Heat and the First Law of Thermodynamics 12

Our focus in this chapter will be on the concept of internal energy, energy transfer, the first law of thermodynamics, and some applications of this law. The first law of thermodynamics expresses the general principle of conservation of energy. According to this law, an energy transfer to or from a system by either heat or work can change the internal energy of the system.

12.1 Heat and Thermal Energy

It is important to make a major distinction between *heat* and *internal energy* (*thermal energy*).

Internal energy is all the energy of a system that is associated with its microscopic constituents. Internal energy includes kinetic energy of random translational, rotational, and vibrational motion of molecules, potential energy of molecules and between molecules.

Heat is defined as the *transfer of energy* from one system to another due to a temperature difference between them.

12.1.1 Units of Heat, The Mechanical Equivalent of Heat

Previously, heat was measured in terms of its ability to raise the temperature of water. Thus, the **calorie** (cal), in cgs units, was defined as the amount of heat required to raise the temperature of 1 g of water from 14.5 to 15.5 °C. The

H. A. Radi and J. O. Rasmussen, *Principles of Physics*,
Undergraduate Lecture Notes in Physics, DOI: 10.1007/978-3-642-23026-4_12,

'Calorie' with a capital C, used by nutritionists, is a kilocalorie (1 Cal = 1 kcal = 10^3 cal).The **British Thermal Unit** (Btu) was also defined as the amount of heat required to raise the temperature of 1 lb of water from 63 to 64 °F. Since heat is now known as transferred energy, the SI unit for it is the **joule** (J).

In a famous experiment, see Fig. 12.1, Joule measured the calorie (cal) by converting mechanical energy into heat energy, expressed as an increase in water temperature.

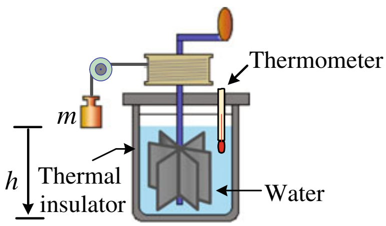


Fig. 12.1 Joule's experiment for measuring the mechanical equivalent of heat from the temperature rise in water

Joule found that the loss in mechanical energy is proportional to the increase in temperature of the water. The proportionality constant was found to be equal to 4,180 J/kg.C°. Hence, 4,180 J of mechanical energy will raise the temperature of 1 kg of water from 14.5 to 15.5 °C. One kilocalorie (1 kcal) is now defined to be exactly 4,186 J without reference to the heating of substance. Thus:

$$\boxed{1\,\text{kcal} = 4{,}186\,\text{J}} \tag{12.1}$$

The relations among the various heat units are as follows:

$$\begin{aligned} &1\,\text{J} = 2.389 \times 10^{-4}\,\text{kcal} = 9.478 \times 10^{-4}\,\text{Btu} \\ &\qquad\qquad\text{or} \\ &1\,\text{kcal} = 4{,}186\,\text{J} = 3.968\,\text{Btu} \end{aligned} \tag{12.2}$$

12.1.2 Heat Capacity and Specific Heat

The quantity of heat energy Q required to raise the temperature of an object by some amount ΔT varies from one substance to another.

The **heat capacity** C of an object is defined as:

The Heat Capacity C:
The heat capacity C of an object of a particular material is defined as the amount of heat energy needed to raise the object's temperature by one degree Celsius.

Accordingly, if Q units of heat energy are required to change the temperature by $\Delta T = T_f - T_i$, where T_i and T_f are the initial and final temperatures of the object, then:

$$\boxed{Q = C\,\Delta T}\,, \quad \text{where} \quad \boxed{\Delta T = T_f - T_i} \tag{12.3}$$

Heat capacity C has the unit $\mathrm{J/C^\circ}(\equiv \mathrm{J/K})$ or $\mathrm{kcal/C^\circ}(\equiv \mathrm{kcal/K})$.

The heat capacity for any object is proportional to its mass m. For this reason, we define the "heat capacity per unit mass" or the **specific heat** c which refers to a unit mass of the material of which the object is made. Thus, with $C = m\,c$, Eq. 12.3 becomes:

$$\boxed{Q = m\,c\,\Delta T}\,, \quad \text{where} \quad \boxed{\Delta T = T_f - T_i} \tag{12.4}$$

Specific heat c has the unit:

$$\boxed{\begin{array}{c} \mathrm{J/kg.C^\circ} \equiv \mathrm{J/kg.K} \\ \text{or} \\ \mathrm{kcal/kg.C^\circ} \equiv \mathrm{kcal/kg.K} \end{array}}$$

The specific heat of water at 15 °C and atmospheric pressure is:

$$\boxed{c_{\text{water}} = 4{,}186\,\mathrm{J/kg.K} = 1\,\mathrm{kcal/kg.C^\circ}}$$

Note that, when heat energy is added to objects, Q and ΔT are both positive, i.e. the temperature increases. Likewise, when heat is removed from objects, Q and ΔT are both negative, i.e. the temperature decreases.

In general, specific heat c varies with temperature. However, if temperature intervals are not too big, the temperature variation can be ignored, and c can be treated as a constant. For example, the specific heat of water varies by about 1% from 0 to 100 °C at atmospheric pressure. Table 12.1 presents some specific heat values for various substances, measured at room temperature and atmospheric pressure.

Table 12.1 Specific heat c of some substances at atmospheric pressure and room temperature (20 °C) with few exceptions

Substance	Specific heat	
	J/kg.C°	kcal/kg.C°
Silver	230	0.0564
Copper	390	0.0923
Iron or steel	450	0.107
Aluminum	900	0.215
Brass	380	0.092
Granite	790	0.19
Glass	840	0.20
Ice (−5 °C)	2,100	0.50
Ice (−10 °C)	2,220	0.530
Mercury	140	0.033
Alcohol (Ethyl)	2,400	0.58
Seawater	3,900	0.93
Water (15 °C)	4,186	1
Steam (100 °C)	2,010	0.48

Measuring Specific Heat

Figure 12.2 shows an example of a calorimeter, which is a device used to determine the specific heat of a solid or liquid substance. The substance (represented by a circular object, having a specific heat c_x and mass m_x) is heated up to some known initial temperature T_x, and then placed in a perfectly insulated vessel containing water of specific heat c_w, mass m_w, and initial temperature T_w. If T_f is the final temperature after reaching equilibrium, then $T_w < T_f < T_x$. Using Eq. 12.4, we calculate the heat gained by the water to be $Q = m_w\, c_w\, (T_f - T_w)$, and calculate the heat energy lost by the object to be $-Q = m_x\, c_x\, (T_f - T_x)$.

Assuming that the entire system does not lose or gain any heat from its surrounding, then the heat gained by the water must equal the heat lost by the object. That is:

$$Q = m_w\, c_w\, (T_f - T_w) = -m_x\, c_x\, (T_f - T_x) \tag{12.5}$$

Solving for c_x gives:

$$c_x = c_w \frac{m_w}{m_x} \frac{(T_f - T_w)}{(T_x - T_f)}, \qquad (T_w < T_f < T_x) \tag{12.6}$$

When calculating c_x, we neglected heat exchange with the vessel, which is acceptable when the mass of the water is considerably larger than that of the vessel, and when the vessel has a negligible specific heat.

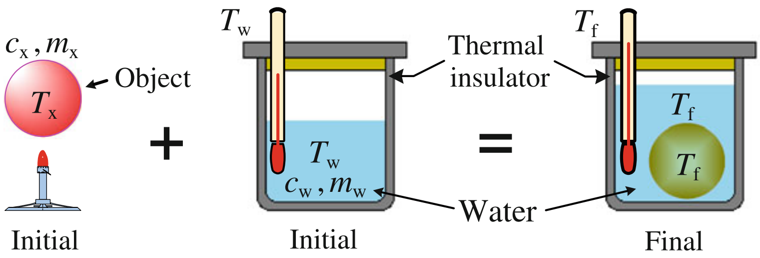


Fig. 12.2 In the method of mixtures, a calorimeter filled with water is used to find the specific heat of unknown heated objects

Example 12.1

The specific heat of zinc is 352 J/kg.C° for temperatures near 25 °C. Determine the amount of heat required to raise the temperature of 0.5 kg zinc from 20 to 30 °C. Take the specific heat to be constant in that temperature range.

Solution: The given values are $c = 352$ J/kg.C°, $m = 0.5$ kg, $T_i = 20$ °C, and $T_f = 30$ °C. The temperature change has the following magnitude:

$$\Delta T = T_f - T_i = 30\,°\text{C} - 20\,°\text{C} = 10\,\text{C}°$$

Using Eq. 12.4 we find the amount of heat required as follows:

$$Q = m\,c\,\Delta T = (0.5\,\text{kg})(352\,\text{J/kg.C}°)(10\,\text{C}°) = 1{,}760\,\text{J}$$

Example 12.2

A steel metal object of mass 0.05 kg is heated to 225 °C and then dropped into a vessel containing 0.55 kg of water initially at 18 °C. When equilibrium is reached, the temperature of the mixture is 20 °C. Find the specific heat of the metal.

Solution: For the steel metal object, we are given $m_x = 0.05$ kg and $T_x = 225$ °C, but its specific heat c_x is unknown. For water, the known values are $m_w = 0.55$ kg, $T_w = 18$ °C, and $c_w = 4{,}186$ J/kg.C° (Table 12.1). For the mixture, the equilibrium temperature occurs at $T_f = 20$ °C. Since the heat gained by the water is equal in magnitude to the heat lost by the steel, see Eq. 12.6 and Fig. 12.2, then we must have:

$$m_w\, c_w\, (T_f - T_w) = -m_x\, c_x\, (T_f - T_x), \quad (T_w < T_f < T_x)$$

Solving for c_x we get:

$$\begin{aligned} c_x &= c_w \frac{m_w}{m_x} \frac{(T_f - T_w)}{(T_x - T_f)} \\ &= (4{,}186\ \text{J/kg.C}^\circ) \frac{(0.55\,\text{kg})}{(0.05\,\text{kg})} \frac{(20\,^\circ\text{C} - 18\,^\circ\text{C})}{(225\,^\circ\text{C} - 20\,^\circ\text{C})} \\ &= 449\,\text{J/kg.C}^\circ \end{aligned}$$

12.1.3 Latent Heat

When heat energy is transferred from one substance to another, the temperature of the substance often changes. However, there are situations in which the transfer of energy does not change the temperature. Instead, the substance may change from one form to another. Such a change is commonly referred to as a **phase change or phase transition**, see Sect. 13.4 and especially Fig. 13.10.

We consider the following two main common phase changes:

1. A phase change from solid to liquid (as ice melting) and from liquid to gas (as water boiling), where heat energy is absorbed while the temperature remains constant.
2. A phase change from gas to liquid (as steam condensing) and from liquid to solid (as water freezing), where heat energy is released while the temperature remains constant.

The amount of heat energy per unit mass, L, that must be transferred when a substance completely undergoes a phase change *without changing temperature* is called the **latent heat** (literally, the "hidden" heat). If a quantity Q of heat energy transfer is required to change the phase of a pure substance of a mass m, then $L = Q/m$ characterizes an important thermal property of that substance. That is:

$$\boxed{Q = \pm m L} \tag{12.7}$$

A positive sign is used in this equation when energy enters the system, causing melting or vaporization of the substance, while a negative sign corresponds to energy leaving the system such that the substance condenses or solidifies.

When a substance experiences a phase change from solid to liquid by absorbing heat, the heat of transformation is called the **latent heat of fusion** L_F, see Fig. 12.3. When the substance releases heat and experiences a phase change from liquid back to solid, the heat of transformation is called the **latent heat of solidification** and is numerically equal to the latent heat of fusion, see Fig. 12.3. In the case of water at its normal melting or freezing temperature, we have:

$$L_F = 3.33 \times 10^5 \text{ J/kg} = 79.5 \text{ kcal/kg} = 6.01 \times 10^6 \text{ J/kmol} \tag{12.8}$$

When a substance experiences a phase change from liquid to gas by absorbing heat, the heat of transformation is called the **latent heat of vaporization** L_V, see Fig. 12.3. When the gas releases heat and experiences a phase change from gas back to liquid, the heat of transformation is called the **latent heat of condensation** and is numerically equal to the latent heat of vaporization, see Fig. 12.3. For water at its normal boiling and condensation temperatures, we have:

$$L_V = 2.256 \times 10^6 \text{ J/kg} = 539 \text{ kcal/kg} = 40.7 \times 10^3 \text{ J/kmol} \tag{12.9}$$

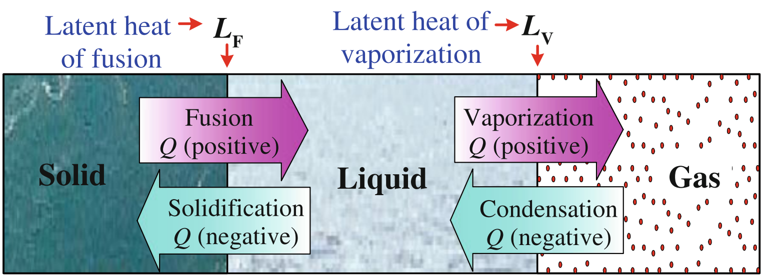


Fig. 12.3 A sketch showing heat of fusion/vaporization (positive Q) as well as heat of condensation/solidification (negative Q)

Phase changes can be described in terms of a rearrangement of molecules when heat energy is added or removed from a substance. Consider, for example, the solid-to-liquid phase change. The molecules in the solid are strongly attracted to each other. As thermal energy is absorbed, the molecules usually move further apart and their potential energy increases. (Water-ice is an exception where there is shrinkage.) This leads to no change in the average kinetic energy of the molecules during the melting process, which involves molecules moving from fixed lattice positions to a random liquid state, the temperature stays constant. The latent heat of fusion is equal to the work done in separating the molecules during the melting process and hence breaking their bonds and transforming the substance from the ordered solid phase into the disordered liquid phase.

Now, we consider the liquid to gas phase change. The attractive forces between molecules in liquid form are stronger than in gas form because the average distance between molecules is smaller in the liquid state. As described in the solid-to-liquid phase transition, work must be done against these attractive forces. The latent heat of vaporization is the amount of energy added to the molecules in liquid form to accomplish this.

Table 12.2 gives some latent heats of various substances.

Table 12.2 Latent heats of fusion and vaporization (approximates)

Substance	Melting		Boiling	
	Melting point °C	Latent heat of fusion J/kg	Boiling point °C	Latent heat of vaporization J/kg
Helium	−270	5.23×10^3	−269	2.09×10^4
Nitrogen	−210	2.55×10^4	−196	2.01×10^5
Oxygen	−219	1.38×10^4	−183	2.13×10^5
Water	0	3.33×10^5	100	2.26×10^6
Sulfur	119	3.81×10^4	445	3.26×10^5
Lead	327	2.45×10^4	1,750	8.70×10^5
Aluminum	660	3.97×10^5	2,450	1.14×10^7
Silver	961	8.82×10^4	2,193	2.33×10^6
Gold	1,063	6.44×10^4	2,660	1.58×10^6
Copper	1,083	1.34×10^5	1,187	5.06×10^6
Silicon	1,410	1.65×10^6	2,447	1.06×10^7

To understand the role of latent heat in phase changes, we calculate the energy required to convert 1 g of ice at −50 °C into steam at 150 °C. Figure 12.4 shows the results obtained when energy is added gradually to 1 g of ice. The red curve of the figure is divided into the following five stages:

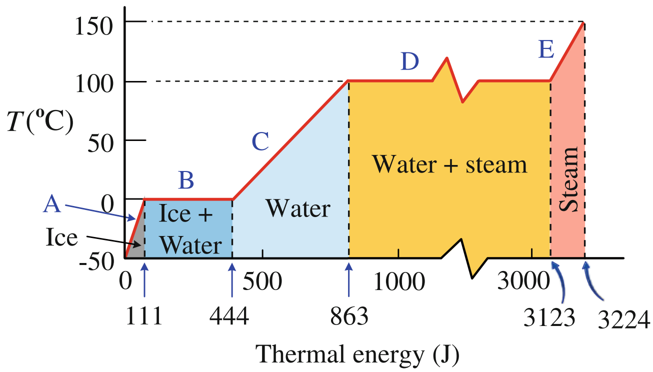


Fig. 12.4 Temperature as a function of the thermal energy added gradually to convert 1 g of ice at −50 °C into steam at 150 °C

Stage A—*Changing the temperature of ice from* −50 *to* 0 °C:

With a specific heat of ice $c_i = 2{,}220$ J/kg.C°, the amount of heat added Q_A is:

$$Q_A = m_i c_i \Delta T = (1 \times 10^{-3}\,\text{kg})(2{,}220\,\text{J/kg.C°})(50\,\text{C°}) = 111\,\text{J}$$

Stage B—*Ice-water mixture remains at* 0 °C *(even heat is added)*:

With a latent heat of fusion $L_F = 3.33 \times 10^5$ J/kg, the amount of heatadded Q_B until all of the ice melts is:

$$Q_B = m\,L_F = (1 \times 10^{-3}\,\text{kg})(3.33 \times 10^5\,\text{J/kg}) = 3.33 \times 10^2\,\text{J}$$

Stage C—*Changing the temperature of water from* 0 *to* 100 °C:

With a specific heat of water $c_w = 4{,}186$ J/kg.C°, the amount of heat added Q_C is:

$$Q_C = m_w c_w \Delta T = (1 \times 10^{-3}\,\text{kg})(4{,}186\,\text{J/kg.C°})(100\,\text{C°}) \simeq 419\,\text{J}$$

Stage D—*Water-steam mixture remains at* 100 °C *(even heat is added)*:

With a latent heat of vaporization $L_V = 2.26 \times 10^6$ J/kg, the amount of heat added Q_D until all of the water evaporates is:

$$Q_D = m\,L_V = (1 \times 10^{-3}\,\text{kg})(2.26 \times 10^6\,\text{J/kg}) = 2.26 \times 10^3\,\text{J}$$

Stage E—*Changing the temperature of steam from* 100 *to* 150 °C:

With a specific heat of steam $c_s = 2{,}010$ J/kg.C°, the amount of heat added Q_E is:

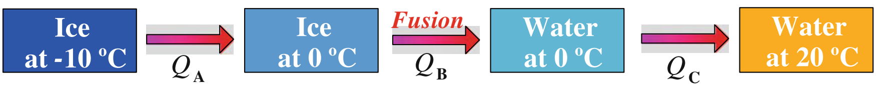
$$Q_E = m_s c_s \Delta T = (1 \times 10^{-3}\,\text{kg})(2{,}010\,\text{J/kg.C°})(50\,\text{C°}) \simeq 101\,\text{J}$$

The total heat added to change 1 g of ice at −50 °C to steam at 150 °C is $Q_{tot} = 3{,}224$ J. That is, if we cool 1 g of steam at 150 °C until we have ice at −50 °C, we must remove 3,224 J of heat.

Example 12.3

Find the quantity of heat required to convert ice of mass 500 g at −10 °C into water at 20 °C. The specific heat of ice is $c_i = 2{,}220$ J/kg.C°, the latent heat of fusion is $L_F = 3.33 \times 10^5$ J/kg, and the specific heat of water is $c_w = 4{,}186$ J/kg.C°.

Solution: The ice gains heat throughout the following three stages.



In stage A we raise the temperature of ice from −10 to 0 °C. Using Eq. 12.4 we get:

$$Q_A = m_i c_i\,\Delta T = (0.5\,\text{kg})(2{,}220\,\text{J/kg.C°})(10\,\text{C°}) = 11{,}100\,\text{J} = 11.1\,\text{kJ}$$

In stage B we melt the 500 g of ice at constant temperature (0 °C) by supplying the latent heat of fusion. Using Eq. 12.7 we get:

$$Q_B = m\,L_F = (0.5\,\text{kg})(3.33 \times 10^5\,\text{J/kg}) = 166{,}500\,\text{J} = 166.5\,\text{kJ}$$

In stage C we raise the temperature of water from 0 to 20 °C. Using Eq. 12.4 we get:

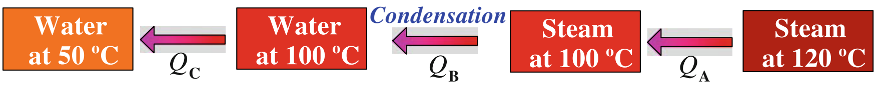
$$Q_C = m_w c_w\,\Delta T = (0.5\,\text{kg})(4{,}186\,\text{J/kg.C}^\circ)(20\,\text{C}^\circ) = 41{,}860\,\text{J} = 41.86\,\text{kJ}$$

Note that $Q_B > Q_C > Q_A$ and the total required heat is $Q_{tot} = 219.46\,\text{kJ}$.

Example 12.4

A glass beaker of water is at 20 °C. The beaker has a mass $m_g = 200$ g with specific heat $c_g = 840$ J/kg.C° and contains water of mass $m_w = 300$ g with specific heat $c_w = 4{,}186$ J/kg.C°. A quantity of steam initially at 120 °C is used to warm the system to 50 °C. If the specific heat of steam is $c_s = 2{,}010$ J/kg.C° and latent heat of vaporization is $L_V = 2.26 \times 10^6$ J/kg, what is the mass of the steam?

Solution: The heat lost by the steam equals the heat gained by both beaker and water. The steam loses heat over the stages shown below.



In the first stage, the steam is cooled from 120 to 100 °C, i.e. $\Delta T = T_f - T_i = 100\,°\text{C} - 120\,°\text{C} = -20\,\text{C}^\circ$. The heat liberated in this stage by the unknown mass m_s of steam is:

$$Q_A = m_s c_s\,\Delta T = m_s(2{,}010\,\text{J/kg.C}^\circ)(-20\,\text{C}^\circ) = -m_s(40{,}200\,\text{J/kg})$$

In the second stage, the steam is condensed to water at 100 °C. Since the latent heat of condensation equals the latent heat of vaporization, we use Eq. 12.7 to find the heat liberated as follows:

$$Q_B = -m_s\,L_V = -m_s(2.26 \times 10^6\,\text{J/kg})$$

In the last stage the temperature of water is reduced from 100 to 50 °C. This liberates an amount of heat given by:

$$Q_C = m_s c_w\,\Delta T = m_s(4{,}186\,\text{J/kg.C}^\circ)(-50\,\text{C}^\circ) = -m_s(209{,}300\,\text{J/kg})$$

The heat lost is thus $Q_{lost} = Q_A + Q_B + Q_C = -m_s(2{,}509{,}500\,\text{J/kg})$. The heat gained by the beaker and water system from 20 to 50 C° is:

$$\begin{aligned} Q_{gained} &= m_w c_w\,\Delta T + m_g c_g\,\Delta T = (m_w c_w + m_g c_g)\Delta T \\ &= [(0.3\,\text{kg})(4{,}186\,\text{J/kg.C}^\circ) + (0.2\,\text{kg})(840\,\text{J/kg.C}^\circ)](30\,\text{C}^\circ) \\ &= 42{,}714\,\text{J} \end{aligned}$$

If we equate the magnitude of heat lost by the steam, $|Q_{\text{lost}}|$, with the heat gained by the beaker and water system, Q_{gained}, we get:

$$m_s = 42{,}714\,\text{J}/(2{,}509{,}500\,\text{J/kg}) = 0.017\,\text{kg} = 17\,\text{g}$$

12.2 Heat and Work

In thermodynamics, when an isolated system is in thermal equilibrium internally, we describe its macroscopic state with the variables P, V, and T to represent pressure, volume and temperature. For such a system, we describe its microscopic state of internal energy with the variable E_{int} (some other textbooks use the symbol U).

Let us assume our system consists of gas confined to a cylinder with insulated walls and a movable frictionless piston of area A, as shown in Fig. 12.5. The cylinder rests on a heat reservoir whose temperature T is controlled by a knob. At equilibrium, the upward force on the piston due to the pressure of the confined gas is equal to the weight of the load on the top of the piston.

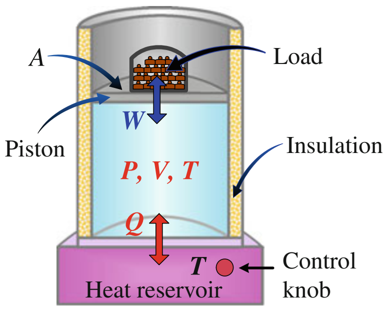


Fig. 12.5 Gas confined to a cylinder with a movable frictionless piston. The gas can do work W to raise or lower the piston. By regulating the temperature T of the thermal reservoir, by means of a control knob, a quantity of heat Q can be added or removed from the gas

Consider that we start at an *initial state* i, where the system is described to have pressure P_i, volume V_i, and temperature T_i. We then change the system to a *final state* f, described to have pressure P_f, volume V_f, and temperature T_f. The process of changing the system from the initial state to the final state is a **thermodynamic process**. During such a process, work is done *by* the system to raise the piston (positive work[1]) or lower it (negative work). In addition, heat may be transferred into the system from the thermal reservoir (positive heat) or vice versa. We assume that the state of the gas changes **quasi-statically**, i.e. slowly enough to allow the system to remain essentially in a thermodynamic equilibrium at all times.

Now, assume we reduce the load from the piston in such a way that the piston will move upward through a differential displacement $d\vec{s}$ with *almost* constant upward force $\vec{F}$, as shown in Fig. 12.6. From the definition of pressure, we have $F = PA$, where A is the area of the piston. The differential work dW done *by* the gas during the displacement is:

$$dW = \vec{F} \bullet d\vec{s} = F\,ds = P\,A\,ds$$

Since $A\,ds$ is the differential change in the volume of the gas dV (i.e. $dV = A\,ds$), we can express the work done by the gas as follows:

$$\boxed{dW = P\,dV} \tag{12.10}$$

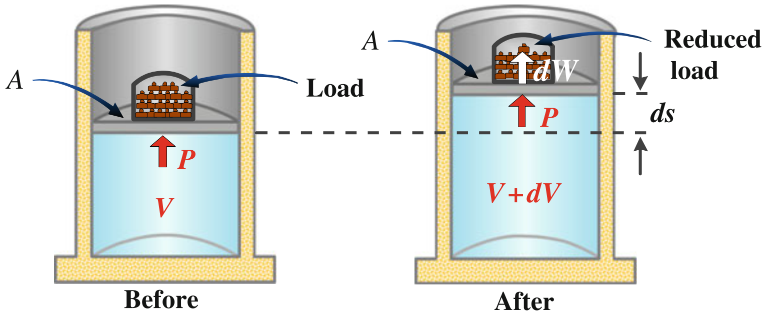


Fig. 12.6 A confined gas in a cylinder at pressure P does work dW on a free piston as the gas expands from volume V to volume $V + dV$ because of a decreased load

If the gas expands, as in Fig. 12.7, then dV is positive and the work done by the gas is positive, whereas if the gas is compressed, dV is negative, indicating that the work done by the gas is negative (which can be interpreted as work done *on* the gas). When we remove an appreciable amount of load from the piston, the volume of the gas changes from V_i to V_f, and the total work done by the gas is:

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[1] For historical reasons, we choose W to represent the work done by the system. In other parts of the text, W is the work done on the system. This difference affects only the sign of W.

$$W = \int dW = \int_{V_i}^{V_f} P\,dV \tag{12.11}$$

During the change in volume of the gas, the pressure and temperature of the gas may also change. To evaluate the integral in the last equation, we need to know how the pressure varies with volume. For example, Fig. 12.7 indicates that the work done by the gas is represented by the area under the *PV* diagram of the figure.

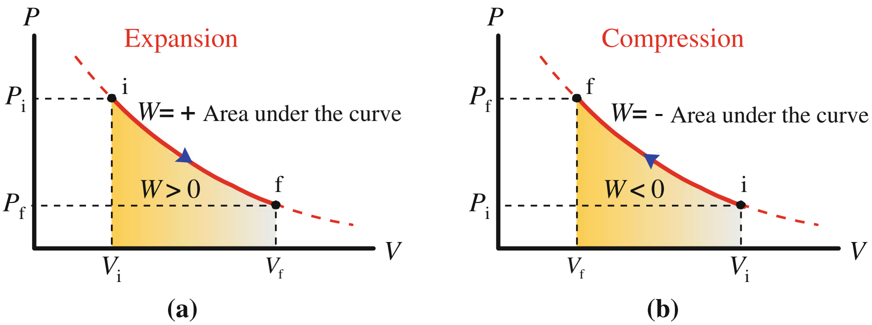


Fig. 12.7 The figure shows a gas that goes from an initial state i to a final state f by means of a thermodynamic process. (a) When the gas expands, the work done by the gas is positive and equals the area under the *PV* curve. (b) Similar to (a), except that the gas is compressed and the work done by the gas is negative

As seen from Fig. 12.7, the total work done during the expansion or compression of the gas depends on the specific path taken from the initial state i to the final state f.

In Fig. 12.8, we illustrate this important point further by considering several different paths for the gas along the *PV* curve, from state i to state f, regardless of how we achieve each path.

Path a—The gas expands from V_i to V_f while the pressure decreases from P_i to P_f. The work done by the gas along this path is positive and represented by the colored area under the curve between i and f.
Path b—The gas first expands from V_i to V_f at constant pressure P_i, and then its pressure is reduced to P_f at constant volume V_f. The work done along this path is $P_i\ (V_f - V_i)$.
Path c—The pressure of the gas is first reduced from P_i to P_f by cooling at a constant volume V_i, and then allowing the gas to expand from V_i to V_f at constant pressure P_f. The work done along this path is $P_f\ (V_f - V_i)$.

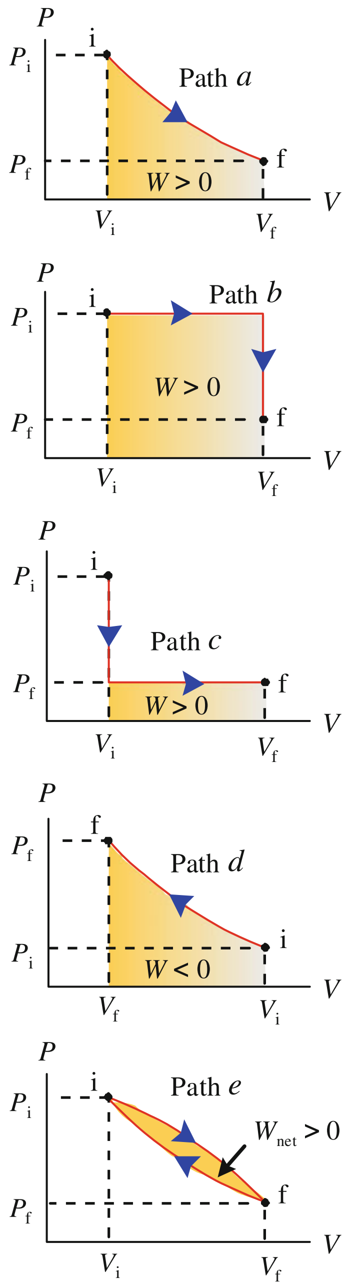


Fig. 12.8 The gas of Fig. 12.5 goes from an initial state i to a final state f by means of several different thermodynamic processes

Path d—The gas is compressed from V_{i} to V_{f} while the pressure increases from P_{i} to P_{f}. The work done along this path is the negative of the colored area under the curve.

Path e—The net work done by the system during a closed cycle is the sum of the positive work done during the expansion and the negative work done during the compression. Here, the net work done by the gas is positive and is represented by the enclosed area between the two curves.

From the graphs of Fig. 12.8, we see that W could be small or large depending on the thermodynamic path between i and f. Thus:

> Spotlight
> The net work done by a system W *depends* on the thermodynamic process (or the path) chosen between its initial and final states.

In a similar manner, we also find that the heat energy transfer Q into or out of a system depends on the thermodynamic process. This can be demonstrated for an ideal gas as shown in Fig. 12.9.

In Fig. 12.9a, the piston is held at a position where the gas is at its initial pressure P_i, volume V_i, and temperature T_i. When the force holding the piston is reduced slightly, the piston rises very slowly to a final pressure P_f and final volume V_f, i.e. the gas is doing work W on the piston. During this expansion process, heat energy Q is transferred from the reservoir to the gas to maintain a constant temperature T_i.

In Fig. 12.9b, the thermally insulated gas has the same initial state as in Fig. 12.9a, but with a membrane replacing the piston. When the membrane is broken, the gas expands *rapidly* into the vacuum until it acquires a pressure P_f and volume V_f. In this case, the gas does no work, i.e. $W = 0$, and no heat is transferred, i.e. $Q = 0$.

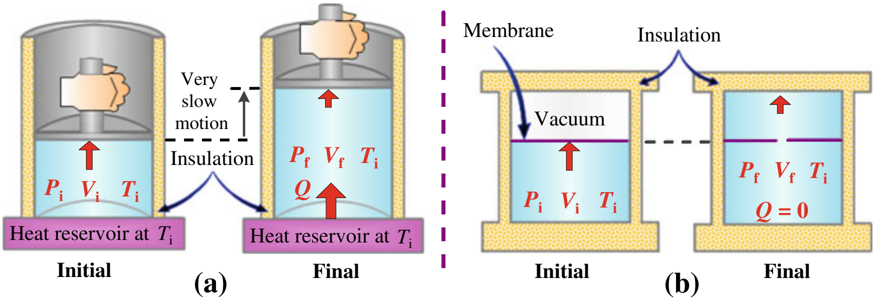


Fig. 12.9 (a) An ideal gas at temperature T_i expands slowly while absorbing heat energy Q from a reservoir in order to maintain its constant temperature T_i. (b) An ideal gas expands rapidly into an evacuated chamber after a membrane is broken

In both parts of Fig. 12.9, the initial and final states of the ideal gas are identical, although the path is different. In part (a) of the figure the gas does work W on the piston, and heat energy Q is transferred slowly to the gas from the reservoir. In part (b) of the figure the work done by the gas is zero and no heat energy is transferred. Thus:

Spotlight
The heat energy transfer Q *depends* on the thermodynamic process (or the path) chosen between the initial and final states of a system.

Finally, we conclude that neither the work done nor the heat energy are independently conserved during a thermodynamic process between the initial and final states of a system.

12.3 The First Law of Thermodynamics

In Chap. 6, we discussed the principle of conservation of energy as applied to systems that are *not isolated,* and we expressed this principle in Eq. 6.61, namely $W = \Delta E_{\mathrm{tot}} = \Delta K + \Delta U + \Delta E_{\mathrm{int}}$. In this chapter, we assume that there are no changes in kinetic energy and potential energy of the system as a whole; that is, $\Delta K = \Delta U = 0$ and hence $W = \Delta E_{\mathrm{tot}} = \Delta E_{\mathrm{int}}$. Moreover, before this chapter, the term *work* and the symbol W always meant the work done *on* a system. But starting from Eq. 12.10 and continuing to the rest of this chapter, we focus on the work done *by* a system. Thus, we replace the symbol W by $-W$ and Eq. 6.61 becomes $-W = \Delta E_{\mathrm{tot}} = \Delta E_{\mathrm{int}}$. If we need to account for the transfer of heat energy Q that is added (if Q positive) or taken (if Q negative) from the system, then we add Q to the left hand side of this equation and arrive at the following thermodynamic equation:

$$\boxed{\Delta E_{\mathrm{int}} = Q - W} \quad \text{(The first law of thermodynamics)} \tag{12.12}$$

As we saw, W and Q are path-dependent, yet a surprising experimental discovery was found: *The quantity $Q - W$ is the same for all thermodynamic processes*. It depends only on the initial and final states of the system and is *path-independent*. Equation 12.12 is known as the **first law of thermodynamics**. This law states that a change in internal energy in a system can occur as a result of energy transfer by heat or by work, or by both. If the thermodynamic system undergoes only a differential change, we can write the first law as:

$$\boxed{dE_{\text{int}} = dQ - dW} \quad \text{(The first law of thermodynamics)} \tag{12.13}$$

Spotlight
The internal energy E_{int} of a system increases if energy is added via heat Q and decreases if energy is lost via work W done *by* the system.

Some special cases of the first law of thermodynamics are as follows

1. Isolated Systems

Consider a system that is not interacting with its surroundings. In this case, no energy transfer by heat takes place, i.e. $Q = 0$, and the value of the work done by the system is zero, i.e. $W = 0$. Then, from the first law we have $\Delta E_{\text{int}} = 0$. Thus, we conclude that the internal energy of an isolated system remains constant.

$$\boxed{E_{\text{int}} = \text{constant}} \quad \text{(Isolated system)} \tag{12.14}$$

2. Cyclic Processes

Consider a non-isolated system that is taken through a cyclic process, i.e. a process that starts and ends at the same state. In this case, the change in the internal energy must again be zero, i.e. $\Delta E_{\text{int}} = 0$. Then, from the first law we have:

$$\boxed{\Delta E_{\text{int}} = 0} \quad \text{and} \quad \boxed{Q = W} \quad \text{(Cyclic process)} \tag{12.15}$$

On the PV curve, a cyclic process appears as a closed curve as shown in path (e) of Fig. 12.8. For this clockwise cyclic path, the net work done by the system (and Q) equals the area enclosed by the path.

12.4 Applications of the First Law of Thermodynamics

The first law of thermodynamics relates the changes in internal energy of a system to transfers of energy by work W or heat Q, or both. In this section, we consider applications of the first law in processes in which certain restrictions are imposed.

1. Adiabatic Process

An adiabatic process is one that occurs so *rapidly* or occurs in *thermally insulated systems* during which no transfer of heat energy enters or leaves the system, i.e. $Q = 0$. With this restriction and the application of the first law of thermodynamics to an adiabatic process, we get:

$$\boxed{Q = 0} \quad \text{and} \quad \boxed{\Delta E_{\text{int}} = -W} \quad \text{(Adiabatic process)} \tag{12.16}$$

Figure 12.10 shows an idealized adiabatic process. Heat cannot enter or leave the system because of the insulation. The only way of transferring energy to the system is by work. We see in this figure that if a gas is compressed adiabatically such that W is negative, then ΔE_{int} is positive and hence the temperature of the gas increases. Conversely, if a gas expands adiabatically such that W is positive, then ΔE_{int} is negative, and hence the temperature of the gas decreases.

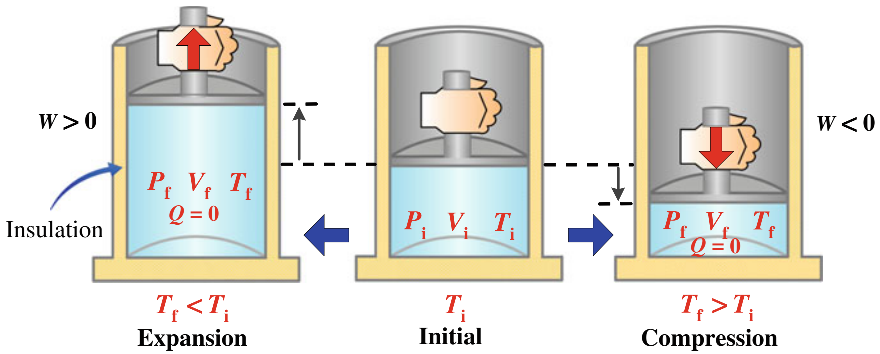


Fig. 12.10 An adiabatic compression/expansion is carried out for an ideal gas leading to an increase/decrease in internal energy

Adiabatic processes have a very important role in mechanical engineering. Some of the common examples include the approximately adiabatic compression/expansion of a mixture of gasoline vapor and air that takes place during operation of a combustion engine, leading to a temperature increase/decrease.

2. Adiabatic Free Expansion Process

The free expansion process is an adiabatic process, i.e. $Q = 0$, in which no work is done on or by the system, i.e. $W = 0$. Thus, with these restrictions and the application of the first law we have:

$$\boxed{Q = W = 0} \quad \text{and} \quad \boxed{\Delta E_{\text{int}} = 0} \quad \text{(Free expansion)} \tag{12.17}$$

Figure 12.11 shows how such an expansion can be carried out. An ideal gas in thermal equilibrium is initially confined by a closed valve to one-half of an insulated chamber; the other half is evacuated. When we open the valve, the gas expands freely to fill both halves of the chamber. No heat is transferred to or from the gas because of the insulation. No work is done by the gas because it rushes into vacuum, during which its motion is unopposed by any counteracting pressure.

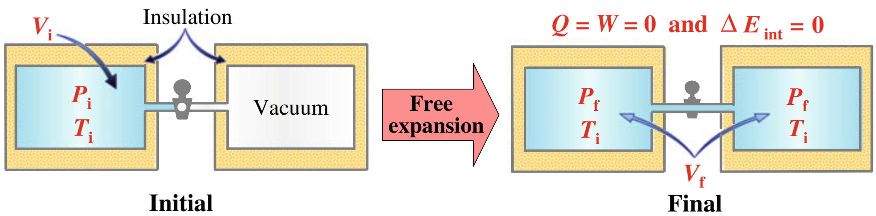


Fig. 12.11 In a free expansion process there will be no change in internal energy or temperature between the initial and final states

A free expansion differs from any other thermodynamic process since it cannot be performed slowly in a controlled way. As a result, at any given instant *during* the sudden expansion, the gas is not in thermal equilibrium and its pressure is not the same everywhere.

3. Isobaric Process

An isobaric process is one that takes place at constant pressure. In general, the first law of thermodynamics does not assume any special values for the isobaric process; that is, Q, W, and ΔE_{int} are all non-zero.

Assume the piston of Fig. 12.12 is free to move in such a way that it is always in equilibrium under the effect of the net force from a gas pushing upwards and the weight of the piston plus the force due to atmospheric pressure pushing downwards. Then, an isobaric process could be established by transferring heat energy Q to or from the gas by any mechanism. This transfer causes the gas to expand or contract depending on the sign of Q. In the PV diagram of Fig. 12.8, the first process in path (b) and the second process in path (c) are examples of isobaric processes.

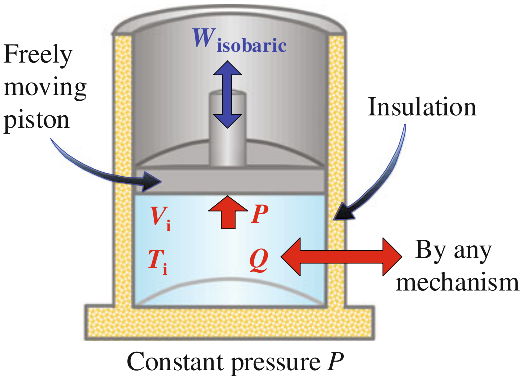


Fig. 12.12 An isobaric process could be achieved by transferring heat energy to a gas enclosed by a freely moving piston to attain a constant pressure

The work done by the gas as it expands or contracts in this isobaric process could be obtained from Eq. 12.11, after removing the constant pressure from the integral, as follows:

$$W_{\text{isobaric}} = P(V_f - V_i) \quad \text{(Isobaric process)} \tag{12.18}$$

4. Isovolumetric Process

An isovolumetric process is one that takes place at constant volume. In the PV diagram of Fig. 12.8, the second process in path (b) and the first process in path (c) are examples of isovolumetric processes.

Assume the piston of Fig. 12.13 is clamped to a fixed position to ensure an isovolumetric process. In such a process, the value of the work done by the gas is zero, i.e. $W = 0$, because the volume does not change. Thus, with this restriction and the application of the first law of thermodynamics to an isovolumetric process, we get:

$$W = 0 \quad \text{and} \quad \Delta E_{\text{int}} = Q \quad \text{(Isovolumetric process)} \tag{12.19}$$

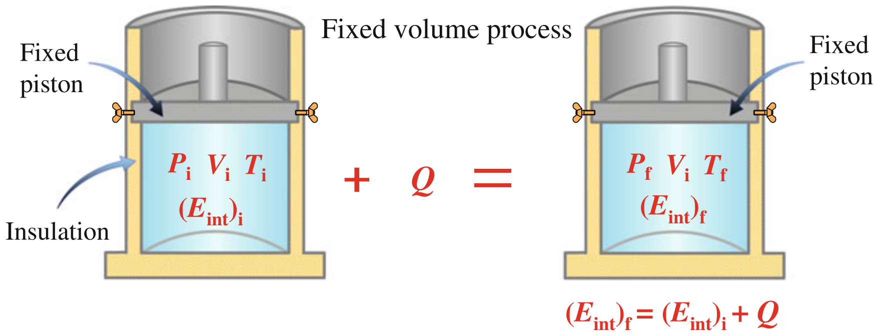


Fig. 12.13 An isovolumetric process could be achieved by fixing the piston's position. The pressure increases, and all the transferred heat energy remains in the system as an increase in its internal energy

This expression specifies that if energy is added by heat to a system kept at constant volume, then all of the transferred energy remains in the system as an increase in its internal energy, and hence, temperature. For example, when a closed metallic can is thrown into a fire, energy enters the gas in the can by the conduction of heat through the metal walls of the can. The temperature, and thus the pressure, in the can increases until the can possibly explodes, hence the warning label on such cans.

5. Isothermal Process

An isothermal process is one that takes place at constant temperature. This process can be established by putting a gas container in contact with a constant-temperature reservoir. If we plot P versus V at constant temperature for an ideal gas described by Eq. 11.8, the plot yields a hyperbolic curve called an *isotherm*. In Chap. 13, we will prove that the internal energy of an ideal gas is a function of temperature only. Consequently, in an isothermal process involving an ideal gas we must have $\Delta E_{int} = 0$. Therefore, for an isothermal process, we conclude from the first law that the energy transfer Q must be equal to the work done by the gas W. That is:

$$\boxed{\Delta E_{int} = 0} \quad \text{and} \quad \boxed{Q = W} \quad \text{(Isothermal process)} \tag{12.20}$$

Any energy that enters the system by heat is transferred out of the system by work; as a result, no change in the internal energy of the system occurs in an isothermal process.

Suppose that an ideal gas is allowed to expand at constant temperature as described by the PV diagram in Fig. 12.14. According to Eq. 11.10, the curve is a hyperbola with the equation $PV = \text{constant}$.

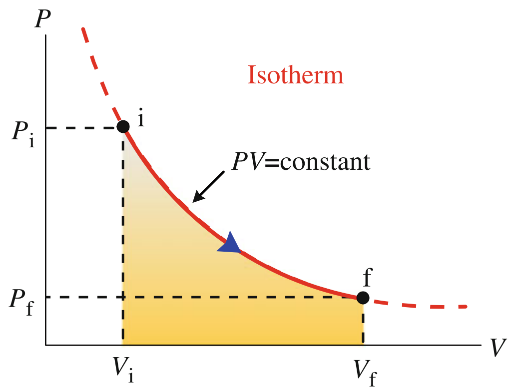


Fig. 12.14 The PV diagram for an isothermal expansion of an ideal gas from initial state i to a final state f

Let us calculate the work done by the gas in the isothermal expansion from state i to state f, as shown in Fig. 12.14. Because the gas is ideal and the process is quasi-static, we can use the expression $PV = nRT$ for each point on the path. Therefore, we have:

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV \tag{12.21}$$

Since T is constant and also n and R are constants, then they can be moved from the integral sign. Thus:

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \left| \ln V \right|_{V_i}^{V_f} \tag{12.22}$$

where we used $\int dV/V = \ln V$ to evaluate the last integral. Thus:

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \tag{12.23}$$

If the gas expands, the work W equals the positive of the shaded area under the PV curve shown in Fig. 12.14; this is because $\ln(V_f/V_i) > 0$. If the gas is compressed, $V_f < V_i$, then $\ln(V_f/V_i) < 0$ and the work done is the negative of the area under the PV curve.

Table 12.3 summarizes the characteristics of the previous processes.

Table 12.3 The first law of thermodynamics in five special cases

Process	Restriction	Consequence
Adiabatic	$Q = 0$	$\Delta E_{\text{int}} = -W$
Free expansion	$Q = W = 0$	$\Delta E_{\text{int}} = 0$
Isobaric	P = constant	$W_{\text{isobaric}} = P(V_f - V_i)$
Isovolumetric	V = constant, $W = 0$	$\Delta E_{\text{int}} = Q$
Isothermal (ideal gas)	T = constant, $\Delta E_{\text{int}} = 0$	$Q = W = nRT\ln(V_f/V_i)$

Example 12.5

At a constant pressure of 1 atm and a temperature of 0 °C, the heat fusion of ice is $L_F = 3.33 \times 10^5$ J/kg, the density of ice is $\rho_i = 920\,\text{kg/m}^3$, and the density of liquid water is $\rho_w = 1{,}000\,\text{kg/m}^3$. (a) Find the work W done by 1 kg of ice that melts completely to water. (b) Find the change in internal energy of this process.

Solution: (a) The initial volume of ice is:

$$V_i = m/\rho_i = (1\,\text{kg})/(920\,\text{kg/m}^3) = 1.087 \times 10^{-3}\,\text{m}^3$$

The final volume of ice after it melts completely to water is:

$$V_w = m/\rho_w = (1\,\text{kg})/(1{,}000\,\text{kg/m}^3) = 10^{-3}\,\text{m}^3$$

The work done by 1 kg of ice that melts completely to water under constant pressure of 1 atm (1.01×10^5 Pa) and temperature of 0 °C, is:

$$\begin{aligned} W &= \int_{V_i}^{V_w} P\,dV = P(V_w - V_i) \\ &= (1.01 \times 10^5\,\text{Pa})(10^{-3}\,\text{m}^3 - 1.087 \times 10^{-3}\,\text{m}^3) = -8.787\,\text{J} \simeq -8.8\,\text{J} \end{aligned}$$

The minus sign appears because ice contracts when it melts.

(b) The heat energy transferred to change the phase of 1 kg of ice to water is:

$$Q = m\,L_F = (1\,\text{kg})(3.33 \times 10^5\,\text{J/kg}) = 3.33 \times 10^5\,\text{J}$$

Thus, from the first law of thermodynamics, we can find the change in internal energy of this process as follows:

$$\Delta E_{\text{int}} = Q - W = 3.33 \times 10^5\,\text{J} + 8.8\,\text{J} = 3.330088 \times 10^5\,\text{J}$$

We see from parts (a) and (b) that $|W|$ is less than 0.003% of Q in this process, i.e. $|W| \ll Q$. That is, the mechanical energy is negligible in comparison to the heat of fusion. So, all the added heat of fusion shows up as an increase in the internal energy.

Example 12.6

At a constant pressure of 1 atm, a movable piston encloses 1 kg of water with a volume of $10^{-3}\,\text{m}^3$ and a temperature of 100 °C, see Fig. 12.15. Heat is added from a reservoir until the liquid water changes completely into steam of volume $1.671\,\text{m}^3$, see the figure. (a) How much work is done by the system (water + steam) during the boiling process? (b) How much heat energy is added to the system? (c) What is the change in the internal energy of the system?

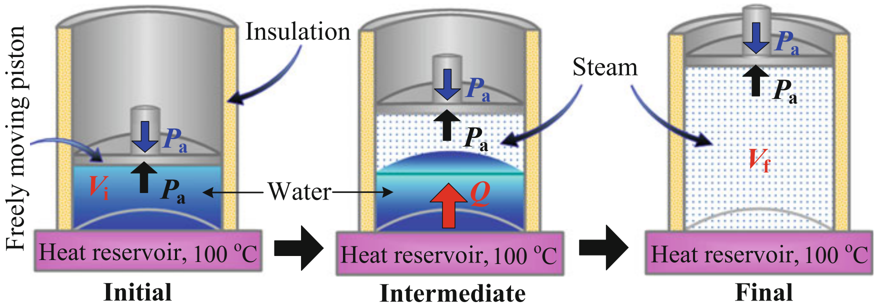


Fig. 12.15

Solution: (a) The work done by 1 kg of water that is converted completely into steam under a constant pressure of 1 atm (1.01×10^5 Pa) and a constant temperature of 100 °C, is:

$$W = \int_{V_i}^{V_f} P\,dV = P(V_f - V_i) = (1.01 \times 10^5\,\text{Pa})(1.671\,\text{m}^3 - 10^{-3}\,\text{m}^3) = 169\,\text{kJ}$$

(b) Since the heat of vaporization of water at atmospheric pressure is 2.26×10^6 J/kg, the heat energy required to change the phase of 1 kg of water to steam will be:

$$Q = m\,L_V = (1\,\text{kg})(2.26 \times 10^6\,\text{J/kg}) = 2{,}260\,\text{kJ}$$

(c) From the first law of thermodynamics, we can find the change in internal energy of this process as follows:

$$\Delta E_{int} = Q - W = 2.26 \times 10^6\,\text{J} - 1.69 \times 10^5\,\text{J} = 2{,}091\,\text{kJ}$$

We see that about 92.5% of the heat energy goes into internal energy while the remaining 7.5% goes into external work.

Example 12.7

An aluminum rod of mass 1 kg is heated from 25 to 55 °C at constant atmospheric pressure, see Fig. 12.16. The aluminum rod has a density ρ of 2.7×10^3 kg/m^3, a coefficient of volume expansion β of 7.2×10^{-5} (C°)$^{-1}$ and a specific heat c of 900 J/kg.C°. (a) How much work is done by the rod? (b) How much heat is transferred to the rod? (c) Quantify the rod's internal energy change.

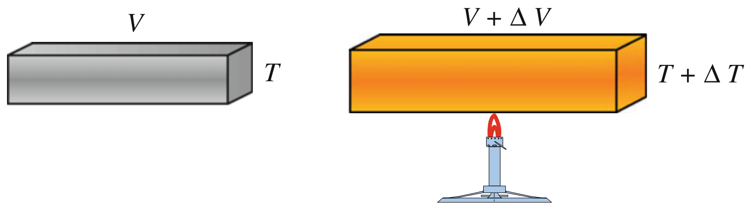


Fig. 12.16

Solution: (a) The initial volume of the aluminum rod is given by:

$$V = \frac{m}{\rho} = \frac{1\,\text{kg}}{2.7 \times 10^3\,\text{kg/m}^3} = 3.704 \times 10^{-4}\,\text{m}^3$$

Using the change in temperature $\Delta T = T_f - T_i = 55\,°\text{C} - 25\,°\text{C} = 30\,\text{C}°$, the change in the rod's volume can be obtained from Eq. 11.5 as follows:

$$\Delta V = \beta\, V \Delta T$$
$$= (7.2 \times 10^{-5}\,(\mathrm{C}^\circ)^{-1})(3.704 \times 10^{-4}\,\mathrm{m}^3)(30\,\mathrm{C}^\circ) = 8 \times 10^{-7}\,\mathrm{m}^3$$

Since the expansion is carried out at a constant pressure, the work done by the aluminum rod is:

$$W = \int_{V_\mathrm{i}}^{V_\mathrm{f}} P\,dV = P(V_\mathrm{f} - V_\mathrm{i}) = P\Delta V$$
$$= (1.01 \times 10^5\,\mathrm{Pa})(8 \times 10^{-7}\,\mathrm{m}^3) = 8.08 \times 10^{-2}\,\mathrm{J}$$

(b) We use the specific heat value in Eq. 12.4 to calculate the amount of heat transferred to the rod as follows:

$$Q = m\,c\,\Delta T = (1\,\mathrm{kg})(900\,\mathrm{J/kg.C}^\circ)(30\,\mathrm{C}^\circ) = 2.7 \times 10^4\,\mathrm{J}$$

(c) From the first law of thermodynamics, we can find the change in internal energy of this process as follows:

$$\Delta E_\mathrm{int} = Q - W = 2.7 \times 10^4\,\mathrm{J} - 8.09 \times 10^{-2}\,\mathrm{J} = 2.699 \times 10^4\,\mathrm{J}$$

We notice that almost all of the heat energy goes towards increasing the internal energy of the aluminum rod. The fraction of heat energy that is used as work against the atmospheric pressure is only about $4 \times 10^{-4}\%$. Therefore, in thermal expansion of solids, the amount of energy that goes into work is usually neglected.

Example 12.8

Find the work done by 1 kmol of an ideal gas that is kept at a constant temperature of 27 °C in an expansion process from 2 to 5 L.

Solution: Rewriting these values and the gas constant R, we have:

$$n = 1\,\mathrm{kmol}$$
$$R = 8.314 \times 10^3\,\mathrm{J/kmol.K}$$
$$T = 27\,^\circ\mathrm{C} = 27 + 273 = 300\,\mathrm{K}$$
$$V_\mathrm{i} = 2\,\mathrm{L} = 2{,}000\,\mathrm{cm}^3 = 2 \times 10^{-3}\,\mathrm{m}^3$$
$$V_\mathrm{f} = 5\,\mathrm{L} = 5{,}000\,\mathrm{cm}^3 = 5 \times 10^{-3}\,\mathrm{m}^3$$

Since this process is isothermal, the work done by the ideal gas is given by Eq. 12.23. Substitution in this equation results in:

$$\begin{aligned} W &= n\,RT\,\ln\left(\frac{V_\mathrm{f}}{V_\mathrm{i}}\right) \\ &= (1\,\mathrm{kmol})(8.314\times 10^3\,\mathrm{J/kmol.K})(300\,\mathrm{K})\ \ln\left(\frac{5}{2}\right) = 2.29\times 10^6\,\mathrm{J} \end{aligned}$$

This means that the heat energy Q that must be given to the ideal gas from the reservoir to keep its temperature $T = 27\,°\mathrm{C}$ is also 2.29×10^6 J.

12.5 Heat Transfer

We discussed the transfer of heat energy between a system and its surroundings, but we did not describe *how that transfer takes place and at what rate*. The three common energy-transfer mechanisms that are responsible for changing the internal energy state of a system are:

1. Conduction:
   The flow of heat that reduces the temperature difference between two materials.
2. Convection:
   The flow of heat in liquids or gases that carries heat from one place to another if the liquids or gases are free to move.
3. Radiation:
   The transfer of energy in the form of electromagnetic waves from objects that have temperatures greater than absolute zero. The transfer of *heat* energy from one location to another is by infrared radiation.

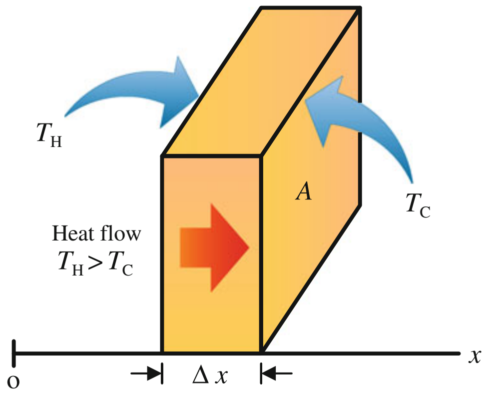
In this section we focus only on the first mechanism, leaving the other two mechanisms for other thermodynamic studies.

Thermal Conduction in One Dimension (Plain Walls)

In thermal conduction, heat transfer can be represented on the atomic scale as an exchange of kinetic energy between microscopic particles (molecules, atoms, and electrons) in which less energetic particles gain energy in collisions with more energetic particles. By this method, heat energy is transferred from the hot parts of an object to its cold parts.

Consider the flow of heat along the x-axis between the faces of a slab of a material of thickness Δx and face area A, as shown in Fig. 12.17. Assume the opposite faces are maintained at different temperatures T_H and T_C, where $T_H > T_C$. Let $\Delta T = T_C - T_H$ denote the *change* in temperature that is maintained along the thickness Δx. The temperature *difference* $T_H - T_C = -\Delta T$ is what gives rise to heat flow.

Fig. 12.17 Linear heat transfer through a conducting slab of face area A and thickness Δx, when the opposite faces are at different temperatures, T_H and T_C



Let ΔQ be the heat energy that is transferred through the slab from its hot face to its cold face, in a time interval Δt. Let $H = \Delta Q/\Delta t$ denote the rate of heat flow across the slab (H is measured in watts). Experiments show that H should be directly proportional to the face area A, the temperature difference $-\Delta T = T_H - T_C > 0$, and inversely proportional to the thickness Δx. That is:

$$H = \frac{\Delta Q}{\Delta t} \propto -A\frac{\Delta T}{\Delta x}$$

or

$$\boxed{H = \frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}} \tag{12.24}$$

where k is a proportionality constant that has the SI unit W/m.C° and is called the **thermal conductivity** of the material. For a slab of differential thickness dx and differential temperature difference dT, we can write what is called the **law of heat conduction** as follows:

$$\boxed{H = -kA\frac{dT}{dx}} \tag{12.25}$$

where dT/dx is known as the temperature gradient. The minus sign in Eq. 12.25 is due to the fact that heat energy flows in the direction of decreasing temperature.

Now, consider a long uniform rod of length L, as shown in Fig. 12.18. The rod is insulated so that thermal energy cannot enter nor escape from its surface except at its ends, which are in thermal contact with heat reservoirs having temperatures T_H and T_C, where $T_H > T_C$.

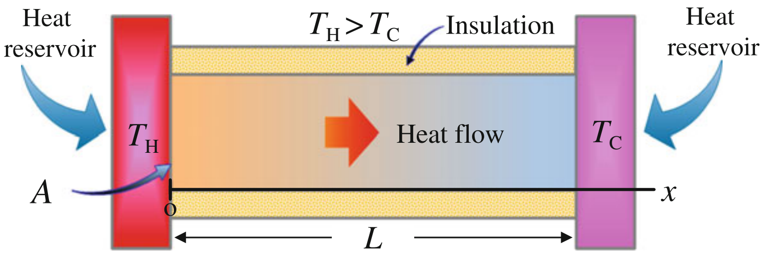


Fig. 12.18 Conduction of heat through a uniform conducting, insulated rod of length L and face area A, where the opposite faces are at different temperatures, T_H and T_C ($T_H > T_C$)

When a steady-state has been reached, the temperature at each point along the rod is constant in time. In such a case, the temperature gradient is the same everywhere along the rod and is given by:

$$\frac{dT}{dx} = \frac{T_C - T_H}{L} \tag{12.26}$$

Thus, the rate of heat flow becomes:

$$\boxed{H = kA\,\frac{T_H - T_C}{L}} \tag{12.27}$$

The thermal conductivity k is a constant that depends on the material of the rod. Large values of k indicate that a material is a good thermal conductor, and vice versa. Table 12.4 displays the thermal conductivities of some common metals, gases, and building materials.

Table 12.4 Thermal conductivity of some substances around normal room temperature.

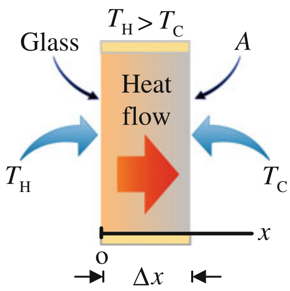
Substance	Thermal conductivity W/m.C°
Metals	
Stainless steel	14
Lead	35
Aluminum	238
Gold	314
Copper	401
Silver	427
Gases	
Air (dry)	0.026
Helium	0.15
Hydrogen	0.18
Building materials	
Foam	0.024
Rock wool	0.043
Fiberglass	0.048
Asbestos	0.08
Wood	0.08
Rubber	0.2
Glass	0.8
Concrete	0.8
Window glass	1.0
Steel	18

These values are approximate because k depends on the temperature

Example 12.9

A glass window measures $1\,\text{m} \times 1.5\,\text{m} \times 0.5\,\text{cm}$ and has a thermal conductivity of $0.8\,\text{W/m.C}^{\circ}$. The temperature of the inner surface of the glass is $T_H = 20\,^{\circ}\text{C}$, while the temperature for the outer surface is $T_C = -15\,^{\circ}\text{C}$, see Fig. 12.19. (a) Calculate the rate of heat flow by conduction through the window. (b) If the inner face of the window is taken to be at $x = 0$, see the figure, then find the temperature of the glass as a function of x.

Fig. 12.19



Solution: (a) The thickness of the glass is $\Delta x = 0.5\,\text{cm} = 5 \times 10^{-3}$ m and the change in temperature is $\Delta T = T_C - T_H = -15\,°\text{C} - 20\,°\text{C} = -35\,\text{C}°$. Using Eq. 12.24 we get:

$$H = -kA\frac{\Delta T}{\Delta x} = -(0.8\,\text{W/m.C}°)(1\,\text{m} \times 1.5\,\text{m})\frac{(-35\,\text{C}°)}{5 \times 10^{-3}\,\text{m}} = 8{,}400\,\text{W}$$

This enormous rate of heat flow by conduction shows that glass is not a very good insulator. The rate of heat flow through a glass window can be reduced substantially by using two layers of glass with a thin air layer between them. This is called *double glazing*.

(b) The temperature gradient for the window is given by:

$$\frac{dT}{dx} = \frac{\Delta T}{\Delta x} = \frac{(-35\,\text{C}°)}{5 \times 10^{-3}\,\text{m}} = -7{,}000\,\text{C}°/\text{m}$$

This equation can be integrated to give:

$$\int_{T_H}^{T} dT = (-7{,}000\,\text{C}°/\text{m})\int_0^x dx \quad \Rightarrow \quad T - T_H = (-7{,}000\,\text{C}°/\text{m})\,(x - 0)$$

Thus: $$T = 20\,°\text{C} - (7{,}000\,\text{C}°/\text{m})\,x$$

We can check whether this gives the correct temperature of $-15\,°\text{C}$ for the outer surface as follows:

$$T = 20\,°\text{C} - (7{,}000\,\text{C}°/\text{m})\,(5 \times 10^{-3}\,\text{m}) = -15\,°\text{C}$$

Example 12.10

Figure 12.20 shows two slabs of thickness L_1 and L_2, thermal conductivities k_1 and k_2, and an equal surface area A. The temperatures at the outer faces of the slabs are T_H and T_C, where $T_H > T_C$. In a steady-state condition, find: (a) the interface temperature T, when $T_H = 25\,°C$, $T_C = -5\,°C$, $L_2 = 2L_1$, and $k_2 = 4k_1$, and (b) the rate of heat transfer by conduction through the slabs.

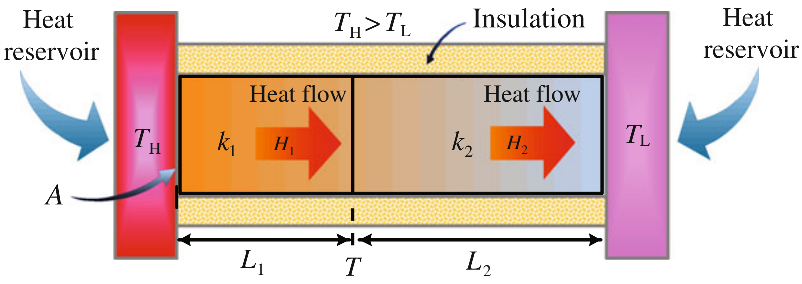


Fig. 12.20

Solution: (a) If T is the temperature at the interface, then the rate of heat flow through the two slabs is:

$$H_1 = k_1 A \frac{T_H - T}{L_1}, \quad \text{and} \quad H_2 = k_2 A \frac{T - T_C}{L_2}$$

When a steady-state is reached, these two rates must be equal, that is:

$$k_1 A \frac{T_H - T}{L_1} = k_2 A \frac{T - T_C}{L_2}$$

Solving for T gives:

$$T = \frac{k_1 L_2 T_H + k_2 L_1 T_C}{k_1 L_2 + k_2 L_1}$$

Inserting the given relations and the known temperatures gives:

$$T = \frac{2k_1 L_1 T_H + 4k_1 L_1 T_C}{2k_1 L_1 + 4k_1 L_1} = \tfrac{1}{6}(2T_H + 4T_C) = \tfrac{1}{6}[2(25\,°C) + 4(-5\,°C)] = 5\,°C$$

(b) The expression of the rate of heat flow by conduction will be:

$$H = H_1 = H_2 = \frac{A\,(T_H - T_C)}{(L_1/k_1) + (L_2/k_2)} = \frac{10\,A\,k_1}{L_1}$$

Home Insulation

Insulation is important in building houses, since it helps limit heat loss and hence keeps homes at a comfortable temperature with less cost, see Fig. 12.21. Good insulation requires many insulation slabs.

Fig. 12.21 In houses, heat is conducted from the inside to the outside more rapidly where insulation is poor. Thus, houses should be well insulated especially in the attic to minimize heat loss

For a compound slab containing several materials of thicknesses $L_1,\ L_2, \ldots$ and thermal conductivities $k_1,\ k_2, \ldots$, we can perform similar steps as in Example 12.10 to show that the rate of heat transfer at a steady-state will take the form:

$$H = \frac{A\,(T_{\rm H} - T_{\rm C})}{\sum\limits_n L_n/k_n}, \qquad (n = 1, 2, \ldots) \tag{12.28}$$

In the engineering practice, the term L/k for a particular substance is referred to as the R value of the material, and Eq. 12.28 takes the following form:

$$H = \frac{A\,(T_{\rm H} - T_{\rm C})}{\sum\limits_n R_n}, \quad (n = 1, 2, \ldots) \tag{12.29}$$

where $R_n = L_n/k_n$. If a wall contains three slabs of insulation, then we can find the value of R for the wall by adding the values of R for each slab. Table 12.5 lists the R-values for common building materials.

Table 12.5 The *R*-values of some common building materials

Material	Thickness (cm)	R-value ($m^2.C°/W$)
Hardwood siding	2	0.185
Wood shingles	1.3	0.111
Brick	10	0.704
Fiberglass batting	8	1.918
Fiberglass board	2.5	0.766
Cellulose fiber	2.5	0.651
Flat glass	0.3	0.151
Insulating glass	0.3	0.318
Air space	10	0.178
Drywall	1.5	0.095
Sheathing	1.5	0.233

Thermal Conduction in Two Dimensions (Cylindrical Shells)

We can apply the law of heat conduction to situations where heat flows in two dimensions by varying the area in consideration.

As an example, consider a steam pipe in which heat flows radially outwards. This type of heat flow is called *cylindrical heat flow* and is illustrated geometrically in Fig. 12.22.

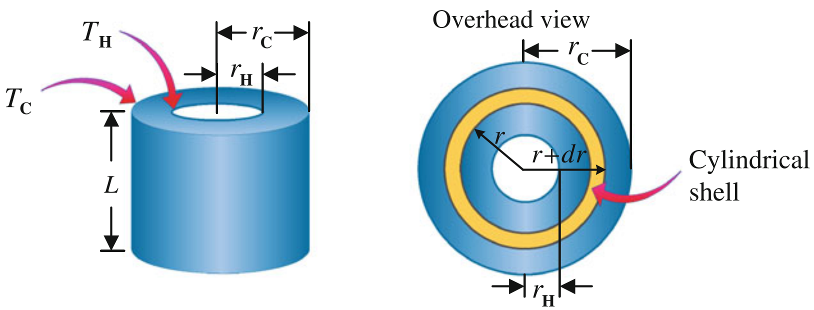


Fig. 12.22 Geometry for heat flow in a cylinder of length *L*. *Left* The inner and outer radii and temperatures are r_H, r_C, T_H, and T_C, respectively. *Right* A cylindrical shell has a radius *r* and thickness *dr*

Conceptually, we can divide a cylindrical pipe of length *L* into a series of thin concentric cylindrical shells. The rate of heat flow through a cylindrical shell of radius *r* and thickness *dr* is given by:

$$H = -kA\frac{dT}{dr} \tag{12.30}$$

where A is the surface area of the cylindrical shell and is given by:

$$A = 2\pi rL \tag{12.31}$$

Thus, for cylindrical heat flow, the law of heat conduction becomes:

$$H = -2\pi k\,Lr\frac{dT}{dr} \tag{12.32}$$

For steady-state conditions H remains constant, and we can find how T varies with r by rearranging Eq. 12.32 as follows:

$$dT = -\frac{H}{2\pi kL}\frac{dr}{r} \tag{12.33}$$

We can now integrate this equation from the initial radius r_H (where the temperature is T_H) to some arbitrary radius r where the temperature is $T \equiv T(r)$ as follows:

$$\int_{T_H}^{T} dT = -\frac{H}{2\pi kL}\int_{r_H}^{r}\frac{dr}{r} \tag{12.34}$$

Thus:

$$\boxed{T - T_H = -\frac{H}{2\pi kL}\ln\left(\frac{r}{r_H}\right)} \tag{12.35}$$

This result shows that for cylindrical heat flow the temperature decreases logarithmically with an increasing r.

The rate of heat flow through the pipe section that has inner and outer radii r_H and r_C, and inner and outer temperatures T_H and T_C, is given by letting $T = T_C$ and $r = r_C$ in Eq. 12.35. That is:

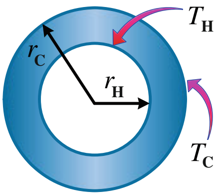
$$\boxed{H = \frac{2\pi kL\,(T_H - T_C)}{\ln\,(r_C/r_H)}} \tag{12.36}$$

Example 12.11

A stainless-steel pipe has inner and outer radii of 2 and 2.5 cm, respectively. The pipe carries hot water at a temperature of $T_H = 60\,°C$ and has a thermal conductivity of 19 W/m.C°. The pipe's outer surface temperature is $T_C = 56\,°C$, see Fig. 12.23. (a) What is the rate of heat flow per unit length of the pipe?

(b) When an additional cylindrical insulator of thermal conductivity of $0.2\ \mathrm{W/m.C^\circ}$ is used, what is the thickness required to reduce heat loss by a factor of 10 and achieve an outer temperature of $37\,^\circ\mathrm{C}$?

Fig. 12.23



Solution: (a) The temperature difference is:

$$T_H - T_C = 60\,^\circ\mathrm{C} - 56\,^\circ\mathrm{C} = 4\,\mathrm{C}^\circ$$

This value is used in Eq. 12.36 to get the value of the rate of heat flow per unit length, H/L, as:

$$\frac{H}{L} = \frac{2\pi k(T_H - T_C)}{\ln(r_C/r_H)} = \frac{2\pi(19\,\mathrm{W/m.C^\circ})(4\,\mathrm{C^\circ})}{\ln(2.5\,\mathrm{cm}/2\,\mathrm{cm})} = 2{,}140\,\mathrm{W/m}$$

This great rate of heat flow per unit length shows that stainless steel is not a very good material to use alone for an isolated hot-water pipe.

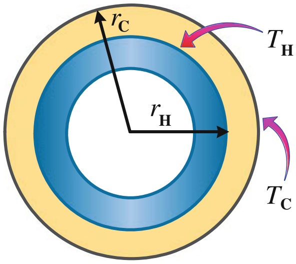
(b) As far as the stainless-steel pipe is concerned, a reduction in H/L by a factor of 10 requires that the temperature difference between the inner and outer surfaces be reduced by the same factor. Thus, the original $4\,\mathrm{C}^\circ$ difference is reduced to $0.4\,\mathrm{C}^\circ$. Hence, the inner surface of the cylindrical insulator will be at $T_H = 59.6\,^\circ\mathrm{C}$ and its outer surface temperature will be at $T_C = 37\,^\circ\mathrm{C}$, i.e. $T_H - T_C = 59.6\,^\circ\mathrm{C} - 37\,^\circ\mathrm{C} = 22.6\,\mathrm{C}^\circ$, see Fig. 12.24. In addition H/L will be reduced to 214 W/m. Solving Eq. 12.36 again for $\ln(r_C/r_H)$, where $r_H = 2.5$ cm, we get:

$$\ln(r_C/r_H) = \frac{2\pi k(T_H - T_C)}{H/L} = \frac{2\pi(0.2\,\mathrm{W/m.C^\circ})(22.6\,\mathrm{C^\circ})}{214\,\mathrm{W/m}} = 0.133$$

$$\text{Thus:}\quad r_C/r_H = \exp(0.133) = 1.142 \quad\Rightarrow\quad r_C = 1.142 \times r_H = 2.9\,\mathrm{cm}$$

The required insulation thickness is $r_C - r_H = 2.9\,\mathrm{cm} - 2.5\,\mathrm{cm} = 0.4\,\mathrm{cm}$.

Fig. 12.24



12.6 Exercises

Section 12.1 Heat and Thermal Energy

Subsection 12.1.1 Units of Heat, The Mechanical Equivalent of Heat

(1) A room is lighted by a 200 W light bulb. A 200 W of power is the rate at which the bulb converts electrical energy into heat and visible light. Assuming that 90% of the energy is converted into heat, how much heat is added to the room in 4 h?

(2) Suppose your mass is 70 kg and you ate a 250 kcal meal. To compensate, you decided to lose an equivalent amount of energy by climbing the stairs of a building. What is the total height that you must climb?

Subsection 12.1.2 Heat Capacity and Specific Heat

(3) 159.2 g of water is initially at 15 °C. To what temperature will this quantity of water rise when 1,000 J of energy is supplied?

(4) The brakes of a 1,500 kg car are used to decelerate its speed from 72 km/h to rest. How many joules and kilocalories are generated during the stopping process?

(5) How many calories of heat are required to raise the temperature of 4 kg of iron of specific heat 448 J/kg.C° from 20 to 40 °C?

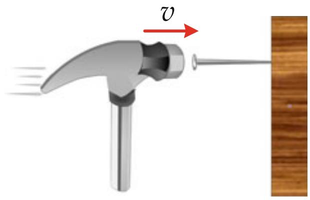
(6) The water cooling system (radiator) of a car holds 20 L of water. How much heat does the radiator absorb if its temperature rises from 20 to 95 °C?

(7) The specific heat of aluminum is 900 J/kg.C°. (a) What is the heat capacity of 5 kg of aluminum? (b) How much heat must be added to 5 kg of aluminum to raise its temperature from 27 to 37 °C?

(8) What is the specific heat of a 4 kg material when its temperature increases from 27 to 37 °C after 18 kJ of heat is added?

(9) A hammer head of mass 1.5 kg strikes an iron nail of mass 15 g that has a specific heat 450 J/kg.C°. The hammer has a speed $v = 6\,\mathrm{m/s}$ just before striking the nail and then comes to rest after the impact, see Fig. 12.25. Assume that all the energy of the hammer goes into heating the nail during the strike. What is the rise in temperature of the nail?

Fig. 12.25 See Exercise (9)



(10) What is the final equilibrium temperature when 20 g of milk at 10 °C is added to 200 g of tea at 100 °C? (Assume that the specific heat of milk, tea, and water are all the same, and neglect the heat capacity of the container).

(11) A 2 kg metallic object is heated to 500 °C and then dropped into a bucket containing 20 kg of water initially at 20 °C. When equilibrium is reached, the temperature of the mixture is 70 °C. What is the specific heat of the metal? (Neglect the heat capacity of the container).

(12) In an experiment where the specific heat of aluminum is measured using the method of mixtures, see Fig. 12.26, a student obtains the following data:

Mass of aluminum: $m_\mathrm{x} = 0.2\,\mathrm{kg}$

Initial temperature of aluminum: $T_\mathrm{x} = 27\,°\mathrm{C}$

Mass of water: $m_\mathrm{w} = 0.4\,\mathrm{kg}$

Specific heat of water: $c_\mathrm{w} = 4{,}186\,\mathrm{J/kg.C°}$

Mass of calorimeter: $m_\mathrm{c} = 0.04\,\mathrm{kg}$

Initial temperature of water and calorimeter: $T_\mathrm{i} = 70\,°\mathrm{C}$

Specific heat of the calorimeter: $c_\mathrm{c} = 630\,\mathrm{J/kg.C°}$

Final temperature of the mixture: $T_\mathrm{f} = 66.4\,°\mathrm{C}$

Use these data to determine the specific heat of aluminum c_x.

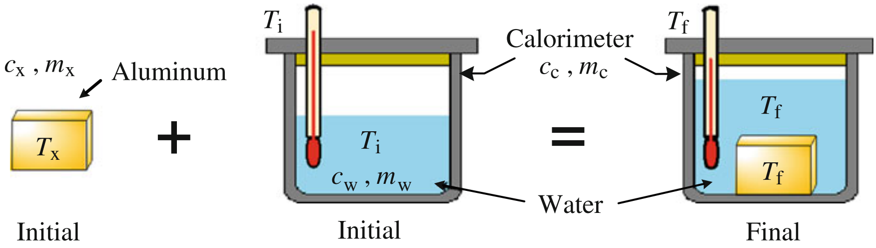


Fig. 12.26 See Exercise (12)

Subsection 12.1.3 Latent Heat

(13) Aluminum has a melting temperature of 660 °C, latent heat of fusion of 3.97×10^5 J/kg, and specific heat of 900 J/kg.C°. How much heat is needed to melt 15 kg of aluminum that is initially at 27 °C?

(14) A runner loses 150 kcal of heat in 15 min by evaporating water from his skin. The latent heat of vaporization of water at room temperature is 2.26×10^6 J/kg. How much water has been lost?

(15) Follow similar steps like the calculations done in Fig. 12.4 to find the energy required to change a 50 g ice cube from the ice state at −10 °C to the steam state at 110 °C.

(16) A 150 g of ice is enclosed in a thermally insulated container. What is the mass of steam at 100 °C that must be mixed with the ice to produce liquid water at 50 °C. (For the ice and steam, use the constants of Tables 12.1 and 12.2)

(17) A 100 g block of ice at 0 °C is added to 400 g of water at 30 °C. Assuming we have a perfectly insulated calorimeter for this mixture, what will be its final temperature when all of the ice has melted?

(18) A copper calorimeter has a mass $m_c = 100$ g. The calorimeter contains water of mass $m_w = 500$ g at a temperature of 20 °C. How much steam must be condensed into water if the final temperature of the mixture is to reach 50 °C? Assume the specific heat of copper is $c_c = 840$ J/kg.C°, the specific heat of water is $c_w = 4{,}186$ J/kg.C°, and the latent heat of condensation of steam is $L_V = 2.26 \times 10^6$ J/kg.

(19) A 60 kg ice skater glides from a speed $v_i = 10$ m/s to a speed $v_f = 4$ m/s on ice at 0 °C, see Fig. 12.27. Assume that 80% of the heat generated by friction is

absorbed by the ice and all of the melted ice stays at 0 °C. The latent heat of fusion of ice is 3.33×10^5 J/kg. How much ice melts?

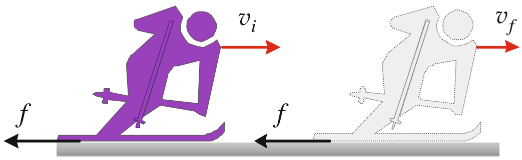


Fig. 12.27 See Exercise (19)

Section 12.2 Heat and Work (Take 1 atm $\simeq 10^5$ Pa)

(20) An ideal gas is enclosed in a container at a pressure of 2 atm and has a volume of 3 m^3. What is the work done by the gas if: (a) the gas expands at a constant pressure to three times its initial volume? (b) the gas is compressed at a constant pressure to one half of its initial volume?

(21) (a) An ideal gas is taken from an initial state i to a final state f, as shown in Fig. 12.28. Find the work done by the gas along the three paths iaf, if, and ibf. (b) Answer part (a) if the gas is taken from f to i.

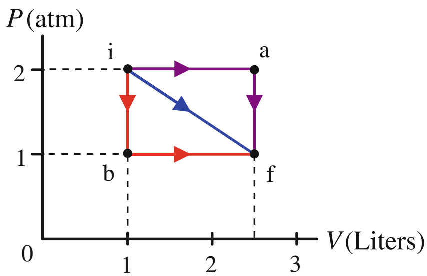
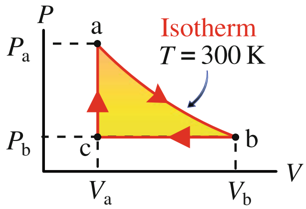


Fig. 12.28 See Exercise (21)

Section 12.4 Applications of the First Law of Thermodynamics

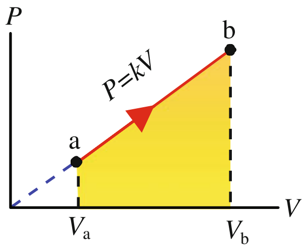
(22) An ideal gas of 2 kmol is carried around the thermodynamic cycle as shown in Fig. 12.29. The cycle consists of three parts; the isothermal expansion ab at $T = 300$ K an isobaric compression bc, and an isovolumetric increase in pressure ca. (a) When $P_a = 4$ atm and $P_b = 1$ atm, then find the work done by the gas per cycle. (b) Answer part (a) when the direction of the cycle is reversed.

Fig. 12.29 See Exercise (22)



(23) An ideal gas expands from an initial volume $V_a = 0.5\,\text{m}^3$ to a final volume $V_b = 1.5\,\text{m}^3$ in a quasi-static process for which $P = kV$, where $k = 2.5\,\text{atm/m}^3$, see Fig. 12.30. How much work was done by the expanding gas?

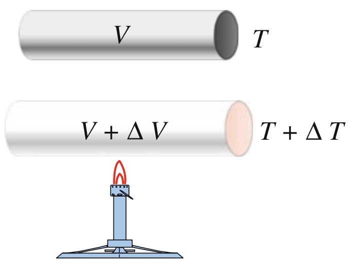
Fig. 12.30 See Exercise (23)



(24) An amount of work of 100 J is done on a system, and 100 cal of heat are extracted from it. In light of the first law of thermodynamics, what are the values (including algebraic signs) of: (a) W, (b) Q, and (c) ΔE_{int}?

(25) A cylindrical steel rod of mass 3.9 kg is heated from $T = 27\,°\text{C}$ to $T + \Delta T = 37\,°\text{C}$ at a constant atmospheric pressure, see Fig. 12.31. The rod has a density ρ of $7.8 \times 10^3\,\text{kg/m}^3$, a coefficient of volume expansion β of $3.3 \times 10^{-6}\,(\text{C}°)^{-1}$, and a specific heat c of 450 J/kg.C°. (a) How much work is done by the rod? (b) How much heat is transferred to the rod? (c) What is the change in the rod's internal energy?

Fig. 12.31 See Exercise (25)



(26) An ideal helium gas of 1 kmol is carried around the thermodynamic cycle as shown in Fig. 12.32. The path ab is isothermal, with $P_a = 2$ atm, $P_b = 1$ atm, and $V_a = 22.4\,\text{m}^3$. (a) What are the values of T_a, V_b, and T_c? (b) How much work is done by the gas in this cycle?

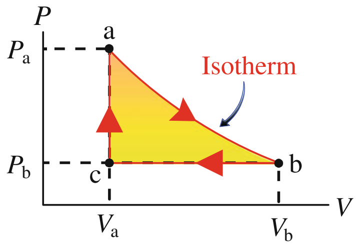


Fig. 12.32 See Exercise (26)

(27) An ideal gas of one kmol does 4,000 J of work as it expands isothermally to a volume of $12 \times 10^{-3}\,\text{m}^3$ that has a final pressure of 2 atm. (a) What is the temperature of the gas? (b) What is the initial volume of the gas?

(28) A fluid is carried through the cycle abcd as shown in Fig. 12.33. How much work (in kilojoules) is done by the fluid during: (a) the isobaric expansion ab, (b) the isovolumetric process bc, and (c) the isobaric compression cd? (d) What is the net amount of heat transferred to work during the cycle abcd?

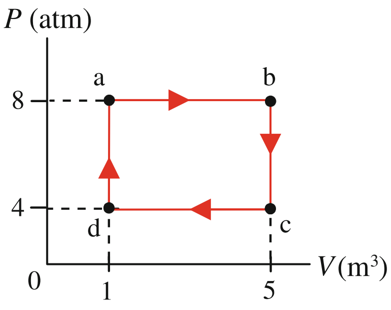


Fig. 12.33 See Exercise (28)

(29) At a constant pressure of 2 atm ($2\,P_a$), the boiling point of water is 120 °C, and its heat of vaporization is $L_V = 2.20 \times 10^6$ J/kg. Under these conditions, assume a movable piston encloses 1 kg of water with a volume of $V_i = 10^{-3}\,\text{m}^3$, see the left part of Fig. 12.34. Heat is added from a reservoir until the liquid water changes completely into steam of volume $V_f = 0.824\,\text{m}^3$, see the right part of Fig. 12.34. (a) How much work is done by the system (water + steam)

during the boiling process? (b) How much heat energy is added to the system? (c) What is the change in the internal energy of the system?

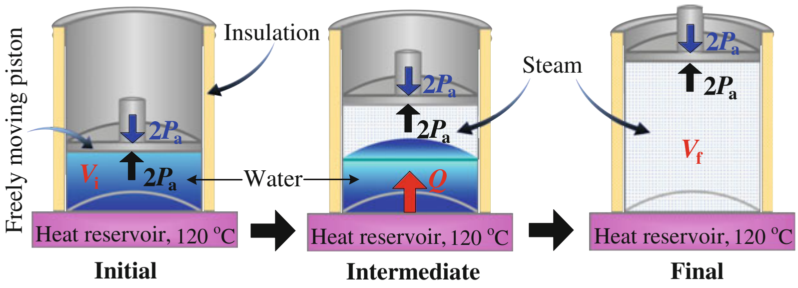
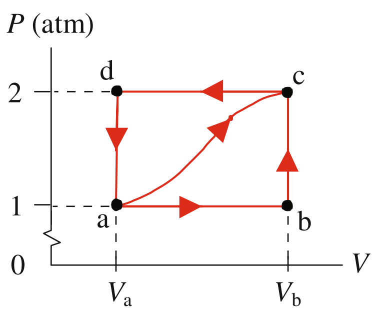


Fig. 12.34 See Exercise (29)

(30) An ideal gas has an initial temperature of 27 °C and an initial volume of 1 m^3. An isobaric expansion of the gas to a new volume of 3 m^3 is achieved by adding 9,500 J of heat at a constant pressure of 3,000 Pa. (a) Determine the work done by the gas during expansion. (b) What is the change in the internal energy of the gas? (c) Find the final temperature of the gas.

(31) An ideal gas is taken from a to c along the curved path in Fig. 12.35. Along this path, the work done by the gas is $W_{ac} = 15$ J and the heat added to the gas is $Q_{ac} = 43$ J. In addition, the work done along path abc is $W_{abc} = 34$ J. (a) What is the change in internal energy of the gas ΔE_{ac} for path ac? (b) What is Q_{abc} for path abc? (c) What is W_{cda} for path cda? (d) What is Q_{cda} for path cda?

Fig. 12.35 See Exercise (31)



(32) Helium with an initial volume of 10^{-3} m^3 and an initial pressure of 10 atm expands to a final volume of 1 m^3. The relationship between pressure and

volume during this expansion process is kept PV = constant by supplying heat at a constant temperature. (a) Calculate the value of the constant. (b) Find the final pressure. (c) Determine the work done by the helium during the expansion. (d) How much heat was absorbed by the expanding helium?

Section 12.5 Heat Transfer

(33) The thermal conductivity of a special type of Pyrex glass at $0\,°C$ is 3×10^{-3} cal/cm.C°.s. (a) Express this quantity in W/m.C° and in Btu/ft.F°.h. (b) What is the R value of a 1 cm sheet of Pyrex?

(34) A slab of a thermal insulator has a cross section of $0.1\ m^2$, a thickness of 2 cm, and thermal conductivity of 0.1 J/m.s.C°. If the temperature difference between the opposite faces of the insulator is 100 C°, how much heat flows through the slab in 24 h?

(35) Consider the slab of copper shown in Fig. 12.36, where $A = 9 \times 10^{-3}\ m^2$, $L = 0.25$ m, and the thermal conductivity is 400 W/m.C°. In the steady-state condition, the temperature of the hot surface is $T_H = 125\,°C$, and the temperature for the cold surface is $T_C = 10\,°C$. Find the rate of heat transfer through the slab.

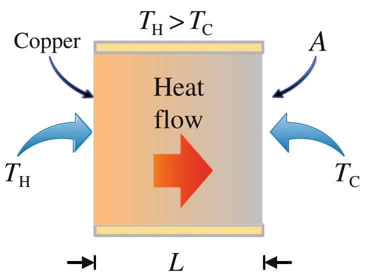
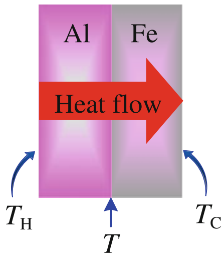


Fig. 12.36 See Exercise (35)

(36) A pair of metal plates having equal areas and equal thicknesses is in thermal contact, as shown in Fig. 12.37. One plate is made of aluminum, and the other is made of iron. Assume that the thermal conductivity of aluminum k_A is exactly three times that of iron k_F. The outer face of the iron plate is maintained at $T_C = 0\,°C$, while the outer face of the aluminum plate is maintained at $T_H = 60\,°C$. In a steady-state condition, find the interface temperature T and the relation that gives the rate of heat transfer by conduction through the slabs.

Fig. 12.37 See Exercise (36)



(37) Bricks and insulation are used to construct the walls of a house. The insulation has R_1-value of 0.095 $m^2.C°/W$. The bricks have an R_2-value of 0.704 $m^2.C°/W$, see Fig. 12.38. In the steady-state condition, the temperature inside the house is $T_H = 24\,°C$ and the outside temperature is $T_C = 10\,°C$. Find the rate of heat loss through such a wall, if its area is 20 m^2.

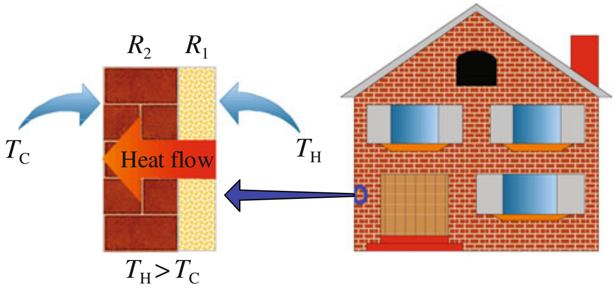


Fig. 12.38 See Exercise (37)

(38) A pipe made of steel has inner and outer radii of 2.5 and 3 cm respectively. The pipe carries hot water at a temperature $T_H = 70\,C°$ and has a thermal conductivity of 14 W/m.C°. The pipe's outer surface temperature is $T_C = 60\,°C$, see the left part of Fig. 12.39. (a) What is the rate of heat flow per unit length of the pipe? (b) When an additional cylindrical insulator of thermal conductivity of 0.2 W/m.C° is used, see the right part of Fig. 12.39, what is the thickness required to reduce heat loss by a factor of 10 and achieve an outer temperature of 30 °C?

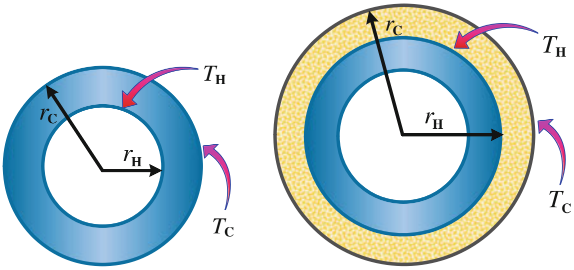
(39) Show that the rate of heat that flows radially outwards in a spherically symmetric system is governed by the equation:

$$H = -4\pi r^2 k \frac{dT}{dr}$$

where r is the distance from the center of the source to the point where the temperature is T. When the inner and outer radii and temperatures are r_H, r_C, T_H, and T_C, respectively, show that:

$$H = \frac{4\pi k(T_H - T_C)}{(1/r_H - 1/r_C)}$$

Fig. 12.39 See Exercise (38)



(40) An insulating spherical container has a total inner surface area of $1.5\,\text{m}^2$ and a thickness of 5 cm. A 60 W (assumed to be a point source) electric bulb inside the container is used to maintain a constant temperature difference $T_H - T_C = 100\,\text{C}^\circ$ between the inside and the outside of the container, see Fig. 12.40. What is the thermal conductivity k of the insulating material?

Fig. 12.40 See Exercise (40)