg/s}$$

Therefore, the value of the time τ is given by:

$$\tau = \frac{m}{b} = \frac{1.5 \times 10^{-3}\,\text{kg}}{0.6\ \text{kg/s}} = 2.5 \times 10^{-3}\ \text{s} = 2.5\ \text{ms}$$

We set $v = 0.9\,v_t$ in the resulting formula of part (a), and we perform the following steps, to find the corresponding time t:

$$0.9 = 1 - e^{-t/\tau} \quad \Rightarrow \quad e^{-t/\tau} = 0.1 \quad \Rightarrow \quad t = -\tau \ln(0.1)$$

Thus: $$t = -(2.5 \times 10^{-3}\,\text{s})(-2.303) = 5.76 \times 10^{-3}\,\text{s} = 5.76\ \text{ms}$$

5.4 Exercises

Section 5.3 Applications to Newton's Laws

(Take $g = 10\ \text{m/s}^2$ in all the following exercises unless g is given)

(1) A 10 g bullet accelerates from rest to 500 m/s in a gun barrel of length 10 cm, see Fig. 5.18. Find the accelerating force (assuming it constant).

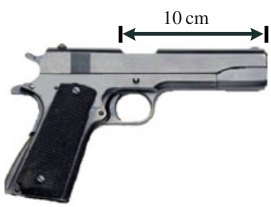


Fig. 5.18 See Exercise (1)

(2) A horizontal cable pulls a golf cart of mass 400 kg along a horizontal track. As in Fig. 5.19, the tension in the cable is 800 N. (a) Starting from rest, how long will it take the cart to reach a speed of 10 m/s? (b) Find the distance covered during this time.

Fig. 5.19 See Exercise (2)

(3) A block of mass 2 kg is accelerated by the two forces $\vec{F}_1 = 8\vec{i} + 3\vec{j}$ and $\vec{F}_2 = -5\vec{i} - 7\vec{j}$, (all units in newtons). (a) What is the net force on the block in unit vector notation, and what is its magnitude and direction? (b) What is the magnitude and direction of the acceleration?

(4) Assume only five forces are acting in the xy plane on a block of mass 4 kg as shown in Fig. 5.20. (a) Taking $\sin\theta = 4/5$ and $\cos\theta = 3/5$, find the block's acceleration in unit-vector notation. (b) Find the acceleration's magnitude and direction.

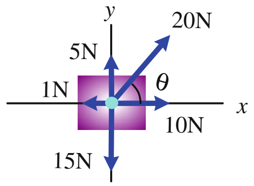


Fig. 5.20 See Exercise (4)

(5) Consider a block of weight W hanging from three ropes as shown in Fig. 5.21. (a) At what angle θ will the magnitude of the tensions T_2 and T_3 each be equal to the weight W. (b) Is this angle independent of W?

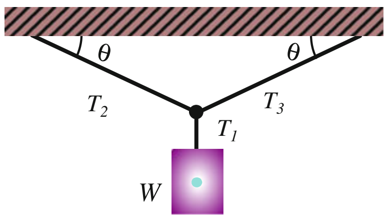


Fig. 5.21 See Exercise (5)

(6) A traffic light of mass $m = 10$ kg is suspended over a road as shown in Fig. 5.22. The ropes are connected to the top of two vertical and identical posts at an angle $\theta = 65°$. Find the magnitude of the tension in all three cables.

(7) A block of mass 20 kg is suspended from the ceiling and the wall by three cords tied together as shown in Fig. 5.23. Find the magnitude of the tensions T_1, T_2, and T_3 in the ropes.

(8) After applying its brakes on a dry road, a 1,000 kg car moving at $v_\circ = 40$ m/s requires a minimum distance d of 50 m to come to a complete stop *without skidding*, see Fig. 5.24. (a) Find the car's acceleration. (b) Find the frictional force exerted on the car by the road. (c) Find the coefficient of static friction.

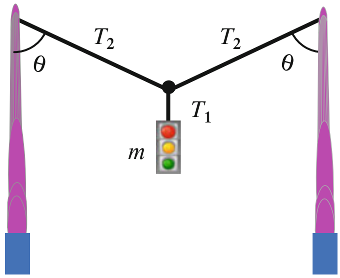


Fig. 5.22 See Exercise (6)

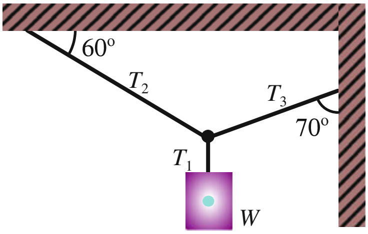


Fig. 5.23 See Exercise (7)

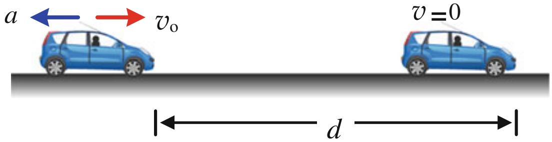


Fig. 5.24 See Exercise (8)

(9) A small sphere of mass m is attached to one end of a massless thread. The other end of that thread is fixed in the roof of a truck when it is at rest. Take $g = 9.8\,\text{m/s}^2$. (a) What angle θ does the thread make with the vertical when the truck has a constant acceleration $a = 1.5\,\text{m/s}^2$, see Fig. 5.25? (b) Find θ when the truck is moving with a constant velocity of magnitude $v = 100\,\text{km/h}$?

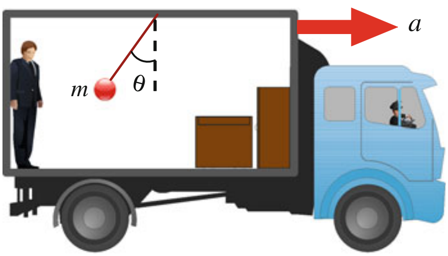
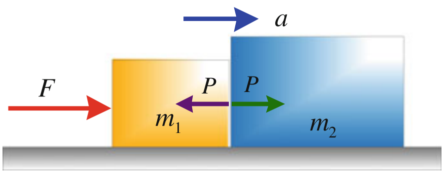


Fig. 5.25 See Exercise (9)

(10) A small object is hanging by a thread from the rearview mirror of a sports car. The car accelerates uniformly from rest to 90 km/h in 10 s. What angle θ does the thread make with the vertical?

(11) Two blocks of masses $m_1 = 4\,\text{kg}$ and $m_2 = 12\,\text{kg}$ are in contact on a smooth horizontal surface. A horizontal force of magnitude $F = 12$ N pushes them as shown in Fig. 5.26. (a) Find the magnitude of the acceleration of the system. (b) Find the magnitude of the force P that block m_1 exerts on block m_2.

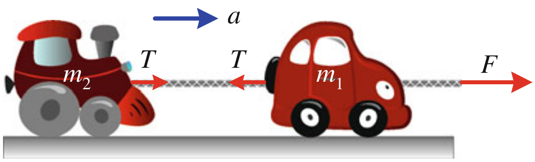
Fig. 5.26 See Exercise (11)



(12) Repeat exercise 11 if block m_2 is before block m_1 and the same force acts on m_2.

(13) Two toys of masses $m_1 = 40$ g and $m_2 = 120$ g are connected by a massless rope and lie on a horizontal frictionless surface as shown in Fig. 5.27. The toys are pulled to the right by a horizontal force of magnitude $F = 0.04\,\text{N}$. (a) Find the acceleration of the system. (b) Find the magnitude of the tension force T in the connecting rope.

Fig. 5.27 See Exercise (13)

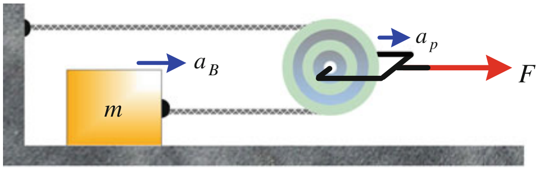


(14) When the surface of exercise 13 is rough, it is found that the toys move with a constant velocity. Find the common coefficient of kinetic friction μ_k between the toys and the surface.

(15) In Fig. 5.28, the pulley is assumed massless and frictionless and rotates freely about its axle. The block has a mass $m = 6\,\text{kg}$ and the pulley is pulled to the right by a horizontal force of magnitude $F = 24\,\text{N}$. As the block moves a distance s_B in time t, the pulley moves half that distance in the same time t, i.e., $s_P = s_B/2$. (a) Find the acceleration ratio a_P/s_B. (b) Find the magnitudes of the tension in the cord and the acceleration of the block if there is no friction between the

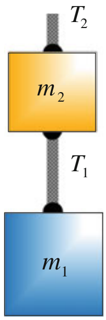
block and the surface. (c) Answer part (b) assuming that the kinetic friction between the block and the surface is $\mu_k = 0.1$.

Fig. 5.28 See Exercise (15)



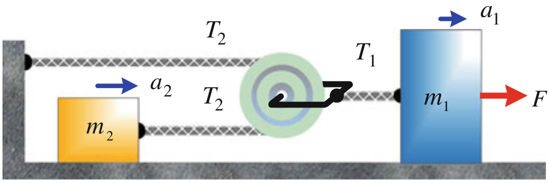
(16) A block of mass $m_1 = 6$ kg is hanging by a massless cord connected to another block of mass $m_2 = 4$ kg, which is also hanging by a massless cord, as shown in Fig. 5.29. (a) What is the tension in the cords when the system is at rest? (b) What is the tension in the cords when the two blocks are pulled up by the upper cord with an acceleration of 2 m/s^2?

Fig. 5.29 See Exercise (16)



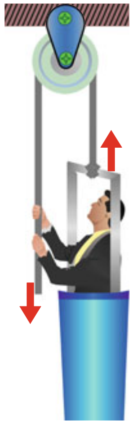
(17) In Fig. 5.30, the pulley is assumed massless and frictionless and rotates freely about its axle. The blocks have masses $m_1 = 40$ g and $m_2 = 20$ g, and block m_1 is pulled to the right by a horizontal force of magnitude $F = 0.03$ N. Find the magnitude of the acceleration of block m_2 and the tension in the cord if the surface is frictionless.

Fig. 5.30 See Exercise (17)



(18) Figure 5.31 shows a bucket connected to a massless rope, which runs over a massless frictionless pulley. A man standing inside the bucket pulls the rope downwards in order to raise himself upwards. The mass of the man is 80 kg and the mass of the bucket is 20 kg. (a) How hard must the man pull the rope for him and the bucket to ascend at a constant speed? (b) Calculate the force that is needed for an upward acceleration of 1.2 m/s^2. (c) Using the man's free-body diagram, find the normal force exerted on the man by the bucket in parts (a) and (b).

Fig. 5.31 See Exercise (18)



(19) A block of mass $m_1 = 2$ kg rests on the top of a second block of mass $m_2 = 8$ kg, as shown in Fig. 5.32. The left sides of the two blocks ate connected by a massless cord, which runs over a fixed massless frictionless pulley. The right side of block m_2 is pulled to the right by a horizontal force of magnitude F. How large must F such that block m_2 accelerates at 2 m/s^2?

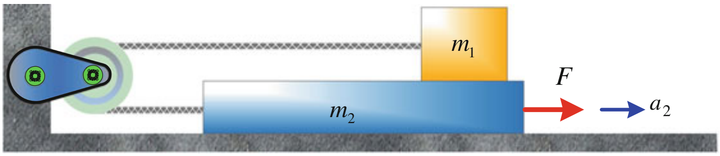
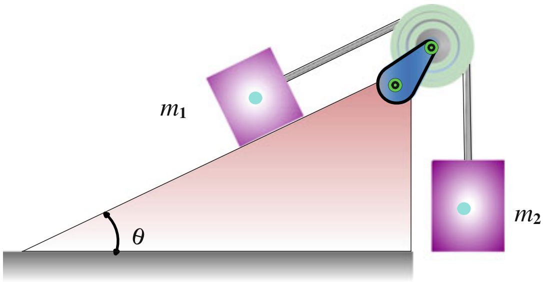


Fig. 5.32 See Exercise (19)

(20) Repeat exercise 19, this time assuming that the coefficient of kinetic friction between the surfaces at the top and bottom of the block m_2 is 0.5.

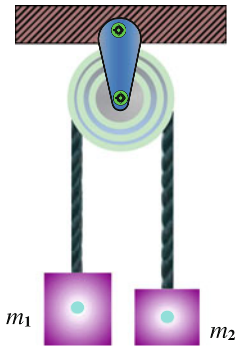
(21) A block of mass $m_1 = 4$ kg lies on a frictionless inclined plane of angle $\theta = 30°$. This block is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 6$ kg hanging vertically, as shown in Fig. 5.33. (a) For each block, find the magnitude and direction of its acceleration. (b) What is the magnitude of the tension in the cord? (c) Repeat parts (a) and (b) after replacing each block by the other.

Fig. 5.33 See Exercise (21)



(22) The Atwood's machine in Fig. 5.34 consists of two masses $m_1 = 6$ kg and $m_2 = 4$ kg that are connected by a light long cord that passes over a massless, frictionless pulley. Find the magnitude and direction of the acceleration of the two masses and the tension in the cord.

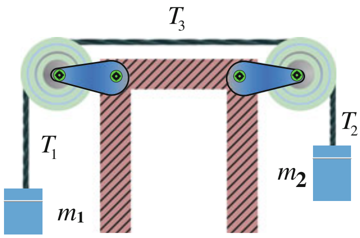
Fig. 5.34 See Exercise (22)



(23) If the two blocks in exercise 22 are initially at rest and are 2.25 m above the ground, find the speed of m_1 before hitting the ground.

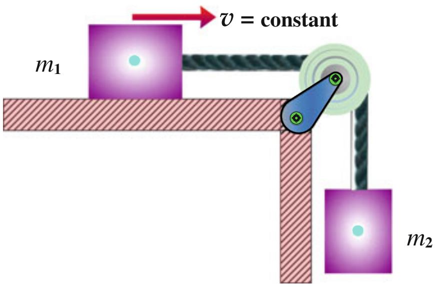
(24) Two blocks having masses m_1 and m_2 ($m_2 > m_1$) are connected to each other by a light non-stretchable cord that passes over two identical massless frictionless pulleys, which rotate freely about their axles, as shown in Fig. 5.35. (a) Find the acceleration of each block and the tensions T_1, T_2, and T_3 in the cord.

Fig. 5.35 See Exercise (24)



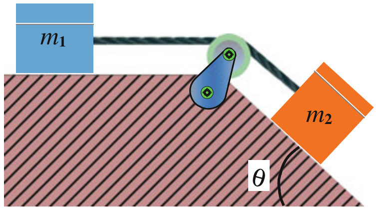
(25) A block of mass $m_1 = 6\,\text{kg}$ lying on a rough horizontal surface is connected to a second block of mass $m_2 = 3\,\text{kg}$ by a light non-stretchable cord over a massless, frictionless pulley as shown in Fig. 5.36. Assuming the two blocks move with constant speed, find the coefficient of kinetic friction between m_1 and the rough horizontal surface, and find the tension in the cord.

Fig. 5.36 See Exercise (25)



(26) A block of mass m_1 located on a horizontal frictionless surface is connected by a light non-stretchable cord that passes over a massless frictionless pulley to a second block of mass m_2, which is allowed to move on an inclined frictionless plane of angle θ, as shown in Fig. 5.37. Find the acceleration of the two blocks and the tension in the cord when $m_1 = 2\,\text{kg}$, $m_2 = 6\,\text{kg}$, $\sin\theta = 4/5$, and $\cos\theta = 3/5$.

Fig. 5.37 See Exercise (26)



(27) Repeat exercise 26, this time assuming that the coefficient of kinetic friction for the two blocks on the horizontal and inclined planes are $\mu_{k1} = 0.3$ and $\mu_{k2} = 0.5$, respectively.

(28) A locomotive engine is pulling three cars behind it, see Fig. 5.38. Assume that the engine has mass m, and that each car also has mass m. If the driving force that is generated by the engine has a magnitude F, find the tension in the coupling between the cars in terms of F. Generalize this result when the number of the locomotive engine plus the cars is n.

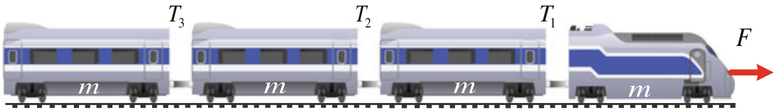
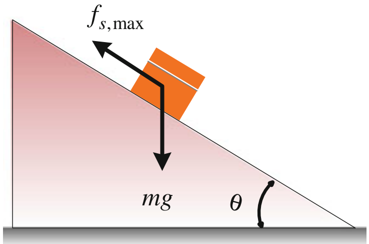


Fig. 5.38 See Exercise (28)

(29) A block is on the verge of skidding on a rough inclined plane of angle $\theta = \theta_c = 30°$, as shown in Fig. 5.39. (a) Find the value of the coefficient of static friction μ_s. (b) After we increase θ further to allow the block to accelerate and then decrease θ again to the value $\theta = \theta' = 20°$ to allow the block to move with constant speed, find the coefficient of kinetic friction μ_k.

Fig. 5.39 See Exercise (29)



(30) A box of mass $m = 200$ kg is pushed at a constant speed up an inclined frictionless ramp of angle $\theta = 30°$ by a horizontal force $\vec{F}$, as shown in Fig. 5.40. (a) What is the magnitude of the horizontal force? (b) What is the magnitude of the force exerted by the ramp on the box?

(31) A block of mass $m = 100$ kg is placed on an incline of angle $\theta = 25°$, see Fig. 5.41. What force is required to pull it up at a constant speed if the coefficient of kinetic friction between the block and the incline is 0.2?

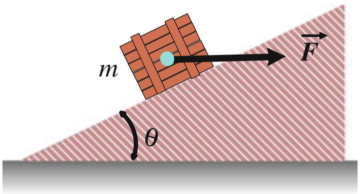


Fig. 5.40 See Exercise (30)

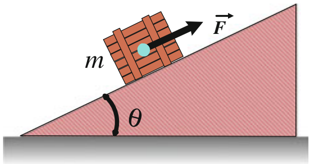


Fig. 5.41 See Exercise (31)

(32) Two blocks of masses $m_1 = 4$ kg and $m_2 = 8$ kg are connected by a massless rope and slide down an inclined plane of angle $\theta = 30°$, see Fig. 5.42. The coefficient of kinetic friction is $\mu_{k1} = 0.25$ between block m_1 and the plane, and $\mu_{k2} = 0.45$ between block m_2 and the plane. Find the acceleration of each block and the tension in the rope.

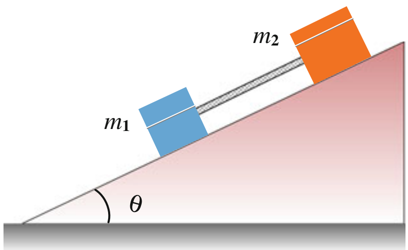


Fig. 5.42 See Exercise (32)

(33) What happens if the two blocks in exercise 31 are reversed such that block m_1 is located behind block m_2 on the plane?

(34) A ball of mass $m = 1.6$ kg hangs by a thread from the roof of an elevator. The thread can withstand only a tension force of 20 N. When the system accelerates upwards, see Fig. 5.43, the thread breaks. Within what range is the elevator accelerating?

(35) Assume that while ascending, an elevator has the same magnitude of acceleration a during starting (accelerating) and stopping (decelerating), see Fig. 5.44. A person stands on a scale in that elevator, and hence pushes downwards on the scale. The scale also reacts with an upward normal force. The maximum

and minimum scale readings are $N_{acc} = 591$ N and $N_{dec} = 391$ N, respectively. (a) Find the person's weight and mass. (b) Find the magnitude of the elevator's acceleration.

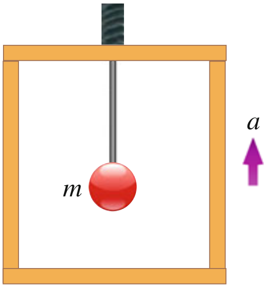


Fig. 5.43 See Exercise (34)

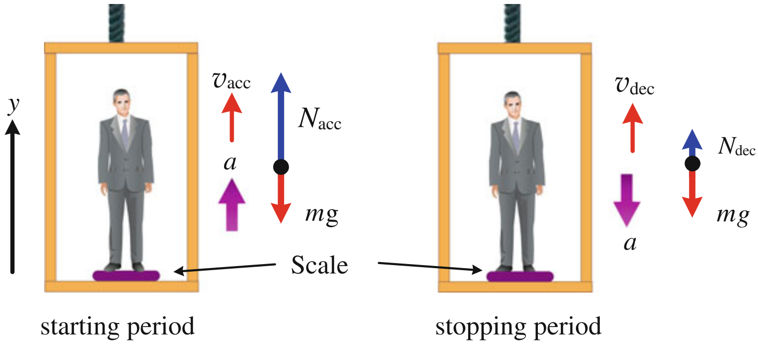


Fig. 5.44 See Exercise (35)

(36) Repeat exercise 35, this time assuming descending situation.

(37) A small sphere of mass $m = 2$ g is released from rest in a large vessel filled with oil. The sphere reaches a terminal speed of $v_t = 3$ cm/s, see Fig. 5.45. Assume that the resistive drag force is given by Eq. 5.11. Find the time it takes the sphere to reach a speed of 0.99 v_t. [Use $v = v_t(1 - e^{-t/\tau})$, where $\tau = m/b$ is the time constant]

(38) A spherical rain drop of radius $r = 2$ mm falls from a cloud at height $h = 1{,}000$ m above the ground. Assume that the drag coefficient C for the drop is 0.7, and that the density of water ρ_w is 1,000 kg/m^3, and that the density of air ρ_a is 1.2 kg/m^3. (a) What is the terminal speed of the drop? (b) If there is no drag force, what is the speed of the drop when it reaches the ground?

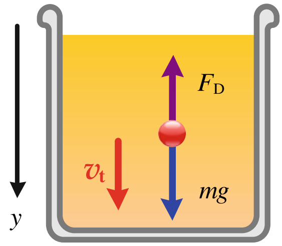


Fig. 5.45 See Exercise (37)

(39) A canonical pendulum consists of a bob of mass m attached to the end of a cord of length ℓ. The bob whirls around in a horizontal circle of radius r at a constant speed v while the cord always makes an angle θ with the vertical, see Fig. 5.46. Show that the bob's speed v and period T (the time for one complete revolution) are given by:

$$v = \sqrt{rg\tan\theta} = \sqrt{\ell g\sin\theta\tan\theta},$$

$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$$

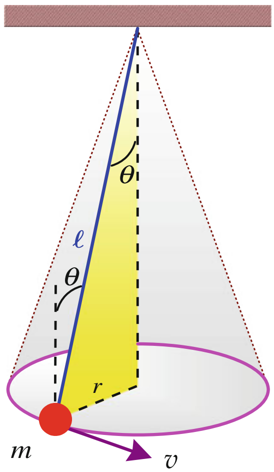


Fig. 5.46 See Exercise (39)

(40) What condition must be imposed on the relationship that governs the period of a canonical pendulum in order to reach to the period of a simple pendulum?