Magnetic Fields

25

It is of common knowledge that every magnet attracts pieces of iron and has two *poles: a north* pole (N) and a *south* pole (S). In addition, given two magnets, like poles (N–N or S–S) repel each other, and opposite poles (N–S) attract each other. Moreover, if we cut a magnet in half, we do not obtain isolated north and south poles. Instead, we get two magnets, each with its own north and south pole.

In 1819, Oersted observed the deflection of a pivoted magnet when it was in the vicinity of a current-carrying wire. Now, it is known that all magnetic phenomena result from forces arising from electric charges in motion. Based on these forces, the concept of a **magnetic field** was introduced as a mechanism for exerting a magnetic force on a moving charge. This is similar to the concept of an electric field surrounding an electric charge. That is, in the region of space around any moving charge, a magnetic field is established (as well as an electric field), and this magnetic field can exert a force on a second moving charge. Consequently, all atoms can exhibit magnetic effects, due to the motion of their electrons about their nuclei.

In this chapter, we discuss forces that act on moving charges as well as forces that act on current-carrying conductors in the presence of a magnetic field. We postpone discussing the sources of such fields.

25.1 Magnetic Force on a Moving Charge

A magnetic field exists at a particular point in space if a force is exerted on a moving charge at that point. The magnetic field, like the electric field, is a vector quantity and historically is denoted by the symbol $\vec{B}$. We can define the magnetic field $\vec{B}$ at any point in terms of the magnetic force $\vec{F}_B$ exerted by the field on a test charge q

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moving with a velocity $\vec{v}$. If the smaller angle between the two vectors $\vec{B}$ and $\vec{v}$ is denoted by θ, then experiments show that:

- $F_B \propto |q| vB \sin\theta$
- $\vec{F}_B$ has the direction of $\vec{v} \times \vec{B}$ if q is positive
- $\vec{F}_B$ has the direction of $-\vec{v} \times \vec{B}$ if q is negative

In vector form, these results can be written as follows:

$$\vec{F}_B = q\,\vec{v} \times \vec{B} = q \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \tag{25.1}$$

Therefore, the magnitude of the magnetic force on q is:

$$F_B = |q|\, vB \sin\theta \tag{25.2}$$

To find the direction of $\vec{v} \times \vec{B}$ and the direction of $\vec{F}_B$ for both positive and negative q, we use the right-hand rule, as shown in Fig. 25.1.

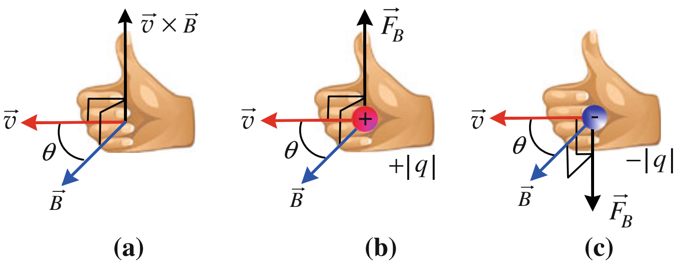


Fig. 25.1 (a) With the right-hand rule, the direction of the thumb points in the direction of $\vec{v} \times \vec{B}$ when the fingers curl $\vec{v}$ into $\vec{B}$. (b) When q is positive, the direction of $\vec{F}_B$ has the same sign as $\vec{v} \times \vec{B}$. (c) When q is negative, the directions of $\vec{F}_B$ is opposite to $\vec{v} \times \vec{B}$

Equation 25.1 indicates that:

- $F_B = 0$ (when $\vec{v} \,//\, \vec{B}$ and, of course, when $v = 0$)
- $F_B|_{\max} = q\,vB$ (when $\vec{v} \perp \vec{B}$)
- $\vec{F}_B \perp \vec{v}$ at all times, (hence $\vec{B}$ changes only the direction of $\vec{v}$)

From Eq. 25.1, we see that the SI unit for B is newton per coulomb-meter per second, which is called **tesla** (T). With the use of the SI unit: 1 ampere is 1 coulomb per second, so we have:

$$1\,\text{T} = 1\frac{\text{N}}{\text{C.m/s}} = 1\frac{\text{N}}{\text{A.m}} \tag{25.3}$$

An earlier non-SI unit of B, still in common use, is *gauss* (G), and is related to tesla through the conversion formula:

$$1\,\text{T} = 10^4\,\text{G} \tag{25.4}$$

Table 25.1 lists some approximate values of B in a few situations.

Table 25.1 Some approximate values of the magnetic fields

Source of the field	Value of B(T)
New kind of neutron star called a "Magnetar"	10^{11}
Neutron star	10^8
Superconducting magnet	30
Strong magnet	2
Medical MRI unit	1.5
Small bar magnet	10^{-2}
Surface of the earth	10^{-4}
Inside human brain	10^{-13}
Smallest value in a magnetically shielded room	10^{-14}

For convenience, we label the magnetic field coming out of the page by black dots (or blue dots), as shown in Fig. 25.2a and the magnetic field going into the page by black crosses (or blue crosses), as shown in Fig. 25.2b. The same approach is used for both $\vec{v}$ and I but sometimes with different colors.

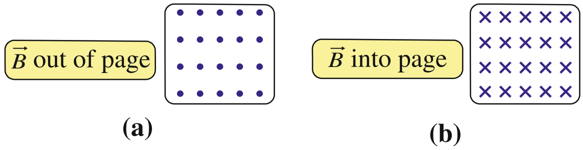
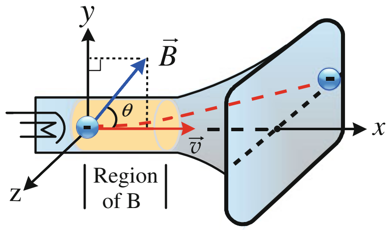


Fig. 25.2 Magnetic field lines: (a) coming out of the page are indicated by *dots*, (b) going into the page are indicated by *crosses*

Example 25.1

An electron in a television tube moves along the x-axis with a speed v of 10^7 m/s, see the sketch in Fig. 25.3. A uniform magnetic field in the xy plane has a magnitude 0.02 T and is directed at an angle of 30° from the x-axis. (a) Calculate the magnitude of the magnetic force on the electron. (b) Find the vector expression of the magnetic force on the electron in terms of the unit vectors $\vec{\text{i}}$, $\vec{\text{j}}$, and $\vec{\text{k}}$ along x, y, and z axes.

Fig. 25.3



Solution: (a) using Eq. 25.2 we find that:

$$F_B = |q|\, vB\sin\theta = (1.6\times 10^{-19}\,\text{C})(10^7\,\text{m/s})(0.02\,\text{T})(\sin 30°) = 1.6\times 10^{-14}\,\text{N}$$

(b) We first express the velocity and the magnetic field in terms of the unit vectors $\vec{\text{i}}$, $\vec{\text{j}}$, and $\vec{\text{k}}$ as follows:

$$\vec{v} = (10^7\,\vec{\text{i}}\,)\,\text{m/s}$$

$$\begin{aligned}\vec{B} &= B\cos\theta\,\vec{\text{i}} + B\sin\theta\,\vec{\text{j}}\\ &= [(0.02)(\cos 30°)\,\vec{\text{i}} + (0.02)(\sin 30°)\,\vec{\text{j}}\,]\,\text{T}\\ &= (0.017\,\vec{\text{i}} + 0.01\,\vec{\text{j}}\,)\,\text{T}\end{aligned}$$

We use Eq. 25.1 to find the force on the electron as follows:

$$\vec{F}_B = q\,\vec{v}\times\vec{B} = q\begin{vmatrix}\vec{\text{i}} & \vec{\text{j}} & \vec{\text{k}}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix} = (-e)\begin{vmatrix}\vec{\text{i}} & \vec{\text{j}} & \vec{\text{k}}\\ 10^7\,\text{m/s} & 0 & 0\\ 0.017\,\text{T} & 0.01\,\text{T} & 0\end{vmatrix}$$

$$\begin{aligned}&= (-e)\Big[(0)\,\vec{\text{i}} - (0)\,\vec{\text{j}} + (10^7\,\text{m/s})(0.01\,\text{T})\,\vec{\text{k}}\Big]\\ &= (-1.6\times 10^{-19}\,\text{C})(10^5\,\text{T m/s})\,\vec{\text{k}}\\ &= -(1.6\times 10^{-14}\,\text{N})\,\vec{\text{k}}\end{aligned}$$

The magnetic force on the electron $\vec{F}_B$ has a magnitude that agrees with the result of part (a) and is directed along the negative z-axis.

25.2 Motion of a Charged Particle in a Uniform Magnetic Field

The fact that $\vec{F}_B \perp \vec{v}$ indicates that the magnetic field $\vec{B}$ does not work on the *charged* particle. Therefore, $\vec{F}_B$ never changes the magnitude of $\vec{v}$, but only changes its direction.

Let us consider a uniform magnetic field (coming out of the page). Now assume a positively charged particle q moving with an initial velocity vector $\vec{v}$ perpendicular to the field, as shown in Fig. 25.4. As the direction of the particle's velocity changes in response to the magnetic force, the new $\vec{F}_B$ at the new location remains perpendicular to the new direction of the particle. As a result, the path of the particle is a circle of radius r. The particle rotates in a clockwise sense if its charge is positive, as shown in Fig. 25.4, and in a counterclockwise sense if the charge is negative.

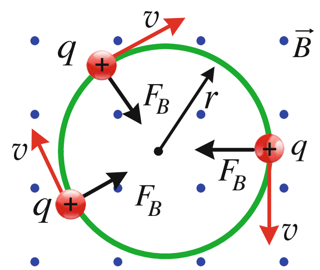


Fig. 25.4 When the initial velocity of a positively charged particle is perpendicular to the magnetic field, the particle's orbit is a *circle*

When we equate the magnitude of the magnetic force, $F_B = qvB$, to the product of the mass of the particle m and the magnitude of the centripetal acceleration, we get:

$$F_B = qvB = m \times \frac{v^2}{r} \tag{25.5}$$

Solving for r, we get:

$$r = \frac{mv}{qB} \tag{25.6}$$

That is, the radius of curvature is proportional to the magnitude of the momentum mv of the particle and inversely proportional to the *magnitude of the charge* and to the magnitude of the magnetic field.

The period of the motion $T = 2\pi r/v$, the frequency $f = 1/T$, and the angular frequency $\omega = 2\pi/T$, can be written as:

$$T = \frac{2\pi m}{qB} \tag{25.7}$$

$$f = \frac{qB}{2\pi m} \tag{25.8}$$

$$\omega = \frac{qB}{m} \tag{25.9}$$

These equations show that T, f, and ω are independent of the speed v of the particle and the radius r of the orbit.

If the velocity of the charged particle has two components, one perpendicular ($v_\perp$) to the uniform magnetic field and the other parallel ($v_\parallel$) to it, then the particle will move in a helical path about the direction of the magnetic field $\vec{B}$. For example, if $\vec{B}$ is along the x-axis, the perpendicular component $v_\perp$ (in the yz plane) determines the radius of the helix $r = mv_\perp/qB$, while the parallel component determines the distance between the turns of the helix (the pitch) $p = v_\parallel T$, see Fig. 25.5.

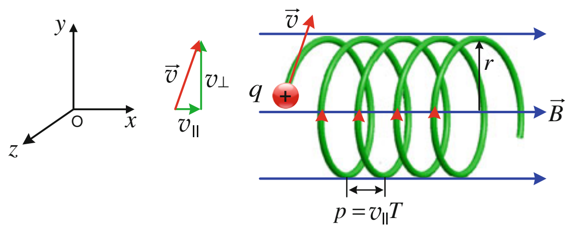


Fig. 25.5 When the initial velocity of a positively charged particle has a component parallel to the magnetic field $\vec{B}$, the particle will move in a helical path about the direction of the field

Example 25.2

A proton of mass $m = 1.67 \times 10^{-27}$ kg and charge $q = e = 1.6 \times 10^{-19}$ C is moving in a circular orbit of radius $r = 20$ cm perpendicular to a uniform magnetic field of magnitude $B = 0.25$ T. (a) Find the period of the proton. (b) Find the speed of the proton. (c) Find the magnitude of the magnetic force on the proton.

Solution: (a) From Eq. 25.7, we have:

$$T = \frac{2\pi m}{eB} = \frac{2\pi(1.67 \times 10^{-27}\,\text{kg})}{(1.6 \times 10^{-19}\,\text{C})(0.25\,\text{T})} = 2.6 \times 10^{-7}\,\text{s}$$

(b) Using the relation $T = 2\pi r/v$ [or Eq. 25.6], we have:

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.2\,\text{m})}{2.6 \times 10^{-7}\,\text{s}} = 4.8 \times 10^{6}\,\text{m/s}$$

(c) From the relation $F_B = |q|vB \sin 90°$, we have:

$$F_B = evB = (1.6 \times 10^{-19}\,\text{C})(4.8 \times 10^{6}\,\text{m/s})(0.25\,\text{T}) = 1.9 \times 10^{-13}\,\text{N}$$

Example 25.3

An electron of mass $m = 9.11 \times 10^{-31}$ kg is moving with a speed $v = 2.8 \times 10^{6}$ m/s. The electron enters a uniform magnetic field of magnitude $B = 5 \times 10^{-4}$ T when the angle between $\vec{v}$ and $\vec{B}$ is 60°. Find the radius and pitch of the helical path taken by the electron.

Solution: The components $v_\perp$ and $v_\parallel$ with respect to $\vec{B}$ are:

$$v_\perp = v \sin\theta = (2.8 \times 10^{6}\,\text{m/s}) \sin 60° = 2.42 \times 10^{6}\,\text{m/s}$$
$$v_\parallel = v \cos\theta = (2.8 \times 10^{6}\,\text{m/s}) \cos 60° = 1.40 \times 10^{6}\,\text{m/s}$$

Using the relations $r = mv_\perp/qB$ and $p = v_\parallel T$, we have:

$$r = \frac{mv_\perp}{eB} = \frac{(9.11 \times 10^{-31}\,\text{kg})(2.42 \times 10^{6}\,\text{m/s})}{(1.6 \times 10^{-19}\,\text{C})(5 \times 10^{-4}\,\text{T})} = 0.0276\,\text{m} = 2.76\,\text{cm}$$

$$p = v_\parallel T = v_\parallel \frac{2\pi r}{v_\perp} = \frac{2\pi(0.0276\,\text{m})(1.4 \times 10^{6}\,\text{m/s})}{(2.42 \times 10^{6}\,\text{m/s})} = 0.1003\,\text{m} = 10.03\,\text{cm}$$

25.3 Charged Particles in an Electric and Magnetic Fields

In the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$, the total force $\vec{F}$ exerted on a charge q moving with velocity $\vec{v}$ is:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{25.10}$$

which is often called the **Lorentz force**.

25.3.1 Velocity Selector

Sometimes it is required to select charged particles moving only with same constant velocity. This can be achieved by applying an upward electric field $\vec{E}$ perpendicular to a magnetic field $\vec{B}$ coming out of the page, as shown in Fig. 25.6. In this figure a positive charge q passes from the source through slits S_1 and S_2 and moves to the right in a straight line with velocity $\vec{v}$. Consequently, the electric force $q\vec{E}$ points upwards with a magnitude qE, while the magnetic force $q\vec{v} \times \vec{B}$ points downwards with a magnitude qvB.

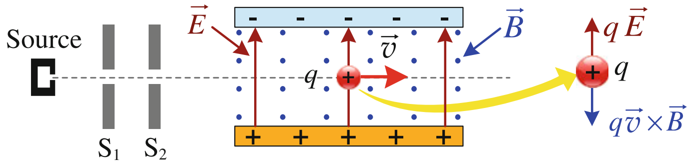


Fig. 25.6 In a velocity selector, the magnetic field $\vec{B}$, electric field $\vec{E}$, and the velocity $\vec{v}$ of the charged particle are perpendicular to each other. When the magnetic force $q\vec{v} \times \vec{B}$ cancels the electric force $q\vec{E}$, the charged particle will move in a *straight line*

If we choose the values of $\vec{E}$ and $\vec{B}$ such that $qE = q\,vB$, then:

$$v = \frac{E}{B} \tag{25.11}$$

and the particle will continue moving in a horizontal straight line through the region of the fields. For the chosen values of $\vec{E}$ and $\vec{B}$, all particles with speeds greater than $v = E/B$ will move downwards, while all particles with speeds less than $v = E/B$ will move upwards.

25.3.2 The Mass Spectrometer

A mass spectrometer is an instrument used to measure the mass or the mass-to-charge ratio for charged particles (or ions). The mass spectrometer of Fig. 25.7 has a source of charged particles behind S_1, and these particles pass through S_1 and S_2 into a velocity selector like the one shown in Fig. 25.6. Particles that have a speed of $v = E/B$ pass through slit S_3 and enter a *deflecting chamber* of uniform magnetic field $\vec{B}'$ that has the direction of $\vec{B}$ in the velocity selector. In this region the particles move in a circular path of radius r.

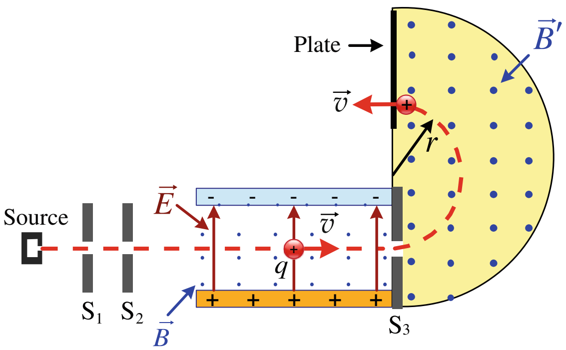


Fig. 25.7 The schematic drawing of a mass spectrometer. Positively charged particles from the source enter the velocity selector and then into a region where the magnetic field $\vec{B}'$ causes the particle to move in a *semicircle* of radius r before striking a plate

From Eq. 25.6, the mass m can be expressed as follows:

$$m = \frac{qB'r}{v} \tag{25.12}$$

Then we use $v = E/B$, to calculate the ratio m/q as follows:

$$\frac{m}{q} = \frac{BB'r}{E} \tag{25.13}$$

If the charge q is known, then the mass m of the charged particle can be calculated in terms of B, B', E, and r.

25.3.3 The Hall Effect

In 1879, Edwin Hall showed that when a current I passes through a strip of metal which is placed perpendicular to a magnetic field $\vec{B}$, a potential difference is established in a direction perpendicular to both I and $\vec{B}$. This phenomenon is known as *Hall effect*.

Figure 25.8a shows a thin flat strip of copper connected to a battery. Electrons flow with drift speed v_d opposite to the conventional current I. In Fig. 25.8b we show that when we apply to the strip a magnetic field $\vec{B}$ (into the page), electrons experience an upward transverse magnetic force $\vec{F}_M = q\vec{v}_d \times \vec{B} = -e\vec{v}_d \times \vec{B}$ and are deflected from their previous course. Because electrons cannot escape from the strip, negative charges accumulate on its upper side, leaving a net positive charge on its lower side. This separation of charges produces an upward transverse *Hall electric field* $\vec{E}_H$ that exerts a downward electric force on the electrons $\vec{F}_E = q\vec{E}_H = -e\vec{E}_H$. Charges accumulate, and $\vec{E}_H$ increases, until the electric force finally cancels the magnetic force and equilibrium is established.

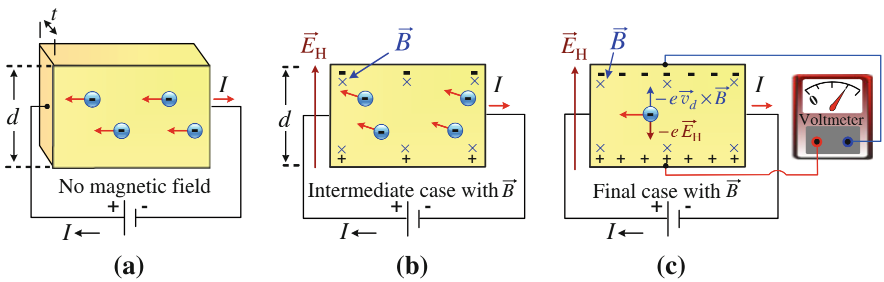


Fig. 25.8 (a) A conductor carrying a current I. (b) The situation immediately after applying the magnetic field into the page. Electrons experience an upward magnetic force $\vec{F}_M$, accumulate on the top surface, which creates an upward electric field that produces a downward electric force $\vec{F}_E$. (c) $\vec{F}_E$ cancels $\vec{F}_M$ at equilibrium

Equating the electric and magnetic forces on an electron gives:

$$eE_H = ev_d B \quad \Rightarrow \quad E_H = v_d B \tag{25.14}$$

When d is the width of the strip, the potential difference ΔV_H, called the *Hall voltage*, across the strip is related to electric field E_H by:

$$\Delta V_H = E_H d \tag{25.15}$$

From Eq. 24.6, the drift speed v_d is related to the current I by:

$$I = nev_d A \tag{25.16}$$

where $A = td$ is the cross-sectional area of the strip. Substituting with E_H from Eq. 25.15 and v_d from Eq. 25.16 into Eq. 25.14, we get $\Delta V_H = IB/net$. Usually this result is written as:

$$\Delta V_H = R_H \frac{IB}{t} \quad \text{where} \quad R_H = \frac{1}{ne} \tag{25.17}$$

where $R_H = 1/ne$ is the **Hall coefficient**. Equation 25.17 can be used to measure the magnitude of the magnetic fields and give information about the sign of the charge carriers and their density.

Example 25.4

The value of the Hall coefficient R_H for a copper strip is 5.4×10^{-11} m^3/C. The strip is 2 mm wide and 0.05 mm thick and carries a current $I = 100$ mA in a magnetic field $B = 1$ T, see Fig. 25.8. (a) How large is the Hall voltage across the strip? (b) Find the magnitude of the Hall electric field.

Solution: (a) From Eq. 25.17, we have:

$$\Delta V_H = R_H \frac{IB}{t} = 5.4 \times 10^{-11}\,\text{m}^3/\text{C} \frac{(100 \times 10^{-3}\,\text{A})(1\,\text{T})}{0.05 \times 10^{-3}\,\text{m}} = 1.08 \times 10^{-7}\,\text{V}$$

A Hall voltage of 0.108 μV needs a sensitive measuring instrument.

(b) From Eq. 25.15, we have:

$$E_H = \frac{\Delta V_H}{d} = \frac{1.08 \times 10^{-7}\,\text{V}}{2 \times 10^{-3}\,\text{m}} = 5.4 \times 10^{-5}\,\text{V/m}$$

25.4 Magnetic Force on a Current-Carrying Conductor

A net flow of charges through a wire is represented by a current. Since a magnetic field exerts a force on a moving charge, then one should expect that it should exert a force on a wire carrying a current.

- Figure 25.9a showns a horizontal flexible conducting wire carrying no current. In the presence of a uniform magnetic field $\vec{B}$ directed out of the page, the wire stays horizontal.
- However, when the wire carries a current in the left direction, as shown in Fig. 25.9b, the wire deflects upwards.
- Now, if the current direction is reversed, as shown in Fig. 25.9c, the wire deflects downwards.

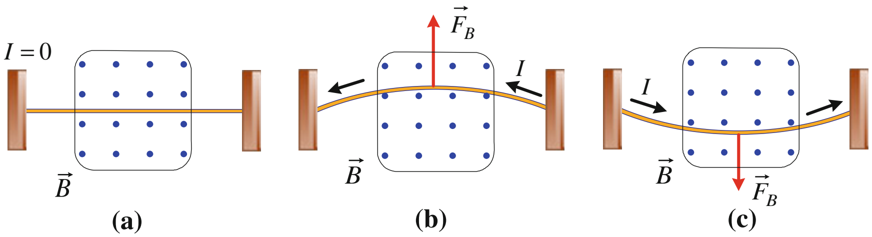


Fig. 25.9 A flexible wire is suspended horizontally and passes through a region of uniform magnetic field. (a) Without current in the wire, the wire stays horizontal. (b) With a left current, the deflection is *upwards*. (c) With a right current, the deflection is *downwards*

Figure 25.10 shows a segment of a horizontal straight wire of length L and cross-sectional area A, carrying a current I to the left in a uniform magnetic field $\vec{B}$ out of the page. First, we consider a conducting electron of charge $q = -e$ drifting to the right (opposite to the conventional left current I) with a drift speed v_d. According to Eq. 25.2, the magnetic force on this electron has a magnitude $ev_d B$ and is directed upwards.

To find the magnitude of the total *upward force* on this segment of wire, we multiply the force on one electron by the total number of conducting electrons in the segment, which is nAL, where n is the number of electrons per unit volume. Thus:

$$F_B = (ev_d B)nAL$$

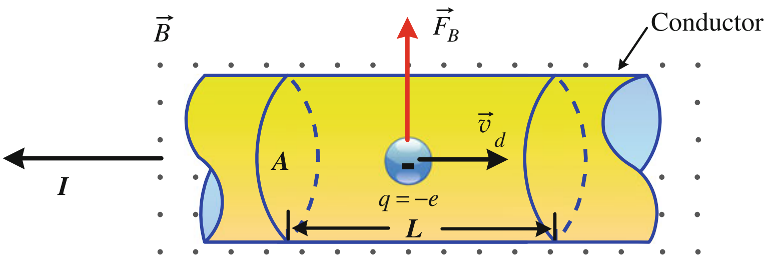


Fig. 25.10 Force on a moving charge in a current-carrying conductor. The current direction is to the left, which means that the electrons drift to the right. A magnetic field out of the page causes the electrons and the wire to be deflected *upwards*

From Eq. 25.16, the current in the wire is $I = nev_d A$. Then, the magnitude of the total upward force on this segment of wire will be:

$$F_B = ILB \tag{25.18}$$

When the uniform magnetic field $\vec{B}$ is not perpendicular to the straight wire, the magnetic force is given by a generalization of Eq. 25.18 as follows:

$$\vec{F}_B = I\,\vec{L} \times \vec{B} \tag{25.19}$$

where $\vec{L}$ is a *length vector* that points in the direction of the conventional current I.

If the wire is not straight, we consider a small straight segment of length ds and apply Eq. 25.19 to calculate the differential force:

$$d\vec{F}_B = I\,d\vec{s} \times \vec{B} \tag{25.20}$$

To calculate the total force on a wire of arbitrary shape, as shown in Fig. 25.11a, we integrate Eq. 25.20 over the length of the wire as follows:

$$\vec{F}_B = \int_a^b d\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B} \tag{25.21}$$

where the current I runs from one endpoint a to another endpoint b.

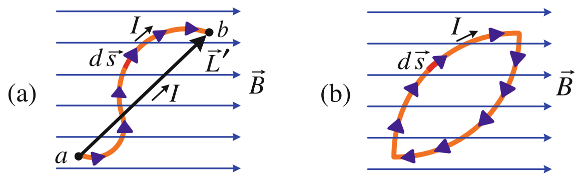


Fig. 25.11 (a) $\vec{F}_B$ on any curved wire carrying a current I in a uniform magnetic field is equal to the magnetic force on a straight wire of length L' from a to b. (b) $\vec{F}_B$ on a closed loop is zero

When the magnetic field is uniform, we take $\vec{B}$ outside the integrand of Eq. 25.21. Therefore, this equation reduces to:

$$\vec{F}_B = I \left(\int_a^b d\vec{s} \right) \times \vec{B} \tag{25.22}$$

When we integrate over $\vec{s}$, we get $\int_a^b d\vec{s} = \vec{L}'$, where $\vec{L}'$ is a length vector directed from a to b. Therefore, Eq. 25.21 becomes:

$$\vec{F}_B = I\,\vec{L}' \times \vec{B} \tag{25.23}$$

For a closed loop, see Fig. 25.11b, $\oint d\vec{s} = 0$ and hence $\vec{F}_B = 0$.

Therefore, in a uniform magnetic field, we conclude that:

- The net magnetic force on any curved wire carrying a current I flowing from one endpoint a to another endpoint b is the same as that for a straight wire carrying the same current from a to b.
- The net magnetic force on any closed loop of a wire carrying a current I is zero.

Example 25.5

A conducting wire has a *linear density* $\rho = 40 \times 10^{-3}$ kg/m and carries a current $I = 20$ A. Assume a magnetic field $\vec{B}$ perpendicular to the wire; find the minimum B and its direction in order to suspend the wire (that is to balance its weight) when the wire: (a) is in a horizontally straight configuration of a length L, (b) is bent into an upward vertical semicircular arc of radius R.

Solution: (a) Figure 25.12 shows the situations for both cases, with a selected direction of I. For a minimum magnetic field, the magnetic force must be upwards in both cases as shown in Fig. 25.12.

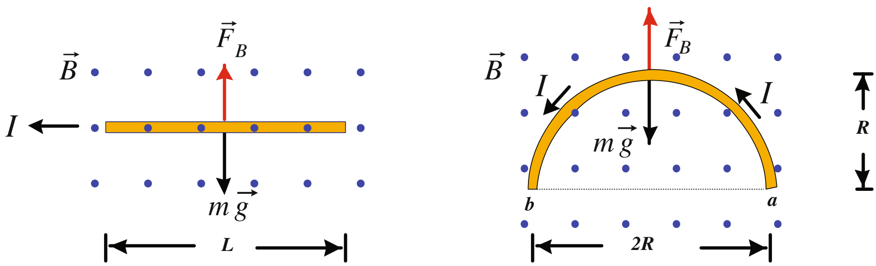


Fig. 25.12

In order to suspend the straight wire, the magnetic force F_B must equal to the wire's weight mg. Since $F_B = ILB$ and $m = \rho L$, we have:

$$F_B = mg \quad \Rightarrow \quad ILB = mg \quad \Rightarrow \quad ILB = \rho L g$$

Thus: $$B = \frac{\rho g}{I} = \frac{(40 \times 10^{-3}\,\text{kg/m})(10\,\text{m/s}^2)}{20\,\text{A}} = 0.02\,\text{T}$$

which is about 200 times the strength of the earth's magnetic field.

(b) The magnetic force F_B on a semicircular wire of radius R carrying a current I flowing from the one endpoint a to another endpoint b is the same as the magnetic force exerted on a straight wire having length $L' = 2R$ carrying the same current from a to b. That is $F_B = I(2R)B$. Since $m = \rho(\pi R)$ and F_B must equal mg, then:

$$2IRB = \pi \rho R g$$

Thus: $$B = \frac{\pi \rho g}{2I} = \frac{\pi \times (40 \times 10^{-3}\,\text{kg/m})(10\,\text{m/s}^2)}{2 \times 20\,\text{A}} = 0.0314\,\text{T}$$

Loudspeakers

The electrical output of a radio or TV set is connected to the leads of a device referred to as a loudspeaker, which converts electrical energy to sound energy. A loudspeaker has a permanent magnet that exerts a force on a current-carrying conductor. Those leads of the speaker are connected internally to a coil that is attached to the speaker cone, which is made of stiff cardboard that can move freely back and forth in front of the magnet, see Fig. 25.13.

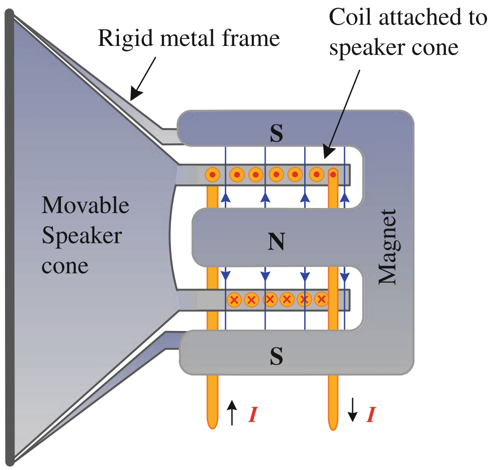


Fig. 25.13 A sketch showing a cross-sectional view of a typical loudspeaker, where both the coil and the speaker cone can move back and forth freely due to the magnetic force exerted by the permanent magnet on the current-carrying coil

When a current representing an audio signal flows through the coil, the magnetic field produced by the magnet will exert a force on the coil. As the current varies with the frequency of the audio signal, the coil and the speaker cone will move back and forth with the same frequency. This movement causes compressions and expansions of the air adjacent to the cone and consequently produces sound waves. As the electrical input to the speaker varies, the frequency and intensity of the generated sound waves also change to match.

25.5 Torque on a Current Loop

Most electric motors operate on the principle that a magnetic field exerts a torque on a loop of a current-carrying conductor. This torque has the ability to rotate the loop about a fixed rotational axis.

Consider a rectangular loop of two short sides ① and ② each of length a and two long sides ③ and ④ each of length b. The loop carries a current I in the presence of uniform magnetic field $\vec{B}$ which is always perpendicular to the long sides ③ and ④, and free to rotate about the axis OO', see Fig. 25.14.

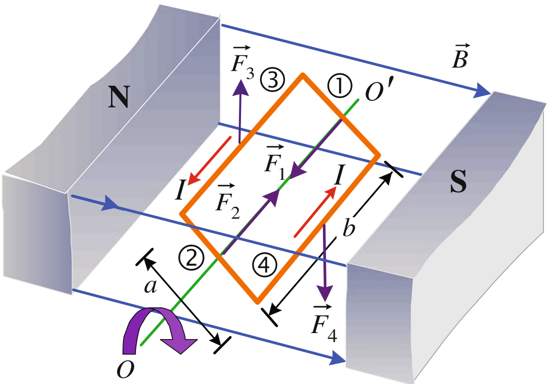


Fig. 25.14 A rectangular loop carrying a current I that can rotate freely about the axis OO' in the presence of a uniform magnetic field

In Fig. 25.14, we notice the following:

- The magnetic forces $\vec{F}_1$ and $\vec{F}_2$ on the short sides ① and ② cancel each other and *produce no torque*, since they pass through a common origin.
- The magnetic forces $\vec{F}_3$ and $\vec{F}_4$ on the long sides ③ and ④ cancel each other, but *produce a torque* about the rotational axis OO'.

We assume that $\vec{B}$ makes an angle $0 \le \theta \le 90°$ with the vector area $\vec{A}$, which is a vector perpendicular to the plane of the loop and has a magnitude equal to the area of the loop, see the side view of the loop shown in Fig. 25.15.

In Fig. 25.15, the side ③ is represented by a circle and the current passing through it is represented by a red dot, while the side ④ has the current represented by a red cross. From Eq. 25.19, the magnitudes of $\vec{F}_3$ and $\vec{F}_4$ are the same and given by:

$$F_3 = F_4 = IbB \tag{25.24}$$

The moment arm of F_3 and F_4 about O is $(a/2)\sin\theta$. Thus, the magnitude of the net torque about the rotational axis OO' is:

$$\begin{aligned} \tau &= F_3(a/2)\sin\theta + F_4(a/2)\sin\theta \\ &= [F_3 + F_4](a/2)\sin\theta = [2IbB](a/2)\sin\theta \\ &= IAB\sin\theta \end{aligned} \tag{25.25}$$

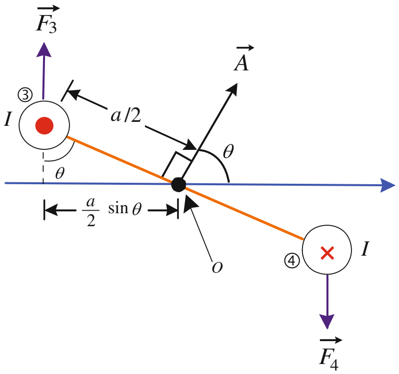


Fig. 25.15 A side view of the loop showing the two forces $\vec{F}_3$ and $\vec{F}_4$ that produce a torque on the current loop about point O

where $A = ab$ is the area of the loop. This equation shows that $\tau_{max} = IAB$ when $\vec{B}$ is perpendicular to the normal of the loop ($\theta = 90°$), and $\tau_{min} = 0$ when $\vec{B}$ is parallel to the normal to the plane of the loop ($\theta = 0$).

The direction of the torque exerted on the loop can be expressed in terms of the vector area as follows:

$$\vec{\tau} = I\vec{A} \times \vec{B} \qquad (\tau = IAB\sin\theta) \tag{25.26}$$

The product $I\vec{A}$ is defined as the magnetic dipole moment $\vec{\mu}$ (or simply the magnetic moment) of the loop and has the SI unit ampere-meter ^{2} (A.m^2). Thus:

$$\vec{\mu} = I\vec{A} \quad \text{(Single loop)} \tag{25.27}$$

If we replace the single loop of current with a *coil* of N loops, or turns, then the magnetic dipole moment $\vec{\mu}$ of the coil will be given by:

$$\vec{\mu} = NI\vec{A} \quad \text{(Coil of } N \text{ loops)} \tag{25.28}$$

Using this definition, Eq. 25.26 can be written as:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{25.29}$$

We can determine the direction of $\vec{A}$ and $\vec{\mu}$ by using the right-hand rule, which is described in Fig. 25.16.

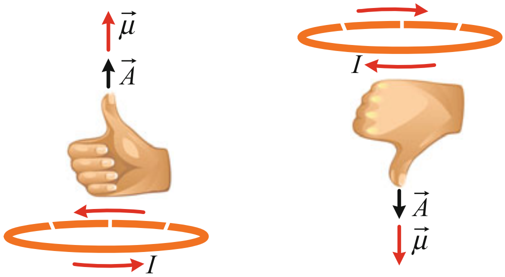


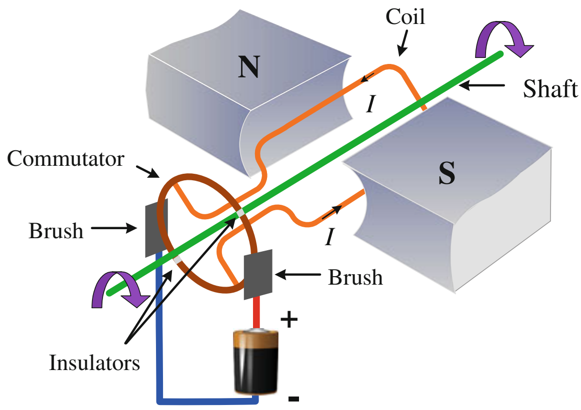
Fig. 25.16 Using the right-hand rule for determining the direction of $\vec{A}$ and $\vec{\mu}$ for a loop of wire carrying a current I

25.5.1 Electric Motors

A motor is an apparatus that converts electrical energy into rotational energy. A battery-powered motor uses the principle of torque exerted on a coil of wire wound onto a shaft that rotates 360°.

In order to allow the coil to continue rotating, the current through the coil must reverse the direction just as the coil reaches its vertical position. As shown in Fig. 25.17, several components are required to achieve this reversal. First, an electric connection is made using two brushes. These are contacts usually made of graphite. Second, a ring that is split into two halves, called a split-ring commutator. Brushes make contact with the commutator and allow current to flow into the coil. As the coil rotates, so does the commutator, which is arranged so that each of its halves changes brushes just as the coil reaches the vertical position. Changing brushes reverses the direction of the current in the coil. As a result, the direction of the force on each side of the coil is reversed and the coil continues to rotate. This process repeats at each half-turn, causing the coil to spin in the magnetic field.

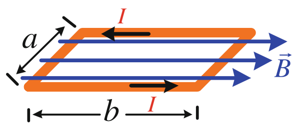
Fig. 25.17 The split-ring commutators in an electric motor allow the current in the wire coil to change direction and thus enable the coil in the motor to rotate continuously



Example 25.6

A rectangular coil of sides $a = 4$ cm and $b = 8$ cm consists of $N = 75$ turns of wire and carries a current $I = 10$ mA. A magnetic field of magnitude $B = 0.2$ T is applied parallel to the plane of the coil, see Fig. 25.18. (a) Find the magnitude of the magnetic dipole moment of the coil. (b) What is the magnitude of the torque acting on the coil?

Fig. 25.18



Solution: (a) Using Eq. 25.28, we have:

$$\mu = NIA = (75)(10 \times 10^{-3}\ \text{A})[(4 \times 10^{-2}\ \text{m})(8 \times 10^{-2}\ \text{m})] = 2.4 \times 10^{-3}\ \text{A.m}^2$$

(b) Since $\vec{B}$ is perpendicular to $\vec{\mu}$, then Eq. 25.29 gives:

$$\tau = \mu B \sin 90° = (2.4 \times 10^{-3}\ \text{A.m}^2)(0.2\ \text{T}) = 4.8 \times 10^{-4}\ \text{N.m}$$

25.5.2 Galvanometers

The basic component of analog ammeters, voltmeters, and ohmmeters is a galvanometer. Figure 25.19 displays the main features of a type of galvanometer called the D'Arsonval galvanometer. It consists of a coil of wire that has N loops, each of cross-sectional area A. That coil is attached to a pointer and a spring. The coil is also suspended so that it can rotate freely in a radial magnetic field produced by a circular cross-sectional permanent magnet.

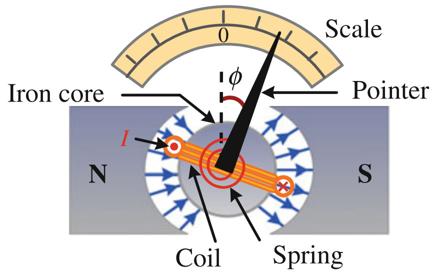


Fig. 25.19 Sketch of the structure of a moving-coil galvanometer

When a current I flows through the coil, the magnetic field exerts a torque on the coil given by Eq. 25.29, and this torque has a magnitude given by:

$$\tau = \mu B = NIAB \tag{25.30}$$

This torque is opposed by the torque τ_s exerted by the spring, which is approximately proportional to the coil deflecting angle ϕ. That is:

$$\tau_s = k\,\phi \tag{25.31}$$

where k is the stiffness constant of the spring. When the pointer is in equilibrium, we have $\tau_s = \tau$, and we get:

$$\phi = \frac{NAB}{k} I \quad \text{or} \quad \phi \propto I \tag{25.32}$$

Thus, the angular deflection ϕ of the pointer is directly proportional to the current I in the coil.

25.6 Non-Uniform Magnetic Fields

One of the useful types of non-uniform magnetic fields is the “magnetic bottle” shown in Fig. 25.20a. Such magnetic bottles can be used to trap charged particles, because the magnetic field is strong at the ends and weak in the middle. Charged particles spiral along the field lines back and forth almost indefinitely if they do not collide.

Therefore, this magnetic bottle can be used to confine a *plasma* (a gas consisting of electrons and ions). Such a confinement can help control nuclear fusion, a process that could supply us with energy indefinitely.

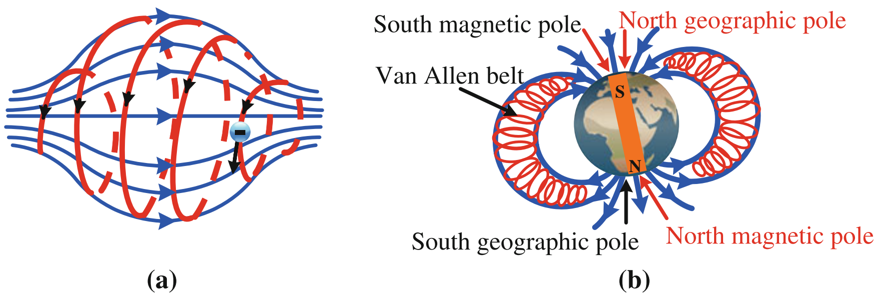


Fig. 25.20 (a) Trapping of charged particles in a non-uniform magnetic bottle. (b) A sketch of the Van Allen belt, which consists of charged particles trapped by Earth's non-uniform magnetic field

The Earth behaves like a gigantic magnet. Its north magnetic pole is actually near the geographic south pole, and its south magnetic pole is near the geographic north pole, see Fig. 25.20b. This non-uniform magnetic field traps charged particles (mostly electrons and protons) in a region of space known as *Van Allen belt*. In this belt, charged particles spiral around the field lines from pole to pole in a period of few seconds. The sun and stars are the sources of these particles (called *cosmic rays*). Most cosmic rays are deflected by the Earth's magnetic field and never reach the atmosphere. When some particles of the Van Allen belt are close to the poles, they collide with the atoms of the atmosphere causing them to emit light (Aurora Borealis or Aurora Australis).

25.7 Exercises

Section 25.1 Magnetic Force on a Moving Charge

(1) For each of the moving charges shown in Fig. 25.21, find the direction of the magnetic force, taking $\vec{v}$ to be the velocity of the particle and $\vec{B}$ to be the magnetic field.

(2) Consider a uniform magnetic field directed vertically up along the page of this paper. In which direction does an electron deflect if its velocity is directed: (a) into the paper, (b) up along the paper, (c) to the left, and (d) out of the paper.

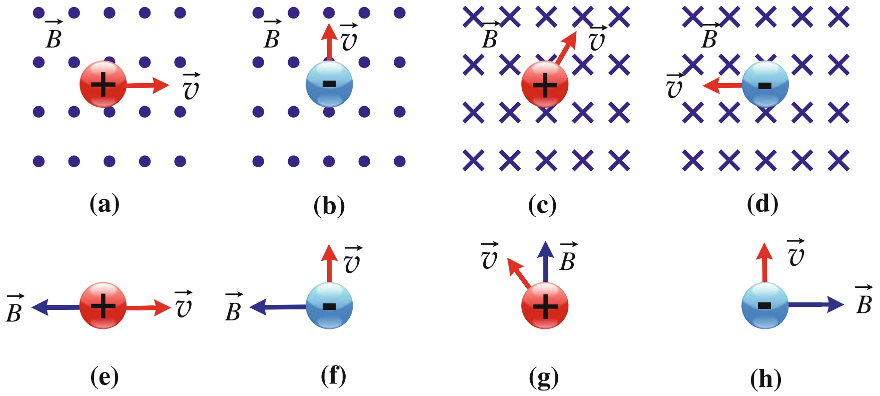


Fig. 25.21 See Exercise (1)

(3) When moving with a speed of 10^7 m/s in a magnetic field of magnitude 1.5 T, an electron experiences a magnetic force of magnitude 10^{-12} N. What is the angle between the electron's velocity and the field at this instant?

(4) A proton that has a velocity $\vec{v} = (3 \times 10^6\ \vec{i} + 4 \times 10^6\ \vec{j}\,)$ (m/s) moves through a magnetic field $\vec{B} = (0.3\ \vec{i} + 0.02\ \text{T}\ \vec{j}\,)$ (T). Find the vector magnetic force exerted by the field on the proton, and then find the magnitude and direction of this force.

(5) Near the Earth's surface at the equator, the magnetic and electric fields are about 50 μT due North and 100 N/C downwards, respectively. Find the net force on an electron traveling with velocity 10^7 m/s due East.

Section 25.2 Motion of a Charged Particle in a Uniform Magnetic Field

(6) In a uniform magnetic field of magnitude of 10^{-4} T, an ion that has a charge $q = +2e$ completes two revolutions in 1.51 ms. Find the mass and the type of the ion.

(7) A proton travels with a speed of 8×10^7 m/s perpendicular to a uniform magnetic field of magnitude 5 T. (a) What is the radius of the proton's circular path? (b) What is the period of the motion? (c) Find the magnitude of the magnetic force on the proton.

(8) An alpha particle has a charge $q = 2\,e$ and mass $m \simeq 4\,m_p$, where m_p is the mass of a proton. The alpha particle has a kinetic energy of 5 MeV and enters a uniform magnetic field of 1.5 T directed perpendicular to its velocity. (a) Find the speed

of the alpha particle. (b) Find the magnetic force acting on the particle due to the field. (c) Find the radius of the particle's path. (d) Find the acceleration of the particle due the magnetic force.

(9) An electron of speed 5×10^6 m/s enters a uniform magnetic field of magnitude 0.01 T at an angle of 36.87°. (a) Determine the radius of the electron's helical path. (b) Determine the period of one helical path. (c) Determine the pitch of the electron's helical path.

(10) Figure 25.22 shows a region of uniform magnetic field $\vec{B}$ of magnitude 0.5 T which extends for a width $W = 0.4$ m. Consider a proton moving with a velocity $\vec{v}$ of magnitude 3×10^7 m/s, where $\vec{v}$ is perpendicular to $\vec{B}$. If the incident angle $\theta_\circ$ at the lower boundary is 60°, the proton emerges from the lower boundary as shown in the left part of the figure. However, if the incident angle $\theta_\circ$ at the lower boundary is 0°, the proton emerges from the upper boundary as shown in the right part of the figure. (a) At what angle θ and distance d does the proton exit from the lower boundary? (b) At what angle θ and distance d does the proton exit from the upper boundary? (c) At what critical incident angle $\theta_\circ$ does the proton barely touch the upper boundary?

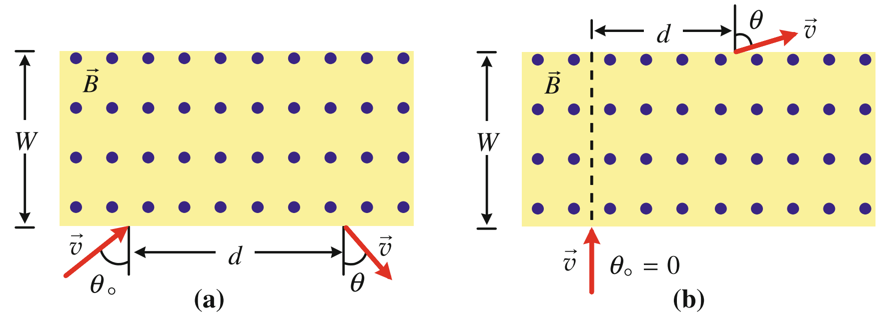


Fig. 25.22 See Exercise (10)

Section 25.3 Charged Particles in Electric and Magnetic Fields

(11) A uniform magnetic field of magnitude 0.02 T is perpendicular to a uniform electric field of magnitude 750 V/m. What is the speed of an electron that goes undeflected when moving perpendicular to both fields?

(12) Assume that a 1 keV electron travels in a uniform electric field $\vec{E} = 385\,\vec{j}$ (kV/m) and a uniform magnetic field $\vec{B} = B_z\,\vec{k}$. Find the value of B_z such that

the electron would have a velocity $\vec{v} = v_x\ \vec{i}$ and would move undeflected in the presence of the two fields.

(13) Figure 25.23 shows the path of an electron in a region of uniform magnetic field. Each of the plates is uniformly charged. (a) Which plate is at the higher electric potential for each pair? (b) What is the direction of the magnetic field in this region? (c) For both pairs of plates, if the magnitude of the electric field between the plates is 6×10^4 V/m and the magnitude of the magnetic field is 2 mT, find the radius of the two semicircles.

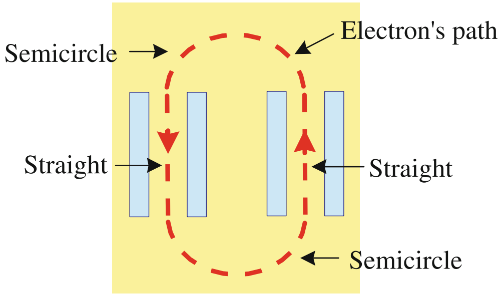


Fig. 25.23 See Exercise (13)

(14) In the mass spectrometer shown schematically in Fig. 25.6, the magnitude of the electric and magnetic fields in the velocity-selector region are 3 kV/m and 40 mT, respectively. The magnitude of the magnetic field in the deflecting chamber is 75 mT. (a) What is the speed of ions in the velocity selector? (b) What is the radius of the path in the deflecting chamber for a singly-charged ion having a mass of 6.49×10^{-26} kg?

(15) Two single ions of the boron isotopes (of masses 10 u and 11 u) are studied in the mass spectrometer shown schematically in Fig. 25.6. Assume that the values $B = B' = 250$ mT and $E = 60$ kV/m are used in this experiment. (a) What is the speed of the ions in the velocity selector? (b) What is the spacing between the marks produced on the photographic plate by the ions of boron?

(16) A strip of copper of thickness $t = 0.4$ mm and width $d = 5$ mm is placed in a uniform magnetic field $\vec{B}$ of magnitude 1.5 T perpendicular to the strip, see Fig. 25.24. When a current $I = 20$ A passes though the strip, a Hall potential difference ΔV_H is generated across the width of the strip. The number of charge carriers per unit volume for copper is 8.47×10^{28} electrons/m^3. (a) Find the Hall coefficient R_H for the copper strip. (b) How large is the Hall voltage ΔV_H across the strip? (c) Find the magnitude of the Hall electric field E_H.

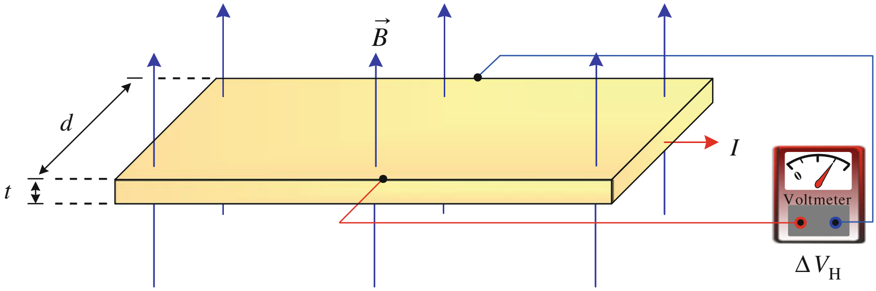
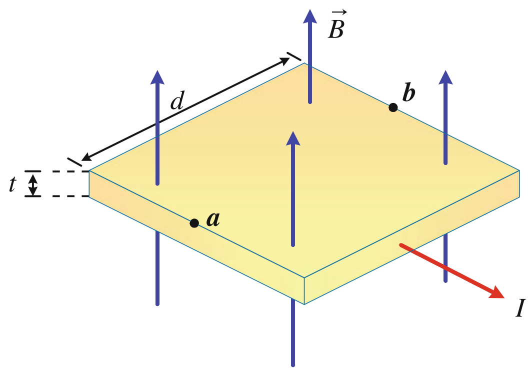


Fig. 25.24 See Exercise (16)

(17) A silver slab of thickness $t = 1.5$ mm and width $d = 2.5$ mm carries a current $I = 4$ A in a region in which there is a uniform magnetic field $\vec{B}$ of magnitude 1.25 T perpendicular to the slab. The Hall voltage ΔV_H across the slab is found to be 0.356 μV. (a) Calculate the density of the charge carriers in the slab. (b) Compare your answer in part (a) to the density of atoms in the silver slab, which has a density $\rho = 10.5 \times 10^3$ kg/m^3 and a molar mass $M = 107.9$ kg/kmol. What is the conclusion that you can find from this comparison? (c) Find the magnitude of the Hall electric field E_H.

(18) A metal strip of thickness $t = 1$ mm and width $d = 2$ cm carries a current $I = 12.5$ A in a region in which there is a uniform magnetic field $\vec{B}$ of magnitude 1.6 T perpendicular to the strip, as shown in Fig. 25.25. The Hall voltage ΔV_H across the strip is measured to be 2.135 μV. (a) Calculate the drift speed of the electrons in the strip. (b) Find the density of the charge carriers in the strip. (c) Which point is at the higher potential, a or b?

Fig. 25.25 See Exercise (18)

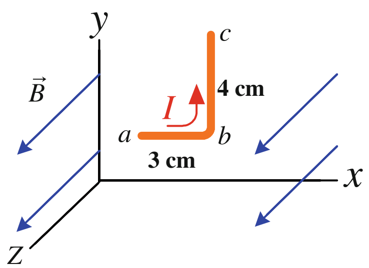


Section 25.4 Magnetic Force on a Current-Carrying Conductor

(19) A 1.5 m long straight stiff wire carries a current of 2 A and makes an angle 30° with a uniform magnetic field of 0.35 T. Find the magnitude of the force on the wire.

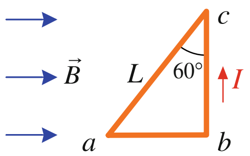
(20) The L-shaped wire shown in Fig. 25.26 lies in the *xy* plane. In the presence of a uniform magnetic field $\vec{B} = 1.5\ \vec{k}$ (T), the wire carries a current of 2.5 A from point a to point c. (a) Find the net force exerted on the wire. (b) Show that this net force is the same as if the wire were a straight segment from point a to point c.

Fig. 25.26 See Exercise (20)



(21) For the circuit shown in Fig. 25.27, find the magnitude and direction of the force on each side, and find the resultant force.

Fig. 25.27 See Exercise (21)



(22) A straight horizontal wire has a length $L = 20$ cm and mass $m = 0.02$ kg. The wire is hung by connecting it by massless flexible leads to an emf source. A uniform magnetic field of magnitude $B = 1.6$ T is perpendicular to the wire, as shown in Fig. 25.28. Find the necessary current needed to suspend the wire and hence remove the tension in the flexible wire.

(23) If $B = 0.2$ T and $I = 5$ A in Fig. 25.29, find the force exerted on each segment of the wire.

Fig. 25.28 See Exercise (22)

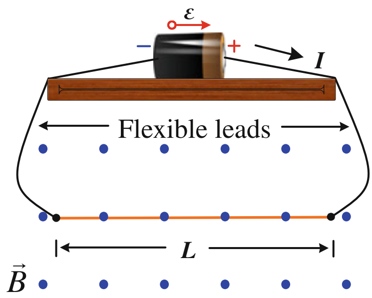
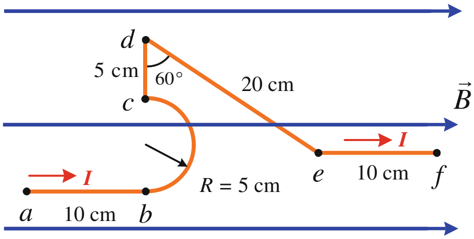
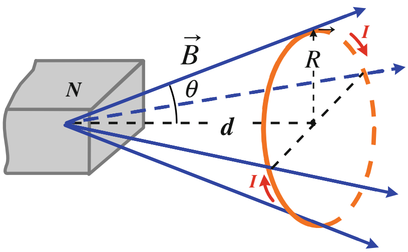


Fig. 25.29 See Exercise (23)



(24) A circular loop of wire has a radius R and carries a current I. The loop is placed in a magnetic field whose lines seem to diverge from a point on the perpendicular axis of the circular loop and at a distance d from its center, see Fig. 25.30. Find the total force on the loop.

Fig. 25.30 See Exercise (24)



Section 25.5 Torque on a Current Loop

(25) A circular coil of $N = 40$ turns has a radius $r = 5$ cm and carries a current $I = 2$ A. The coil is placed in a uniform magnetic field of 0.5 T so that the normal to the coil makes an angle $\theta = 30°$ with the direction of $\vec{B}$. (a) What is the magnitude of the magnetic moment of the coil? (b) What is the magnitude of the torque exerted on the coil?

(26) For the current loop shown in the figure of exercise 21, find: (a) the magnitude and direction of the loop's magnetic moment. (b) the magnitude of the torque on the loop and the direction in which it will rotate.

(27) What is the maximum torque exerted on a 400-turn circular coil of radius 0.5 cm placed in a uniform magnetic field of magnitude 0.2 T if it carries a current of 1.5 A?

(28) A small, stiff, circular loop of radius R and mass m carries a current I. The loop lies horizontally on a rough flat table in the presence of a horizontal magnetic field of magnitude B. (a) What is the required minimum value of B so that one edge of the loop will lift off the table? (b) What is the required value of B so that one edge of the loop will lift off the table through an angle θ?

(29) The 240-turn rectangular coil shown in Fig. 25.31 carries a current of 1.5 A in a uniform magnetic field of $B = 0.25\,\text{T}$. Find the magnitude of the torque on the loop and the direction in which it will rotate.

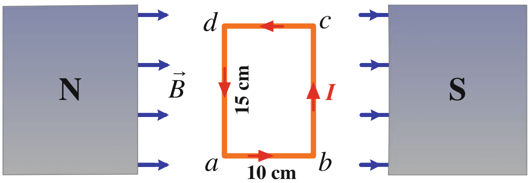


Fig. 25.31 See Exercise (29)

(30) A rectangular 100-turn coil carries a current $I = 1.75$ A and has sides $a = 40$ cm and $b = 30$ cm. The coil is hinged along the y-axis, so that its plane makes an angle $\theta = 73°$ with the x-axis as shown in Fig. 25.32. (a) What is the magnitude of the magnetic moment $\vec{\mu}$ of the coil? (b) What angle does the vector $\vec{\mu}$ make with the x-axis. (c) In the presence of a uniform magnetic field $\vec{B} = 0.8\,\vec{i}$ (T), what is the magnitude of the torque exerted on the coil and what is the expected direction of the coil's rotation?

(31) A current $I = 0.75$ A flows in a quarter of a single circular loop of wire that has a radius $R = 5$ cm. The loop lies in the xy plane and is hinged along the y-axis, so that it can rotate about this axis, see Fig. 25.33. (a) What is the magnitude of the magnetic moment $\vec{\mu}$ of the coil? (b) Express the vector $\vec{\mu}$ in terms of unit vectors. (c) When a uniform magnetic field $\vec{B} = [0.2\,\vec{i} + 0.3\,\vec{j} + 0.4\,\vec{k}]$ (T) is applied to the loop, express the torque acting on the coil in terms of unit vectors? In which direction will the loop rotate?

Fig. 25.32 See Exercise (30)

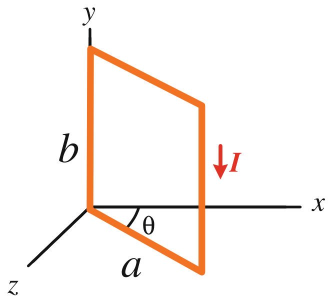
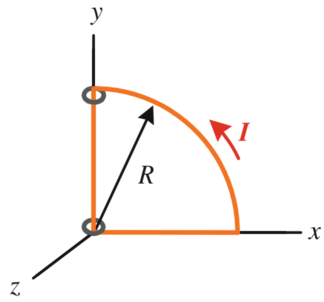
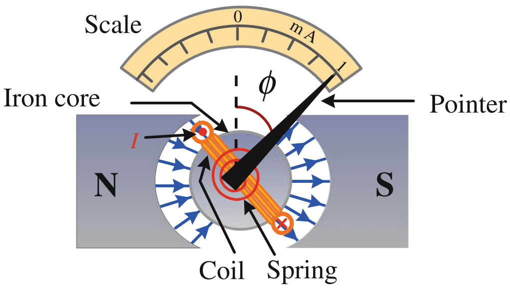


Fig. 25.33 See Exercise (31)



(32) The coil of the galvanometer shown in Fig. 25.34 has $N = 35$ turns where the dimensions of each rectangular turn are 2 cm by 2.5 cm. For any position of the coil, its plane is parallel to the magnetic field which has the value $B = 0.4$ T. The galvanometer has a spring with a stiffness constant $k = 5 \times 10^{-6}$ N.m/rad and gives a full-scale deflection if the current I going through it is 1 mA. What is the full-scale deflection angle ϕ in radians and degrees?

Fig. 25.34 See Exercise (32)



(33) Assume that the Earth's magnetic field at the equator is uniform and northerly directed at all points with a magnitude 5×10^{-5} T and that it extends out by

Earth's diameter (i.e. by 1.28×10^4 km). (a) Find the speed and time that a singly-ionized uranium atom ($m = 238$ u, $q = +e$) would take to circulate the Earth 20 km above the surface at the equator. (b) A cosmic-ray proton traveling with a speed of 2.5×10^7 m/s is heading directly towards the center of the Earth in the plane of the Earth's equator. Estimate the radius of the proton's path. Will the proton hit the Earth?