Thermal Properties of Matter 11

In this chapter, we introduce a physical quantity known as **temperature**, which is one of the seven SI base quantities. Temperature is associated with our sense of *hot* and *cold*. Physicists and engineers measure temperature more objectively using the **Kelvin scale**, which is independent of the properties of any substance. We will study the effect of temperature on matter; solid, liquid, and gas.

11.1 Temperature

The Kelvin Scale

The limiting temperature of a body is taken as the zero of the Kelvin scale, and called the *absolute zero*.

To set up the Kelvin temperature scale, we select a standard fixed point and give it a standard fixed point temperature. We select the **triple point of water**, where liquid water, solid ice, and water vapor can coexist in thermal equilibrium at only specific values of pressure and temperature. By international agreement, at water vapor pressure of 4.58 mm Hg, the temperature of this mixture has been assigned a value 273.16 Kelvins, written as 273.16 K. That is:

$$T_3 = 273.16 \text{ K} \qquad \text{(Triple-point temperature)} \qquad (11.1)$$

where the subscript 3 denotes the triple point. This agreement sets the size of the kelvin as 1/273.16 of the difference between absolute zero and the triple-point temperature of water. This scale is used mostly in basic scientific calculations and studies.

H. A. Radi and J. O. Rasmussen, *Principles of Physics*,
Undergraduate Lecture Notes in Physics, DOI: 10.1007/978-3-642-23026-4_11,

On the Kelvin scale, measurements show that the lowest reached temperature is $\sim 10^{-10}$ K, and the freezing and boiling (at 1 atm. Pressure) temperature points are:

$$T_{ice} = 273.15\,\text{K} \quad \text{and} \quad T_{steam} = 373.15\,\text{K}$$

The Celsius Scale

The symbol °C stands for **degrees Celsius**. The *size* of 1 °C on the Celsius scale is the same as the *size* of 1 K on the Kelvin scale. However, the zero of the Celsius scale is shifted by 273.15° with respect to the absolute zero of the Kelvin scale. On the Celsius scale, any temperature value T_C is related to its Kelvin equivalent T by:

$$T_C = T - 273.15, \;\; (^\circ\text{C}) \quad \text{or} \quad T = T_C + 273.15, \;\; (\text{K}) \tag{11.2}$$

For example, the Celsius temperature of the triple point of water is 0.01 °C, because $T_3 = 273.16$ K and the ice point (273.15 K) corresponds to 0.00 °C, and the steam point (373.15 K) corresponds to 100.00 °C. Note that we do not use a degree mark in reporting Kelvin temperatures.

The Fahrenheit Scale

The symbol °F stands for **degrees Fahrenheit**. The Fahrenheit scale has a smaller degree size than the Celsius and has a different zero (the freezing point of a certain concentration of salt water). The relation between the Celsius and Fahrenheit scales is:

$$T_F = \tfrac{9}{5}T_C + 32 \quad (^\circ\text{F}) \tag{11.3}$$

Accordingly, 9 degrees on the Fahrenheit scale equals 5 degrees on the Celsius scale. Moreover, 0 °C = 32 °F and 100 °C = 212 °F. Table 11.1 shows some corresponding temperatures and Fig. 11.1 compares graphically the Kelvin, Celsius, and Fahrenheit scales.

Table 11.1 Some corresponding temperatures

Temperature	K	°C	°F
Boiling point of water (at 1 atm.)	373.15	100	212
Normal body temperature (average)	310.15	37	98.6
Triple point of water	273.16	0.01	32.02
Freezing point of water	273.15	0	32
Triple point of hydrogen	13.81	−259.34	−434.82
Absolute zero	0	−273.15	−459.67

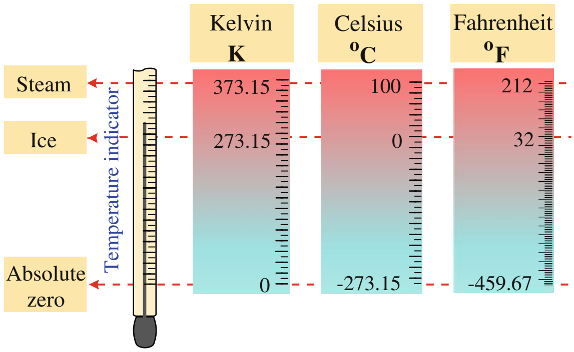


Fig. 11.1 The Kelvin, Celsius, and Fahrenheit temperature scales

Example 11.1

The normal boiling point of nitrogen is −195.75 °C. (a) What is this temperature in Kelvin and in Fahrenheit? (b) If the temperature changes from −195.75 °C to −100 °C, find the change in the temperature on the Fahrenheit scale.

Solution: (a) Substituting $T_C = -195.75\,°C$ into Eq. 11.2, we get:

$$T = T_C + 273.15 = -195.75 + 273.15 = 77.4 \text{ K}$$

Also, from Eq. 11.3, we get:

$$T_F = \tfrac{9}{5} T_C + 32 = \tfrac{9}{5} \times (-195.75) + 32 = -320.35\,°F$$

Thus, −195.75 °C, 77.4 K, and −320.35 °F are equivalent temperatures on different scales.

(b) For a change $\Delta T_C = [-100\,°C - (-195.75\,°C)] = 95.75\,C°$, we use Eq. 11.3 to find the change in temperature on the Fahrenheit scale as:

$$\Delta T_{\rm F} = \tfrac{9}{5}\Delta T_{\rm C} = \tfrac{9}{5}[-100 - (-195.75)] = 172.35\,{\rm F}^{\circ}$$

Thus, a change $95.75\,{\rm C}^{\circ} = 172.35\,{\rm F}^{\circ}$, where the notations C° and F° refer to temperature difference, not to be confused with actual temperatures, which are written in terms of symbols °C and °F.

11.2 Thermal Expansion of Solids and Liquids

Most bodies expand as their temperatures increase. This phenomenon plays an important role in numerous engineering applications, such as the joints in buildings, highways, railroad tracks, bridges . . . etc. Such thermal expansion is not always desirable.

Microscopically, thermal expansion arises from the change in the separation between the constituent atoms or molecules of the solid. To understand this, we consider a crystalline solid of a regular array of atoms or molecules held together by electrical forces. A mechanical model can be used to imagine the electrical interaction between the atoms or molecules, as shown in Fig. 11.2. At an ordinary temperature, the average spacing between the atoms is of the order of 10^{-10} m, and they vibrate about their equilibrium positions with an amplitude of about 10^{-11} m and a frequency of about 10^{13} Hz. As the temperature increases, the atoms vibrate with larger amplitudes and the average separation between them increases.

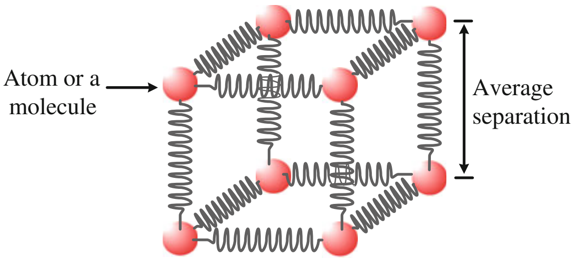


Fig. 11.2 A mechanical model representing the average spacing in a unit cell of crystalline solid at a given instant. Neighboring atoms or molecules (*red spheres*) are imagined to be attached to each other by elastic stiff springs, which represent the inter-atomic electric forces

If the thermal expansion of an object is sufficiently small compared to its initial dimensions, then the change in any dimension (length, width, or thickness) is, to a good approximation, a linear function of the temperature.

11.2.1 Linear Expansion

If a rod of length L and temperature T experiences a small change in temperature ΔT, its length changes by an amount ΔL, see Fig. 11.3. For a sufficiently small change ΔT, experiments show that ΔL is proportional to both L and ΔT. We introduce a proportionally coefficient α for the solid and write:

$$\Delta L = \alpha L \Delta T \tag{11.4}$$

where the proportionality constant α is called the **coefficient of linear expansion** for a given material. Note that Eq. 11.4 describes an expansion when ΔT is positive and a contraction when ΔT is negative. If we set[1] $\Delta T = 1\text{C}^\circ$, we see from Eq. 11.4 that α represents the fractional change in length $(\Delta L/L)$ per one degree change in temperature. Thus the unit of α is (degree^{-1}). An α value of $24 \times 10^{-6}\ (\text{C}^\circ)^{-1}$ means that the length L of an object changes by 24 parts per million for every Celsius degree change (C°) in temperature.

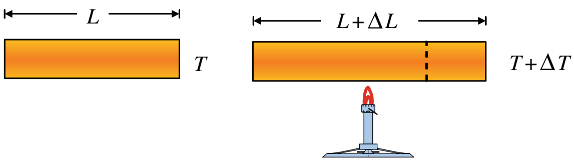


Fig. 11.3 The length L of the rod will increase by ΔL when its temperature changes from T to $T + \Delta T$. The expansion is exaggerated in the figure

Generally, the coefficient of linear expansion α varies with temperature, but this variation is negligible over the temperature range of most everyday measurements. Table 11.2 depicts some values of α.

Table 11.2 Coefficients of linear expansion α for some materials at room temperature (approximate)

Material Name	$\alpha\ (\text{C}^\circ)^{-1}$	Material Name	$\alpha\ (\text{C}^\circ)^{-1}$
Fused quartz	0.5×10^{-6}	Concrete	12×10^{-6}
Diamond	1.2×10^{-6}	Copper	17×10^{-6}
Glass (Pyrex)	3.2×10^{-6}	Brass & bronze	19×10^{-6}
Glass (ordinary)	9×10^{-6}	Aluminum	24×10^{-6}
Steel	11×10^{-6}	Lead	29×10^{-6}

[1] As in Example 11.1, temperature changes are expressed in units of Celsius degrees, abbreviated C° which should not to be confused with actual temperatures, written with the symbol °C and read degree Celsius.

Example 11.2

A steel rod has a length $L = 8$ m and radius $r = 1.5$ cm when the temperature is 20 °C. Take $\alpha = 11 \times 10^{-6}\,(\text{C}^\circ)^{-1}$ and Young's modulus of the rod to be $Y = 200 \times 10^9\ \text{N/m}^2$ (a) What is its length on a hot day when the temperature is 50 °C? (b) If the rod's ends were originally fixed, then find the compression force on the rod?

Solution: (a) From Eq. 11.4, we can find the increase ΔL when the change in temperature is $\Delta T_C = 50\,°\text{C} - 20\,°\text{C} = 30\,\text{C}^\circ$ as follows:

$$\Delta L = \alpha L \Delta T = [11 \times 10^{-6} (\text{C}^\circ)^{-1}](8\ \text{m})(30\ \text{C}^\circ) = 2.64 \times 10^{-3}\ \text{m} = 2.64\ \text{mm}$$

Therefore, the rod's new length at 50 °C is 8.00264 m.

(b) If the rod is not allowed to expand, we then calculate what force would be required to compress the rod by the amount 2.64×10^{-3} m. From the definition of Young's modulus $Y = (F_\perp/A)/(\Delta L/L)$:

$$\begin{aligned} F_\perp &= \frac{AY\Delta L}{L} = \frac{\pi r^2 Y \Delta L}{L} \\ &= \frac{(3.14)(1.5 \times 10^{-2}\ \text{m})^2 (200 \times 10^9\ \text{N/m}^2)(2.64 \times 10^{-3}\ \text{m})}{(8\ \text{m})} \\ &= 4.7 \times 10^4\ \text{N} \end{aligned}$$

Note that this answer is independent of the length L.

11.2.2 Volume Expansion

Not only does the length of an object increase with temperature, but its area and volume change as well. The change in volume ΔV at a constant pressure is proportional to the original volume V and to the change in temperature ΔT according to the following relation:

$$\Delta V = \beta\, V \Delta T \tag{11.5}$$

where the proportionality constant β is called the **coefficient of volume expansion** for a given solid or liquid. Setting $\Delta T = 1\ \text{C}^\circ$ in Eq. 11.5, we see that β is numerically equal to the fractional change in volume $(\Delta V/V)$ per one degree change in temperature. Thus, like α, the unit of β is (deg^{-1}). For example, a β value of

24×10^{-3} $(C°)^{-1}$ means that the volume V of a solid or liquid changes by 24 parts in 10^3 for every Celsius degree change (C°) in temperature.

An **isotropic solid** is a solid that has a coefficient of linear expansion that is equal in all directions. Accordingly, for an isotropic solid, the coefficient of volume expansion is approximately three times the linear expansion coefficient, i.e. $\beta = 3\alpha$. Table 11.3 depicts some values of β.

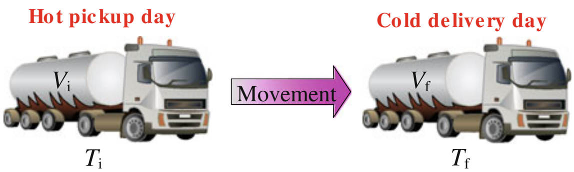
Table 11.3 Coefficients of volume expansion β for some materials at room temperature (approximate)

Material Name	β $(C°)^{-1}$	Material Name	β $(C°)^{-1}$
Alcohol, ethyl	1.12×10^{-4}	Water	6.3×10^{-4}
Benzene	1.24×10^{-4}	Turpentine	9×10^{-4}
Acetone	1.5×10^{-4}	Gasoline	9.6×10^{-4}
Mercury	1.82×10^{-4}	Air	3.67×10^{-3}
Glycerin	4.85×10^{-4}	Helium	3.665×10^{-3}

Example 11.3

On a hot day, when the temperature was $T_i = 45$ °C, an oil trucker fully loaded his truck from an oil station with 10,000 gal of gasoline (1 gallon $\simeq$ 3.8 liter). On his way to a delivery city, he encountered cold weather, where the temperature went down to $T_f = 20$ °C, see Fig. 11.4. The coefficients of volume expansion of gasoline and steel are $\beta = 9.6 \times 10^{-4}$ $(C°)^{-1}$ and $\beta = 11 \times 10^{-6}$ $(C°)^{-1}$, respectively. How many gallons did the trucker deliver?

Fig. 11.4



Solution: The change in temperature from the production city to the delivery city is $\Delta T_C = T_f - T_i = 20\,°C - 45\,°C = -25$ C°. From Eq. 11.5, we can find the change in the gasoline volume ΔV as follows:

$$\Delta V = \beta V \Delta T = [9.6 \times 10^{-4}\,(C°)^{-1}](10{,}000\text{ gal})(-25\text{ C°})$$
$$= -240\text{ gal}$$

Thus, the amount of gasoline delivered was:

$$V_f = V_i + \Delta V = 10{,}000\,\text{gal} - 240\,\text{gal}$$
$$= 9{,}760\,\text{gal}$$

The thermal expansion of the volume of the steel tank can also be calculated as follows:

$$\Delta V = \beta V \Delta T = [11 \times 10^{-6}\,(\text{C}°)^{-1}](10{,}000\,\text{gal})(-25\,\text{C}°)$$
$$= -2.75\ \text{gal}$$

This change is very small and has nothing to do with the problem, since the decrease in the gasoline volume is much bigger than that of the steel. Question: Who paid for the missing gasoline?

The most common liquid, water, does not behave like other liquids, see Fig. 11.5a. Above 4 °C, water expands as its temperature rises, and thus its density decreases as shown in the Fig. 11.5b. Between 0 °C and 4 °C, however, water contracts as its temperature increases, and thus its density increases. Hence, the density of water reaches a maximum value of 1,000 kg/m^3 at 4 °C.

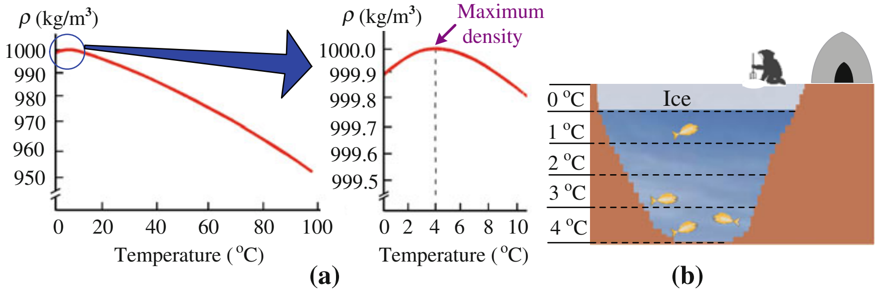


Fig. 11.5 (a) The density of water versus temperature at atmospheric pressure. The maximum density of 10^3 kg/m^3 occurs at 4 °C. (b) The warmer water from 1 to 4 degrees stays below the ice because it is more dense than the ice

This unusual thermal expansion behavior of water explains why a pond or lake frezes only at its surface. As the water on the surface is cooled towards the freezing point, it becomes denser (heavier) than the water below it and sinks to the bottom. Warmer, less dense (lighter) water rises upwards to take its place and this in turn is also cooled down. The water only stops circulating this way when it has all cooled to 4 °C (the maximum density). Further cooling below 4 °C makes the water on the surface less dense than the water below it, so it stays on the surface until it freezes. In time, ice continues to build up at the surface, and the denser warmer water at the bottom is unlikely to cool any further because it does not circulate, and water near the bottom remains at 4 °C. The water temperature stabilizes as shown in Fig. 11.5b. Fish can survive by staying in the warmer deeper water.

11.3 The Ideal Gas

Let us examine the basic thermal properties of gases from an elementary point of view. To do that, we will consider the properties of a gas of mass m confined within a container of volume V at absolute pressure P and temperature T. The relation that interrelates these quantities, the *equation of state,* is complicated. However, if the gas is maintained at a very low pressure (or density), this equation is found experimentally to be quite simple, and this keeps the mathematics relatively simple. This model is known as the **ideal gas model** and the low-pressure gas is commonly referred to as an **ideal gas**. Most gases at room temperature and atmospheric pressure behave as ideal gases.

Equation of State of an Ideal Gas

One mole (1 mol) of a substance is the amount of substance that contains as many particles as in exactly 12 g of the isotope carbon-12. Although the **mole** is one of the seven SI base units, it is convenient to introduce the **One kilomole** (1 kmol) of a substance as the amount of substance that contains as many particles as in exactly 12 kilograms of the isotope carbon-12. Thus:

> One kilomole (1 kmol):
> One kmol is the *number* of atoms in a 12 kg sample of pure carbon-12.

This number is called **Avogadro's number**, N_A, after A. Avogadro, who suggested that *all gases* contain the same number of particles (atoms or molecules) when they occupy the same volume under the same conditions of pressure and temperature. Avogadro's number is determined experimentally to be:

$$N_A = 6.022 \times 10^{26} \text{ particles/kmol} \tag{11.6}$$

Depending on the kind of study, atoms or molecules will replace the term "particles".

In addition, the **molar mass** of each chemical element is defined as:

Molar Mass (M):
The molar mass M of a chemical element is its atomic mass expressed in g/mol or equivalently in kg/kmol.

For example, the mass of one ^{12}C atom is 12 u (12 atomic mass units, see Chap. 1); then the *molar mass* M of ^{12}C is 12 kg/kmol and contains N_A atoms. For a molecular substance or chemical compound, we add up the molar mass from its molecular formula. Thus, the *molar mass* M of nitrogen gas (N_2) is 28 kg/kmol and it consists of N_A molecules; this is because the mass of one nitrogen atom is 14 u.

Now, for an ideal gas of mass m (in kg) confined to a container of volume V (in m^3) at a pressure P (in Pa) and temperature T (in K), it is convenient to express the amount of the gas in terms of the **number of kilomoles** n, see Fig. 11.6. This number is related to the gas mass m and its molar mass M through the expression:

$$n = \frac{m}{M} \qquad (m \text{ in kg and } M \text{ in kg/kmol}) \tag{11.7}$$

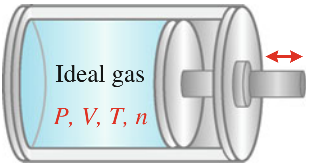


Fig. 11.6 An ideal gas defined by P, V, T, and n is contained in a cylinder with a movable piston to allow the volume to be varied

Moreover, we consider that the volume of the container can be varied, and hence the gas volume, by means of the movable piston of Fig. 11.6.

For this system, experiments show that at low densities, all gases tend to obey the following **equation of state** (which is known as the **ideal gas law**):

$$PV = nRT \qquad \text{(The ideal gas law)} \tag{11.8}$$

where n is the number of kilomoles of gas present and R is the **gas constant** which is determined from experiments to have the same value for all gases, namely:

$$R = \begin{cases} 8.314 \times 10^3 \text{ J/kmol.K} & \text{if } n \text{ is the number of kmol} \\ 8.314 \text{ J/mol.K} & \text{if } n \text{ is the number of mol} \end{cases} \tag{11.9}$$

Using this value of R and the equation of state, Eq. 11.8, we can find the volume occupied by 1 kmol (kilomolar volume) of any ideal gas at the *standard temperature and pressure* (**STP**), which means temperature of 0 °C (273.15 K) and atmospheric pressure (1 atm), as follows:

$$V = \frac{nRT}{P} = \frac{(1\ \text{kmol})(8.314 \times 10^3\ \text{J/kmol.K})(273.15\ \text{K})}{(1.013 \times 10^5\ \text{Pa})} = 22.42\ \text{m}^3 = 22{,}420\ \text{L}$$

where $1\ \text{L} \equiv 1\ \text{liter} = 10^3\ \text{cm}^3 = 10^{-3}\ \text{m}^3$. Thus:

> Volume of 1 kmol:
> One kmol of any ideal gas at atmospheric pressure and at 0 °C occupies a space of $22.42\ \text{m}^3 = 22{,}420$ L.

We can express the ideal gas law in terms of the total number of molecules N by using the fact that N equals the product of the number of kmol n and Avogadro's number N_A, i.e. $N = n\,N_A$. Thus:

$$PV = nRT = \frac{N}{N_A}RT = N\frac{R}{N_A}T$$

or,

$$PV = Nk_B T \tag{11.10}$$

where k_B is called Boltzmann's constant, which has the value:

$$k_B = \frac{R}{N_A} = \frac{8{,}314\ \text{J/kmol.K}}{6.022 \times 10^{26}\ \text{molecules/kmol}} = 1.38 \times 10^{-23}\ \text{J/K} \tag{11.11}$$

Equation 11.10 indicates that the pressure of a fixed volume of gas depends only on the temperature and the number of molecules in that volume.

Example 11.4

According to the periodic table of elements, see Appendix C, the molar mass of copper is $M(\text{Cu}) = 63.546$ kg/kmol. Use this information to find the mass of one atom.

Solution: The molar mass of $^{63.5}$Cu is $M(\text{Cu}) = 63.546$ kg/kmol and contains $N_A = 6.022 \times 10^{26}$ atoms/kmol. The mass of 1 atom is then:

$$\text{Mass of one Cu atom} = \frac{M(\text{Cu})}{N_A} = \frac{63.546\,\text{kg/kmol}}{6.022 \times 10^{26}\,\text{atoms/kmol}}$$
$$= 1.059 \times 10^{-25}\,\text{kg/atom}$$

Example 11.5

The main constituents of air are nitrogen molecules of molar mass $M(N_2) = 28\,\text{kg/kmol}$ and oxygen molecules of molar mass $M(O_2) = 32\,\text{kg/kmol}$ with approximate proportions of 80% and 20%, respectively. Using the ideal gas law, find the mass of air in a volume of $50\,\text{cm}^3$ at a pressure of 700 torr and temperature of 20 °C.

Solution: The molar mass of air can be obtained from the ratios of the two gases as follows:

$$M(\text{air}) = 0.8\,M(N_2) + 0.2\,M(O_2)$$
$$= 0.8\ (28\ \text{kg/kmol}) + 0.2\ (32\ \text{kg/kmol}) = 28.8\ \text{kg/kmol}$$

The volume, pressure, and temperature values can be written as:

$$V = 50\ \text{cm}^3 = 5 \times 10^{-5}\ \text{m}^3$$
$$P = 700\ \text{torr} = 700\ \text{torr} \times \frac{1\ \text{atm}}{760\ \text{torr}} \times \frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}} = 9.3 \times 10^4\ \text{Pa}$$
$$T = 20\,^\circ\text{C} = 20 + 273 = 293\,\text{K} \quad \text{(From now on, we ignore the 0.15 K)}$$

We can use the ideal-gas equation $PV = nRT$ with $n = m/M(\text{air})$, where m is the mass of air under consideration, to find m as follows:

$$m = \frac{PVM(\text{air})}{RT} = \frac{(9.3 \times 10^4\ \text{Pa})(5 \times 10^{-5}\ \text{m}^3)(28.8\ \text{kg/kmol})}{(8.314 \times 10^3\ \text{J/kmol.K})(293\ \text{K})} = 5.5 \times 10^{-5}\ \text{kg}$$

Example 11.6

(a) How many molecules are there in $1\,\text{cm}^3$ of air at room temperature (27 °C)? (b) How many kilomoles of air are in that volume? (c) The best vacuum that can be produced corresponds to a pressure of about 10^{-16} atm. How many molecules remain in 1 cm^3?

Solution: The number of molecules in 1 cm^3 can be calculated from the ideal gas equation $PV = Nk_BT$. (a) Rewriting the quantities given, we have:

$$V = 1\ cm^3 = 10^{-6}\ m^3$$
$$P = 1\ atm \simeq 10^5\ Pa$$
$$T = 27\,°C = 27 + 273 = 300\ K$$

$$\text{Thus:}\quad N = \frac{PV}{k_BT} = \frac{(10^5\ Pa)(10^{-6}\ m^3)}{(1.38 \times 10^{-23}\ J/K)(300\ K)} = 2.4 \times 10^{19}\ \text{molecules}$$

(b) We use the ideal gas equation $PV = nRT$ to calculate the number of kilo-moles as follows:

$$n = \frac{PV}{RT} = \frac{(10^5\ Pa)(10^{-6}\ m^3)}{(8.314 \times 10^3\ J/kmol.K)(300\ K)} = 4 \times 10^{-8}\ kmol$$

Also, we can use the relation $N = n\,N_A$ to get n as follows:

$$n = \frac{N}{N_A} = \frac{2.4 \times 10^{19}\ \text{molecules}}{6.022 \times 10^{26}\ \text{molecules/kmol}} = 4 \times 10^{-8}\ kmol$$

(c) Rewriting the quantities given, we have:

$$V = 1\ cm^3 = 10^{-6}\ m^3$$
$$P = 10^{-16}\ atm \simeq 10^{-11}\ Pa$$
$$T = 27\,°C = 27 + 273 = 300\ K$$

$$\text{Thus:}\qquad N = \frac{PV}{k_BT} = \frac{(10^{-11}\ Pa)(10^{-6}\ m^3)}{(1.38 \times 10^{-23}\ J/K)(300\ K)} = 2{,}415\ \text{molecules}$$

There are still a large number of molecules left in this 1 cm^3 vacuum.

Example 11.7

A metal barrel is filled with air and is closed firmly when the pressure is 1 atm and the temperature is 20 °C. On a hot sunny day, the barrel's temperature rises to 60 °C while its volume remains almost the same. Find the final pressure inside the barrel.

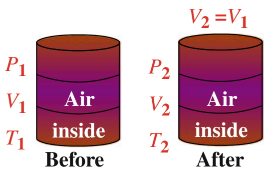
Solution: We mark the initial state of air with P_1, V_1, T_1 and final state with P_2, V_2, T_2, see Fig. 11.7. If no air escapes from the barrel, the number of moles of air n remains constant. Therefore, using the ideal gas law $PV = nRT$ in the initial and final states and the fact that $V_2 = V_1$, we get:

$$nR = \frac{P_1\,V_1}{T_1} = \frac{P_2\,V_2}{T_2} \quad \Rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \Rightarrow \quad P_2 = P_1\frac{T_2}{T_1}$$

The quantities given are: $\begin{cases} P_1 = 1\text{ atm} = 1.01 \times 10^5\text{ Pa} \\ T_1 = 20\,°\text{C} = 20 + 273 = 293\text{ K} \\ T_2 = 60\,°\text{C} = 60 + 273 = 333\text{ K} \end{cases}$

Thus: $$P_2 = (1\,\text{atm})\frac{333\text{ K}}{293\text{ K}} = 1.14\text{ atm} = 1.15 \times 10^5\text{ Pa}$$

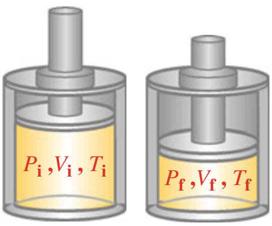
Fig. 11.7



Example 11.8

The initial volume, pressure, and temperature of helium gas trapped in a container with a movable piston are $2 \times 10^{-3}\text{ m}^3$, 150 kPa, and 300 K, respectively; see Fig. 11.8. If the volume is decreased to $1.5 \times 10^{-3}\text{ m}^3$ and the pressure increases to 300 kPa find the final temperature of the gas, assuming it behaves like an ideal gas.

Fig. 11.8



Solution: The initial state of helium is P_{i}, V_{i}, T_{i} and final state is P_{f}, V_{f}, T_{f}. With the use of the ideal gas law $PV = nRT$, we get:

$$\frac{P_{\text{i}}\ V_{\text{i}}}{T_{\text{i}}} = \frac{P_{\text{f}}\ V_{\text{f}}}{T_{\text{f}}} \quad \Rightarrow \quad T_{\text{f}} = T_{\text{i}}\frac{P_{\text{f}}\ V_{\text{f}}}{P_{\text{i}}\ V_{\text{i}}}$$
$$= (300\ \text{K})\frac{(300\ \text{kPa})(1.5 \times 10^{-3}\ \text{m}^3)}{(150\ \text{kPa})(2 \times 10^{-3}\ \text{m}^3)} = 450\ \text{K}$$

11.4 Exercises

Section 11.1 Temperature

(1) Convert the temperatures −30 °C, 10 °C, and 50 °C to Kelvin and Fahrenheit.

(2) Express the normal human body temperature, 37 °C, and the sun's surface temperature, ∼6000 °C, in Fahrenheit and Kelvin.

(3) A Celsius thermometer indicates a temperature of −40 °C. (a) What Fahrenheit and Kelvin temperatures correspond to this Celsius temperature? (b) If the temperature changes from −40 °C to +10 °C, find the change in temperature on the Fahrenheit scale.

(4) The normal melting point of gold is 1064.5 °C and its boiling point is 2660 °C. (a) Convert these two values to the Fahrenheit and Kelvin scales. (b) Find the difference between those two values in Celsius. (c) Repeat (b) using the Kelvin scale.

(5) The height of an alcohol column in an alcohol thermometer has a length 12 cm at 0 °C and a length 22 cm at 100 °C. Assume that the temperature and the length of the alcohol thermometer are linearly related. What is the temperature that the thermometer will measure if the alcohol column has a length 12.5 cm?

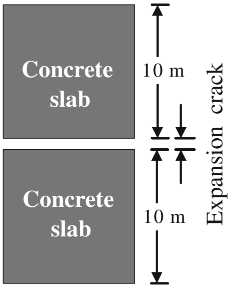
Section 11.2 Thermal Expansion of Solids and Liquids

(6) The Eiffel tower is built from iron and it is about 324 m high. Its coefficient of linear expansion is approximately $12 \times 10^{-6}\ (\text{C}^\circ)^{-1}$ and assumed constant. What is the increase in the tower's length when the temperature changes from 0 °C in winter to 30 °C?

(7) A copper rod is 8 m long at 20 °C and has a coefficient of linear expansion $\alpha = 17 \times 10^{-6}\,(\mathrm{C}^{\circ})^{-1}$. What is the increase in the rod's length when it is heated to 40 °C?

(8) A road is built from concrete slabs, each of 10 m long when formed at 10 °C, see Fig. 11.9. How wide should the expansion cracks between the slabs be at 10 °C to prevent road buckling if the range of temperature changes from −5 °C in winter to +40 °C in summer? The coefficient of linear expansion for concrete is $\alpha = 12 \times 10^{-6}\,(\mathrm{C}^{\circ})^{-1}$.

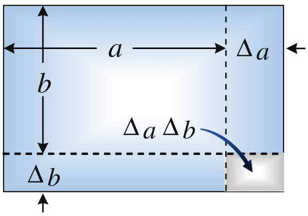
Fig. 11.9 See Exercise (8)



(9) An iron steam pipe is 100 m long at 0 °C and has a coefficient of linear expansion $\alpha = 10 \times 10^{-6}\,(\mathrm{C}^{\circ})^{-1}$. What will be its length when heated to 100 °C?

(10) An ordinary glass window has a coefficient of linear expansion $\alpha = 9 \times 10^{-6}\,(\mathrm{C}^{\circ})^{-1}$. At 20 °C the sides a and b have the values 1 m and 0.8 m respectively, see Fig. 11.10. By how much does the area increase when its temperature rises to 40 °C?

Fig. 11.10 See Exercise (10)



(11) A steel tape measure has a coefficient of linear expansion $\alpha = 12 \times 10^{-6}\,(\mathrm{C}^{\circ})^{-1}$ and is calibrated at 20 °C. On a cold day when the temperature is −20 °C, what will be the percentage error for a reading made using this tape measure?

(12) A bar of length $L = 4$ m and linear expansion $\alpha = 25 \times 10^{-6}$ $(\text{C}^\circ)^{-1}$ has a crack at its center. The ends of the bars are fixed as shown in Fig. 11.11. As a result of a temperature rise of 40 C°, the bar buckles upwards, see Fig. 11.11. Find the vertical rise d of the bar's center.

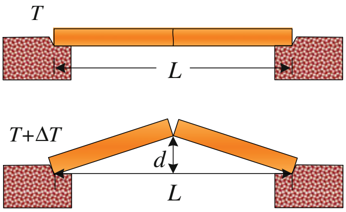


Fig. 11.11 See Exercise (12)

(13) A composite rod of length L is made from two different rods of lengths L_1 and L_2 with linear expansion coefficients of α_1 and α_2, respectively, see Fig. 11.12. (a) Show that the coefficient of linear expansion α for this composite rod is given by $\alpha = (\alpha_1 L_1 + \alpha_2 L_2)/L$. (b) Using the linear expansion coefficients of steel and brass given in Table 11.2, find L_1 and L_2 in the case where $L = 0.8$ m and $\alpha = 14 \times 10^{-6}$ $(\text{C}^\circ)^{-1}$.

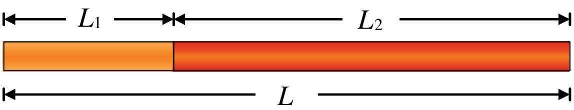


Fig. 11.12 See Exercise (13)

(14) A homogeneous metal ring of temperature T has inner and outer radii a and b, respectively. As the metal ring is heated to a temperature of $T + \Delta T$, its inner and outer radii increase linearly to $a + \Delta a$ and $b + \Delta b$ respectively, see Fig. 11.13. Show that the heating has no effect on the ratio between the inner and the outer radii.

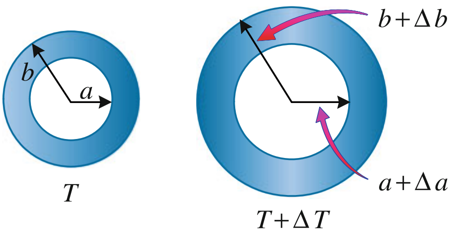


Fig. 11.13 See Exercise (14)

(15) A spherical brass plug has a diameter d of 10 cm at $T = 150\,\text{C}^\circ$ and has a coefficient of linear expansion $\alpha = 19 \times 10^{-6}\,(\text{C}^\circ)^{-1}$, see Fig. 11.14. At what temperature will its diameter be 9.950 cm?

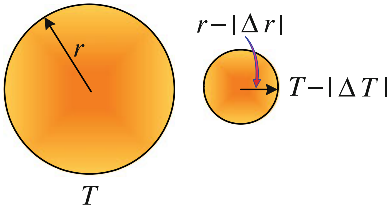


Fig. 11.14 See Exercise (15)

(16) Two rods of the same diameter, one made of brass of length $L_1 = 25$ cm, and the other rod made of steel of length $L_2 = 50$ cm, are placed end-to-end and pinned to two rigid supports, see Fig. 11.15. The Young's modulus for the brass and steel rods are $Y_1 = 100 \times 10^9\,\text{N/m}^2$ and $Y_2 = 200 \times 10^9\,\text{N/m}^2$ respectively, and their respective coefficients of linear expansion are $\alpha_1 = 18 \times 10^{-6}\,(\text{C}^\circ)^{-1}$ and $\alpha_2 = 12 \times 10^{-6}\,(\text{C}^\circ)^{-1}$. The two rods are heated until the rise in temperature becomes $\Delta T = 40\,\text{C}^\circ$. What is the stress in each rod?

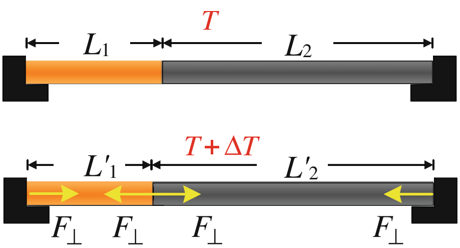


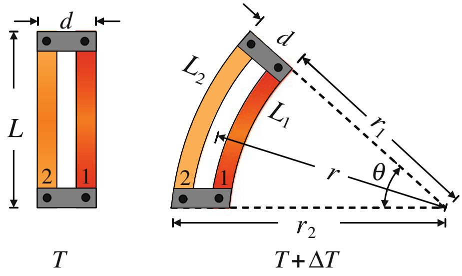
Fig. 11.15 See Exercise (16)

(17) Two parallel metal bars with the same length L and negligible width, but different linear expansion coefficients α_1 and α_2, are fixed at a distance d apart, see Fig. 11.16. When their temperature changes by ΔT, they will bend into two circular arcs intercepting at an angle θ as shown in Fig. 11.16. Find their mean radius of curvature r.

(18) Find the change in volume of an aluminum sphere that has a radius of 5 cm when it is heated from 0 °C to 300 °C. Assume that the coefficient of volume expansion is $\beta = 7.2 \times 10^{-5}\,(\text{C}^\circ)^{-1}$.

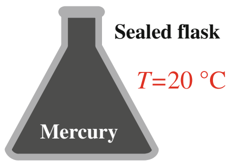
(19) A glass flask holds $50\,\text{cm}^3$ at a temperature of 20 °C. What is its capacity at 30 °C? Assume the coefficient of volume expansion of this glass flask is $2.7 \times 10^{-5}\,(\text{C}^\circ)^{-1}$.

Fig. 11.16 See Exercise (17)



(20) A flask is completely filled with mercury at 20 °C and is sealed off, see Fig. 11.17. Ignore the expansion of the glass and assume that the bulk modulus of mercury is $B = 2.5 \times 10^9\ \text{N/m}^2$ and its coefficient of volume expansion is $\beta = 1.82 \times 10^{-4}\ (\text{C}°)^{-1}$. Find the change in pressure inside the flask when it is heated to 100 °C.

Fig. 11.17 See Exercise (20)



(21) A glass flask of volume 200 cm^3 is filled with mercury when the temperature is $T = 20$ °C, see Fig. 11.18. The coefficient of volume expansion of the glass and mercury are $\beta = 1.2 \times 10^{-5}\ (\text{C}°)^{-1}$ and $\beta = 18 \times 10^{-5}\ (\text{C}°)^{-1}$ respectively. How much mercury will overflow when the temperature of the flask is raised to 100 °C?

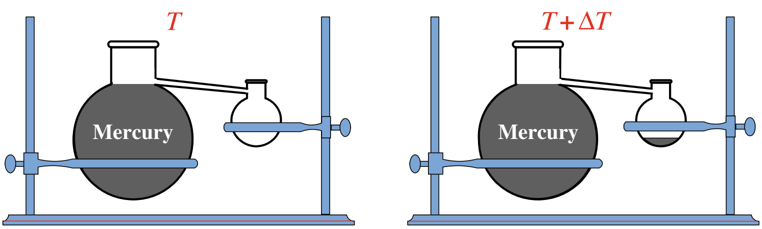


Fig. 11.18 See Exercise (21) (Take 1 atm $\simeq 10^5$ Pa unless specified)

Section 11.3 The Ideal Gas

(22) Find the density of nitrogen (N_2) and oxygen (O_2) at STP assuming they behave like an ideal gas.

(23) A tank contains $0.5\,\text{m}^3$ of nitrogen at a pressure of $1.5 \times 10^5\,\text{N/m}^2$ and a temperature of 27 °C. (a) What will be the pressure if the volume is increased to $5.0\,\text{m}^3$ and the temperature is raised to 327 °C? (b) Answer part (a) if the volume remains constant.

(24) A tank contains nitrogen N_2 at an absolute pressure of 2.5 atm. What will be the pressure of an equal mass of CO_2 that replaces the nitrogen at the same temperature?

(25) A tire is filled with air at 27 °C in a normal day to a gauge pressure of 2 atm. Then its temperature reaches 40 °C in a hot day. What fraction of the original air must be removed if the original pressure is to be restored?

(26) A 1,000 L container holds 50 kg of argon gas at 27 °C. The molar mass of argon is $M = 40$ kg/kmol. What is the pressure of the gas?

(27) A bubble of air rises from the bottom of a lake, where the pressure is 3 atm and the temperature is 7 °C, to the surface, where the pressure is 1 atm and the temperature is 27 °C, see Fig. 11.19. What is the ratio of the volume of the bubble just as it reaches the surface to its volume at the bottom?

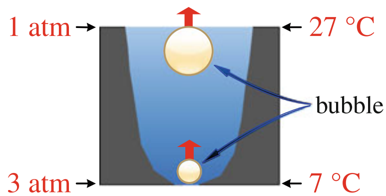
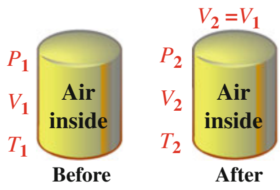


Fig. 11.19 See Exercise (27)

(28) (a) How many molecules are there in 1 L of air at a temperature of 27 °C? (b) How many kilomoles of air are in that volume? (c) The best vacuum that can be produced corresponds to a pressure of about 10^{-16} atm. How many molecules remain in 1 L?

(29) A cylindrical metallic container is filled with air and is closed firmly when the pressure is $P_i = 1$ atm and the temperature is $T_i = 27$ °C. In a very hot sunny day, the container's temperature rises to $T_f = 70$ °C while its volume remains almost the same, see Fig. 11.20. Find the final pressure inside the container.

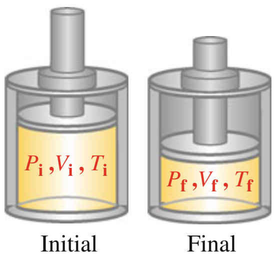
Fig. 11.20 See Exercise (29)



(30) The main constituents of air are nitrogen molecules of molar mass $M(N_2) = 28$ kg/kmol and oxygen molecules of molar mass $M(O_2) = 32$ kg/kmol with approximate ratios of 80 and 20%, respectively. Using the ideal gas law, find the mass of air in a volume of 1 L at atmospheric pressure and temperature of 27 °C.

(31) The initial volume, pressure, and temperature of helium gas trapped in a container with a movable piston are $V_i = 3$ L, $P_i = 150$ kPa, and $T_i = 300$ K, respectively, see Fig. 11.21. If the volume is decreased to $V_f = 2.5$ L and the pressure increases to $P_f = 300$ kPa, find the final temperature of the gas assuming that it behaves like an ideal gas.

Fig. 11.21 See Exercise (31)



(32) The volume of an oxygen tank is 50 L. As oxygen is withdrawn from the tank, the pressure of the remaining gas in the tank drops from 20 atm to 8 atm, and the temperature also drops from 30 to 10 °C. (a) How many kilograms of oxygen were originally in the tank? (b) How many kilograms of oxygen were withdrawn from the tank? (c) What volume would be occupied by the oxygen that withdrawn from the tank at a pressure of 1 atm and a temperature of 27 °C?

(33) A balloon filled with helium is left free on the surface of the ground when the temperature is 27 °C. When the balloon reaches an altitude of 3,000 m, where the temperature is 5 °C and the pressure is 0.65 atm, how will its volume compare to the original volume on the ground?

(34) The density of water vapor at exactly 100 °C and $1\,\text{atm} = 1.013 \times 10^5$ Pa is $\rho = 0.598\,\text{kg/m}^3$. Calculate the density of water vapor, with a molecular mass $M = 18$ kg/kmol, from the ideal gas law. Why would you expect a difference?

(35) An empty room of volume V contains air having a molar mass M. At atmospheric pressure P_a, the mass and temperature of the room are initially m_i and T_i, respectively. Assuming that the room is maintained at atmospheric pressure while its temperature is increased to T_f, show that the final mass of air left in the room, m_f, will be given by:

$$m_f = m_i - \frac{P_a V M}{R}\left(\frac{1}{T_i} - \frac{1}{T_f}\right).$$