Electron-Electron Interactions: Screening

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14.1 Introduction

In this chapter we will investigate how the presence of other charges and dipoles influences the charge-charge interaction.

Consider, for example, a net charge introduced into a semiconductor, and consider how the electrons in the conduction band react to it. The net charge could be, for example, the charge of the ionized Si impurity in GaAs or P impurity in Si. One can ask what is the electric field that the carriers see? Do they see the full Coulomb field of the ionized impurity or a reduced field? We know from elementary electromagnetic theory that the presence of an insulating medium around a charge partially screens the charge and introduces the bound electron permittivity term ε_b in the Coulomb potential, for example. We also saw in Chap. 10 how one can derive the permittivity of bound charges ε_b in a medium. But what about the presence of free electrons? What happens to the net field in a metal where we have both bound and free charges? We saw how the free carriers change the total permittivity of a system and how this can be incorporated in the optical properties of solids, but we did not look at the consequences for carrier-carrier and carrier-charge interactions. The free carriers were given a classical Drude treatment which is adequate for optics, but we did not investigate how the medium affects the net interaction between the charges themselves. To answer this question, we have to start from a first principle point of view and give the problem a quantum mechanical treatment. Let us examine these questions from a fundamental point of view following closely the book by J. Ziman (see references). We start by applying a general potential $V(r, t)$ to a medium where:

$$V(\bar{r}, t) = V_0 e^{ik'\cdot\vec{r}} e^{i\omega t} e^{\alpha t} \tag{14.1}$$

We have allowed the field to grow slowly to its full value with a time constant α in order not to cause large deviations from equilibrium. Now we go back and use time-dependent perturbation theory as we did before for bound electrons in the previous

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chapter and consider the first-order change in the wavefunction of a Bloch electron in a solid that this potential introduces (Ziman 1964):

$$\Psi_{\vec{k}}(\vec{r},t) = \left|\vec{k}\right\rangle + b_{\vec{k}+\vec{k'}}(t)\left|\vec{k}+\vec{k'}\right\rangle \tag{14.2}$$

whereas in Eq. (10.42), we have from perturbation theory:

$$b_{\vec{k}+\vec{k'}}(t) = \frac{\left\langle \vec{k}\left|V(\vec{k'},\vec{r},t)\right|\vec{k}+\vec{k'}\right\rangle}{E_{\vec{k}} - E_{\vec{k}+\vec{k'}} + \hbar\omega - i\hbar\alpha} \tag{14.3}$$

$$b_{\vec{k}+\vec{k'}}(t) = \frac{V_0 e^{i\omega t} e^{\alpha t}}{E_{\vec{k}} - E_{\vec{k}+\vec{k'}} + \hbar\omega - i\hbar\alpha} \tag{14.4}$$

The new wavefunction implies also a new charge distribution. Thus, we can compute by considering the deviation from the unperturbed distribution:

$$\delta\rho(\vec{r},t) = q\sum_{\vec{k}}\left\{\left|\Psi_{\vec{k}}(r,t)\right|^2 - 1\right\} \tag{14.5}$$

Substituting from Eq. (14.4) and keeping only terms in first order, we find:

$$\delta\rho(\vec{r},t) = q\sum_{\vec{k}}\left\{b_{\vec{k}+\vec{k'}}(t)e^{i\vec{k}.\vec{r}} + b^*_{\vec{k}+\vec{k'}}(t)e^{-i\vec{k'}.\vec{r}}\right\} \tag{14.6}$$

In practice, it is more convenient to work with a real perturbation, so let us write instead:

$$\delta V(\vec{r},t) = V_0 e^{i\vec{k'}.\vec{r}} e^{i\omega t} e^{\alpha t} + V_0^* e^{-i\vec{k'}.\vec{r}} e^{-i\omega t} e^{\alpha t} \tag{14.7}$$

Then it follows by substituting Eq. (14.7) into Eq. (14.3) and then Eq. (14.5) that:

$$\delta\rho = q\sum_{\vec{k}}\left\{\frac{V_0}{E(\vec{k}) - E(\vec{k}+\vec{k'}) + \hbar\omega - i\hbar\alpha} + \frac{V_0}{E(\vec{k}) - E(\vec{k}-\vec{k'}) - \hbar\omega + i\hbar\alpha}\right\}$$
$$\times e^{i\vec{k'}.\vec{r}} e^{i\omega t} + cc \tag{14.8}$$

The next step is to generalize this expression taking into account the fact that the initial states must be occupied, and the final states, to which the electrons are moved to by the perturbation, must be empty to find the charge density change:

$$\delta\rho = qV_0 \sum_{\vec{k}} \left\{ \frac{f_0\left(\vec{k}\right) - f_0\left(\vec{k} + \overline{k'}\right)}{E\left(\vec{k}\right) - E\left(\vec{k} + \vec{k'}\right) + \hbar\omega - i\hbar\alpha} \right\} e^{i\vec{k'}\cdot\vec{r}} e^{i\omega t} + cc \tag{14.9}$$

This is a new charge distribution caused by the application of the perturbation, so it also produces a new potential, which must be a solution of the Poisson equation:

$$\begin{aligned} \nabla^2(\delta\Phi) &= -4\pi q\delta\rho \\ \delta\Phi &= \Phi_0 e^{i\vec{k'}\cdot\vec{r}} e^{i\omega t} + cc \end{aligned} \tag{14.10}$$

where we have assumed that the potential has the same time and spatial variation as the perturbation. Now we substitute Eq. (14.10) and evaluate the ∇^2 operator to find:

$$\begin{aligned} -k'^2\Phi &= -4\pi q^2 V_0 \sum_{\vec{k}} \frac{f\left(\vec{k}\right) - f\left(\vec{k} + \vec{k'}\right)}{E\left(\vec{k}\right) - E\left(\vec{k} + \vec{k'}\right) + \hbar\omega - i\hbar\alpha} \\ \Phi_0 &= 4\pi q^2 \frac{V_0}{k'^2} \sum_{\vec{k}} \frac{f\left(\vec{k}\right) - f\left(\vec{k} + \vec{k'}\right)}{E\left(\vec{k}\right) - E\left(\vec{k} + \vec{k'}\right) + \hbar\omega - i\hbar\alpha} \end{aligned} \tag{14.11}$$

So we see that the perturbation has produced a reaction, a new internal potential. But this reaction is itself a similar perturbation, so the calculation should really be self-consistent and take this internal response into account right from the start. In other words, the total perturbation acting on the electrons is not just the external potential but also the internal response that the external one has generated. We now have the total perturbation:

$$\delta U\left(\vec{r}, t\right) = \delta V\left(\vec{r}, t\right) + \delta\Phi\left(\vec{r}, t\right) \tag{14.12}$$

And if we assume that the external potential has the form given by Eq. (14.7):

$$U = V + \frac{4\pi q^2}{k'^2} \sum_{\vec{k}} \left\{ \frac{f\left(\vec{k}\right) - f\left(\vec{k} + \vec{k'}\right)}{E\left(\vec{k}\right) - E\left(\vec{k} + \overline{k'}\right) + \hbar\omega - i\hbar\alpha} \right\} U \tag{14.13}$$

or in other words:

$$U = \frac{V}{\varepsilon\left(\vec{k'}, \omega\right)} \tag{14.14}$$

where:

$$\varepsilon\left(\vec{k'}, \omega\right) = 1 + \frac{4\pi q^2}{k'^2} \sum_{\vec{k}} \left\{ \frac{f\left(\vec{k}\right) - f\left(\vec{k} + \vec{k'}\right)}{E\left(\vec{k} + \vec{k'}\right) - E\left(\vec{k}\right) - \hbar\omega + i\hbar\alpha} \right\} \tag{14.15}$$

This important formula is known as Lindhard's expression. The applied potential is $V(\vec{r},t)$, but the potential seen by the carriers is modified or screened by the medium to give $U(\vec{r},t)$ where:

$$V(\vec{r},t) = \iint d\vec{k'}\, d\omega e^{ik'.\vec{r}} e^{i\omega t} V(\vec{k'},\omega) \tag{14.16}$$

$$U(\vec{r},t) = \iint \frac{V(\vec{k'},\omega)}{\varepsilon(\vec{k'},\omega)} d\vec{k'}\, d\omega e^{i\vec{k'}.\vec{r}} e^{i\omega t} \tag{14.17}$$

14.2 Static Response

In order to appreciate the significance of this formula, consider the situation where the applied field is time independent, so we need to study $\varepsilon(k',0)$, i.e., at zero frequency. To do that we look at the limit $\vec{k'} \to 0$ in Eq. (14.15) where the denominator is largest and write:

$$\begin{aligned} E(\vec{k}+\vec{k'}) - E(\vec{k}) &= \vec{k'}.\nabla_{\vec{k}} E(\vec{k}) \\ f(\vec{k}) - f(\vec{k}+\vec{k'}) &= -\vec{k'}.\frac{\partial f_{\vec{k}}}{\partial E_{\vec{k}}} \nabla_{\vec{k}} E(\vec{k}) \end{aligned} \tag{14.18}$$

And in Eq. (14.15), we have:

$$\varepsilon(\vec{k'},0) \to 1 + \frac{q^2}{\varepsilon_0 k'^2} \int \frac{\vec{k'} \cdot \nabla_{\vec{k}} E(\vec{k})}{\vec{k'} \cdot \nabla_{\vec{k}} E(\vec{k})} \left(-\frac{\partial f}{\partial E}\right) d\vec{k} \tag{14.19}$$

$$\varepsilon(\vec{k'},0) = 1 + \frac{q^2}{\varepsilon_0 k'^2} \int \left(-\frac{\partial f}{\partial E}\right) g_V(E) dE \tag{14.20}$$

$$\varepsilon(\vec{k'},0) = 1 + \frac{\lambda_s^2}{k'^2} \tag{14.21}$$

where g_V is the density of states per unit volume. If we remember that $\left(-\frac{\partial f}{\partial E}\right)$ is at low temperatures almost a delta function at the Fermi energy, then this gives us:

$$\lambda_s = \frac{q^2 g_V(E_F)}{\varepsilon_0} \tag{14.22}$$

But the general result at any temperature follows from Eq. (14.20) and Eq. (14.21).

Now assume that the external potential is, for example, caused by an impurity with a Coulomb potential:

$$V(r) = \frac{q^2}{4\pi\varepsilon_0 r} \tag{14.23}$$

In Fourier space the bare Coulomb potential gives:

$$V(k') = \frac{q^2}{4\pi\varepsilon_0 k'^2} \tag{14.24}$$

so that the net potential seen by the other carriers in Fourier space is:

$$U = \left\{ \frac{q^2}{1 + \frac{\lambda_s^2}{k'^2}} \right\} \frac{1}{4\pi\varepsilon_0 k'^2} = \frac{q^2}{4\pi\varepsilon_0 \left(\lambda_s^2 + k'^2\right)} \tag{14.25}$$

And in real space this transforms back to:

$$U(r) = \frac{q^2}{4\pi\varepsilon_0 r} \exp[-\lambda_s r] \tag{14.26}$$

Now we understand that the quantity λ_s is an inverse screening length and depends on the magnitude of the density of states at the Fermi level. The density of states at the Fermi level is only finite when we have free carriers, i.e., when we have a finite conductivity at $T = 0$. In a metal, the screening length can be as short as $1/\lambda_s \sim 0.1$ nm. In a doped semiconductor, the screening length can be 100 times longer than that. Note that Eq. (14.21) is an approximation, and Eq. (14.26) is only valid at longer distances than the screening length. The exact evaluation and spatial dependence of the potential are quite a bit more complicated than that. For our purposes, however, the simple exponential result which is valid at long distances $r \gg 1/\lambda_s$ is good enough.

14.3 Screening in a Semiconductor

Now let us consider how charges are screened in a medium such as a semiconductor, where there are no free charges at low temperatures. For this purpose it is convenient to now explicitly index the bands so we have:

$$\varepsilon\left(\vec{k'},\omega\right) = 1 + \frac{4\pi q^2}{k'^2} \sum_{\vec{k};k',m} \left\{ \frac{\left| \left\langle \vec{k}, g \left| e^{i\vec{k'}\cdot\vec{r}} \right| \vec{k'}, m \right\rangle \right|^2 \left[f_g\left(\vec{k}\right) - f_m\left(\vec{k}+\vec{k'}\right) \right]}{\varepsilon_g\left(\vec{k}\right) - \varepsilon_m\left(\vec{k}+\vec{k'}\right) + \hbar\omega - i\hbar\alpha} \right\} \tag{14.27}$$

where g is the valence bands and m the conduction bands. The first allowed process in the sum is from the valence band to the conduction band, so we have for small $\vec{k'}$:

$$\varepsilon_m\left(\vec{k}+\vec{k'}\right) - \varepsilon_g\left(\vec{k}\right) \sim E_{\text{gap}} \tag{14.28}$$

In order to proceed further, we use the sum rule:

$$\sum_n (E_n - E_s) \left| \left\langle n \left| e^{i\vec{k}\cdot\vec{r}} \right| s \right\rangle \right|^2 = \frac{\hbar^2 k^2}{2m} \tag{14.29}$$

which follows from the commutator $[x,p] = i\hbar$. Then we use the approximation Eq. (14.28), assuming this is true for all k' to find:

$$\sum_m \left| \left\langle \vec{k}, g \left| e^{i\vec{k'}\cdot\vec{r}} \right| \vec{k}+\vec{k'}, m \right\rangle \right|^2 \sim \frac{\hbar^2 k'^2}{2mE_{\text{gap}}} \tag{14.30}$$

which then gives when substituted back into Eq. (14.27):

$$\varepsilon(k',0) = 1 + \frac{q^2}{\varepsilon_0 k'^2} \frac{n_v}{E_{\text{gap}}} \frac{\hbar^2 k'^2}{2m} \frac{2}{E_{\text{gap}}} \tag{14.31}$$

$$\varepsilon(k',0) = 1 + \frac{q^2}{\varepsilon_0} \frac{n_v}{E_{\text{gap}}} \frac{\hbar^2}{m} \frac{1}{E_{\text{gap}}} \tag{14.32}$$

$$\varepsilon(k',0) = 1 + \left(\frac{\hbar\omega_p}{E_{\text{gap}}} \right)^2 \tag{14.33}$$

where n_v is the density of electrons in the valence band and where we have defined the plasma frequency in the valence band as:

$$\omega_p = \left\{ \frac{n_v q^2}{m\varepsilon_0} \right\}^{1/2} \tag{14.34}$$

Going back to Eq.(13.14) again, and looking at the effective Coulomb potential, transforming back to real space, we now have:

$$V(r) = \frac{q^2}{4\pi\varepsilon_0\left[1 + \left(\frac{\hbar\omega_p}{E_G}\right)^2\right]r} \tag{14.35}$$

which, in contrast to Eq. (14.21), gives us a constant permittivity and keeps the long range nature of the Coulomb potential. The effective permittivity produced by the polarization of the valence band in the absence of free carriers is therefore:

$$\varepsilon_s = 1 + \left(\frac{\hbar\omega_p}{E_g}\right)^2 \tag{14.36}$$

The presence of free charges has a drastic effect on the screening as can be seen by comparing Eqs. (14.36) and (14.26). These results are some of the most important in solid-state physics.

According to Eq. (14.36), we have for direct bandgap zinc-blende materials, a valence band electron density which is the atom density, because we have four valence electrons per atom and thus a scaling of the permittivity with the energy gap. The smaller the gap, the larger the permittivity. Table 14.1 shows the experimental

Table 14.1 Table of important semiconductor parameters (see also Appendix A.4)

Semiconductor		Bandgap energy (eV)		Band	ε
		300 K	0 K		
Element	C	5.47	5.48	Indirect	5.7
	Si	1.12	1.17	Indirect	11.9
	Ge	0.66	0.74	Indirect	16.0
	Sn		0.082	Direct	
IV–IV	α-SiC	2.996	3.03	Indirect	10.0
III–V	BN	~7.5		Indirect	7.1
	GaN	3.36	3.50	Direct	12.2
	GaP	2.26	2.34	Indirect	11.1
	BP	2.0			
	AlSb	1.58	1.68	Indirect	14.4
	GaAs	1.42	1.52	Direct	13.1
	InP	1.35	1.42	Direct	12.4
	GaSb	0.72	0.81	Direct	15.7
	InAs	0.36	0.42	Direct	14.6
	InSb	0.17	0.23	Direct	17.7
II–VI	ZnS	3.68	3.84	Direct	5.2
	ZnO	3.35	3.42	Direct	9.0
	CdS	2.42	2.56	Direct	5.4
	CdSe	1.70	1.85	Direct	10.0
	CdTe	1.56		Direct	10.2
IV–VI	PbS	0.41	0.286	Indirect	17.0
	PbTe	0.31	0.19	Indirect	30.0

values for different semiconductors. Although there is a clear relationship, it is not as pronounced as that given by Eq. (14.36). There are two main reasons: the first one is that we have used quite a strong approximation in deriving Eq. (14.36). In particular the nature of the variations in the Bloch functions and effective masses has not been properly included in the evaluation of the permittivity and plasma frequencies. All that is left to distinguish the materials is the energy gap and the plasma frequency which itself also depends on the effective mass. The variations in the plasma frequency should of course also be included. Within the Kane model of Chap. 5, small m^* implies small bandgap and from Eq. (14.34) large plasma frequency, so the scaling with the energy gap is apparently even stronger. But the fact that this strong dependence is not observed has to do with the strong approximation we used to derive Eq. (14.33) which neglects the effect of the effective mass matrix elements. The second reason is that we have neglected exchange and correlation effects in discussing the electronic structure and assumed that one-body band theory is enough. Deriving the energy gap of semiconductors without these corrections turns out to be impossible. So it is not surprising that this simple scheme does not fully reproduce the experimental trend.

14.4 Screening in a 2-Dimensional System

The Lindhard function Eq. (14.15) is sensitive to the dimensionality of the system. For example, in a 2D free electron gas, Eq. (14.15) can be shown to become:

$$\varepsilon\left(\vec{k'},0\right) = 1 + \frac{q_{2d}}{k'}\left\{1 - \left[1 - \left(\frac{2k_F}{k'}\right)^2\right]^{1/2}\right\} \rightarrow k' > 2k_F \tag{14.37}$$

where:

$$q_{2d} = \frac{mq^2}{2\pi\varepsilon_0\varepsilon_b\hbar^2} = \frac{2}{a_{B,e}} \tag{14.38}$$

$$a_{B,e} = \frac{\pi\hbar^2\varepsilon_b\varepsilon_0}{mq^2} \tag{14.39}$$

and where we have assumed that the bound electrons give a constant permittivity ε_b or ε_s. This expression is actually very close to the classical 2D Thomas-Fermi function:

$$\varepsilon_{TF}\left(\vec{k'},0\right) = 1 + \frac{q_{2d}}{k'} \tag{14.40}$$

which gives us the screened Coulomb potential:

$$V_{2d}\left(\vec{r}\right)_{\mathrm{TF}} = \frac{q^2}{4\pi\varepsilon_0\varepsilon_b}\frac{q_{2d}(1+q_{2d}d)}{\left(rq_{2d}\right)^3} \tag{14.41}$$

and thus a cubic power law distance dependence. The quantum mechanical form Eq. (14.37) is more difficult to evaluate in real space, but at long distances, it has the interesting oscillatory structure:

$$V_{2d}(r)_{\mathrm{QM}} = -\frac{q^2}{4\pi\varepsilon_0\varepsilon_b}\frac{4\left(k_f\right)^2}{\left(2k_{\mathrm{F}}+q_{2d}\right)^2}\frac{\sin\left(2k_{\mathrm{F}}r\right)}{2k_{\mathrm{F}}r^2} \tag{14.42}$$

We note that the quantum mechanical result depends explicitly on the magnitude of the Fermi wave vector k_{F}, but the classical Thomas-Fermi result does not. The reader should also make a note of the very different screening properties of a 3D and 2D electron gases. This is very important in nanotechnology. The lower the dimensionality, the more ineffective the screening becomes. In one dimension, the Lindhard approximation is not accurate, so we have not discussed it here. One consequence is that in nanostructures, the effect of electron-electron interactions on the electronic transport and optical properties is much more significant than in the bulk. This has implications for engineering because it implies that physicists and engineers can use electron-electron interactions as an engineering design tool to find novel device functionalities.

14.5 Plasmon Modes

Consider again the 3D permittivity Eq. (14.15), and this time the very high-frequency limit:

$$\hbar\omega > \varepsilon\left(\bar{k}+\vec{k'}\right) - \varepsilon\left(\vec{k}\right) \tag{14.43}$$

with:

$$\varepsilon\left(\vec{k'},\omega\right) = 1 + \frac{q^2}{\varepsilon_0 k'^2}\sum_{\vec{k}}\left\{\frac{2f\left(\vec{k}\right)\left(E_{\bar{k}} - E_{\vec{k}+\vec{k'}}\right)}{-\left(E_{\vec{k}+\vec{k'}} - E_{\vec{k}}\right)^2 + (\hbar\omega)^2}\right\} \tag{14.44}$$

where Eq. (14.44) becomes after expanding for small $\vec{k'}$:

$$\varepsilon\left(\vec{k'},\omega\right) = 1 + \frac{q^2}{\varepsilon_0 k'^2}\sum_{\vec{k}}\frac{f_0\left(\vec{k}\right)}{(\hbar\omega)^2}\left(-k'^2\frac{\partial^2\varepsilon\left(\vec{k}\right)}{\partial k^2}\right) \tag{14.45}$$

giving:

$$\varepsilon(\omega,0)=1-\frac{\omega_p^2}{\omega^2}\rightarrow\omega_p^2=\frac{q^2n}{m\varepsilon_0}(\text{MKS}) \tag{14.46}$$

The reader should note that we have encountered this result before when we were analyzing the permittivity of the electron gas in Drude theory; see Eq. (14.37).

If we do the same analysis in 2D, we obtain a different plasma dispersion. We quote here the result which is (q = electron charge):

$$\omega_{p,2D}\left(k_p\right)=\sqrt{\frac{q^2n_{2D}k_p}{2\varepsilon_0\varepsilon_b m}} \tag{14.47}$$

We note that the zero wavevector value of the 2D plasmon dispersion is zero in contrast to the 3D result. Plasmon modes are very much geometry and system size dependent. The general plasmon dispersion relation can be obtained from the requirement that there be longitudinal mode solutions in the Maxwell equations and thus that the wavevector and frequency-dependent permittivity be:

$$\varepsilon\left(\vec{k},\omega\right)=0 \tag{14.48}$$

14.6 Surface Plasmons

Plasmon modes or, in other words, the collective oscillations of electron clouds in bulk 2D systems and nanoparticles are the so-called surface plasmons. This is a field of great current interest, and the reader is referred to the specialized literature on this topic available from Internet searches. The point is that, normally, in solid-state physics, we consider the properties and response behavior of single charges. But in general and especially when we apply a time-dependent electric field, we also have to take account of the fact that all the free electrons experience the same stimulation and therefore produce internal and external responses which are the result of the motion of many charges. This collective response can be very much larger than the response of a single particle and enhance the total electric field seen by individual charges. Thus if we place a charge near a small spherical metal particle and then apply an oscillating field to the system, the charge sees not only the applied field but also the field produced by the collective motion or response of the free electrons in the metallic nanoparticle (see Pinchuk et al. 2004, in the references). This additional field would not be very significant until the frequency of the applied stimulation reaches the plasma frequency of the nanoparticle. When this happens, i.e., at resonance, the collective response becomes very large and can be many orders of magnitude bigger than the original stimulating field. Clearly these types of processes imply many novel applications. One can, in this way, enhance local fields by using surface plasmon amplifiers, by many orders of magnitude, and thus is a very topical

field in modern solid-state engineering. The reader is referred to the specialized literature on the subject.

14.7 Summary

In this chapter we have in some sense completed the work we started in Chap. 10 and investigated how bound electrons and free electrons change the electric fields that charges produce in solids. This gave rise to the concept of screening which we have in part already encountered in elementary electromagnetic theory. We derived the important Lindhard function. We noted how drastically a Coulomb potential is modified at long range by the presence of free electrons. This is ultimately one of the reasons which one-body approximations work so well in solids. Then we discovered that the screening is strongly dimensionality dependent becoming less and less effective the lower the dimensionality of the system. This has a strong impact on "nanotechnology" and makes electron-electron interaction a serious design tool. For example, a single trapped charge can block an entire current path in a thin enough wire. Removing the charge will open the channel again. We also briefly touched on the exciting new field of "surface plasmon" research and development and urged the interested reader to consult the specialized literature.

Problems

1. Explain what is meant by screening of electrical potentials. Explain the difference between the screening properties of metals and insulators. If in a solid the density of states at the Fermi level $g_v(E_F)$ is $10^{26}/m^3eV$, what is the screening length? If we lower the temperature and the solid turns into an insulator or wide bandgap semiconductor, what happens to the screening length?
2. What is a plasmon? What is the plasma frequency of a 3D metal for which the electron density is $n_c = 10^{27}/m^3$. If you were asked to choose materials or design a system for which the plasmon frequency is in the regime of $\hbar\omega_p \sim 0.5$ eV, what would you choose? What is the free electron density needed to obtain such a plasmon frequency?

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