Chapter 26
More About the Quantum Well Laser

We continue the discussion of subbands with a description of wave functions and energy bands of electrons in a quantum well. We also show how light holes modify the gain profile. Furthermore, we discuss the influence of inhomogeneous broadening on the properties of a quantum well laser.

26.1 Electron Subbands

Electrons can move freely in the GaAs plane (y, z plane) of a GaAs quantum well (Fig. 26.1, left). The motion perpendicular to the plane, along the x direction, is spatially limited. The potential energy $E_{\text{pot}}(x, y, z)$ of electrons (Fig. 26.1, center) is equal to the conduction band profile $E_{\text{c}}(x, y, z)$.

We treat the GaAs layer as an infinitely extended layer. The Schrödinger equation of an electron in the GaAs quantum layer (quantum film) has the form

$$\left[-\frac{\hbar^2}{2m_{\text{e}}}\nabla^2 + E_{\text{pot}}(x)\right]\Psi = \text{i}\hbar\frac{\partial\Psi}{\partial t}. \tag{26.1}$$

We assume that the effective mass of an electron in a quantum well is the same as for bulk GaAs ($m_{\text{e}} = 0.07\, m_0$). To determine the wave function Ψ, we use an ansatz of stationary states,

$$\Psi(x, y, z, t) = \psi(x, y, z)\text{e}^{-\text{i}(E/\hbar)t}. \tag{26.2}$$

E is the energy of a stationary state. We obtain

$$\left[-\frac{\hbar^2}{2m_{\text{e}}}\nabla^2 + E_{\text{pot}}(x)\right]\psi = E\psi. \tag{26.3}$$

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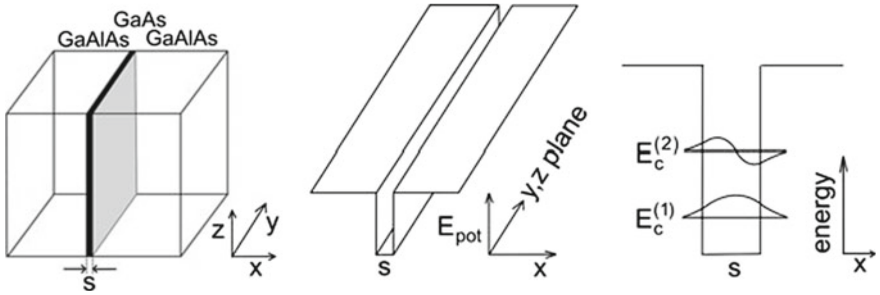

Fig. 26.1 Quantum well

The ansatz

$$\psi = \chi(x)\,\eta(y,z), \tag{26.4}$$

leads to the differential equation

$$\left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2} + E_{pot}(x)\right]\eta\chi - \frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\eta\chi = E_{tot}\,\eta\chi. \tag{26.5}$$

By dividing by $\eta\chi$, we obtain

$$\frac{1}{\chi}\left(-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2} + E_{pot}(x)\right)\chi + \frac{1}{\eta}\left(-\frac{\hbar^2}{2m_e}\right)\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\eta = E_{tot}. \tag{26.6}$$

The two terms on the left side must have constant values. The total energy is given by

$$E_{tot} = E_\perp + E_{||}, \tag{26.7}$$

where

- $E_\perp$ is the energy of electron motion perpendicular to the layer and
- $E_{||}$ is the energy of electron motion along the film plane.

The Schrödinger equation describing motion along the layer plane is

$$-\frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\eta = E_{||}\eta. \tag{26.8}$$

As a solution, we obtain the wave function

$$\eta = C e^{i\boldsymbol{k}_{||}\boldsymbol{r}_{||}}. \tag{26.9}$$

C is a constant and

- $\boldsymbol{k}_{||} = (k_y, k_z) = \boldsymbol{k}$ is the wave vector parallel to the film plane and
- $\boldsymbol{r}_{||} = (y, z)$ is a location in the plane of the quantum layer.

The differential equation yields the energy

$$E_{||} = \frac{\hbar^2}{2m_\mathrm{e}} k^2. \tag{26.10}$$

The Schrödinger equation of the electron motion perpendicular to the quantum layer,

$$\left[-\frac{\hbar^2}{2m_\mathrm{e}} \frac{\partial^2}{\partial x^2} + E_\mathrm{pot}(x) \right] \chi(x) \;=\; E_\perp \chi(x), \tag{26.11}$$

is the equation of an electron in a one-dimensional square well potential. The energy eigenvalues of a quantum well with infinitely high walls are given by

$$E_\mathrm{c}^{(n)} \;=\; \frac{\pi^2 \hbar^2}{2m_\mathrm{e} s^2} n^2; \quad n = 1,\ 2, \ldots, \tag{26.12}$$

where s is the thickness of the quantum film. The quantized motion leads to discrete energy eigenvalues $E_\mathrm{c}^1, E_\mathrm{c}^2, \ldots$. The energy E_c^1 is the *zero point energy* of the electron in the quantum well. The wave functions $\chi_n(x)$ are cosine and sine functions within the film and are zero at the borders of the film and outside the film.

The energy values of wells with walls of finite height are smaller than in the case that the potential walls are infinitely high. The wave functions are cosine and sine shaped within the well and decrease exponentially outside the quantum well (Fig. 26.1, right).

The zero point energy of a quantum well with infinitely high walls varies as $1/s^2$ (Fig. 26.2, solid line). A quantum well, like a GaAs quantum well has walls of finite height. Therefore, the zero point energy is smaller (Fig. 26.2, dashed) but decreases also strongly with increasing layer thickness. A calculation of the zero point energy in the case that the potential walls have finite height is possible by applying appropriate boundary conditions taking into account the different effective masses of GaAs and AlAs (Sect. 30.5).

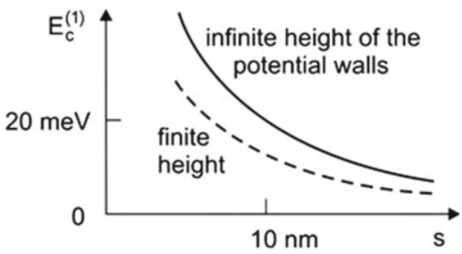

Fig. 26.2 Zero point energy of an electron ($m_\mathrm{e} = 0.07\, m_0$) in a one-dimensional square well potential

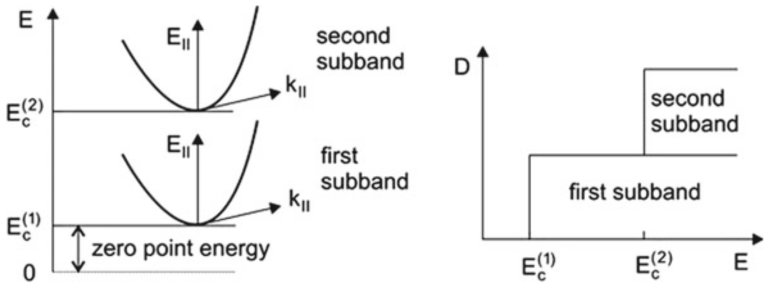

Fig. 26.3 Subbands of electrons in a quantum film

The energy of an electron is given by

$$E = E_{\mathrm{c}}^{(\mathrm{n})} + E_{||}, \tag{26.13}$$

where n is the number of a subband. The conduction band of a quantum layer consists of *electron subbands*. Figure 26.3 (left) shows the zero point energy and the first and the second electron subband. The energy of the motion perpendicular to the layer is discrete while the motion within the quantum layer corresponds to the motion of a free-electron. The density of states in a subband (Fig. 26.3, right) is a constant (Problem 27.1):

$$D_c^{2\mathrm{D}}(\epsilon) = \frac{m_{\mathrm{e}}}{\pi \hbar^2}. \tag{26.14}$$

The total density of states is the sum of the densities of states in the different subbands.

That the density of states of electrons in a quantum film is independent of the thickness of the film is plausible: the film thickness determines the zero point energy, which is due to the lateral confinement of an electron, while the dispersion relation for electrons in a two-dimensional semiconductor determines the propagation along the plane.

The *quantum confinement* of an electron in a quantum film has consequences:

- *Subbands*.
- *Discrete energy values* for motion perpendicular to the quantum layer.
- *Zero point energy* ($E_{\mathrm{c}}^{(1)}$). The value of $E_{\mathrm{c}}^{(1)}$ depends on the thickness s of the quantum film.
- The electrons move freely along the plane of the quantum film.
- *The depth of the quantum well* depends on the composition of the GaAlAs layers.
- The wave functions χ_1 and χ_2 have cosine and sine shapes within the GaAs film material and extend into the confinement material GaAlAs. Their amplitudes decrease exponentially with the distance from the GaAs film.

26.2 Hole Subbands

GaAs and other group III–V semiconductors have three hole bands (Fig. 26.4): the heavy hole band; the light hole band; the split-off band. In a laser diode, the split-off band is completely populated and does not play any role. However, the light holes ($m_{\mathrm{lh}} \sim 0.08\, m_0$) influence the gain coefficient curves.

In a quantum well, the zero point energy of a heavy hole,

$$E_{\mathrm{v}}^{(1)} = \frac{\pi^2 \hbar^2}{2 m_{\mathrm{h}} s^2}, \tag{26.15}$$

is by the factor $m_{\mathrm{h}}/m_{\mathrm{e}}$ smaller than the zero point energy of an electron in the conduction band while the zero point energy $E_{\mathrm{v,lh}}^{(1)}$ of the light hole is comparable with the zero point energy of a conduction band electron since $m_{\mathrm{lh}} \sim m_{\mathrm{e}}$. There is an energy range, between $E_{\mathrm{v}}^{(1)}$ and $E_{\mathrm{v,lh}}^{(1)}$, without light hole energy levels. The density of states of light holes is much smaller than that of heavy holes.

The conduction band states of GaAs have their origin in s-like hybrid states composed of s-states of Ga and As atoms. The s-like hybrid states overlap spatially. This leads to electron waves extended over the whole crystal and to the conduction band (Sect. 30.2); the dispersion relation $E(\boldsymbol{k})$ characterizes the states of the conduction band. The valence band states stem from hybrid states composed of p-states (i.e., p_x, p_y, p_z states) of Ga and As atoms. The three p-state components give rise to three different energy bands and dispersion relations—i.e., a wave function of a valence band state can assume, for the same k vector, three different energy values. At the Brillouin zone center ($k = 0$), two of the dispersion curves have the same energy value indicating energy degeneracy; however, the two dispersion curves have completely different shapes described by the different effective masses,

$$m_{\mathrm{h}} \equiv m_{\mathrm{hh}} = (\mathrm{d}^2 E_{\mathrm{hh}}/\mathrm{d}^2 k)_{k=0}, \tag{26.16}$$

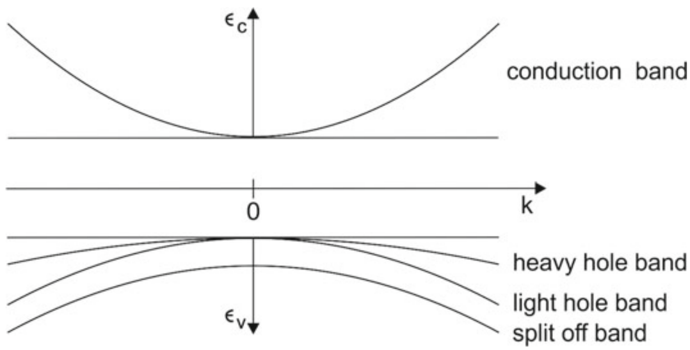

Fig. 26.4 Energy bands of electrons in GaAs: conduction band; heavy hole band; light hole band; split off band

$$m_{\mathrm{lh}} = (\mathrm{d}^2 E_{\mathrm{lh}}/\mathrm{d}^2 k)_{k=0}. \tag{26.17}$$

The third band, the split off band, is shifted to smaller energies relative to the two other valence bands due to spin-orbit interaction.

26.3 Modification of the Gain Characteristic by Light Holes

The quasi-Fermi energy of the electrons in the conduction band is determined by the density of nonequilibrium electrons in the electron subband. The quasi-Fermi energy of the electrons in the hole subbands follows from the condition

$$\int f_1 (D_{\mathrm{v}}^{2\mathrm{D}} + D_{\mathrm{v,lh}}^{2\mathrm{D}}) \mathrm{d}E = N^{2\mathrm{D}}, \tag{26.18}$$

where $D_{\mathrm{v}}^{2\mathrm{D}}$ is the density of states of heavy holes, $D_{\mathrm{v,lh}}^{2\mathrm{D}}$ is the density of states of light holes and f_1 is the Fermi function of the electrons in the heavy hole and light hole subbands. The equation yields the quasi-Fermi energy of the valence band electrons.

In our earlier treatment of quantum wells, we ignored the light hole band. Now, we discuss, qualitatively, the modification of a quantum well laser at room temperature that is due to the light hole band. Figure 26.5 shows (qualitatively) the occupation number difference $f_2 - f_1$ for a GaAs quantum well at room temperature, with $N^{2\mathrm{D}} = 2 \times 10^{16}\ \mathrm{m}^{-2}$. In comparison with the case of a single hole subband, we have a different situation:

- There are two peaks in the gain characteristic H^{2D}, which is proportional to $f_2 - f_1$.
- The gain curve has a larger width and the maximum of the gain characteristic has a smaller value.
- The actual threshold current density is larger because the holes are distributed over two subbands. Since the density of states of the light holes is smaller than the density of states of the heavy holes, the increase of the threshold current density for laser oscillation is less than a factor of two.

The theoretical expressions presented in Chap. 21 are suited to perform a quantitative analysis. However, we do not deepen the discussion of the gain characteristic.

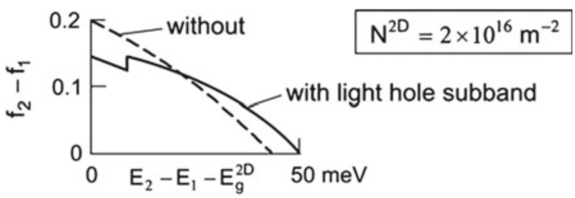

Fig. 26.5 Occupation number difference for a GaAs quantum well at room temperature

26.4 Gap Energy of a Quantum Well

Taking into account the zero point energy of electrons and holes, we find that the gap energy of a quantum well of infinitely high potential walls is equal to

$$E_{\mathrm{g}}^{\mathrm{2D}} = E_{\mathrm{g}} + \frac{\pi^2 \hbar^2}{2 m_r s^2}. \tag{26.19}$$

Example GaAs quantum film (thickness $s = 10\,\mathrm{nm}$) at room temperature ($\hbar = 1.04 \times 10^{-34}\,\mathrm{J\,s}$; $m_0 = 0.9 \times 10^{-30}\,\mathrm{kg}$).

- $E_{\mathrm{g}} = 1.42\,\mathrm{eV}$ = gap energy of bulk GaAs.
- $E_{\mathrm{g}}^{\mathrm{2D}} = E_{\mathrm{g}} + 61\,\mathrm{meV}$; energy gaps of a GaAs quantum well in the case of infinitely high walls.
- $\nu_g = 344\,\mathrm{THz}$; $\lambda_g = 872\,\mathrm{nm}$.
- $\nu_g^{\mathrm{2D}} = 358\,\mathrm{THz}$; $\lambda_g^{\mathrm{2D}} = 838\,\mathrm{nm}$.

The actual zero point energy is smaller because of the finite height of the walls. Through the choice of the composition of the quantum film and of the barrier material as well as of s, different values of the gap energy $E_{\mathrm{g}}^{\mathrm{2D}}$ ($> E_{\mathrm{g}}$) can be realized .

26.5 Temperature Dependence of the Threshold Current Density of a GaAs Quantum Well Laser

The electrons occupy energy levels in the electron subband in the range from $E_{\mathrm{c}}^{\mathrm{2D}}$ to E_{Fc}, with a spread of kT. The holes in the heavy hole subband occupy energy levels between $E_{\mathrm{v}}^{\mathrm{2D}}$ and E_{Fv}. With increasing temperature, the energy distribution of the electrons in the electron subband broadens. The energy distribution of the holes in the heavy hole subbands broadens too. It follows that the threshold current of a quantum well laser operating at room temperature is much larger than the threshold current of a quantum well laser operating at low temperature.

26.6 Gain Mediated by a Quantum Well with Inhomogeneous Well Thickness

When the thickness of a quantum film is different at different positions of the film, interband transitions are inhomogeneously broadened. Both the zero point energies $E_{\mathrm{c}}^{(1)}$ and $E_{\mathrm{v}}^{(1)}$ show variations. We obtain an energy gap distribution of a width $\Delta E_{\mathrm{g}}^{\mathrm{2D}}$ that is given by the relation

$$\frac{\Delta E_{\mathrm{g}}^{2\mathrm{D}}}{E_{\mathrm{c}}^{(1)} + E_{\mathrm{v}}^{(1)}} = -\frac{2\Delta s}{s}, \tag{26.20}$$

where Δs is an average variation of the thickness.
Example GaAs quantum well of thickness $s = 10\,\mathrm{nm}$ and an inaccuracy of the well thickness of 0.1 nm. We obtain $E_{\mathrm{c}}^{(1)} + E_{\mathrm{v}}^{(1)} \sim 49\,\mathrm{meV}$ and $\Delta E_{\mathrm{g}}^{2\mathrm{D}} \sim 1\,\mathrm{meV}$. This is, for a GaAs quantum well at room temperature, smaller than the broadening (10 meV) that is due to the inelastic scattering of the electrons at phonons.

26.7 Tunability of a Quantum Well Laser

A single-mode quantum well laser operating at room temperature emits radiation at a laser frequency that is determined by the gain characteristic and the resonance frequency of the laser resonator. The position of the energy gap, the frequency of the maximum of the gain characteristic, and the refractive index of a semiconductor depend on temperature. Therefore, the frequency of laser oscillation depends on temperature. The temperature of a laser diode changes if the temperature of the surrounding or if the current through the diode is varied. A shift of several percent of the frequency of a quantum well laser can be achieved. Tuning over a small frequency range is possible by the use of an external resonator (Sect. 25.3).

26.8 Anisotropy of a Quantum Well

The quantum theory of the optical transitions in a quantum well shows that transitions in which heavy holes are involved are only allowed if the electric field vector of the electromagnetic field lies in the plane of the quantum well. If light holes are involved, transitions for the field vector parallel and perpendicular to the quantum well are allowed too.

References [187–192].

Problems

26.1 Two-dimensional density of states. Determine the density of states of a two-dimensional electron gas.

26.2 Subpicosecond quantum well laser. Is it possible to generate subpicosecond pulses with a quantum well laser? Divide the procedure of answering this question into three parts.

(a) Is it in principle possible to generate subpicosecond pulses with a quantum well laser?
(b) Is it possible to use a quantum well laser of 1 mm length, supposed that the reflector on one side is a SESAM reflector?
(c) Discuss a semiconductor laser that uses an external broadband reflector.