Chapter 22
GaAs Quantum Well Laser

As an example of a bipolar semiconductor laser, we treat the GaAs quantum well laser (wavelength around 800 nm). In later chapters, we will study quantum well lasers consisting of other materials and bipolar lasers of other types.

We describe a quantum well by an electron subband and a hole subband (the heavy hole subband); we will, in a later chapter (Chap. 26), slightly modify the description of a quantum well laser by taking into account another hole subband (the light hole subband).

To characterize an active quantum well, we calculate the quasi-Fermi energies of electrons and holes; because the densities of states of electrons and holes have constant (energy-independent) values, we obtain analytic expressions of the quasi-Fermi energies. We consider a GaAs quantum well (at low temperature and at room temperature) carrying nonequilibrium electrons of different densities N^{2D}. We determine the quasi-Fermi energy, the occupation number difference $f_2 - f_1$ and the two-dimensional gain characteristic H^{2D}. We discuss modal growth and gain coefficients. We introduce the material gain coefficient; the material gain coefficient corresponds to a three-dimensional description of the quantum film, but with a two-dimensional density of states.

The quantum well laser consists of a heterostructure composed of at least five semiconductor layers. These have the tasks: to form a quantum well; to provide a light guide effect; to allow for injection of electrons and holes into the quantum well by means of a current. We describe the principle and the design of the edge emitting GaAs quantum well laser. We derive the laser threshold condition and determine the threshold current. The solutions to the rate equations of a quantum well laser indicate clamping of the quasi-Fermi energies of the electron and hole gases.

A multi-quantum well laser, containing several quantum wells in parallel, has a larger gain and a larger output power than a quantum well laser containing one quantum well only.

The arrangement of many laser diodes in a linear array or in a stack of arrays results in a high-power semiconductor laser. The radiation of a high-power semiconductor laser is not a single coherent wave but is composed of different coherent waves, which

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permanently change the relative phase to each other. The radiation has, however, a high degree of monochromaticity.

Besides the edge emitting quantum well laser, we discuss the vertical-cavity surface-emitting laser (VCSEL). The importance with respect to applications—of both edge emitting quantum well laser and vertical-cavity surface-emitting laser—has already been discussed (in Sect. 1.4).

We finally point out that the laser radiation of an edge-emitting laser can be polarized, with the electric field vector of the laser radiation lies in the plane of the quantum well. As a last point, we determine the spectrum of luminescence radiation emitted by a quantum well laser in addition to laser radiation.

22.1 GaAs Quantum Well

A GaAs quantum well (Fig. 22.1) consists of a thin GaAs film embedded in GaAlAs. Electrons can assume lower energies in the GaAs layer than in GaAlAs and holes can assume higher energies. Because of the lateral restriction, free-electron motion is only possible along the film plane. The two-dimensional free-electron motion of a conduction band electron is characterized by the *electron subband*. Correspondingly, the two-dimensional free-electron motion of a valence band electron is characterized by a *hole subband*. The gap energy E_g^{2D} of the two-dimensional semiconductor is slightly larger than the gap energy E_g of the corresponding bulk semiconductor because of the zero point energy associated with the electron and the hole confinement.

Example GaAs quantum well (at room temperature).

- $E_g \sim 1.42\,\text{eV}$; gap energy.
- $Ga_{0.85}Al_{0.15}As$; $E_g \sim 1.51\,\text{eV}$ (that is 90 meV larger than for GaAs).
- $Ga_{0.75}Al_{0.25}$ As; $E_g \sim 1.60\,\text{eV}$ (that is 180 meV larger than for GaAs).
- $E_g^{2D} = 1.45\,\text{eV}$ (that is 30 meV larger than for bulk GaAs); two-dimensional gap energy of a quantum well of 10 nm thickness (Sect. 26.4).
- $\nu_g^{2D} = E_g^{2D}/h \sim 359\,\text{THz}$; gap frequency.
- $\lambda_g^{2D} = c/\nu_g^{2D} \sim 836\,\text{nm}$; vacuum wavelength corresponding to the gap frequency.
- $m_e \sim 0.07\, m_0$; $m_0 = 0.92 \times 10^{-30}\,\text{kg}$ = electron mass.

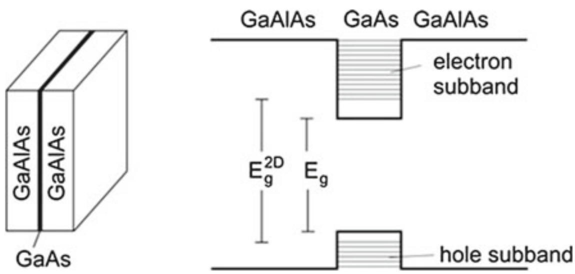

Fig. 22.1 GaAs quantum well

- $m_h = 0.43\, m_0$.
- $m_r \sim 0.06\, m_0$.
- $D_c^{2D} = m_e/(\pi\hbar^2) = 2.0 \times 10^{36}\,\mathrm{J^{-1}\,m^{-2}}$; density of states of electrons in the conduction band.
- $D_v^{2D} = m_e/(\pi\hbar^2) = 12 \times 10^{36}\,\mathrm{J^{-1}\,m^{-2}}$; density of states of holes = density of states of electrons in the valence band.
- $D_r^{2D} = 1.7 \times 10^{36}\,\mathrm{J^{-1}\,m^{-2}}$ = reduced density of states for $E_{21} \geq E_g^{2D}$; *see* (21.81).

22.2 An Active Quantum Well

Injection of electrons into the conduction band (i.e., into the electron subband) and extraction of electrons from the valence band (i.e., injection of holes into the hole subband) results in an active quantum well (Fig. 22.2). An electron in the conduction band has the energy

$$E_c = E_c^{2D} + \epsilon_c, \tag{22.1}$$

where E_c^{2D} is the minimum of the electron subband and ϵ_c is the energy within the conduction band. An electron level in the valence band has the energy

$$E_v = E_v^{2D} - \epsilon_v, \tag{22.2}$$

where E_v^{2D} is the maximum of the hole subband and ϵ_v is the energy within the valence band. The conduction band electrons have a Fermi–Dirac distribution with the quasi-Fermi energy E_{Fc} and the electrons in the valence band have a Fermi–Dirac distribution with the quasi-Fermi energy E_{Fv}. We can write

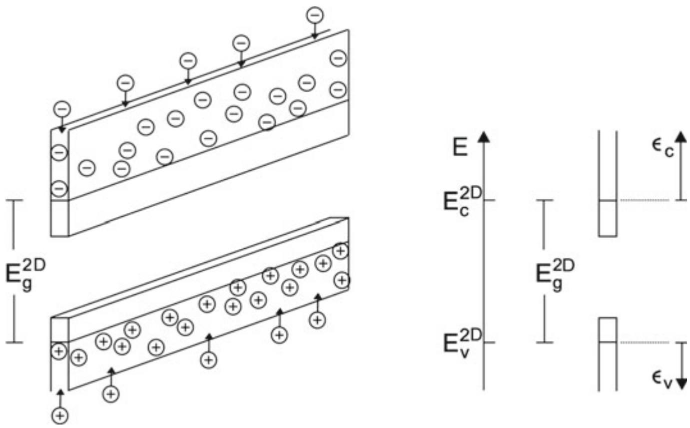

Fig. 22.2 An active quantum well and energy scales

$$E_{\mathrm{Fc}} = E_{\mathrm{c}}^{2\mathrm{D}} + \epsilon_{\mathrm{Fc}}, \tag{22.3}$$

$$E_{\mathrm{Fv}} = E_{\mathrm{v}}^{2\mathrm{D}} - \epsilon_{\mathrm{Fv}}, \tag{22.4}$$

where ϵ_{Fc} is the quasi-Fermi energy of the conduction band electrons relative to the energy $E_{\mathrm{c}}^{2\mathrm{D}}$ of the conduction band minimum and ϵ_{Fv} the quasi-Fermi energy of the valence band electrons relative to the energy $E_{\mathrm{v}}^{2\mathrm{D}}$ of the valence band maximum.

At zero temperature, all conduction band levels between $E_{\mathrm{c}}^{2\mathrm{D}}$ and the quasi-Fermi energy E_{Fc} are occupied while all valence band levels between $E_{\mathrm{v}}^{2\mathrm{D}}$ and E_{Fv} are completely empty. The quasi-Fermi energies at $T = 0$ are

$$E_{\mathrm{Fc}} = E_{\mathrm{c}}^{2\mathrm{D}} + N^{2\mathrm{D}}/D_{\mathrm{c}}^{2\mathrm{D}} = E_{\mathrm{c}}^{2\mathrm{D}} + \epsilon_{\mathrm{Fc}}, \tag{22.5}$$

$$E_{\mathrm{Fv}} = E_{\mathrm{v}}^{2\mathrm{D}} - N^{2\mathrm{D}}/D_{\mathrm{v}}^{2\mathrm{D}} = E_{\mathrm{v}}^{2\mathrm{D}} - \epsilon_{\mathrm{Fv}}. \tag{22.6}$$

E_{Fc} increases linearly with $N^{2\mathrm{D}}$ and E_{Fv} decreases linearly with $N^{2\mathrm{D}}$. Because of the larger density of states in the valence band, the energy range $E_{\mathrm{Fc}} - E_{\mathrm{c}}^{2\mathrm{D}}$ of occupied energy levels in the conduction band is larger than the energy range $E_{\mathrm{v}} - E_{\mathrm{Fv}}$ of empty levels in the valence band.

At finite temperature, the Fermi distributions are broader. The quasi-Fermi energy of the electrons in the conduction band follows from the expression

$$\int f_2 D_{\mathrm{c}}^{2\mathrm{D}} \mathrm{d}E = N^{2\mathrm{D}} \tag{22.7}$$

and the quasi-Fermi energy of the electrons in the valence band from

$$\int (1 - f_1) D_{\mathrm{v}}^{2\mathrm{D}} \mathrm{d}E = N^{2\mathrm{D}}, \tag{22.8}$$

where

$$f_2 = \frac{1}{\exp\left[(E - E_{\mathrm{Fc}})/kT\right] + 1} = \frac{1}{\exp\left[(\epsilon_{\mathrm{c}} - \epsilon_{\mathrm{Fc}})/kT\right] + 1} \tag{22.9}$$

is the quasi-Fermi distribution of the electrons in the conduction band and

$$f_1 = \frac{1}{\exp\left[(E - E_{\mathrm{Fv}})/kT\right] + 1} = \frac{1}{\exp\left[(\epsilon_{\mathrm{Fv}} - \epsilon_{\mathrm{v}})/kT\right] + 1} \tag{22.10}$$

is the quasi-Fermi distribution of the electrons in the valence band. Because the densities of states are constants, the quasi-Fermi energies can be expressed analytically. Taking into account that

$$\int_0^{\infty} \frac{\mathrm{d}x}{1 + \mathrm{e}^{x-a}} = \ln \frac{\mathrm{e}^x}{1 + \mathrm{e}^x} = a + \ln(1 + \mathrm{e}^{-a}), \tag{22.11}$$

we find the quasi-Fermi energies

$$\epsilon_{\rm Fc} = kT \ln\left(-1 + \exp[N^{\rm 2D}/D_{\rm c}^{\rm 2D} kT]\right), \tag{22.12}$$

$$\epsilon_{\rm Fv} = kT \ln\left(-1 + \exp[N^{\rm 2D}/D_{\rm v}^{\rm 2D} kT]\right). \tag{22.13}$$

The difference of the quasi-Fermi energies is given by

$$(E_{\rm c} - E_{\rm Fv})/kT = (\epsilon_{\rm Fv} + \epsilon_{\rm Fc})/kT = \ln\left(-1 + \exp[N^{\rm 2D}/D_{\rm c}^{\rm 2D} kT]\right) + \ln\left(-1 + \exp[N^{\rm 2D}/D_{\rm v}^{\rm 2D} kT]\right). \tag{22.14}$$

We now discuss a GaAs quantum well at room temperature (T = 300 K), which contains different electron densities $N^{\rm 2D}$ (Fig. 22.3). The quasi-Fermi energy $E_{\rm Fc}$ of the electrons in the conduction band has a value of $-\infty$ at zero electron density. With increasing electron density the quasi-Fermi energy increases, reaches the minimum $E_{\rm c}^{\rm 2D}$ of the conduction band (where $E_{\rm Fc} = E_{\rm c}^{\rm 2D}$), and increases further. At large electron density the quasi-Fermi energy $E_{\rm Fc}$ increases proportionally to the electron density. The quasi-Fermi energy $E_{\rm Fv}$ of the electrons in the valence band is $+\infty$ at

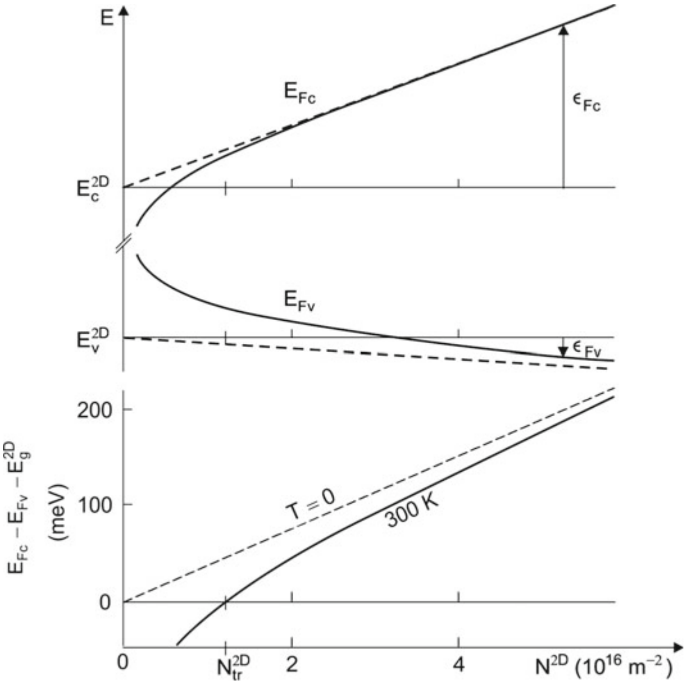

Fig. 22.3 Quasi-Fermi energies and their difference for a GaAs quantum well at room temperature (*solid lines*) and at zero temperature (*dashed*)

$N^{2D} = 0$, decreases with increasing N^{2D}, reaches the maximum of the valence band (where $E_{Fv} = E_v^{2D}$), and decreases further.

The difference of the quasi-Fermi energies (Fig. 22.3, lower part) increases with increasing electron density. The difference is 0 at thetransparency density N_{tr}^{2D} ($=1.2 \times 10^{16}\,m^{-2}$). Gain occurs if $E_{Fc} - E_{Fv} > E_g^{2D}$. The difference of the quasi-Fermi energies reaches a value of 100 meV at an electron density of about $3 \times 10^{16}\ m^{-2}$. For a GaAs quantum well at zero temperature, $E_{Fc} - E_{Fv}$ is always larger than E_g^{2D} (dashed curves).

The energy difference between radiative pair levels is

$$E_2 - E_1 = E_g^{2D} + \epsilon, \tag{22.15}$$

where

$$\epsilon = E_2 - E_1 - E_g^{2D} = \epsilon_c + \epsilon_v \tag{22.16}$$

is the energy difference $E_2 - E_1$ minus the gap energy and where

$$\epsilon_c = \frac{m_r}{m_c}\,\epsilon, \tag{22.17}$$

$$\epsilon_v = \frac{m_r}{m_h}\,\epsilon. \tag{22.18}$$

The difference between the occupation number of the upper level and the occupation number of the lower level is (with $m_h = 6m_e$ for GaAs):

$$f_2(E_2) - f_1(E_1) = f_2(\epsilon_c) - f_1(\epsilon_v) = f_2\left(\frac{6}{7}\epsilon\right) - f_1\left(\frac{1}{7}\epsilon\right). \tag{22.19}$$

To discuss the occupation number difference, we first consider the energy range in which gain occurs, i.e., where $f_2 - f_1 > 0$. Figure 22.4 shows the occupation number difference concerning a GaAs quantum well at room temperature for different electron densities. With increasing electron density N^{2D}, the difference $f_2 - f_1$ increases and approaches, near $E_2 - E_1 = E_g$, at very large N^{2D}, the saturation

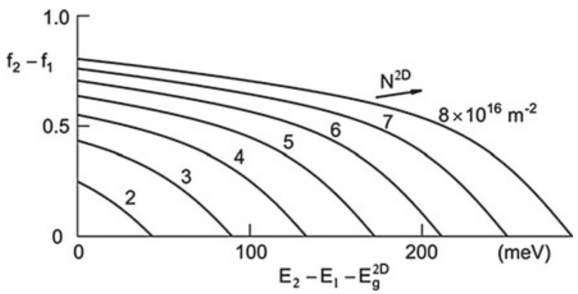

Fig. 22.4 Occupation number difference for a GaAs quantum well at room temperature

value $f_2 - f_1 = 1$. The range of gain increases with increasing electron density. When $f_2 - f_1$ is known, we can determine the different quantities describing gain.

- $H^{2D}(\nu) = (n/c)h\nu\bar{B}_{21}D_r^{2D}(E_{21})(f_2 - f_1) =$ two-dimensional gain characteristic.
- $\alpha(\nu) = (1/a_2)H^{2D}(\nu) = (n/c)h\nu\bar{B}_{21}(D_r^{2D}/a_2)(f_2 - f_1) =$ *modal gain coefficient* (=gain coefficient related to a mode); α depends on the extension of the radiation mode perpendicular to the propagation direction; D_r^{2D}/a_2 is the average density of states of radiative pairs levels within a mode of the radiation.
- $\gamma(\nu) = (c/na_2)h\nu\bar{B}_{21}D_r^{2D}(f_2 - f_1) =$ modal growth coefficient; it also depends on the extension a_2 of the mode perpendicular to the quantum well.

For completeness, we write the gain coefficient in the form

$$\alpha = \alpha_{\text{mat}} \times \Gamma, \tag{22.20}$$

where

$$\alpha_{\text{mat}}(\nu) = \frac{1}{s}H^{2D} = \frac{n}{c}h\nu\bar{B}_{21}\frac{D_r^{2D}}{s}(f_2 - f_1) \tag{22.21}$$

is the *material gain coefficient*, i.e., the gain coefficient of the quantum well that is now described as a three-dimensional system, with the two-dimensional density of states averaged over the quantum well thickness s, and where

$$\Gamma = a_2/s \tag{22.22}$$

is the ratio of the height of a photon mode and the quantum well thickness, sometimes called confinement factor. The material gain coefficient depends on the thickness of the quantum well while the modal gain coefficient is independent of the quantum well thickness but depends on the height of the photon mode.

Example Gain mediated by a GaAs quantum well, with a nonequilibrium electron density $N^{2D} = 2 \times 10^{16}\ \text{m}^{-2}$ corresponding to ($f_2 - f_1 = 0.25$), for radiation of frequency $\nu = \nu_g^{2D} = E_g^{2D}/h$.

- $\nu_g^{2D} = 3.6 \times 10^{14}$ Hz; $n = 3.6$.
- $A_{21} = 3 \times 10^9\ \text{s}^{-1}$; $B_{21} = 2.2 \times 10^{21}\ \text{m}^3\ \text{J}^{-1}\ \text{s}^{-2}$; $\bar{B}_{21} = hB_{21}$.
- $H^{2D} = 1.5 \times 10^5$.
- $\gamma = 7.4 \times 10^{11}\ \text{s}^{-1}$ for $a_2 = 200$ nm.
- $\alpha = 8.9 \times 10^3\ \text{m}^{-1}$ for $a_2 = 200$ nm.
- $\alpha_{\text{mat}} = 1.8 \times 10^5\ \text{m}^{-1}$ for $s = 10$ nm.

We now determine the occupation number difference $f_2 - f_1$ for $\epsilon = 0$ ($E_2 - E_1 = E_g^{2D}$). The occupation number difference (Fig. 22.5) is -1 for $N^{2D} = 0$. With increasing electron (and hole) density, $f_2 - f_1$ increases, becomes zero at the transparency density and increases further. At very large electron density, it approaches $+1$. If the electron density has values near N_{tr}^{2D}, we can approximate the occupation number difference by

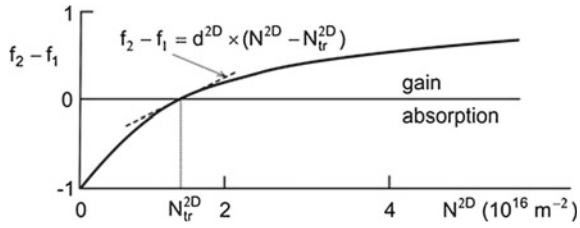

Fig. 22.5 Occupation number difference at the transition energy $E_2 - E_1 = E_g^{2D}$ for a GaAs quantum well at room temperature

$$f_2 - f_1 = d^{2D} \times (N^{2D} - N_{tr}^{2D}), \tag{22.23}$$

where $d^{2D} = 3.8 \times 10^{-17}\,m^2$. The gain characteristic is then given by

$$H^{2D}(\nu_g^{2D}) = h\nu_g^{2D} \bar{B}_{21} D_r^{2D}(E_g^{2D}) d^{2D} \times (N^{2D} - N_{tr}^{2D}). \tag{22.24}$$

The modal gain coefficient is equal to

$$\alpha(\nu_g^{2D}) = \frac{H^{2D}(\nu_g^{2D})}{a_2} = \sigma_{eff}(N_{av} - N_{tr,av}) \tag{22.25}$$

and the modal growth coefficient

$$\gamma(\nu_g^{2D}) = \frac{cH^{2D}(\nu_g^{2D})}{na_2} = b_{eff}(N_{av} - N_{tr,av}), \tag{22.26}$$

where a_2 is the height of the photon mode, $b_{eff} = h\nu \bar{B}_{21} D_r^{2D}(E_{21}) d^{2D}$ is the effective growth constant, $\sigma_{eff} = (n/c) b_{eff}$ is the effective gain cross section, $N_{av} = N^{2D}/a_2$ the average electron density in the photon mode volume and $N_{tr,av} = N_{tr}^{2D}/a_2$ the average transparency density.

Example Gain, mediated by a GaAs quantum well (at room temperature), for radiation of frequency ν_g^{2D}.

- $N^{2D} - N_{tr}^{2D} = 0.1\,N_{tr}^{2D}$.
- $N_{tr}^{2D} = 1.4 \times 10^{16}\,m^{-2}$.
- $N^{2D} - N_{tr}^{2D} = 1.4 \times 10^{15}\,m^{-2}$.
- $d^{2D} = 3.8 \times 10^{-17}\,m^2$; $\sigma_{eff} = 2.7 \times 10^{-19}\,m^2$.
- $H^{2D} = 3.2 \times 10^{-4}$.
- $\gamma = 1.6 \times 10^{11}\,s^{-1}$ for $a_2 = 200\,nm$.
- $\alpha = 1.9 \times 10^3\,m^{-1}$ for $a_2 = 200\,nm$.
- $\alpha_{mat} = 3.8 \times 10^4\,m^{-1}$ for $s = 10\,nm$.

We finally discuss the effect of thermal level broadening of energy levels (Fig. 22.6). Due to inelastic scattering of the electrons at phonons, the gain characteristic broadens and the maximum gain becomes smaller. The change of the gain curve is strongest in the range near the two-dimensional gap energy E_g^{2D}. The transparency

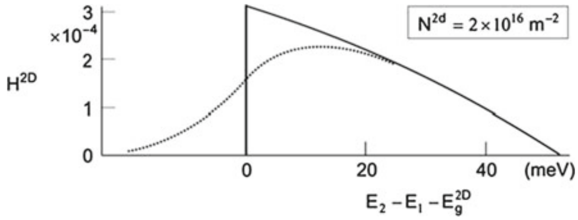

Fig. 22.6 Two-dimensional gain characteristic of a GaAs quantum well at room temperature without and with level broadening due to electron–phonon scattering

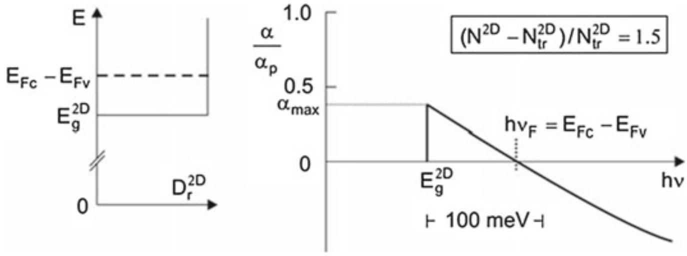

Fig. 22.7 Difference of the quasi-Fermi energies and transparency frequency for a GaAs quantum well at room temperature

density does not depend on the thermal broadening since the population difference $f_2(E_2) - f_1(E_1)$ is determined by the energy difference $E_{21} = E_2 - E_1$, i.e., by the energy difference between the center of level 2 and the center of level 1 rather than by the photon energy $h\nu$. Due to the thermal broadening, the maximum gain mediated by a quantum well at room temperature is reduced by a factor of about two, in comparison with the case that the thermal broadening is not taken into account. (This factor is compensated because of the anisotropy of a quantum well; *see* Sects. 22.8 and 26.8.)

Figure 22.7 shows the density of states of radiative electron-hole pairs and a modal gain coefficient of a GaAs quantum well at room temperature. Gain occurs at quantum energies up to

$$h\nu_F = E_{Fc} - E_{Fv}. \tag{22.27}$$

The gain coefficient (in the case that inelastic scattering is neglected) is given by

$$\alpha(h\nu)/\alpha_p = f_2(E_2) - f_1(E_1), \qquad \text{with} \qquad E = E_2 - E_1 = h\nu, \tag{22.28}$$

and with the peak gain coefficient

$$\alpha_p = n/(ca_2)h\nu \bar{B}_{21} D_r^{2D}(E). \tag{22.29}$$

E is the energy of a radiative electron-hole pair composed of a conduction band electron of energy E_2 and a valence band hole of energy E_1, $D_r^{2D}(E)$ is the density of states of radiative electron-hole pairs, f_2 is the occupation number of the conduction band electrons, and f_1 the occupation number of the valence band electrons. With increasing band filling, i.e., with increasing ν_F, the range of gain increases and the maximum absorption coefficient α_{max} ($< \alpha_p$) increases. If $\nu = \nu_F$, annihilation and creation of electron-hole pairs compensate each other; ν_F is the transparency frequency.

22.3 GaAs Quantum Well Laser

The GaAs quantum well laser (Fig. 22.8) consists of a heterostructure with two different n-doped GaAlAs layers, the GaAs quantum layer, and two different p-doped GaAlAs layers. Under the influence of a voltage, electrons from the n-doped region and holes from the p-doped region drift into the quantum film. Stimulated transitions from occupied levels in the conduction band to empty levels in the valence band give rise to generation of laser radiation. The quantum film has no net charge, the two-dimensional densities of nonequilibrium electrons and nonequilibrium holes are equal. The quantum well laser contains at least five different semiconductor layers. An n-doped GaAs substrate (n^+ GaAs substrate) supports the layers. The layer sequence can be, for instance, the following (beginning with the substrate).

- n^+ GaAs substrate.
- n $Ga_{0.75}Al_{0.25}As$.
- n $Ga_{0.9}Al_{0.1}As$.

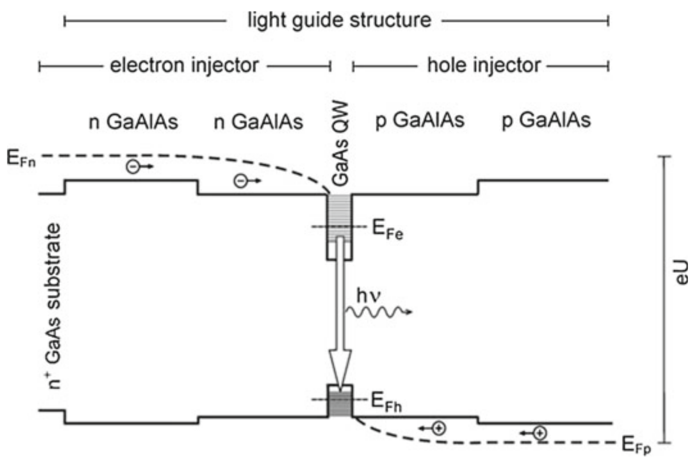

Fig. 22.8 GaAs quantum well laser (*principle*)

- GaAs quantum well (QW).
- p $Ga_{0.9}Al_{0.1}As$.
- p $Ga_{0.75}Al_{0.25}As$.

The layers fulfill the following different tasks.

- *Quantum well* (QW): GaAs.
- *Waveguide*: the $Ga_{0.9}Al_{0.1}As$ and $Ga_{0.75}Al_{0.25}As$ layers together.
- *Electron injector*: both n-doped GaAlAs layers together.
- *Hole injector*: both p-doped GaAlAs layers together.

The heavily doped substrate (n^+ GaAs) and an adjacent epitaxial n^+ GaAs layer are doped with silicon atoms and contain free-electrons of a concentration of (1–2) $\times\ 10^{19}\ m^{-3}$; n^+ indicates a high n-doping concentration. The Fermi level E_{Fn} lies within the conduction band of GaAs. The concentration of excess electrons is smaller (by about two orders of magnitude) in the n GaAlAs layers. The Fermi level of the valence band electrons in the p-doped GaAs layers lies within the valence band.

The photon mode of a laser diode (Fig. 22.9) has submillimeter size (e.g., 100 μm × 0.2 μm × 500 μm). Metal films on top of the heterostructure and on the backside of the substrate serve as electrical contacts. Under the action of a voltage (U), a current (I) is flowing through the heterostructure. Electrons in the n-doped region and holes in the p-doped region carry the current. Electrons and holes recombine within the quantum well. Stimulated electron-hole pair recombination drives the laser oscillation. The laser is an *edge-emitting laser*.

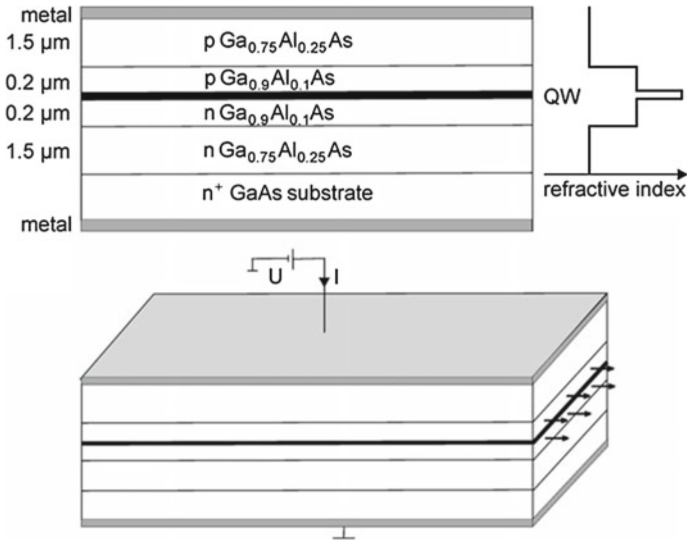

Fig. 22.9 GaAs quantum well laser; five semiconductor layers; refractive index profile; principle of the design

Example of a light guiding structure (at room temperature).

- GaAs; $n = 3.60$ for radiation with $h\nu = 1.42\,\text{eV}$.
- $Ga_{0.9}Al_{0.1}As$; refractive index $n = 3.52$.
- $Ga_{0.75}Al_{0.25}As$; $n = 3.41$.

The refractive is $n(Ga_{1-x}Al_xAs) \approx 3.60 - 0.71x$. Across the heterostructure, the refractive index has the largest value in the very thin GaAs quantum layer. The refractive index is larger in the $Ga_{0.75}Al_{0.25}As$ layers than in the $Ga_{0.9}Al_{0.1}As$ layers. This leads to a light guide effect. The light is concentrated in the $Ga_{0.9}Al_{0.1}As$ layers, which together have an optical thickness between one and two wavelengths. The GaAs film has a thickness (of the order of 10 nm) that is much smaller than the thickness of the adjacent $Ga_{0.9}Al_{0.1}As$ layers. Accordingly, only a small portion of the field overlaps with the quantum well.

The current through a quantum well corresponds to a migration of electrons from the n^+ substrate to the quantum well and, at the same time, to the migration of holes from the p-doped layers to the quantum well. The injection of electrons into the quantum well (*see* Fig. 22.8) leads to a nonequilibrium population of electrons in the electron subband, characterized by the quasi-Fermi energy E_{Fe}. The injection of holes into the quantum well leads to a nonequilibrium population of holes in the hole subband, characterized by the quasi-Fermi energy E_{Fv}. Under the action of a voltage U across the heterostructure, an electron migrating through the heterostructure loses its potential energy eU mainly due to the processes: relaxation within the electron subband; transition to the hole subband by stimulated emission of a photon; and energy necessary for extraction of the electron from the hole subband. The extraction of an electron from the hole subband corresponds to injection of a hole from the p-doped region into the hole subband accompanied with relaxation of the hole. Accordingly, we find the quantum efficiency

$$\eta_q = \frac{(E_{Fn} - E_2) + (E_2 - E_{Fp})}{h\nu} = \frac{F_{Fn} - E_{Fp}}{h\nu}, \tag{22.30}$$

where $h\nu = E_2 - E_1$ and where $E_2 - E_1$ is the energy difference of energy levels that contribute to stimulated emission of radiation at frequency ν, E_{Fn} is the Fermi energy of the n GaAs contact layer, E_{FP} is the Fermi energy of the p GaAs contact layer. The energy levels of energy E_2 belong to the lower part of the electron subband and the energy levels of energy E_1 to the upper part of the hole subband. The quantum efficiency can reach a value larger than 0.9.

The efficiency of a GaAs quantum well laser is

$$\eta = \eta_q \times \eta_{loss}, \tag{22.31}$$

where η_{loss} is an efficiency factor that takes account of loss.

22.4 Threshold Current of a GaAs Quantum Well Laser

To estimate the laser threshold condition of a GaAs quantum well laser at room temperature, we choose the simplest description.

- We ignore thermal broadening of the energy levels. The gain characteristic is then equal to

$$H^{2D}(\nu) = H^{2D}(h\nu) = (n/c)h\nu\bar{B}_{21}D_r^{2D}(E_{21})\,[f_2(E_2) - f(E_1)]\,, \qquad (22.32)$$

  where $E_{21} = E_2 - E_1 = h\nu$.
- We choose $E_{21} = E_g^{2D}$.
- We assume that the threshold density has a value near the transparency density. Then we can write

$$f_2(E_2) - f_1(E_1) = d^{2D} \times (N^{2D} - N_{tr}^{2D}), \qquad (22.33)$$

  where $d^{2D} = 3.8 \times 10^{-17}$ m^2.
- The modal growth coefficient is

$$\gamma = \frac{(n/c)H^{2D}}{a_2} = b_{eff} \times \frac{N^{2D} - N_{tr}^{2D}}{a_2}, \qquad (22.34)$$

  where a_2 is the height of the mode and

$$b_{eff} = h\nu\bar{B}_{21}D_r^{2D}d^{2D} \qquad (22.35)$$

  is the effective growth rate constant.

The photon generation rate at the steady state osciallation of the laser is equal to the photon emission rate,

$$b_{eff}(N_{av,\infty} - N_{tr,av})Z = \frac{Z}{\tau_p}. \qquad (22.36)$$

$N_{av,\infty} = N_\infty^{2D}/a_2$ is the average threshold electron density and $N_{tr,av} = N_{tr}^{2D}/a_2$ the average transparency density in the laser resonator. This leads, with $N_\infty^{2D} = N_{th}^{2D}$ (=threshold density) and $b_{eff} = (c/n)\sigma_{eff}$, to

$$N_{th}^{2D} - N_{tr}^{2D} = \frac{1}{\sigma_{eff} l_p/a_2}, \qquad (22.37)$$

where σ_{eff} is the effective gain cross section and l_p is the photon mean free path in the resonator. The threshold current is, for $N_{th}^{2D} - N_{tr}^{2D} \ll N_{tr}^{2D}$, equal to

$$I_{th} = N_{tr}^{2D} L a_2 e/\tau_{sp} \qquad (22.38)$$

and the threshold current density (with e = elementary charge) is

$$j_{\mathrm{th}} = N_{\mathrm{tr}}^{2\mathrm{D}} e/\tau_{\mathrm{sp}}. \tag{22.39}$$

Example GaAs quantum well laser.

- $h\nu = E_{\mathrm{g}}^{2\mathrm{D}}$.
- $L = 1\,\mathrm{mm}$; $a_1 = 100\,\mu\mathrm{m}$; $a_2 = 0.2\,\mu\mathrm{m}$.
- $l_{\mathrm{p}} = 1.2\,\mathrm{mm}$; $l_{\mathrm{p}}/a_2 \sim 7 \times 10^3$.
- $N_{\mathrm{tr}}^{2\mathrm{D}} = 1.4 \times 10^{16}\ \mathrm{m}^{-2}$.
- $\sigma_{\mathrm{eff}} = 2.7 \times 10^{-19}\,\mathrm{m}^2$.
- $N_{\mathrm{th}}^{2\mathrm{D}} - N_{\mathrm{tr}}^{2\mathrm{D}} = 1 \times 10^{15}\,\mathrm{m}^{-2}$.
- $\alpha_{\mathrm{th}} = 700\,\mathrm{m}^{-1}$.
- $\tau_{\mathrm{sp}} = 2 \times 10^{-9}\,\mathrm{s}$.
- $j_{\mathrm{th}} = 1 \times 10^6\,\mathrm{A\,m}^{-2}$; $I_{\mathrm{th}} = 100\,\mathrm{mA}$.

Because of the small height of the active volume, the ratio l_{p}/a_2 has a large value.

We can write the laser threshold condition in the form

$$\alpha_{\mathrm{th}} l_{\mathrm{p}} = 1, \tag{22.40}$$

where

$$\alpha_{\mathrm{th}} = \sigma_{\mathrm{eff}} \frac{N_{\mathrm{th}}^{2\mathrm{D}} - N_{\mathrm{tr}}^{2\mathrm{D}}}{a_2} \tag{22.41}$$

is the threshold gain coefficient.

Or we can write

$$\alpha_{\mathrm{th}} = \sigma_{\mathrm{eff}} \frac{N_{\mathrm{th}}^{2\mathrm{D}} - N_{\mathrm{tr}}^{2\mathrm{D}}}{s} \times \Gamma = \alpha_{\mathrm{mat,th}} \times \Gamma, \tag{22.42}$$

where we have the quantities:

- s = thickness of a quantum well.
- $N^{2\mathrm{D}}/s$ = electron density within the quantum well described as a three-dimensional system.
- $\Gamma = s/a_2$ = confinement factor.
- $\alpha_{\mathrm{mat,th}}$ = threshold material gain coefficient.

It follows for our example that $\alpha_{\mathrm{th}} \sim 10^3\,\mathrm{m}^{-1}$ and that $\Gamma = 1/20$ and $\alpha_{\mathrm{mat,th}} = 10^4\,\mathrm{m}^{-1}$ at a quantum well thickness of $s = 10\,\mathrm{nm}$.

To determine the threshold current, we have taken into account the loss due to spontaneous emission of radiation. We ignored loss that is due to other processes such as nonradiative transitions of electrons from the conduction band to the valence band and loss of photons within the semiconductor materials.

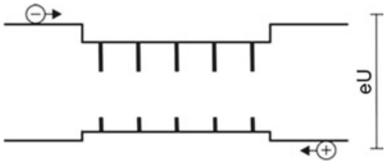

Fig. 22.10 Multi quantum well laser

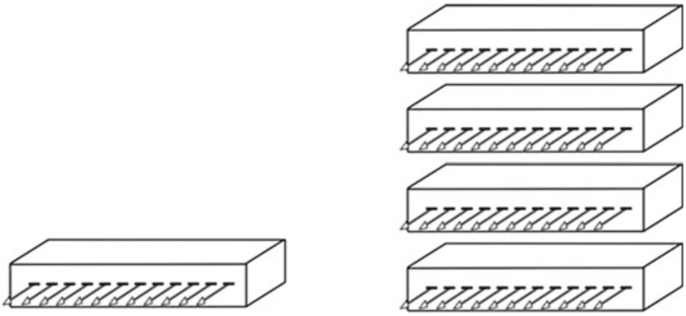

Fig. 22.11 Laser array and laser bar (*high-power semiconductor laser*)

22.5 Multi-Quantum Well Laser

A laser diode can contain (Fig. 22.10) more than one quantum well (e.g., five to ten quantum wells), arranged in parallel. This leads to a larger output power and a smaller threshold current. *The radiation of a multi-quantum well laser is coherent*.

22.6 High-Power Semiconductor Laser

A high-power semiconductor laser consists of laser diodes arranged in an array or as a bar of laser arrays (Fig. 22.11). A laser array contains 10–100 laser diodes. The laser diodes are kept at room temperature by the use of a cooler, which itself is cooled with air or via the mechanical support. Each single laser diode emits coherent radiation. However, the oscillations of different laser diodes are not in phase. Therefore, a high-power semiconductor laser generates a beam of incoherent monochromatic radiation. Depending on the number of arrays, a high-power semiconductor laser produces radiation with a power in the watt to kW range.

A diode array of laser diodes, each with a microlens collimating the radiation, emits radiation that has a divergence of $\sim 10°$ in the plane of the array and $1°$ perpendicular to the plane; without lenses, the divergence is $10°$ in the plane and $40°$ perpendicular to the plane.

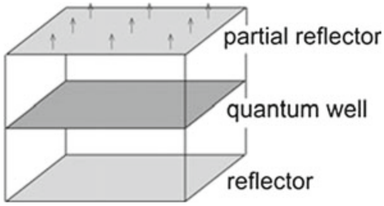

Fig. 22.12 Surface-emitting semiconductor laser

22.7 Vertical-Cavity Surface-Emitting Laser

In a vertical-cavity surface-emitting laser (=VCSEL), the reflector and the output coupling mirror are parallel to the quantum film (Fig. 22.12). The condition of steady state oscillation,

$$b_{\text{eff}}(N_{\text{av},\infty} - N_{\text{tr,av}})Z = \frac{Z}{\tau_{\text{p}}}, \tag{22.43}$$

leads, with $N_{\text{av}} = N^{2\text{D}}/L$, to the threshold condition

$$N_{\text{th}}^{2\text{D}} - N_{\text{tr}}^{2\text{D}} = \frac{1}{\sigma_{\text{eff}} l_{\text{p}}/L}, \tag{22.44}$$

where τ_{p} is the mean lifetime of a photon in the resonator and $l_{\text{p}} = (c/n)\tau_{\text{p}}$ the length of the path of a photon within the resonator. In order to obtain a large ratio l_{p}/L, the quality factor of the laser resonator has to be large. For a high-Q resonator, with a reflector (reflectivity = 1) and a partial reflector (reflectivity R), the threshold condition can be written, with $l_{\text{p}}/L = 1/(1-R)$, in the form

$$N_{\text{th}}^{2\text{D}} - N_{\text{tr}}^{2\text{D}} = \frac{1-R}{\sigma_{\text{eff}}}, \tag{22.45}$$

or

$$1 - R = \left(N_{\text{th}}^{2\text{D}} - N_{\text{tr}}^{2\text{D}}\right)\sigma_{\text{eff}}. \tag{22.46}$$

Example Surface-emitting GaAs quantum well laser, with $N_{\text{th}}^{2\text{D}} \sim 2N_{\text{tr}}^{2\text{D}}$.

- $L = 10\,\mu\text{m}$; $a_1 = 10\,\mu\text{m}$; $a_2 = 10\,\mu\text{m}$.
- $\sigma_{\text{eff}} = 2.7 \times 10^{-19}\,\text{m}^2$.
- $N_{\text{th}}^{2\text{d}} - N_{\text{tr}}^{2\text{D}} = 1.4 \times 10^{16}\,\text{m}^{-2}$.
- $\tau_{\text{sp}} = 4 \times 10^{-9}\,\text{s}$.
- $1 - R = 1 \times 10^{-3}$.
- $j_{\text{th}} = 5 \times 10^5\,\text{A}\,\text{m}^{-2}$.
- $I_{\text{th}} = 50\,\mu\text{A}$.

To describe a case of stronger pumping, we use the laser equation involving the occupation number difference and find

$$(f_2 - f_1)_{\mathrm{th}} = \frac{c/n}{h\nu \bar{B}_{21} d^{2\mathrm{D}} l_{\mathrm{p}}/L}. \tag{22.47}$$

As we have seen, the occupation number difference $f_2 - f_1$ saturates at large electron densities. Therefore, an increase of $N^{2\mathrm{D}}$ to values much larger than a few times $N_{\mathrm{tr}}^{2\mathrm{D}}$ does not lead to noticeably larger values of $f_2 - f_1$ (Problem 22.4).

In comparison with the edge emitting laser, the vertical-cavity surface-emitting laser requires, as shown, a resonator with a high Q factor. The vertical-cavity surface-emitting laser has advantages:

- The radiation is less divergent.
- The size can be much smaller.
- The threshold current can be much smaller.

The lower threshold current results in a smaller heating effect.

22.8 Polarization of Radiation of a Quantum Well Laser

A quantum film is optically anisotropic. More detailed studies show that the Einstein coefficient $B_{21}^{\perp}$, i.e., with the electric field being perpendicular to the film plane, is zero and therefore $B_{21}^{||} = 1.5 B_{21}$. Accordingly, the radiation of an edge emitting bipolar laser—that generates radiation due to recombination of electrons and heavy holes—is polarized and the direction of the electric field of the electromagnetic wave is parallel to the plane of the quantum well. However, emission of radiation of well-defined polarization direction is limited to a narrow frequency range near the two-dimensional gap frequency. Toward higher frequency, light holes (Sect. 26.3) can give rise to generation of radiation of a less defined polarization direction.

It is a further consequence of the anisotropy that the Einstein relations have to be modified: A_{21} is related to an average value between $B_{21}^{||}$ and $B_{21}^{\perp}$.

22.9 Luminescence Radiation from a Quantum Well

Figure 22.13 (solid line) shows a luminescence spectrum calculated by the use of (21.61), modified corresponding to the two-dimensional density of states of electrons and holes in a quantum well, for a GaAs quantum well at room temperature containing electrons in the electron subband of a density of about twice the transparency density. S/S_{max} is equal to the ratio of the luminescence intensity at the frequency ν and the maximum luminescence intensity at the two-dimensional gap frequency

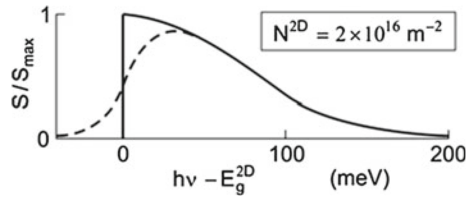

Fig. 22.13 Luminescence radiation from a quantum well

$\nu_g^{2D} = E_g^{2D}/h$, calculated without taking into account thermal level broadening. In comparison, a luminescence curve (dashed), which takes account of thermal level broadening, is wider and the maximum occurs at a larger frequency. The halfwidth of the luminescence curve is larger than kT. The luminescence radiation is emitted into the whole solid angle.

References [1–4, 6, 187–200].

Problems

22.1 Quasi-Fermi energies. A quantum film contains nonequilibrium electrons and holes. Determine the electron density at which the difference of the quasi-Fermi energies is $3kT$ (T = 300 K). [*Hint*: make use of the figure concerning the Fermi energies.]

22.2 Quantum well laser. A GaAs quantum well laser (length 0.5 mm, width 0.2 mm, resonator height 500 nm) contains 3 quantum wells and is operated at room temperature. Estimate the threshold electron density, threshold current density and threshold current. [*Hint*: neglect thermal broadening of the gain curve.]

22.3 Photons in a quantum well laser. A GaAs quantum well laser (length 0.5 mm, width 0.2 mm, resonator height 500 nm) contains 3 quantum wells, is operated at room temperature and emits, in two directions, laser radiation of a power of $P_{out} = 1$ mW into each of the directions.

(a) Determine the photon density in the resonator.
(b) Determine the total photon number Z_{tot} in the resonator.
(c) Compare Z_{tot} with the total number of nonequilibrium electrons in the quantum film.

22.4 Vertical-cavity surface-emitting laser. A vertical-cavity surface-emitting laser contains a GaAs quantum well and another laser contains five quantum wells; $(f_2 - f_1)_{th} = 0.5$ and $\tau_{sp} = 8 \times 10^{-9}$ s. Determine the following quantities:

(a) Threshold reflectivity of the output coupling mirror.
(b) Threshold current.
(c) Threshold current density.