Mechanical Properties of Matter

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The physical states of matter can generally be divided into three broad classes: solids, liquids, and gases, see Fig. 10.1. A solid maintains its shape: it resists the action of external forces that tend to change its shape or volume. Liquids and gases are fluids. A fluid can easily change shape, and flows when subjected to a force. The three states of matter are distinguishable at the microscopic level as follows:
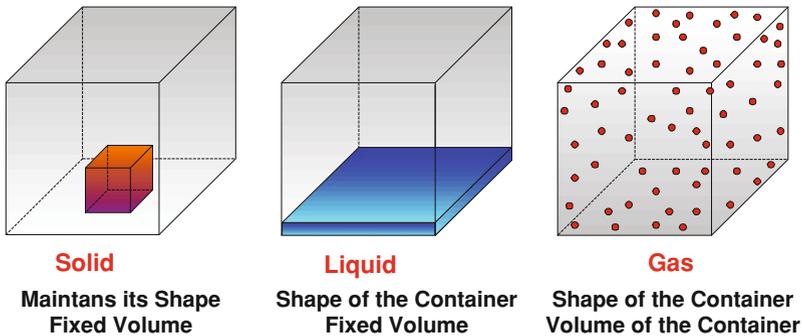

Solid	**Liquid**	**Gas**
Maintans its Shape	Shape of the Container	Shape of the Container
Fixed Volume	Fixed Volume	Volume of the Container

Fig. 10.1 The three states of matter: solid, liquid, and gas

1. A solid is a highly ordered array of atoms or molecules that are bound securely by mutual electrical forces.
2. A liquid is a crowded assembly of mobile atoms or molecules. Each atom or molecule is in contact with several neighbors, but is not bound securely to any of them. As an atom or molecule moves about in a liquid, it collides frequently with its neighbors.

3. A gas consists of atoms or molecules that are far apart and consequently move independently, with no forces keeping them together or pushing them apart. Collisions of atoms or molecules in gases are infrequent in comparison to those in liquids.

10.1   Density and Relative Density

The density (or mass density) of a material is defined as the mass per unit volume. If a mass m is distributed uniformly over a volume V, the density will be given by the following equation:

$$\rho = \frac{m}{V} \tag{10.1}$$

The SI unit of density is kg/m^3. If the mass is not uniformly distributed, then Eq. 10.1 defines the average density. The densities of several materials are listed in Table 10.1.

Table 10.1 Density and relative density comparison (approximates)

Material type	Material name	Density (kg/m^3)	Relative density
Gas	Helium	0.179	1.79×10^{-4}
	Air	1.29	1.29×10^{-3}
	Carbon dioxide	1.98	1.98×10^{-3}
Liquid	Alcohol	7.9×10^2	0.79
	Gasoline	8.6×10^2	0.86
	Water	1×10^3	1
	Mercury	13.6×10^3	13.6
Solid	Glass (common)	$2.4 - 2.8 \times 10^3$	2.5
	Aluminum	2.7×10^3	2.7
	Iron	7.86×10^3	7.86
	Copper	8.92×10^3	8.92
	Silver	10.5×10^3	10.5
	Lead	11.36×10^3	11.36
	Uranium	19.07×10^3	19.07
	Gold	19.3×10^3	19.3

The relative density of a substance tells us how many times more dense the substance is than pure water, see Table 10.1. Sometimes we refer to it as the specific gravity (SG). Thus:

$$SG = \frac{\rho}{\rho_{\text{water}}} \tag{10.2}$$

Example 10.1

Calculate the average density of both the Earth and the Sun, given that the mass and radius of the Earth are $m_E = 5.98 \times 10^{24}$ kg and $R_E = 6.37 \times 10^6$ m, respectively, and the mass and radius of the Sun are $m_S = 1.99 \times 10^{30}$ kg and $R_S = 6.95 \times 10^8$ m, respectively. Compare between the resulting average densities.

Solution: Matter in the Earth and the Sun is not uniform. In spite of this fact, we can use Eq. 10.1 to calculate their average density. Using the given radius, we calculate the volume of the Earth to be:

$$V_E = \tfrac{4}{3}\pi R_E^3 = 1.08 \times 10^{21} \text{ m}^3$$

Thus, the average density of the Earth is:

$$\rho_E = \frac{m_E}{V_E} = \frac{5.98 \times 10^{24} \text{ kg}}{1.08 \times 10^{21} \text{ m}^3} = 5.54 \times 10^3 \text{ kg/m}^3$$

In comparison to water, the average density of the Earth is 5.54 times more dense than water.

Similarly, we use the given radius of the Sun to calculate its volume:

$$V_S = \tfrac{4}{3}\pi R_S^3 = 1.41 \times 10^{27} \text{ m}^3$$

Thus, the average density of the Sun is:

$$\rho_S = \frac{m_S}{V_S} = \frac{1.99 \times 10^{30} \text{ kg}}{1.41 \times 10^{27} \text{ m}^3} = 1.41 \times 10^3 \text{ kg/m}^3$$

In comparison to water, the average density of the Sun is 1.41 times more dense than water. Although the mass and volume of the Earth are much smaller than the mass and volume of the Sun, the average density of the Earth is nearly four times the average density of the Sun. That is:

$$\rho_E \simeq 4\rho_S$$

10.2   Elastic Properties of Solids

All solids are to some extent elastic. This means that we can change their dimensions slightly by pulling, pushing, twisting, and/or compressing them. We shall discuss the elastic properties of solids by introducing the concepts of *stress* and *strain*.

> Stress:
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> Stress is the magnitude of the applied external force that acts perpendicularly on a unit area of the object.

> Strain:
>
> Strain is a measure of the degree of deformation of the object.

It is found that for small stresses, stress is proportional to strain. The *proportionality constant* is called the *elastic modulus* and it depends on the material being deformed, as well as on the nature of the deformation. Therefore:

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{Strain}} \qquad (10.3)$$

This relation is equivalent to **Hooke's law** that states: Stress $\propto$ Strain. In this chapter, we introduce the three *most famous types of deformations* and their elastic moduli:

> Young's Modulus:
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> Measures the resistance of a solid to a change in its length.

> Shear Modulus:
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> Measures the resistance to motion of the planes of solids when sliding over each other.

> Bulk Modulus:
>
> Measures the resistance of a solid (or a liquid) to a change in its volume.

10.2.1 Young's Modulus: Elasticity in Length

Young's Modulus measures the resistance of a solid to a change in its length, which indicates its stiffness. Consider a metallic long rod of original length L and cross-sectional area A. When an external force $F_\perp$ is applied perpendicularly to the cross-sectional area A of a rod, its internal forces resist its distortion. As a final result, the rod attains equilibrium when its length increases to a new length $L + \Delta L$ and the magnitude of the perpendicular external force $F_\perp$ exactly balances the internal forces, see Fig. 10.2a. In light of this, we define the *tensile stress* and the *tensile strain* as follows:

$$\text{Tensile stress} = \frac{F_\perp}{A} \quad (\text{N/m}^2) \qquad (10.4)$$

$$\text{Tensile strain} = \frac{\Delta L}{L} \qquad (10.5)$$

The relation between the tensile stress and the tensile strain is linear when the rod is in its elastic range. When the stress exceeds what is called the *elastic limit*, the rod is permanently distorted and will not return to its original shape after the stress is removed. As the stress is increased even further, the rod will ultimately break, see Fig. 10.2b.
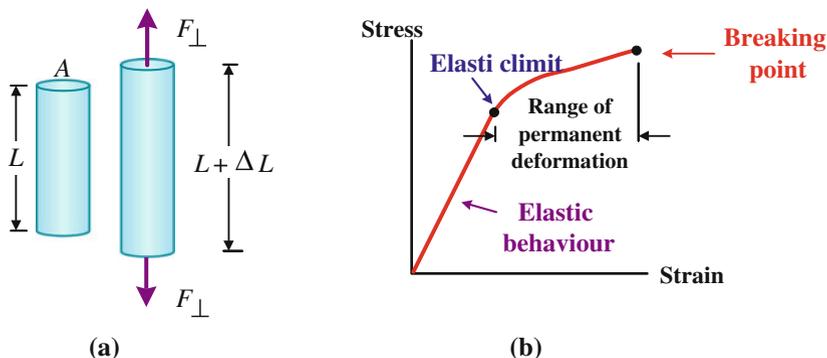

Fig. 10.2 (a) A rod of height L and cross-sectional area A. The rod stretches by an amount ΔL after application of a tensile stress. (b) The stress versus strain curve for an elastic solid

We use Eqs. 10.4 and 10.5 to define *Young's modulus*, Y, as:

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp/A}{\Delta L/L} \quad (\text{N/m}^2) \tag{10.6}$$

This quantity is used to characterize a solid that is stressed under either tension or compression. Table 10.2 depicts some values for Y.

Table 10.2 Young's modulus for different materials (approximates)

Material name	Young's modulus $Y \times (10^9 \text{ N/m}^2)$
Rubber	0.004
Lead	16
Glass	65 – 78
Aluminum	70
Brass	91
Copper	110
Steel	200
Tungsten	350

Example 10.2

A pendulum consists of a big sphere of mass $m = 30$ kg hung from the end of a steel wire that has a length $L = 15$ m, a cross-sectional area $A = 9 \times 10^{-6}$ m^2, and Young's modulus $Y = 200 \times 10^9$ N/m^2. Find the tensile stress on the wire and the increase in its length.

Solution: The applied force on the wire must equal to the weight of the sphere, i.e. $F_\perp = m\,g$. Thus, the tensile stress will be:

$$\text{Tensile stress} = \frac{F_\perp}{A} = \frac{m\,g}{A} = \frac{(30 \text{ kg}) \times (9.8 \text{ m/s}^2)}{9 \times 10^{-6} \text{ m}^2} = 3.27 \times 10^7 \text{ N/m}^2$$

Using the value of the Young's modulus $Y = 20 \times 10^{10}$ N/m^2 and the length of the steel wire before stretching $L = 15$ m, we get:

$$Y = \frac{F_\perp/A}{\Delta L/L} \quad \Rightarrow \quad \Delta L = \frac{F_\perp/A}{Y}L = \frac{3.27 \times 10^7 \text{ N/m}^2}{200 \times 10^9 \text{ N/m}^2} \times 15 \text{ m} = 2.45 \times 10^{-3} \text{ m}$$

Note that this large stress produces a relatively small change in L.

When we carefully study the deformation of the rod, we find that the rod's length L increases by ΔL in the direction of the force while it radius r decreases by $|\Delta r|$, where Δr is negative, in a direction perpendicular to the force, see Fig. 10.3. The tensile strain $\Delta L/L$ of the rod is called the linear strain. The strain $-\Delta r/r$ is called the lateral strain, and Poisson's ratio μ is defined as:

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = -\frac{\Delta r/r}{\Delta L/L} = -\frac{L}{r}\frac{\Delta r}{\Delta L} \quad \Rightarrow \quad \mu = -\frac{L}{r}\frac{dr}{dL} \qquad (10.7)$$

The minus sign is inserted in this definition to make μ positive.
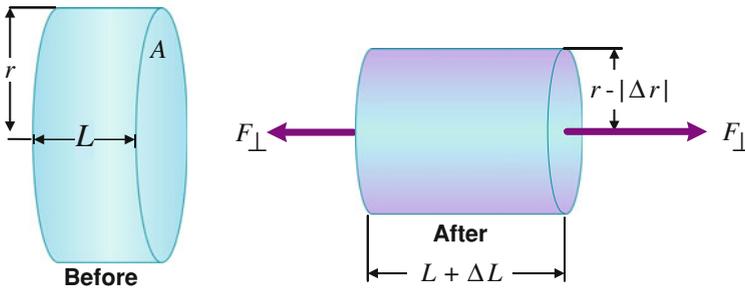
Fig. 10.3   The length of the rod will increase by ΔL and its radius will decrease by Δr (exaggerated scale) after applying a tensile stress $F_\perp/A$

Example 10.3

A cylindrical steel rod has a length of 2 m and a radius of 0.5 cm. A force of magnitude 2×10^4 N is acting normally on each of its ends. Find the change in its length and radius, if the Young's modulus Y is 200×10^9 N/m^2 and the Poisson's ratio μ is 0.25.

Solution: Using $F_\perp = 2 \times 10^4$ N, $A = \pi r^2 = \pi \times (0.5 \times 10^{-2} \text{ m})^2 = 7.9 \times 10^{-5}$ m^2, $L = 2$ m, and $Y = 200 \times 10^9$ N/m^2 in Eq. 10.6, we have:

$$\Delta L = \frac{F_\perp L}{YA} = \frac{(2 \times 10^4 \text{ N})(2 \text{ m})}{(200 \times 10^9 \text{ N/m}^2)(7.9 \times 10^{-5})} = 2.5 \times 10^{-3} \text{ m} = 0.25 \text{ cm}$$

From the definition of the Poisson's ratio μ, we have:

$$\Delta r = -\frac{\mu r \Delta L}{L} = -\frac{0.25 \times (0.5 \times 10^{-2} \text{ m})(2.5 \times 10^{-3} \text{ m})}{(2 \text{ m})} = -1.56 \times 10^{-6} \text{ m}$$

Note that $\Delta L \approx 1{,}600 |\Delta r|$, i.e. $|\Delta r|$ is extremely small compared to ΔL.

10.2.2 Shear Modulus: Elasticity of Shape

Another type of deformation occurs when a solid is subject to a force applied parallel to one of its surfaces while the opposite surface is kept fixed. Figure 10.4 shows a cylindrical rod subjected to a linear or torsional shear stress deforming it by an amount Δx due to a force $F_\parallel$ parallel to the surface area A. As a final result, the shape of the rod will attain equilibrium when the effect of the shear force $F_\parallel$ balances exactly the internal shear forces. For linear shearing, we define the *shearing stress* and the *shearing strain* as follows:

$$\text{Shearing stress} = \frac{\text{Tangential acting force}}{\text{Area of surface being sheared}} = \frac{F_\parallel}{A} \quad (\text{N/m}^2) \qquad (10.8)$$

$$\text{Shearing strain} = \frac{\text{Distance sheared}}{\text{Distance between surfaces}} = \frac{\Delta x}{h} = \tan\theta \simeq \theta \qquad (10.9)$$
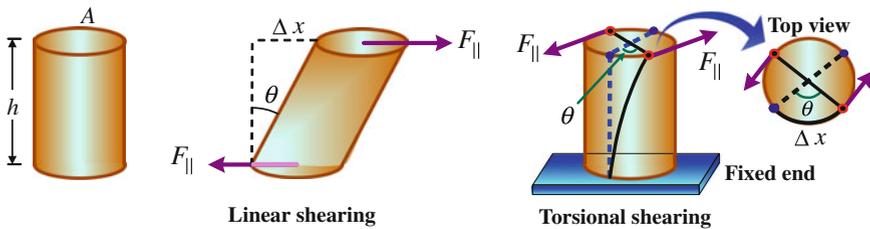
Fig. 10.4 The *left* part shows a cylindrical rod of height h. The *middle* part shows a linear shear where the rod is subject to a shearing force $F_\parallel$ parallel to each of its surface areas. The rod is deformed through an angle θ which is defined as the shearing strain. The *right* part shows a torsional shear when one end of the rod is kept fixed

The approximation $\tan\theta \simeq \theta$ is valid for small strains. We use Eqs. 10.8 and 10.9 to define the *shear modulus*, S, as follows:

$$S = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F_\parallel/A}{\Delta x/h} \quad (\text{N/m}^2) \qquad (10.10)$$

S is also called the modulus of rigidity or the torsion modulus and is significant only for solids. Table 10.3 depicts some values for S.

Table 10.3 Shear modulus for different materials (approximates)

Material name	Shear modulus $S \times (10^9 \text{ N/m}^2)$
Rubber	0.001
Lead	6
Glass	23
Aluminum	23
Brass	36
Copper	42
Steel	80
Tungsten	120

Example 10.4

Assume that the rod in Fig. 10.4 has a cross-sectional area $A = 2 \times 10^{-3} \text{ m}^2$, length $h = 1 \text{ m}$, and is made of brass with a shear modulus $S = 36 \times 10^9 \text{ N/m}^2$. How large should the shear force $F_{\parallel}$ exerted on each edge of the rod be if the displacement Δx is 0.02 cm?

Solution: The shearing stress on each edge is:

$$\text{Shearing stress} = \frac{F_{\parallel}}{A} = \frac{F_{\parallel}}{2 \times 10^{-3} \text{ m}^2} = 500 \, F_{\parallel} \text{ m}^{-2}$$

The shearing strain is:

$$\text{Shearing strain} = \frac{\Delta x}{h} = \frac{2 \times 10^{-4} \text{ m}}{1 \text{ m}} = 2 \times 10^{-4}$$

From the definition of the shearing modulus Eq. 10.10 and the last two results, we have:

$$S = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{500 F_{\parallel} \text{ m}^{-2}}{2 \times 10^{-4}}$$

Using the given shear modulus value for brass, we get:

$$36 \times 10^9 \text{ N/m}^2 = \frac{500 F_{\parallel} \text{ m}^{-2}}{2 \times 10^{-4}}$$

Thus: $F_{\parallel} = (36 \times 10^9 \text{ N/m}^2)(2 \times 10^{-4})/(500 \text{ m}^{-2}) = 14,400 \text{ N} = 1.44 \times 10^4 \text{ N}$

10.2.3 Bulk Modulus: Volume Elasticity

Another type of deformation occurs when an object is subject to an equal increase in normal forces acting on all its faces. For such a study, it is appropriate to define the pressure P as the force acting perpendicularly on a unit area of the object. That is:

$$P = \frac{F_\perp}{A} \quad (\text{N/m}^2) \tag{10.11}$$

Hence, we can study the deformation of an object subject to an equal increase in pressure on all its faces. Figure 10.5 shows a cube of original volume V and face area A under uniform pressure P. When the force $F_\perp$ on each face increases to $F_\perp + \Delta F_\perp$, the pressure will increase to $P + \Delta P$ and consequently the volume V will decrease to $V' = V - |\Delta V|$, where ΔV is negative, see Fig. 10.5.
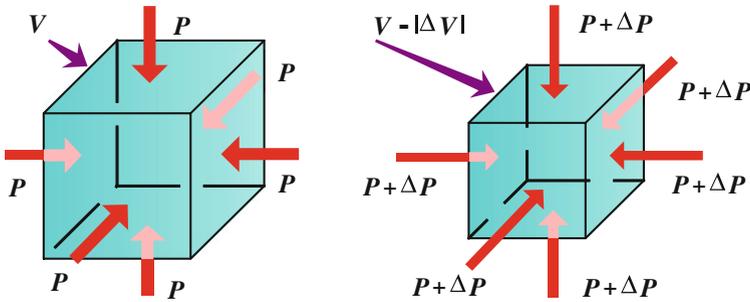
Fig. 10.5 When the uniform pressure P on each face of a cube of volume V increases to $P + \Delta P$, its volume will decrease to $V - |\Delta V|$.

In light of this, we define the *volume stress* and the *volume strain* as:

$$\text{Volume stress} = \Delta P = \frac{\Delta F_\perp}{A} \quad (\text{N/m}^2) \tag{10.12}$$

$$\text{Volume strain} = -\frac{\Delta V}{V} \tag{10.13}$$

We use Eqs. 10.12 and 10.13 to define the *bulk modulus*, B, as:

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = -\frac{\Delta F_\perp / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V} \Rightarrow B = -V \frac{dP}{dV} \tag{10.14}$$

The minus sign is inserted in Eq. 10.13 to make B a positive number, because an increase/decrease of pressure always causes a decrease/increase in volume.

Example 10.5

A sphere of lead has a volume $V = 0.5\,\text{m}^3$ when placed in atmospheric pressure ($P_a \simeq 10^5\,\text{N/m}^2$). The sphere is lowered to a particular depth in the ocean where the water pressure is $P = 10^8\,\text{N/m}^2 = 1{,}000\,P_a$. The bulk modulus B of lead is $8 \times 10^9\,\text{N/m}^2$. (a) What is the change in volume of the sphere? (b) What is the relative density change in lead?

Solution: (a) From the definition of the bulk modulus, we have:

$$B = -\frac{\Delta P}{\Delta V / V}$$

The change in pressure is:

$$\Delta P = P - P_a = 10^8\,\text{N/m}^2 - 10^5\,\text{N/m}^2 = 9.99 \times 10^7\,\text{N/m}^2$$

Using $V = 0.5\,\text{m}^3$, we can find the change in volume as follows:

$$\Delta V = -\frac{V\Delta P}{B} = -\frac{(0.5\,\text{m}^3)(9.99 \times 10^7\,\text{N/m}^2)}{8 \times 10^9\,\text{N/m}^2} = -6.2 \times 10^{-3}\,\text{m}^3$$

The negative sign indicates a decrease in volume.

(b) We can find the new volume V' of lead as follows:

$$V' = V - |\Delta V| = 0.5\,\text{m}^3 - |-6.2 \times 10^{-3}\,\text{m}^3| = 0.4938\,\text{m}^3 \quad \Rightarrow \quad V' = 0.9876\,V$$

If the original density of lead is denoted by $\rho = m/V$, then the new density ρ' will be:

$$\rho' = \frac{m}{V'} = \frac{m}{0.9876\,V} = 1.0126\,\frac{m}{V} = 1.0126\,\rho$$

Thus, a thousand times increase in pressure on the surfaces of a sphere of lead causes a *decrease* in its volume by about 1.3% and consequently an *increase* in density by the *same* percentage.

Table 10.4 depicts some values for the bulk modulus B.

Table 10.4 Bulk modulus for different materials (approximates)

Material name	Bulk modulus $B \times (10^9 \text{ N/m}^2)$
Rubber	3
Lead	8
Glass	37
Aluminum	70
Brass	61
Copper	140
Steel	160
Tungsten	200

10.3   Fluids

What is a Fluid?

Liquids and gases are fluids. The liquid state of any substance always exists at a higher temperature than its solid state. The reason for lumping liquids and gases together and calling them fluids is because neither liquids nor gases (such as liquid water and steam, for example) have a fairly rigid three-dimensional array of atoms/molecules as compared to solids (such as ice, for example).

In contrast to solids, fluids can flow and conform to the boundaries of any container in which they are placed. This is because a fluid cannot sustain a force that is tangent to its surface. In the language of the previous section, a fluid flows because it cannot withstand a shearing stress. On the other hand, a fluid can exert a force in a direction perpendicular to its surface.

Pressure in Fluids

Figure 10.6 shows a pressure device inside a fluid-filled vessel. The device consists of a light piston of area ΔA fitting in a vacuumed cylinder and resting on a light spring. As we insert the device into the fluid, the fluid will compress the piston due to the effect of a normal force of magnitude $\Delta F_\perp$. Using Eq. 10.11, after replacing $F_\perp$ by F, we define the average pressure exerted by the fluid on the piston by the following relation:

$$\overline{P} = \frac{\Delta F}{\Delta A} \quad (\text{N/m}^2) \tag{10.15}$$

Fig. 10.6 A pressure device inside a fluid-filled vessel. The pressure is measured by the relative position of a movable piston in the device
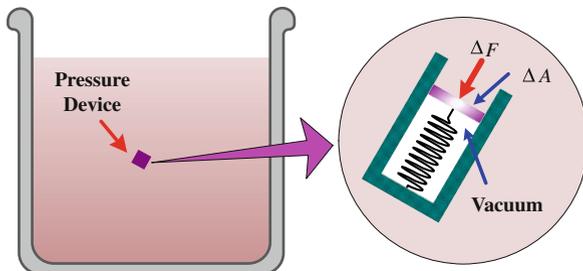
The pressure at any point in the fluid is the limit of the above ratio as ΔA of the piston, centered on that point, approaches zero. That is:

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \quad (\text{N/m}^2) \tag{10.16}$$

It was found by experiment that at a given point in a static fluid, the pressure P given by Eq. 10.16 has the same value no matter how the pressure device is oriented. Moreover, all points at the same depth from a liquid surface have the same value of pressure.

The SI unit of pressure is N/m^2, which is given the special name pascal (Pa). That is:

$$1\,\text{Pa} = 1\,\text{N/m}^2 \tag{10.17}$$

Atmospheric pressure at sea level P_a (abbreviated by atm) is:

$$1\,\text{atm} = 1.01 \times 10^5\,\text{Pa} \simeq 10^5\,\text{Pa} \tag{10.18}$$

The pascal is also related to the torr, the bar, and the pound per square inch, which are other common (non-SI) pressure units. The torr unit (named after Evangelista Torricelli who invented the mercury barometer) was formerly called *millimeter of mercury* (mm Hg). The bar unit is usually used in meteorological sciences. Finally, the pound per square inch (lb/in^2) is often abbreviated by psi. Note that:

$$1\,\text{atm} = \begin{cases} 760\,\text{torr} \\ \quad \text{or} \\ 760\,\text{mm Hg} \end{cases} = 14.7\,\text{psi} \quad \text{and} \quad 1\,\text{bar} = 10^5\,\text{Pa (Exactly)} \tag{10.19}$$

Table 10.5 depicts some approximate pressure values.

Table 10.5 Some approximate pressures

Locations	Pressure (Pa)	Pressure (atm)
Center of the Sun	2×10^{16}	2×10^{11}
Center of the Earth	4×10^{11}	4×10^{6}
Highest laboratory pressure	2×10^{10}	2×10^{5}
Deepest Ocean	1.1×10^{8}	1.1×10^{3}
Automobile tire (excess of 1 atm)	2×10^{5}	2
Atmosphere at sea level	1.0×10^{5}	1
Normal blood pressure (excess of 1 atm)[a]	0.16×10^{5}	0.16
Best laboratory vacuum	10^{-12}	10^{-17}

[a] The systolic pressure that corresponds to 120 mm Hg on the physician's pressure gauge.

To study the mechanics of fluids, we need to deal with:

1. Fluids at rest, or fluid statics (hydrostatics)
2. Fluids in motion, or fluid dynamics (hydrodynamics)

10.4 Fluid Statics

Variation of Pressure with Depth

As indicated in the previous section, all points at the same depth from a liquid surface have the same value of pressure. The variation of pressure P with depth h in a liquid of density ρ open to the atmosphere can be found by considering a small horizontal area dA at that depth, as shown in Fig. 10.7. The force dF that acts downwards on dA must be equal to the weight of the liquid column of height h plus the weight of the atmospheric air column. Accordingly, we have:

Volume of the liquid column $= h \, dA$
Mass of the liquid column $= h \, dA \rho$
Weight of the liquid column $= h \, dA \rho g$
Weight of the atmospheric air column $= P_a \, dA$
Total force dF on the horizontal area dA $= P_a \, dA + h \, dA \rho g$

Thus, from Eq. 10.16, the pressure $P = d F / d A$ at depth h gives:

$$P = P_a + \rho g h \quad \Rightarrow \quad dP = P - P_a = \rho g h \qquad (10.20)$$

Fig. 10.7 The pressure P at a depth h below the surface of a liquid open to the atmosphere is given by $P = P_a + \rho g h$
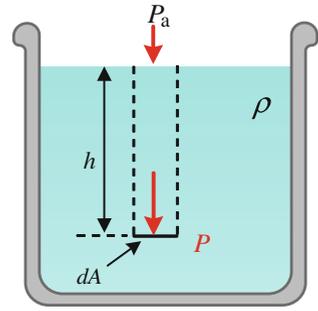
This relation verifies that the pressure is the same at all points having the same depth from a liquid surface. Moreover, the pressure is not affected by the shape of the container, see Fig. 10.8.

Fig. 10.8 The pressure in the liquid is the same at all points having the same depth. The shape of the vessel does not affect the pressure

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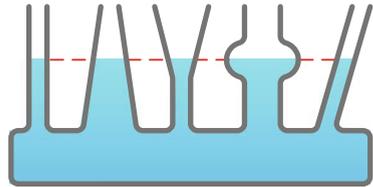
Example 10.6

Find the pressure at depths of 10 m and 10 km in ocean water. Assume $P_a \equiv 1\,\text{atm} \simeq 10^5\,\text{Pa}$, $\rho \simeq 10^3\,\text{kg/m}^3$, and $g \simeq 10\,\text{m/s}^2$.

Solution: The pressure at a depth $h = 10\,\text{m}$ will be:

$$P = P_a + \rho g h = 10^5\,\text{Pa} + (10^3\,\text{kg/m}^3)(10\,\text{m/s}^2)(10\,\text{m})$$
$$= 2 \times 10^5\,\text{Pa} = 2\,\text{atm}$$

The pressure at a depth $h = 10\,\text{km} = 10^4\,\text{m}$ will be:

$$P = P_a + \rho g h = 10^5\,\text{Pa} + (10^3\,\text{kg/m}^3)(10\,\text{m/s}^2)(10^4\,\text{m})$$
$$= 1{,}001 \times 10^5\,\text{Pa} \simeq 1{,}000\,\text{atm}$$

The fact that the pressure in a fluid depends only on depth indicates that any increase in pressure at the liquid surface must be transmitted to every point in the liquid. This fact is known as **Pascal's law** or **Pascal's principle**.

An important application of Pascal's law is the *hydraulic lever* illustrated in Fig. 10.9. Let an external input force of magnitude F_1 be exerted downwards on a small piston of area A_1. The pressure will be transmitted through an incompressible fluid which then exerts an output force F_2 on a larger piston of area A_2, balancing the load.

Fig. 10.9 A hydraulic device used to magnify a force. However, for small strokes (small d_1), the input and output work done is the same
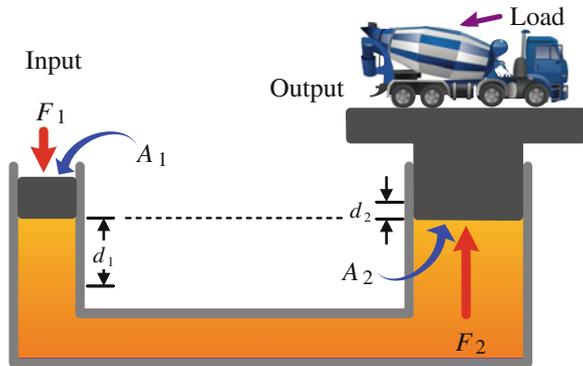
The pressure on both leveled pistons is the same. That is:

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \Rightarrow \quad F_2 = F_1 \frac{A_2}{A_1} \quad \text{(Leveled pistons)} \qquad (10.21)$$

Thus, the force F_2 is larger than F_1 by the multiplying factor A_2/A_1. Hydraulic brakes, car lifts, etc make use of this principle.

When we move the input piston downwards a distance d_1, the output piston moves upwards a distance d_2, such that the same volume V of the incompressible liquid is displaced at both pistons. Then, we get:

$$V = A_1 d_1 = A_2 d_2 \quad \Rightarrow \quad d_2 = d_1 \frac{A_1}{A_2} \qquad (10.22)$$

Thus, for $A_2 > A_1$, the output piston moves a smaller distance than the input piston. On the other hand, for *small values of d_1*, we can use Eq. 6.1 to find the following input/output relationship:

$$W_2 = F_2 d_2 = \left(F_1 \frac{A_2}{A_1} \right) \left(d_1 \frac{A_1}{A_2} \right) = F_1 d_1 = W_1 \quad \text{(for small } d_1 \text{ only)} \qquad (10.23)$$

which shows that the work W_1 done on the input piston by the applied force equals the work W_2 done by the output piston in lifting the load.

Measuring Pressures

The Mercury Barometer

Figure 10.10 shows a very basic **mercury barometer** used to measure *atmospheric pressure*. Here, a long glass tube is first filled with mercury and then inverted with its open end in a container filled with mercury.

Fig. 10.10   A closed-end
mercury barometer
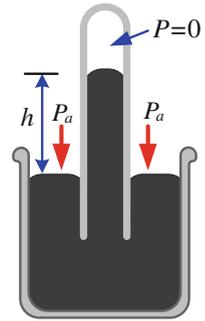
The closed end of the tube is nearly in a state of vacuum, i.e. with $P \simeq 0$. Moreover, the pressure is the same at all points having the same horizontal level in mercury. Therefore, according to Figure 10.10, the atmospheric pressure P_a will be given by: $P_a = \rho g h$, where ρ is the density of mercury and h is the height of the mercury column.

The height of the mercury column is measured only if the barometer is set at a place where $g = 9.80665 \, \text{m/s}^2$ and the temperature of the mercury is 0°C. At this temperature, the mercury has a density $\rho = 13.595 \times 10^3 \, \text{kg/m}^3$ and the height of mercury is measured to be exactly 760 mm. Therefore:

$$\begin{aligned}
P_a &= \rho g h \\
&= (13.595 \times 10^3 \, \text{kg/m}^3)(9.80665 \, \text{m/s}^2)(0.76 \, \text{m}) \quad\quad (10.24) \\
&= 1.013 \times 10^5 \, \text{Pa}
\end{aligned}$$

The Open-Tube Manometer

Figure 10.11 shows a very basic **open-tube manometer** used to measure the gauge *pressure of a gas*. It consists of a U-tube containing a liquid, with one end of the tube connected to a gas tank of pressure P, and the other end open to the atmosphere.

Fig. 10.11 An open-tube manometer for measuring gas pressure
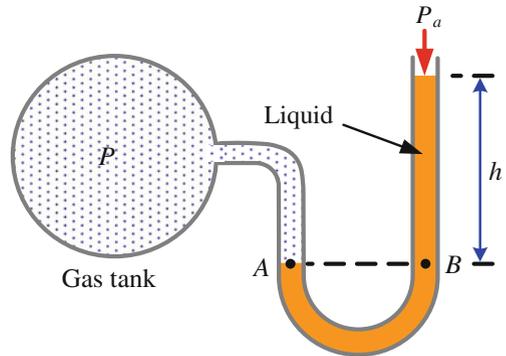
From this figure, we see that the pressure at point A is the unknown pressure P of the gas in the tank. On the other hand, the pressure P_A at point A is equal to the pressure P_B at point B, which equals $P_a + \rho gh$, where ρ is the density of the liquid and h is the height of the liquid column. Since $P_A = P_B$, we have:

$$P = P_a + \rho gh \qquad (10.25)$$

This relation gives us what we call the **absolute pressure** P. In general, the difference between an absolute pressure and an atmospheric pressure is called the **gauge pressure** P_g. That is:

$$P_g = P - P_a = \rho gh \qquad (10.26)$$

The gauge pressure can be positive or negative, depending on whether $P > P_a$ or $P < P_a$. In inflated tires or in the human circulatory system, the absolute pressure is greater than the atmospheric pressure (i.e. $P > P_a$), so the gauge pressure is positive (i.e. $P_g > 0$).

Example 10.7

The U shaped tube shown in Fig. 10.12 contains oil in the right arm and water in the left arm. In static equilibrium, the measurements give $h = 18$ cm and $d = 2$ cm. What is the value of the density of the oil ρ_o?

Fig. 10.12
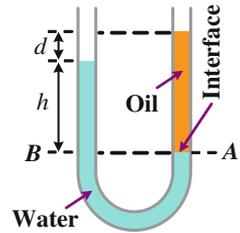
Solution: The pressure P_A at the oil–water interface of the right arm must be equal to the pressure P_B in the left arm at the same level. In the right arm, we use Eq. 10.20 to get:

$$P_A = P_a + \rho_o\, g(h + d)$$

In the left arm, we use the same Eq. 10.20 to get:

$$P_B = P_a + \rho_w g h$$

Since $P_A = P_B$, we equate the last two equations to get:

$$\rho_o = \frac{h}{h + d}\, \rho_w = \frac{18\,\text{cm}}{18\,\text{cm} + 2\,\text{cm}}\, 10^3\,\text{kg/m}^3 = 900\,\text{kg/m}^3$$

Note that the answer does not depend on the atmospheric pressure.

Example 10.8

For the car lift shown in the Fig. 10.13, the pistons on the left and right have areas $25\,\text{cm}^2$ and $750\,\text{cm}^2$ respectively. The car and the right piston have a total weight of 15,000 N. What force must be applied on the left piston (if it has negligible weight)? What pressure will produce this force?

Solution: From Eq. 10.21, we have:

$$F_1 = F_2\frac{A_1}{A_2} = (15,000\,\text{N})\frac{25\,\text{cm}^2}{750\,\text{cm}^2} = 500\,\text{N}$$

Fig. 10.13
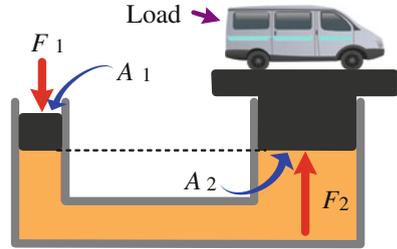
The pressure that produce this force is given by:

$$P = \frac{F_1}{A_1} = \frac{500\,\text{N}}{25 \times 10^{-4}\,\text{m}^2} = 2 \times 10^5\,\text{Pa} \simeq 2\,\text{atm}$$

* **Example 10.9**

(a) A person dives to a depth $h = 50$ cm below the water surface without inhaling first. Find the pressure on his body and on his lungs. (b) Repeat part (a) when he dives to a depth $h = 5$ m. When the diver ignores diving rules and foolishly uses a snorkel tube at that depth, find the pressure on his lungs? Why he is in danger? Assume that $P_a = 1.01 \times 10^5$ Pa, $\rho = 10^3$ kg/m^3, and $g = 9.8$ m/s^2.

Solution: (a) The external pressure on the diver's body will be:

$$P_{\text{Body}} = P_a + \rho g h$$
$$= 1.01 \times 10^5\,\text{Pa} + (10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.5\ \text{m})$$
$$= 1.01 \times 10^5\,\text{Pa} + 4{,}900\,\text{Pa} = 1.059 \times 10^5\,\text{Pa} \simeq 1.05\,\text{atm}$$

The diver's body adjusts to that pressure by a very slight contraction until the internal pressure is in equilibrium with the external pressure. Consequently, his average blood pressure increases, and the average air pressure in his lungs increases until it balances the external pressure. Thus, his lung pressure will be at:

$$P_{\text{Lungs}} \simeq 1.05\,\text{atm}$$

(b) When $h = 5$ m, the external pressure on the diver's body will be:

$$P_{\text{Body}} = P_a + \rho g h$$
$$= 1.01 \times 10^5\,\text{Pa} + (10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(5\ \text{m})$$
$$= 1.01 \times 10^5\,\text{Pa} + 49{,}000\,\text{Pa} = 1.5 \times 10^5\,\text{Pa} \simeq 1.5\,\text{atm}$$

Again, as in part (a), the pressure inside his lungs will be:

$$P_{\text{Lungs}} \simeq 1.5\,\text{atm}$$

If the diver foolishly uses a 5 m snorkel tube, the pressurized air in his lungs will be expelled upwards through the tube to the atmosphere. Consequently, the air pressure in his lungs will drop rapidly from 1.5 to 1 atm. This 0.5 atm pressure difference is sufficient to collapse his lungs and force his still-pressurized blood into them.

Buoyant Forces and Archimedes' Principle

In a swimming pool, you may have noticed that it is relatively easy to carry an object that is totally or partially immersed in the water. This is because you must support only part of the object's weight, while the buoyant force supports the remainder.

This important property of fluids in hydrostatic equilibrium is summarized by Archimedes' principle, which can be stated as follows:

Archimedes' Principle:
A body fully or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.

Let us show that the buoyant force is equal in magnitude to the weight of the displaced fluid. We can do this by considering a cube of fluid of height h (and hence volume $V_f = h^3$) as in Fig. 10.14a.
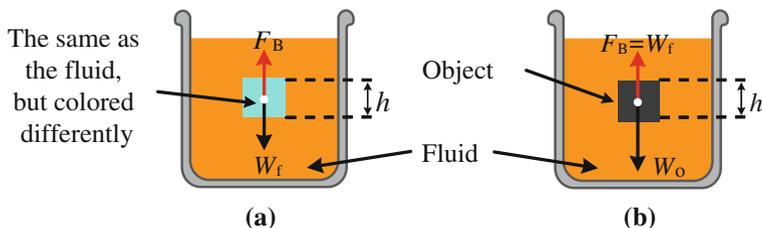
Fig. 10.14 (a) External forces acting on a cube of fluid (colored *blue*). Under equilibrium, the fluid's weight W_f is equal to the buoyant force F_B. (b) A cube of weight W_o is buoyed by a force $F_B = W_f$

The cube of this fluid is in equilibrium under the action of the forces on it. One of the forces is its own weight $\vec{W}_f$. Apparently, the rest of the fluid in the container is buoying up the cube and holding it in equilibrium. Therefore, the magnitude of this buoyant force, F_B, must be exactly equal in magnitude to the weight of the fluid. That is:

$$F_B = W_f \qquad (10.27)$$

Now, imagine we replace the cube of fluid by a *cubical object* of the same dimensions. The fluid surrounding the cube will behave the same way, regardless whether the cube is a fluid or a solid. Therefore, *the buoyant force acting on an object of any density will be equal to the weight of the fluid displaced by the object*, i.e. $F_B = W_f$.

To show this result explicitly, we notice in Fig. 10.14b that the pressure at the bottom of the object is greater than the pressure at the top by $\Delta P = \rho_f g h$, where ρ_f is the density of the fluid. Since the pressure difference ΔP equals the buoyant force per unit area, then $\Delta P = F_B/A$, where $A = h^2$ is the area of one of the cube's faces. Therefore:

$$F_B = \Delta P A = \rho_f g h A = \rho_f V_f g = W_f \qquad (10.28)$$

Consider the object of Fig. 10.14b to be of weight $W_o = \rho_o V_o g$, where ρ_o and V_o are its density and volume, respectively. If the object is totally immersed in a fluid of density ρ_f, the buoyant force will be $F_B = W_f = \rho_f V_f g$, where $V_f = V_o$. Thus, the net force on the object will depend only on ρ_o and ρ_f, see parts of Fig. 10.15a–c.
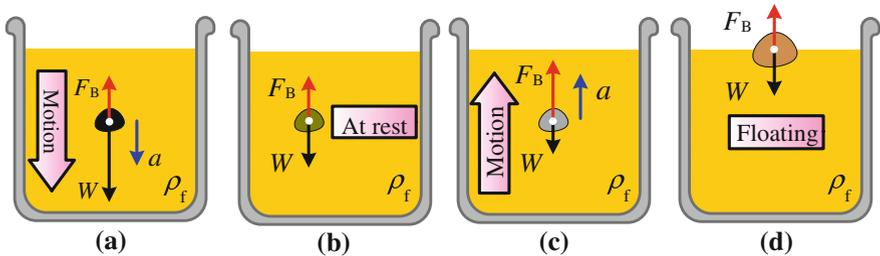
Fig. 10.15 An immersed object of density ρ_o when: (a) $\rho_o > \rho_f$, (b) $\rho_o = \rho_f$, (c) $\rho_o < \rho_f$, and (d) A floating object where $\rho_o V_o = \rho_f V_f$

Now, if that object floats; see Fig. 10.15d, the upward buoyant force will be $F_B = W_f = \rho_f V_f g$, where V_f is the volume of the displaced fluid and $V_f \neq V_o$.

Equilibrium in this case gives:

$$\rho_o V_o = \rho_f V_f \tag{10.29}$$

Example 10.10

A piece of steel has a mass $m_s = 0.5\,\text{kg}$ and a density $\rho_s = 7.8 \times 10^3\,\text{kg/m}^3$. The steel is suspended in air by a string attached to a scale, see Fig. 10.16. After that, the steel is immersed in a container filled with water of density $\rho_w = 10^3\,\text{kg/m}^3$. Find the tension in the string before and after the steel is immersed.

Fig. 10.16
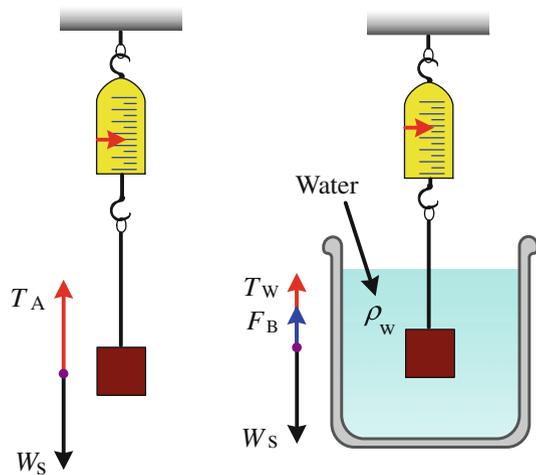
Solution: When the piece of steel is suspended in air, the tension in the string T_a equals the weight $m_s\, g$ of that piece of steel. That is:

$$T_a = m_s\, g = (0.5\,\text{kg})(9.8\,\text{m/s}^2) = 4.9\,\text{N}$$

When the steel is immersed in water, it experiences an upward buoyant force F_B. Thus, the tension in the string will be reduced to a new value T_w, see the figure. Equilibrium in this case gives:

$$T_w + F_B = m_s\, g \quad \Rightarrow \quad T_w = m_s\, g - F_B \quad \Rightarrow \quad T_w = 4.9\,\text{N} - F_B$$

To find F_B, we first calculate the volume of the steel as follows:

$$V_s = \frac{m_s}{\rho_s} = \frac{0.5\,\text{kg}}{7.8 \times 10^3\,\text{kg/m}^3} = 6.4 \times 10^{-5}\,\text{m}^3$$

This volume equals the volume of the displaced water. That is:

$$V_w = 6.4 \times 10^{-5}\,\text{m}^3$$

Since the buoyant force equals the weight of the displaced water, then:

$$F_B = m_w\,g = \rho_w V_w\,g = (10^3\,\text{kg/m}^3)(6.4 \times 10^{-5}\,\text{m}^3)(9.8\,\text{m/s}^2) = 0.63\,\text{N}$$

Therefore, the tension in the string T_w (the apparent weight) will be:

$$T_w = 4.9\,\text{N} - F_B = 4.9\,\text{N} - 0.63\,\text{N} = 4.3\,\text{N}$$

Example 10.11

The approximate density of ice is $\rho_i = 918\,\text{kg/m}^3$ and the approximate density of sea water in which an iceberg floats is $\rho_w = 1{,}020\,\text{kg/m}^3$, see Fig. 10.17. What fraction of the iceberg is beneath the water surface?

Fig. 10.17
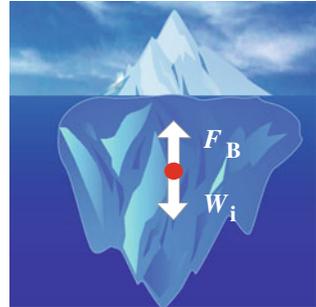
Solution: The iceberg floats, as shown in the figure, due to the effect of an upward buoyant force given by:

$$F_B = W_w = \rho_w\,V_w\,g$$

where V_w is the volume of the displaced water or the volume of the iceberg beneath the water surface. The weight of the iceberg is:

$$W_i = \rho_i \, V_i \, g$$

where V_i is the volume of the iceberg. Equilibrium in this case gives $F_B = W_i$. That is:

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{918 \, \text{kg/m}^3}{1,020 \, \text{kg/m}^3} = 0.90 \quad \text{or} \quad 90\%$$

Thus, 90% of the iceberg lies below water level. This means that, only 10% of an iceberg—its tip is above the surface of the water.

Example 10.12

An object of a known density ρ_o floats three-fourths immersed in a liquid of unknown density ρ_f. Find the density of the liquid.

Solution: We use Eq. 10.29 when $V_f = \frac{3}{4} V_o$. Then:

$$\rho_f = \rho_o \frac{V_o}{V_f} = \rho_o \frac{V_o}{\frac{3}{4} V_o} = \frac{4}{3} \rho_o$$

Note that ρ_f is larger than ρ_o by the reciprocal of the immersed ratio.

Example 10.13

A helium-filled balloon has a volume $V_b = 8 \times 10^3 \, \text{m}^3$ and balloon mass $m_b = 200 \, \text{kg}$, see Fig. 10.18. What is the maximum mass m of a load that keeps the balloon in equilibrium? Neglect the air displaced by the load. Take $\rho_{He} = 0.18 \, \text{kg/m}^3$ to be the density of helium and $\rho_{air} = 1.28 \, \text{kg/m}^3$ to be the density of air.

Fig. 10.18

F_B

$(m_{He} + m_b + m)g$

m
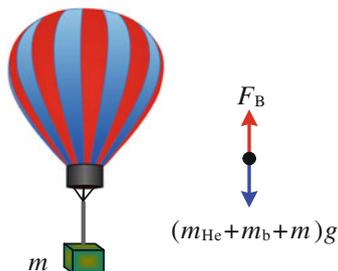
Solution: The volume of the displaced air equals the balloon's volume, i.e. $V_{air} = V_b$. According to Archimedes' principle, the buoyant force is the weight of

the displaced air, i.e.:

$$F_B = W_{air} = \rho_{air} V_{air} g = \rho_{air} V_b g$$

Since the balloon's volume is approximately equal to the volume of helium, i.e. $V_{He} \simeq V_b$, then the weight of the helium is:

$$W_{He} = \rho_{He} V_{He} g = \rho_{He} V_b g$$

At equilibrium, see Fig. 10.18, we have:

$$F_B = W_{He} + W_b + mg \quad \Rightarrow \quad \rho_{air} V_b g = \rho_{He} V_b g + m_b g + m g$$

Thus: $m = (\rho_{air} - \rho_{He}) V_b - m_b$

$$= (1.28 \, \text{kg/m}^3 - 0.18 \, \text{kg/m}^3)(8 \times 10^3 \, \text{m}^3) - 200 \, \text{kg} = 8{,}600 \, \text{kg}$$

10.5   Fluid Dynamics

Ideal Fluids

The motion of a real fluid is very complicated. Instead, we shall discuss the motion of an **ideal fluid** that will obey the following four assumptions:

1. *Steady flow*: The velocity of the fluid at any specific point does not change with time. However, in general the velocity might vary from one point to another.
2. *Incompressible flow*: The density of the fluid does not change with time. That is, the density has a constant uniform value.
3. *Non-viscous flow*: A tiny object can move through the fluid without experiencing a viscous drag force; that is, there is no resistive force due to viscosity.
4. *Irrotational flow*: A tiny object can move through the fluid without rotating about an axis passing through its center of mass.

Streamlines

A **streamline** is the path traced out by a tiny fluid element, called a fluid "*particle*". As the fluid particle moves, its velocity may change in magnitude or in direction or both. However, the velocity of the fluid particle at any point is always tangent to the streamline at that point, see Fig. 10.19a. Streamlines never cross each other because

if they do, a fluid particle could move either way at the cross over point, and the flow
would not be steady.

When air flows around objects, the air particles must avoid the object, see Fig.
10.19b. Conservation of mass sets up the streamlines that the air particles must follow
to avoid the object.

(a)   **(b)**
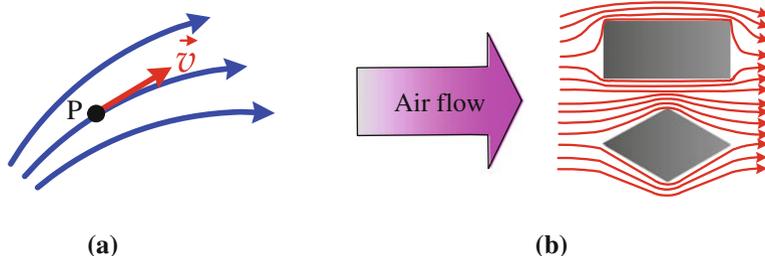
Fig. 10.19 (a) The diagram shows a set of streamlines. A fluid particle P traces out a streamline as
it moves. The velocity vector of the fluid particle is tangent to the streamline at every point. (b) The
streamlines of air flow near two different obstacles

Equation of Continuity

In flows like that of Fig. 10.19b, we consider a fluid flowing through a tube called
a *stream tube*, or *tube of flow*, whose boundary is made up of streamlines, see Fig.
10.20. Such a tube acts like a pipe because any fluid particle entering it cannot escape
through its walls.

Fig. 10.20 A stream tube
formed by the streamlines. The
flow rate of fluid at the cross
sections A_1 and A_2 is the same
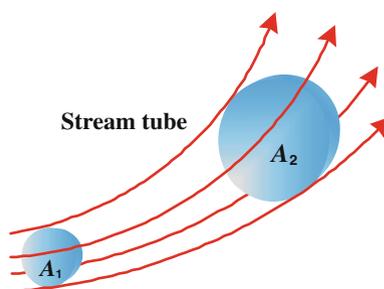
Figure 10.21 shows two cross-sectional areas A_1 and A_2 in a thin stream tube of
fluid of varying cross-sectional areas. The fluid particles are moving steadily through
this stream tube.
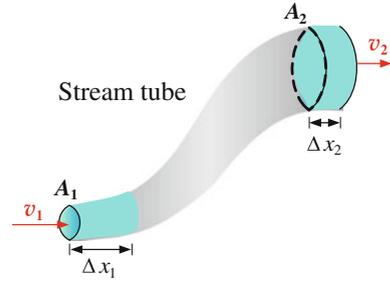
In a small time interval Δt, the fluid at the area A_1 moves a small distance $\Delta x_1 = v_1 \Delta t$. Assuming uniform density over the area A_1, then the mass in the colored segment of Fig. 10.21 is:

$$\Delta m_1 = \rho_1 (A_1 \Delta x_1) = \rho_1 A_1 v_1 \Delta t = \text{Mass into segment}$$

Similarly, the fluid that moves through the area A_2 in the same time interval will be:

$$\Delta m_2 = \rho_2 (A_2 \Delta x_2) = \rho_2 A_2 v_2 \Delta t = \text{Mass out of segment}$$

Since **mass is conserved** and because the flow is steady, the mass that crosses A_1 in time interval Δt must be equal to the mass that crosses A_2 in the same time interval. Then, $\Delta m_1 = \Delta m_2$ and we get:

$$\left\{ \begin{array}{c} \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \\ \text{or} \\ \rho A v = \text{constant} \end{array} \right. \qquad \text{(Steady flow)} \qquad (10.30)$$

This is the *equation of continuity* for a steady flow. Since $\rho A v$ has the dimension of mass/time, it is called the *mass flow rate R_m*, i.e.:

$$R_m = \rho A v = \text{constant} . \qquad (10.31)$$

If we assume the fluid is incompressible, then the density ρ is constant, and the continuity equation reduces to:

$$\left\{ \begin{array}{c} A_1 v_1 = A_2 v_2 \\ \text{or} \\ A v = \text{constant} \end{array} \right. \qquad \text{(Steady and Incompressible flow)} \qquad (10.32)$$

This is another form of the *equation of continuity* for incompressible steady flow. Since Av has the dimension of volume/time, it is called the *volume flow rate* R_V, i.e.:

$$R_V = Av = \text{constant} \tag{10.33}$$

A constant-volume flow rate tells us that the flow is faster in narrower sections of a tube of flow, where the streamlines are close together.

Incompressible steady flow:
The product of the area and the fluid speed at all points along the pipe is constant.

Example 10.14

Water flows in a pipe from a large cross-sectional area $A_1 = 0.5\,\text{m}^2$ with a speed $v_1 = 15\,\text{m/s}$ to a smaller cross-sectional area $A_2 = 0.05\,\text{m}^2$. (a) What is the speed v_2 at which the water leaves the smaller cross section as in the left part of Fig. 10.22? (b) What is the effect of lowering A_2 by 10 m as in the right part of Fig. 10.22?
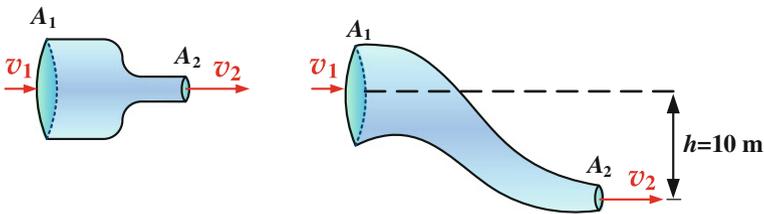
Fig. 10.22

Solution: (a) The flow of water through the pipe in the left part is governed by the continuity equation, or conservation of mass. That is:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

For most liquids, density is essentially constant. Then $A_1 v_1 = A_2 v_2$, and:

$$v_2 = v_1 \frac{A_1}{A_2} = (15\,\text{m/s}) \frac{0.5\,\text{m}^2}{0.05\,\text{m}^2} = 150\,\text{m/s}$$

(b) Since the continuity equation does not depend on altitude, then lowering A_2 by 10 m causes no change to the result.

Example 10.15

The fact that a water stream emerging from a faucet "necks down" as it falls is shown in Fig. 10.23, where $A_1 = 1.8\,\text{cm}^2$, $A_2 = 0.3\,\text{cm}^2$, and $h = 25\,\text{cm}$. What is the water flow rate from the faucet, assuming a steady flow?

Fig. 10.23
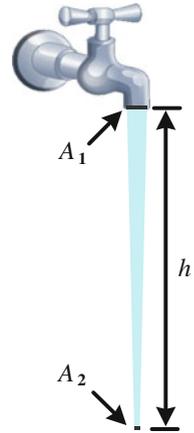
Solution: As water falls from a faucet, its speed increases due to gravity. Because the volume flow rate must be the same at all cross sections, the stream must "neck down".

The flow of water is governed by the continuity Eq. 10.32, that is:

$$A_1 v_1 = A_2 v_2$$

where v_1 and v_2 indicate the speed of the water at the marked levels shown in Fig. 10.23. Since water is falling freely, we must have:

$$v_2^2 = v_1^2 + 2gh$$

Eliminating v_2 from the last two equations, we get:

$$v_1 = \sqrt{\frac{2ghA_2^2}{A_1^2 - A_2^2}} = \sqrt{\frac{2(9.8\ \text{m/s}^2)(0.25\ \text{m})(0.3\ \text{cm}^2)^2}{(1.8\ \text{cm}^2)^2 - (0.3\ \text{cm}^2)^2}} = 0.374\,\text{m/s}$$

The volume flow rate R_V, given by Eq. 10.33, is thus:

$$R_V = A_1 v_1 = (1.8 \times 10^{-4} \text{ m}^2)(0.374 \text{ m/s}) = 6.732 \times 10^{-5} \text{ m}^3/\text{s}$$

Finally, we find the mass-flow rate R_m using Eq. 10.31 to be:

$$R_m = \rho A_1 v_1 = \rho R_V = (10^3 \text{ kg/m}^3)(6.735 \times 10^{-5} \text{ m}^3/\text{s}) = 0.067 \text{ kg/s} \simeq 70 \text{ g/s}$$

Example 10.16

Water flowing from a faucet of cross-sectional area $A = 2 \text{ cm}^2$ is used to fill a bucket of volume $V = 30 \text{ liters} = 30 \times 10^3 \text{ cm}^3$, see Fig. 10.24. What is the speed v at which the water leaves the faucet if it takes exactly 1 minute to fill the bucket?

Fig. 10.24
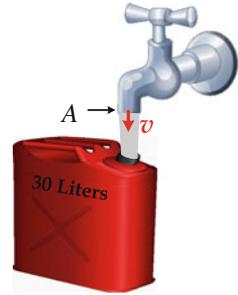
Solution: According to the given information, the volume flow rate is:

$$R_V = 30 \text{ liters/min} = 500 \text{ cm}^3/\text{s}$$

Using Eq. 10.33, $R_V = Av$, we can find the speed v as follows:

$$v = \frac{R_V}{A} = \frac{500 \text{ cm}^3/\text{s}}{2 \text{ cm}^2} = 250 \text{ cm/s} = 2.5 \text{ m/s}$$

Bernoulli's Equation

In static fluids, the pressure is the same at all points on the same horizontal level but increases with depth. This is not generally true when the fluid is in motion. In

the year 1738, Bernoulli derived an expression for an **ideal fluid** (i.e. a fluid that is incompressible, non-viscous and flows in a non-rotational steady manner) that relates the pressure, speed, and elevation within different locations in the fluid.

* Consider a small portion of a tube of flow of an ideal fluid with density ρ through a non-uniform pipe as shown in Fig. 10.25. The width of the tube in this figure is exaggerated for clarity.
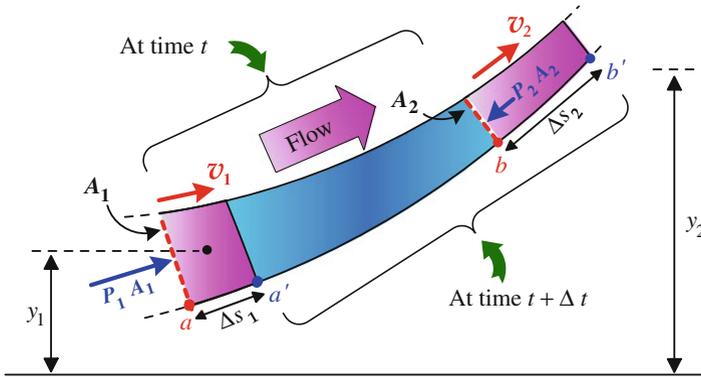
Fig. 10.25 The fluid in a section of length Δs_1 moves to a section of length Δs_2, while the volume of the two sections are the same

Using the information in Fig. 10.25, we perform the following steps:

1. At some initial time t, the fluid lies between two cross sections A_1 and A_2 at two points labeled a and b, respectively.
2. After a time interval Δt, the fluid's ends undergo displacements Δs_1 and Δs_2 to new points labeled a' and b', respectively.
3. The volume of fluid that passes from point a to point a' over a time Δt is equal to the volume of fluid that passes from point b to point b' in the same time interval, that is $\Delta V = A_1 \Delta s_1 = A_2 \Delta s_2$.
4. The force on the cross section A_1 is $P_1 A_1$ and the force on the cross section A_2 is $P_2 A_2$. Thus, the net work done on the fluid by these forces over the time Δt is:

$$W = P_1 A_1 \Delta s_1 - P_2 A_2 \Delta s_2 = (P_1 - P_2)\Delta V \qquad (10.34)$$

where the negative sign in the second term is due to the fact that the fluid force $P_2 A_2$ is opposite to the displacement Δs_2.

5. Part of this work goes into changing the kinetic energy of the fluid, and the other part goes into changing its gravitational potential energy. If Δm is the mass of the

fluid that passes through the tube during the time interval Δt, then $\Delta m = \rho \Delta V$ and the change in kinetic and potential energy will be given by:

$$\Delta K = \tfrac{1}{2}\Delta m v_2^2 - \tfrac{1}{2}\Delta m v_1^2 = \tfrac{1}{2}\rho\Delta V(v_2^2 - v_1^2) \qquad (10.35)$$

$$\Delta U = \Delta m g y_2 - \Delta m g y_1 = \rho\Delta V g(y_2 - y_1) \qquad (10.36)$$

6. We can now apply the work-energy theorem written in the form $W = \Delta K + \Delta U$, where W is the work done by all applied forces and is given by Eq. 10.34. Thus:

$$(P_1 - P_2)\Delta V = \tfrac{1}{2}\rho\Delta V(v_2^2 - v_1^2) + \rho\Delta V g(y_2 - y_1)$$

If we divide both sides by ΔV, we get:

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1) \qquad (10.37)$$

7. Rearranging the terms, we get:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad (10.38)$$

This is the standard form of Bernoulli's equation for non-viscous, incompressible fluids experiencing steady flow. Since the subscripts 1 and 2 refer to any two points along the tube flow, Bernoulli's equation may also written as:

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad (10.39)$$

When the fluid is at rest, $v_1 = v_2 = 0$ and Bernoulli's equation becomes:

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

which is of the same form as Eq. 10.20.

When we take y to be constant, say $y = 0$, so that the fluid does not change elevation as it flows, then Bernoulli's equation becomes:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \quad \text{(Horizontal flow)} \qquad (10.40)$$

This tells us that if the speed of a fluid *increases* as it travels horizontally, then its pressure must *decrease*, and vice versa.

Example 10.17

Gasoline of density $\rho = 860\,\text{kg/m}^3$ flows steadily through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.5 \times 10^{-3}\,\text{m}^2$ to $A_2 = \frac{1}{2}A_1$, see Fig. 10.26. What is the volume flow rate when the pressure difference $P_1 - P_2$ is 5,160 Pa?

Solution: The flow of gasoline is governed by the continuity Eq. 10.32, i.e. $A_1 v_1 = A_2 v_2$. Then for $A_2 = \frac{1}{2}A_1$ we find:

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad A_1 v_1 = \tfrac{1}{2}A_1 v_2 \quad \Rightarrow \quad v_2 = 2v_1$$

Fig. 10.26
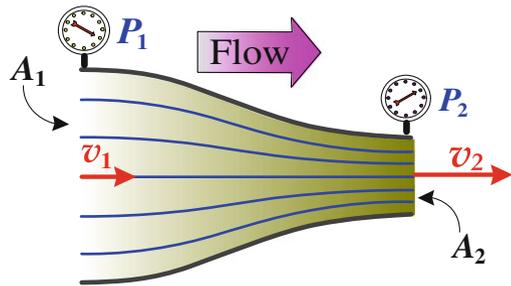
That is, the gasoline speed at the narrower section is twice that at the wider section. Using this result in Bernoulli's equation of a fluid traveling horizontally, see Eq. 10.40, we get:

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) = \tfrac{1}{2}\rho(4v_1^2 - v_1^2) = \tfrac{3}{2}\rho v_1^2$$

Thus, we can find v_1 in terms of ρ and $P_1 - P_2$ as follows:

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{3\rho}} = \sqrt{\frac{2(5{,}160\,\text{Pa})}{3(860\,\text{kg/m}^3)}} = 2\,\text{m/s}$$

Finally, the volume flow rate R_V, given by Eq. 10.33, is thus:

$$R_V = A_1 v_1 = (1.5 \times 10^{-3}\,\text{m}^2)(2\,\text{m/s}) = 3 \times 10^{-3}\,\text{m}^3/\text{s}$$

Example 10.18

Flow Speed from a Reservoir (*Torricelli's Law*). A tank is filled with water to a height $y_1 = 3.5$ m. The tank has a small hole in one of its walls at a height $y_2 = 1.5$ m, see Fig. 10.27. (a) What is the speed v_2 of the water emerging from the hole? (b) What is the horizontal distance x from the base of the tank to the point at which the water stream strikes the floor?

Fig. 10.27
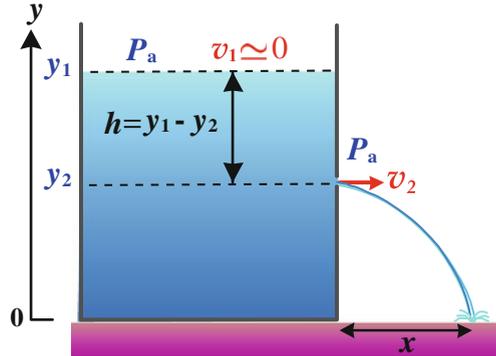
Solution: (a) The pressure at the top of reservoir and at the hole is the atmospheric pressure P_a, because both of them are exposed to the atmosphere. If we assume the tank has a large cross-sectional area A_1 compared to that of the hole's, i.e. $A_1 \gg A_2$, then water at the top of the reservoir will be almost stationary, i.e. $v_1 \simeq 0$. In addition to this, $h = y_1 - y_2$. Using Bernoulli's equation, see Eq. 10.38, we get:

$$P_a + \tfrac{1}{2}\rho \times (0) + \rho g y_1 = P_a + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \Rightarrow \quad v_2^2 = 2gh$$

That is:
$$v_2 = \sqrt{2gh}$$

Thus:
$$v_2 = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(2 \text{ m})} = 6.26 \text{ m/s}$$

The relation $v_2 = \sqrt{2gh}$ is the same as that for an object falling freely from rest through a height h. This is known as *Torricelli's law*.

(b) As in Sect. 4.3, the initial water velocity $\vec{v_o} \equiv \vec{v_2}$ is horizontal (i.e. $\theta_o = 0$) and the initial position is $y_o \equiv y_2 = 1.5$ m. Since water strikes the floor at $y = 0$, then we use $y - y_o = (v_o \sin \theta_o)\, t - \tfrac{1}{2}g\, t^2$ to first find the duration of descent for the water as follows:

$$0 - 1.5\,\text{m} = 0 - \tfrac{1}{2}\,(9.8\,\text{m/s}^2)\,t^2 \quad \Rightarrow \quad t = 0.553\,\text{s}$$

The horizontal distance x covered by the water in that time is:

$$x = (v_\text{o} \cos \theta_\text{o})\,t = (6.26\,\text{m/s})(\cos 0°)(0.553\,\text{s}) = 3.46\,\text{m}$$

Example 10.19

In the Venturi meter of Fig. 10.28, air of density $\rho_\text{air} = 1.3\,\text{kg/m}^3$ flows from left to right through a horizontal pipe of radius $r_1 = 1.25\,\text{cm}$ that necks down to $r_2 = 0.5\,\text{cm}$. The U-shaped tube of the meter contains mercury of density $\rho_\text{mer} = 13.6 \times 10^3\,\text{kg/m}^3$. If the speed of the air entering the meter is $v_1 = 10\,\text{m/s}$, then find the mercury-level difference h between the two arms.
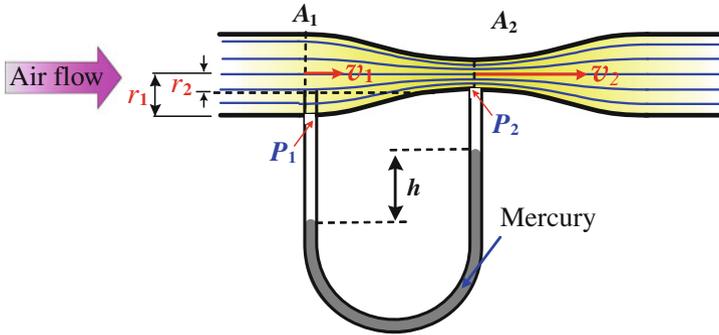
Fig. 10.28

Solution: Since the air moves horizontally, then Bernoulli's equation at the two openings of the U-shaped tube becomes:

$$P_1 - P_2 = \tfrac{1}{2}\rho_\text{air}(v_2^2 - v_1^2)$$

Also, from the continuity Eq. 10.32 we have:

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \left(\frac{r_1}{r_2}\right)^2$$

Substituting with the form of v_2 in the pressure difference, we get:

$$P_1 - P_2 = \tfrac{1}{2}\rho_\text{air}v_1^2\left[\left(\frac{r_1}{r_2}\right)^4 - 1\right]$$

The pressure difference between the two openings of the U-shaped tube produces a mercury-level difference h given by $P_1 - P_2 = \rho_{mer}gh$. Thus, by combining the last result with this equation we get:

$$h = \frac{\rho_{air}v_1^2}{2\rho_{mer}g}\left[\left(\frac{r_1}{r_2}\right)^4 - 1\right] = \frac{(1.3\,\text{kg/m}^3)(10\,\text{m/s})^2}{2(13.6 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)}\left[\left(\frac{1.25\,\text{cm}}{0.5\,\text{cm}}\right)^4 - 1\right]$$

$$= 0.019\,\text{m} = 1.9\,\text{cm}$$

---

* Example 10.8

The sketch in Fig. 10.29 shows a perfume atomizer before and after compressing its bulb. When the bulb is compressed gently, air with density ρ_{air} flows steadily through a narrow tube, reducing the pressure at the position of the vertical tube. Liquid of density ρ_L can rise in this vertical tube and enter the horizontal tube and be sprayed out. If the pressure in the bulb is $P_a + P$, where P_a is the atmospheric pressure, and v_2 is the speed of air in the horizontal tube, then find the pressure formula in the horizontal tube. What must the value of v_2 be in order to raise the liquid to the horizontal tube?
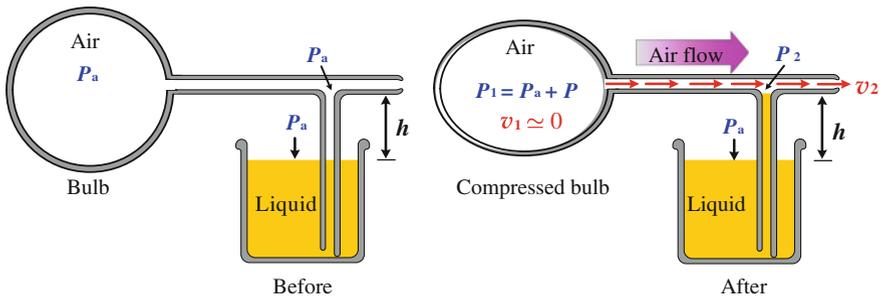
Fig. 10.29

Solution: By applying Bernoulli's equation on the bulb and the narrow horizontal tube we get the following:

$$(P_a + P) + \tfrac{1}{2}\rho \times (0) = P_2 + \tfrac{1}{2}\rho_{air}v_2^2$$

That is:

$$P_2 = P_a + P - \tfrac{1}{2}\rho_{air}v_2^2$$

So, the decrease in P_2 depends on the square of the speed v_2.

When the liquid rises a distance h to the horizontal tube, we have:

$$P_a = P_2 + \rho_L g h$$

By equating the expression of P_2 from the last two results we get:

$$P_2 = P_a + P - \tfrac{1}{2}\rho_{air} v_2^2 = P_a - \rho_L g h$$

Thus:

$$v_2 = \sqrt{\frac{2(P + \rho_L g h)}{\rho_{air}}}$$

Viscosity

Fluids cannot withstand a shearing stress. However, fluids show some degree of *resistance* to shearing motion, and this resistance is called **viscosity**.

The degree of viscosity can be understood by considering a fluid between two sheets of glass where the lower one is kept fixed; see the sketch in Fig. 10.30. It is easier to slide the upper glass if the fluid is oil as opposed to tar because tar has a higher viscosity than oil.
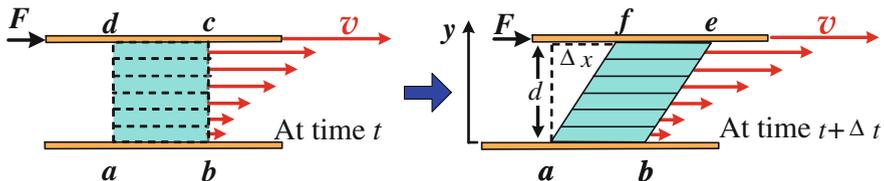
Fig. 10.30 A fluid between two sheets of glass where the lower one is kept fixed while the upper one moves to the right with a speed v under the action of an external force of magnitude F

We can think of fluids as a set of adjacent layers. Thus, a reasonable shearing stress produces smooth relative displacement of adjacent layers in fluids, called *laminar flow*. When we apply a force of magnitude F to the upper glass of area A, it will move to the right with a speed v. As a result of this motion, a portion of the fluid with shape *abcd* will take a new shape *abef* after a short time interval Δt.

According to Sect. 10.2, we can define the shearing stress and the shearing strain on the fluid of Fig. 10.30 as follows:

$$\text{Shearing stress} = \frac{F}{A}, \quad \text{Shearing strain} = \frac{\Delta x}{d} \tag{10.41}$$

Since the upper sheet is moving with speed v, the fluid just beneath it will move with the same speed. Thus, in time Δt, the fluid just beneath the upper sheet moves a distance $\Delta x = v \Delta t$. Accordingly, we define the rate of shear strain as:

$$\text{Rate of shear strain} = \frac{\text{shear strain}}{\Delta t} = \frac{\Delta x/d}{\Delta t} = \frac{v}{d} \tag{10.42}$$

By analogy to the shear modulus in solids, we define in fluids, at a given temperature, the ratio of the shear stress to the *rate of shear strain*. This ratio, η, is known as the **coefficient of viscosity** or simply the **viscosity**:

$$\eta = \frac{F/A}{v/d} = \frac{Fd}{Av} \tag{10.43}$$

The SI unit of viscosity is $N.s/m^2 = Pa.s$ which is called the **poiseuille** (abbreviated by Pl) while in cgs it is $dyne.s/cm^2$ which is called the **poise** (abbreviated by P). Thus, $1\,N.s/m^2 = 1\,Pa.s = 1\,Pl = 10\,P = 10^3\,cP$.

Table 10.6 depicts some viscosity values.

Table 10.6 The viscosity of some fluids at specific temperatures

Fluid	Temperature T (°C)	Viscosity η (N.s/m² = Pl)
Benzene	20	0.07×10^{-3}
Water	100	0.3×10^{-3}
Water	20	1×10^{-3}
Whole blood	37	2.7×10^{-3}
10-wt Motor oil	30	250×10^{-3}
Glycerin	20	830×10^{-3}

* Equation 10.43 is valid only when the velocity of the fluid varies linearly with the perpendicular distance to the fluid velocity. In this case, it is common to say that the velocity gradient is uniform. In case of non-uniform velocity gradient, the viscosity has the general form:

$$\eta = \frac{F/A}{dv/dy} \tag{10.44}$$

Stokes' law

The drag force of a small object that moves with a low speed v through a viscous medium was given in Sect. 5.2 of Chap. 5 by the relation $F_D = bv$, where b is a proportionality constant.

When the small object is a sphere of radius r and it moves with a terminal speed v_t through a viscous medium with viscosity η, it experiences a drag force F_{vis} which by **Stokes' law** has a magnitude:

$$F_{vis} = 6\pi \eta r v_t \tag{10.45}$$

As an application to Stokes' formula, Fig. 10.31 displays the fall of a small metallic spherical ball of volume $V_s = \frac{4}{3}\pi r^3$, density ρ_s, and mass $m_s = V_s \rho_s$ in a viscous liquid of density ρ. The forces that act on the sphere when it reaches its terminal (constant) speed v_t will be:

1. The sphere's weight $W = m_s\,g = \rho_s\,V_s\,g$ (downwards)
2. The buoyant force $F_B = \rho\,V g$ (upwards)
3. The viscous force $F_{vis} = 6\pi \eta r v_t$ (upwards)

Fig. 10.31 A small sphere falling with terminal speed v_t in a liquid of density ρ and viscosity η
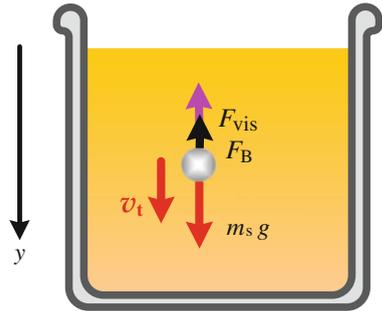
We must equate the volume of the liquid V that was displaced by the sphere with the volume of the falling sphere V_s. Thus:

$$m_s\,g = F_B + F_{vis} \quad \Rightarrow \quad \rho_s\,V_s\,g = \rho\,V_s\,g + 6\pi\,\eta r v_t$$

From this equation we get the following relation for the viscosity η:

$$\eta = \frac{2}{9}\frac{(\rho_s - \rho)r^2 g}{v_t} \tag{10.46}$$

Example 10.21

A steel plate of area $A = 0.2\,\text{m}^2$ is placed over a thin film of lubricant of thickness $d = 0.4\,\text{mm}$ sprayed over the flat horizontal surface of a table, see Fig. 10.32. When connected via a cord that passes over a massless and frictionless pulley to a mass $m = 10\,\text{g}$, the steel plate is found to move with a constant speed $v = 0.05\,\text{m/s}$. Find the viscosity of the lubricant oil.

Fig. 10.32
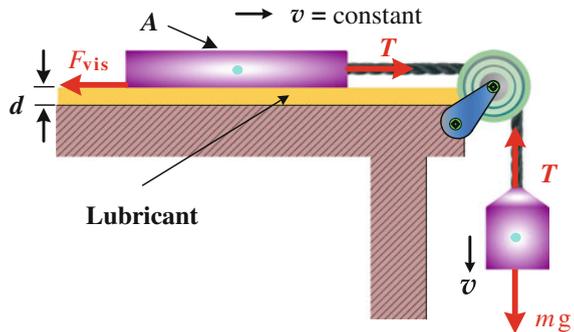
Lubricant

Solution: Since the steel plate moves with a constant speed, its resultant force must be zero. Thus, the magnitude of the tension force must equal the magnitude of viscous force exerted by the lubricant on the plate, i.e. $F_{\text{vis}} = T$. Also, the magnitude of the tension in the cord is equal to the magnitude of the suspended weight, i.e. $T = m\,g$. Thus:

$$F_{\text{vis}} = T = mg = (10 \times 10^{-3}\,\text{kg})(9.8\,\text{m/s}^2) = 9.8 \times 10^{-2}\,\text{N}$$

The layer of lubricant in contact with the horizontal surface of the table is at rest. The lubricant speed increases across the film, reaching a maximum, v, at the layer in contact with the steel plate which moves with speed v. If we assume that the rate of shear strain is constant, i.e. the velocity gradient is uniform, then we can use Eq. 10.43 to evaluate the viscosity as follows:

$$\eta = \frac{F_{\text{vis}}/A}{v/d} = \frac{F_{\text{vis}}\,d}{Av} = \frac{(9.8 \times 10^{-2}\,\text{N})(0.4 \times 10^{-3}\,\text{m})}{(0.2\,\text{m}^2)(0.05\,\text{m/s})}$$
$$= 3.92 \times 10^{-3}\,\text{N.s/m}^2 = 3.92\,\text{cP}$$

Example 10.22

A tiny glass sphere of density $\rho_s = 2.6 \times 10^3 \, \text{kg/m}^3$ falls with a terminal velocity v_t through oil which has a density $\rho = 950 \, \text{kg/m}^3$ and a viscosity coefficient $\eta = 0.2 \, \text{N.s/m}^2$. It is experimentally observed that the sphere drops a distance $d = 20 \, \text{cm}$ between the two points A and B in time $t = 50 \, \text{s}$, see Fig. 10.33. Find the radius r of the glass sphere.

Fig. 10.33
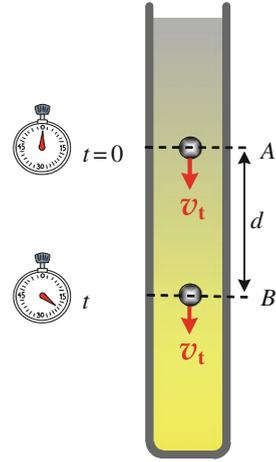
Solution: From experimental observations, the terminal speed of the sphere will be given by:

$$v_t = \frac{d}{t} = \frac{20 \times 10^{-2} \, \text{m}}{50 \, \text{s}} = 4 \times 10^{-3} \, \text{m/s}$$

The forces that act on the sphere when it reaches its terminal (constant) speed v_t will be: the sphere's weight $W = m_s \, g = \rho_s \, V_s \, g$ (downwards), the buoyant force $F_B = \rho \, V g$ (upwards), and the viscous force $F_{\text{vis}} = 6\pi \, \eta r v_t$ (upwards), see Fig. 10.34. The volume of the liquid V that was displaced by the sphere equals the volume of the falling sphere, i.e. $V = V_s = \frac{4}{3}\pi r^3$. Since the sphere moves with constant speed v_t, its resultant force must be zero. Thus:

$$\rho_s \, V_s \, g = \rho \, V_s \, g + 6\pi \, \eta r v_t$$

Solving for the sphere's radius, we obtain:

$$r = \sqrt{\frac{9\eta v_t}{2(\rho_s - \rho)g}} = \sqrt{\frac{9(0.2 \, \text{N.s/m}^2)(4 \times 10^{-3} \, \text{m/s})}{2(2.6 \times 10^3 \, \text{kg/m}^3 - 950 \, \text{kg/m}^3)(9.8 \, \text{m/s}^2)}}$$

$$= 4.7 \times 10^{-4} \, \text{m} = 0.47 \, \text{mm}$$

Fig. 10.34

F_{vis}

F_{B}

v_{t}

$m_s g$
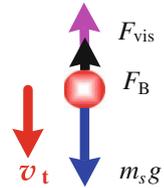
10.6 Exercises

Section 10.1 Density and Relative Density

(1) A solid cube of mass of 0.04 kg has a side of length 1 cm. What is the density and the specific gravity of the cube?

(2) A solid sphere has a radius of 1 cm and a mass of 0.04 kg. What is the density and the specific gravity of the sphere?

(3) Lead bricks like the one in Fig. 10.35 are used to shield people from the hazards of radioactive materials. If the lead density is $\rho = 11.36 \times 10^3$ kg/m^3, then find the mass and weight of such a brick.

Fig. 10.35 See Exercise (3)

5 cm

10 cm

20 cm
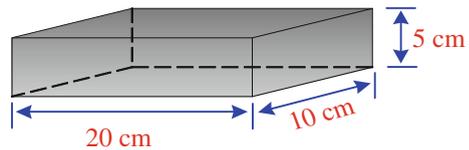
Section 10.2 Elastic Properties of Solids

(4) A mass of 5 kg is suspended from the end of a copper wire that has a diameter of 1 mm. Find the tensile stress on the wire?

(5) A 4 m long structural steel rod with a cross-sectional area of 0.5 cm^2 stretches 1 mm when a mass of 250 kg is hung from its lower end. Find the value of Young's modulus for this steel.

(6) An iron rod 10 m long and 0.5 cm^2 in cross section, stretches 2.5 mm when a mass of 300 kg is hung from its lower end. Find Young's modulus for the iron rod.

(7) A wire has a length $L = 3$ m and a radius $r = 0.75$ cm, see Fig. 10.36. A force acting normally on each of its ends has a magnitude $F_\perp = 9 \times 10^4$ N.

Find the change in the wire's length and radius, when its Young's modulus Y is 190×10^9 N/m^2 and its Poisson's ratio μ is 0.25.

Fig. 10.36 See Exercise (7)
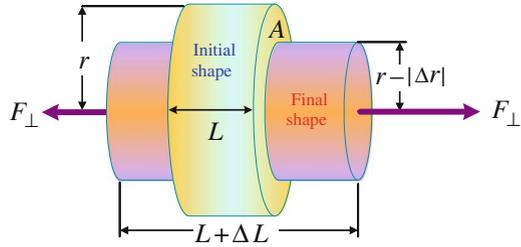
(8) A uniform platform is suspended by four wires, one on each corner. The wires are 2-m long and have a radius of 1 mm, and their material has a Young's modulus $Y = 190 \times 10^9$ N/m^2. How far will the platform drop if an 80 kg load is placed at its center?

(9) A block of gelatin resting on a rough dish has a length $L = 60$ cm, width $d = 40$ cm, and height $h = 20$ cm, see the vertical cross section abcd in Fig. 10.37. A force $F_\parallel = 0.6$ N is applied tangentially to the upper surface, leading to a new shape abef and hence a displacement $\Delta x = 5$ mm for the upper surface relative to the lower one. Find: (a) the shearing stress, (b) the shearing strain, and (c) the shear modulus.
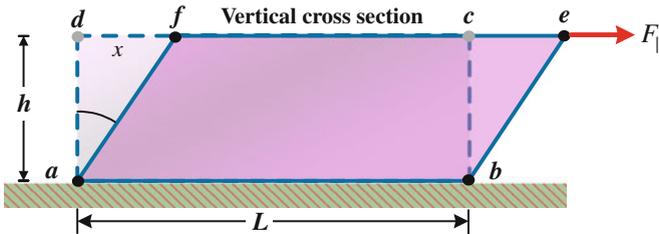
Fig. 10.37 See Exercise (9)

(10) Two parallel but opposite forces, each having a magnitude $F_\parallel = 4 \times 10^3$ N, are applied tangentially to the upper and lower faces of a cubical metal block of side $a = 25$ cm and shear modulus $S = 80 \times 10^9$ N/m^2, see Fig. 10.38. Find the displacement Δx of the upper surface relative to the lower one, and hence find the angle of shear θ.
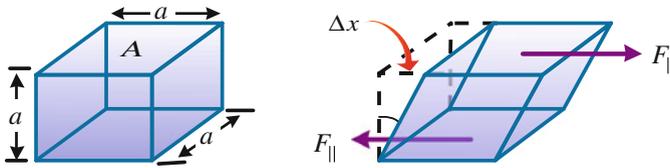
Fig. 10.38 See Exercise (10)

(11) Show that the angle of twist θ (in radians) for a torsional shearing caused by a tangential force $F_\parallel$ on the top of a cylindrical rod of height h, radius R, and shearing modulus S, see Fig. 10.39, is given by:

$$\theta = \frac{F_\parallel h}{\pi R^3 S}$$

Find θ when $F_\parallel = 500\,\text{N}$, $S = 80 \times 10^9\,\text{N/m}^2$, $R = 2.5\,\text{cm}$, and $h = 3\,\text{m}$.

Fig. 10.39 See Exercise (11)
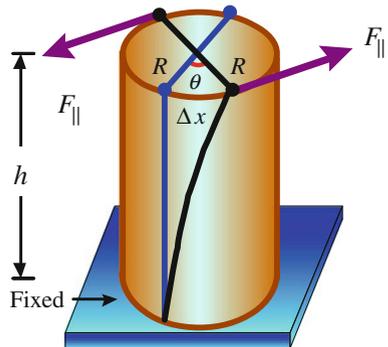
(12) The pressure of the atmosphere around a metal block is reduced to almost zero when the block is placed in vacuum. Find the fractional change in volume if the bulk modulus of the metal is $B = 150 \times 10^9\,\text{N/m}^2$.

(13) The pressure around a cube of copper of side 40 mm is changed by $\Delta P = 2 \times 10^{10}\,\text{N/m}^2$. Find the change in volume if the bulk modulus for copper is $B = 125 \times 10^9\,\text{N/m}^2$.

(14) What increase in pressure is required to decrease the volume of 200 liters of water by 0.004%? Take the bulk modulus of water B to be $2.1 \times 10^9\,\text{N/m}^2$.

(15) In an experiment, 750 cm^3 of water expands to 765 cm^3 when heated. What increase in pressure is required to squeeze the water back to its original volume? (Water bulk modulus is $2 \times 10^9\,\text{N/m}^2$)

Section 10.4 Fluid Statics

(16) A piston that has a cross-sectional area of $8\,\text{cm}^2$ and a mass of $20\,\text{kg}$ holds a compressed gas in a tank as shown in Fig. 10.40. What is the total pressure of the gas in the tank? What would an ordinary pressure gauge inside the tank read?

Fig. 10.40 See Exercise (16)

P_a

Gas
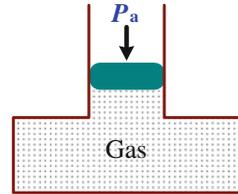
(17) A vessel contains mercury of height $h_\text{m} = 10\,\text{cm}$ and water of height $h_\text{w} = 30\,\text{cm}$, see Fig. 10.41. The density of mercury is $\rho_\text{m} = 13.6 \times 10^3\,\text{kg/m}^3$ and the density of water is $\rho_\text{w} = 1.0 \times 10^3\,\text{kg/m}^3$. Find the pressure exerted by the two liquids on the bottom of the vessel.

Fig. 10.41 See Exercise (17)

Water

h_w

Mercury

h_m
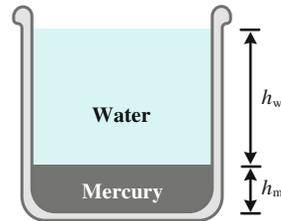
(18) If a water gauge pressure at the ground floor of a building reads $280\,\text{kPa}$, how high will the water rise in the pipes of that building. Take density of water to be $\rho_\text{w} = 1.0 \times 10^3\,\text{kg/m}^3$.

(19) Two liquids are placed in a U-shaped tube as shown in Fig. 10.42. (a) Show that the heights of the liquids above their surface of separation are inversely proportional to their densities. (b) Assume that the two liquids are water and oil where $h_1 = 20\,\text{cm}$ and $h_2 = 25\,\text{cm}$. Find the density of oil if the density of water is $\rho_1 = 1.0 \times 10^3\,\text{kg/m}^3$.

(20) The special manometer shown in Fig. 10.43, uses mercury of density $\rho = 13.6 \times 10^3\,\text{kg/m}^3$. If the atmospheric pressure is $100\,\text{kPa}$ and the height of mercury above the surface of separation at point B is $h = 10\,\text{cm}$, what is the pressure of the gas tank?
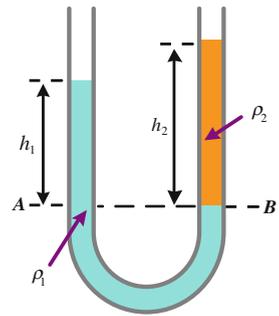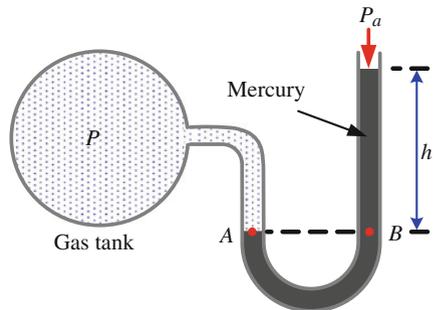
Fig. 10.42 See Exercise (19)

Fig. 10.43 See Exercise (20)

(21) Assume the value of the atmospheric pressure P_a is 1.01×10^5 Pa and water of density $\rho = 10^3$ kg/m^3 is used as a fluid in the barometer of Fig. 10.44. (a) What will be the height of the water column? (b) Repeat part (a) when water is replaced by alcohol of density $\rho = 7.9 \times 10^2$ kg/m^3. Comment on the practicality of your answers.

Fig. 10.44 See Exercise (21)
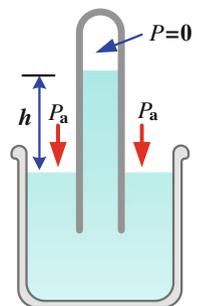
(22) The areas of the car lift pistons, shown in Fig. 10.45, are $A_1 = 25$ cm^2 and $A_2 = 500$ cm^2. The car and the right piston have a total weight of 10^4 N, while the left piston has a negligible weight and is at a height $h = 10$ m with respect

to the right one. The apparatus is filled with oil of density $\rho = 800\,\text{kg/m}^3$. What is the value of the force F_1 needed to keep the system in equilibrium?

Fig. 10.45 See Exercise (22)
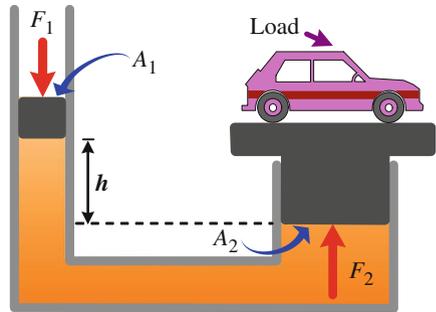
(23) A piece of wood has a mass $m = 0.25\,\text{kg}$ and a density $\rho = 750\,\text{kg/m}^3$. The wood is tied by a string to the bottom of a container of water in order to have the wooden piece fully immersed, see Fig. 10.46. Take the water density to be $\rho_\text{w} = 1.0 \times 10^3\,\text{kg/m}^3$. (a) What is the magnitude of the buoyant force F_B on the wood? (b) What is the magnitude of the tension T in the string?

Fig. 10.46 See Exercise (23)
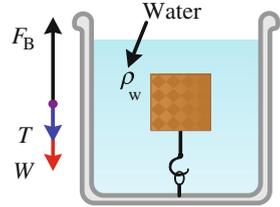
(24) Figure 10.47 shows a metal ball weighing $T = W = 9.5 \times 10^{-2}$ N in air. When the ball is immersed in water of density $\rho_\text{w} = 10^3\,\text{kg/m}^3$, it is found that it has an apparent weight $T_\text{w} = W_\text{w} = 7.0 \times 10^{-2}$ N. Find the density of the metal.

Fig. 10.47 See Exercise (24)
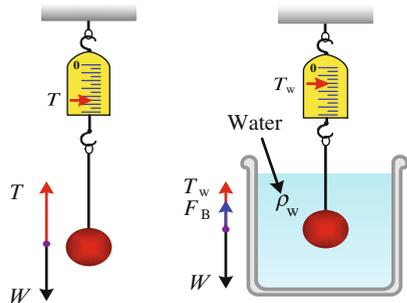
(25) A cubical block of side $a = 0.75$ cm floats in oil of density $\rho_o = 800$ kg/m^3 with one-third of its side out the oil, see Fig. 10.48. (a) What is the magnitude of the buoyant force on the cube? (b) What is the density ρ_b of the block?

Fig. 10.48 See Exercise (25)
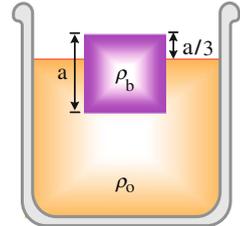
Section 10.5 Fluid Dynamics

(26) A fluid flows in a cylindrical pipe of radius r_1 with a speed v_1. (a) What would be the speed of this fluid at a point where the fluid is confined to a cylindrical part of radius $r_2 = r_1/5$, see the top part of Fig. 10.49. (b) What is the effect of elevating the constriction in the pipe by $h = 10$ m, see the lower part of Fig. 10.49?

Fig. 10.49 See Exercise (26)
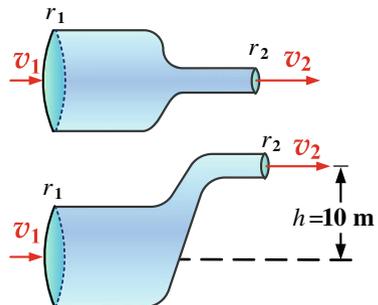
(27) Water with a density of 10^3 kg/m^3 is flowing steadily through a closed-pipe system via the motor M, see Fig. 10.50. At height y_1, the water speed and pressure are $v_1 = 4$ m/s and $P_1 = 30$ kPa, respectively. At height y_2, which is a point higher than y_1 by a height $h = 2$ m, the water speed is $v_2 = 6$ m/s. (a) What is the pressure at y_2? (b) What would be the pressure at y_2 if the water in the closed system was to stop flowing and the pressure at y_1 were 25 kPa?

(28) A large tank is kept full at a height of $h = 4$ m as shown in Fig. 10.51. Take the water density to be $\rho = 1.0 \times 10^3$ kg/m^3. (a) Find the speed v_2 of the jet

of water emerging from a small pipe at the bottom of the tank. (b) If $y_2 = 2$ m, then find the horizontal distance x (from the base of the tank) that the water stream travels before striking the floor.
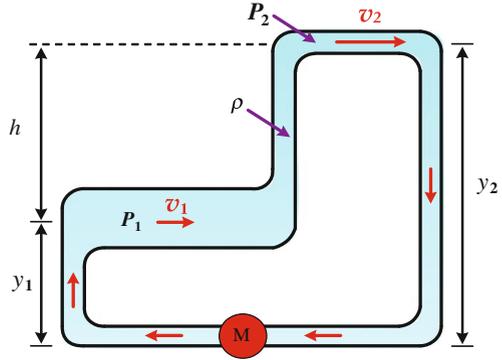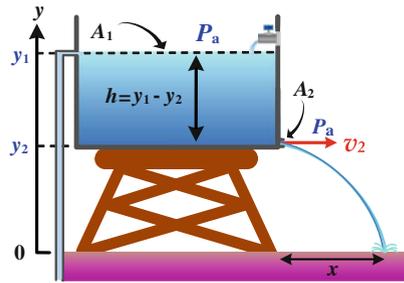
Fig. 10.50 See Exercise (27)

**Fig. 10.51** See Exercise (28)

(29) Assume the faucet in the previous exercise is closed. In terms of the cross-sectional areas A_2 and A_1 of the small pipe and the tank, respectively, show that v_2 and x depend on the variable height h as follows:

$$v_2 = \sqrt{\frac{2gh}{[1 - (A_2/A_1)^2]}} \quad , \quad x = 2\sqrt{\frac{y_2 h}{[1 - (A_2/A_1)^2]}}$$

(30) Figure 10.52 shows a tube of uniform cross section that is filled with water to siphon water at a steady rate from a large vessel. (a) Use Torricelli's approach to find an expression for the speed at level C. (b) Use Bernoulli's equation to find an expression for the pressure at level B in terms of P_a, H, and h. (c) Find the maximum value H_{max} of H for which the siphon will work. (d) Calculate the answers to the previous parts when $\rho_w = 10^3$ kg/m^3, $P_a = 100$ kPa, $H = 2$ m, and $h = 3$ m.

Fig. 10.52 See Exercise (30)
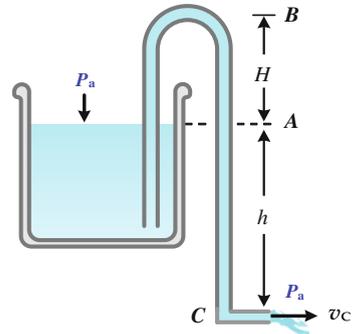
(31) A child tows a thin piece of wood of surface area $A = 200\,\text{cm}^2$ through a water puddle at a constant speed $v = 15\,\text{cm/s}$, see Fig. 10.53. The depth of the water puddle is $d = 2\,\text{mm}$ and its viscosity is $\eta = 1\,\text{cP}$. Assume that the velocity gradient is constant from the bottom to the surface of the water. Find the horizontal force component F exerted by the tow cord.

Fig. 10.53 See Exercise (31)
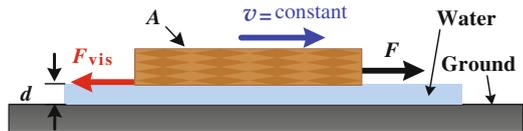
(32) A steel plate of weight $W = 0.5\,\text{N}$ and area $A = 0.2\,\text{m}^2$ is placed over a thin film of lubricant (oil) of thickness $d = 0.2\,\text{mm}$ sprayed over a flat surface inclined at an angle $\theta = 30°$, as shown in Fig. 10.54. What is the value of the constant speed of the plate v assuming that the rate of shear strain, v/d, is constant across the thickness of the film and the viscosity of the oil is $\eta = 0.05\,\text{Pa.s}$.

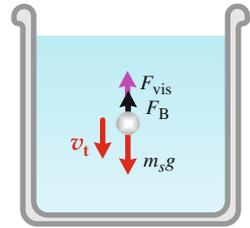**Fig. 10.54** See Exercise (32)
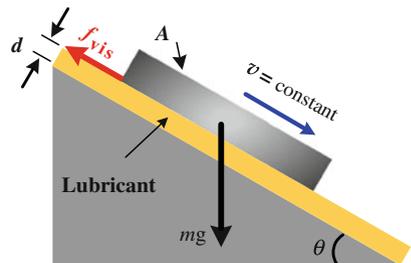
(33) How fast will an aluminum sphere of radius $1\,\text{mm}$ fall through water at $20°\text{C}$ once its terminal speed has been reached, see Fig. 10.55? Assume that the

viscosity of water is $\eta = 8.5\,\mathrm{Pl}$ and that the aluminum sphere's density is $\rho_{\mathrm{s}} = 2.7 \times 10^3\,\mathrm{kg/m^3}$.

Fig. 10.55 See Exercise (33)

(34) The viscous force on a liquid flowing steadily through a cylindrical pipe of length L is given by:

$$F_{\mathrm{vis}} = 4\pi\,\eta L v_{\mathrm{m}}$$

where η is the viscosity of the liquid and v_{m} is the maximum speed of the liquid which occurs along the central axis of the pipe, see Fig. 10.56. If the pressures in the rear and front horizontal segments of the pipe are respectively P_1 and P_2, where $P_1 > P_2$, then show that v_{m} will be given by:

$$v_{\mathrm{m}} = (P_1 - P_2)\,r^2/4\eta L$$
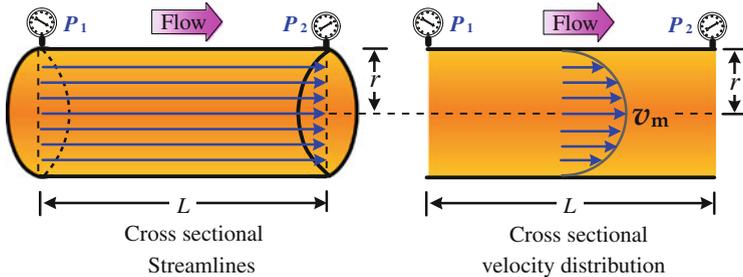
where r is the radius of the cylinder.

Fig. 10.56 See Exercise (34)

(35) Blood of viscosity $\eta = 4 \times 10^{-3}\,\mathrm{PI}$ is passing through a capillary of length $L = 1\,\mathrm{mm}$ and radius $r = 2\,\mu m$. If the speed of this blood as it travels through the center of this capillary is found to be $v_m = 0.66\,\mathrm{mm/s}$, calculate the blood pressure (in pascal and mm Hg) using the result from the previous exercise.