Chapter 21
Probability Theory

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Abstract This chapter covers the epistemic or information-based interpretations of probability: logical, subjective, objective Bayesian, and group level. It explains how these differ from aleatory or world-based interpretations of probability, presents each in detail, and then discusses its strengths and weaknesses.

21.1 The Ubiquity of Probability Talk

Rarely does one get through a day without encountering a reference to probability or one of its relatives. Meteorological reports purport to tell us the chance of rain, and meteorologists take this to reflect a particular kind of probability. Bookies offer betting odds, which we take up according to how probable we take particular events to be. And when we're asked whether we'll attend an event, we often answer "Probably!" or "Probably not!"

Probabilities are regularly appealed to in philosophy as well. For example, it's often taken for granted in informal philosophy that the more probable of two mutually exclusive and jointly exhaustive events is the better one to bet on when offered even odds (to further the end of winning). But this isn't as obvious at it may initially seem; in fact, I hold it to be wrong under some interpretations of probability. And sometimes I encounter discussions in which it is boldly proclaimed that inductive inferences have something to do with probabilities, although what these probabilities are is never discussed. In fact, I have read several papers like this. Whole arguments are constructed on probability claims, but the understanding

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of probability under discussion is never explained. Perhaps an intuitive notion is supposed to be operating? My view is that this will not do for serious philosophy.

Let me try to convince you. Imagine a weather forecaster says that the chance of rain tomorrow is zero, but that it nevertheless rains. Should we be angry with the forecaster? Must her methods be fundamentally flawed? Against intuitive expectations, the answer may lie in the negative if she is operating with a *frequency in the limit* interpretation of probability. All that would be required, for her statement to be correct, is that in identical meteorological circumstances, were these to be repeated infinitely many times, rain would occur the next day with a frequency of zero. To see that this is compatible with rain occurring, consider the following set, with infinitely many members, in which R represents rain and N represents no rain:

$$\{N, R, N, N, R, N, N, N, R, N, N, N, N, R, \ldots\}$$

The set continues in the same pattern, with five Ns to the next R, then six after the subsequent N, and so on. Mathematically, the ratio of Rs to Ns is zero in the limit.

I could spend more space trying to convince you that it's important to understand how probability talk may be interpreted, but this may be poorly spent given that you've already made the effort to consult this piece. So I'll move on to the meat.

21.2 Epistemic Versus Aleatory Interpretations of Probability

Probability is a mathematical notion, and the issue of its interpretation typically arises only when we seek to apply it. In fact, mathematicians have had interpretative difficulties too, especially when it comes to understanding which kinds of arguments for their successful predictions were more fundamental. You can read about this elsewhere; see Hacking [15] and Shafer and Vovk [44]. The best place for us to start is to recognize the most fundamental interpretative rift if we begin with a measure-theoretic view of probability (based, for example, on Kolmogorov's or Popper's axioms of probability).[1] This is between *aleatory* and *epistemic* views.

The basic distinction is not so hard to grasp. Roughly, *aleatory* probabilities are 'out there' in the world; indeed '*alea*' is the Latin word for 'die', although one should not conclude that gambling concerns only *aleatory* probabilities. *Epistemic*

[1] Shafer and Vovk [44] argue that we should not begin by understanding probability in a measure-theoretic way, but instead in a game-theoretic way. As such, their interpretative strategy is different from those considered here (although the Dutch Book argument, which we will cover in due course, is game-theoretic in nature).

probabilities concern us more intimately, and roughly relate to our epistemic states.[2] To give a flavour of how individuals thinking on either side of this divide might apply probabilities, consider the following dialogue:

Philosopher: Here is a normal two-sided coin. What is the probability that it will land on heads when I flip it?

Mr Epistemic: One half.

Ms Aleatory: I don't know.

Mr E: What do you mean, you don't know?

Ms A: It might be biased. We'd need to see the experiment repeated several times before forming a reasonable opinion.

Mr E: Yes, it could be biased! But you've no reason to think it's biased one way, rather than another, so why not just assign the two possibilities the same probability, namely one half?

Ms A: Well that's just guessing.

Mr E: But isn't that a reasonable betting strategy? What odds would you accept on a 'heads' bet? That way we can work out how probable you think it is . . .

Ms A: If I did bet, I'd use my knowledge about similar circumstances, i.e. similar coins being flipped by similar people, and the frequencies from those.

This should be enough to give a flavour of the disagreements that can occur in this dimension. I experience such disagreements frequently, when I pose the same question as the philosopher, in the dialogue, to students.

'*Objective*' is often used in place of '*aleatory*'. I avoid the former term because probabilities might be thought to be 'out there' *in the non-material world* on some epistemic views of probability. Most notably, probabilities may be construed as logical relations between propositions, which might be taken to exist, *qua* abstracta, even in possible worlds containing no beings capable of grasping them.

Now since this chapter appears in a section of the handbook on 'Epistemology', I will cover only the epistemic interpretations of probability in what follows. Although it is true that one of the aleatory interpretations has been used, in the past, in epistemic contexts—specifically, inductive probabilities have been construed, e.g. by Reichenbach, in terms of relative frequencies of truth of consequences given the truth of the premises (in the limit)—this is now widely considered to have been in error. I will not rehearse the arguments here, for lack of space. See Rowbottom ([31], pp. 39–41) for more discussion.

Each of the interpretations covered below has variants. So the interpretative possibility space is much more complex than the normal philosophical taxonomy makes apparent. The best I can do, in what follows, is to provide an overview of the positions on the standard taxonomy and to flag any areas where there are subtleties of interpretative difference that are easy to miss.

[2] I call the two kinds of probability 'world-based' and 'information-based' in Rowbottom [35]; I think this is better terminology, but it's non-standard.

21.3 The Logical Interpretation

The basic idea behind the logical interpretation of probability is that there are degrees of partial entailment, between propositions or groups thereof, in addition to entailment relations.[3] (The talk of 'degrees of partial entailment' is from Carnap [5]; Keynes [21], the architect of the logical interpretation, used 'logical relations'.) Consider a situation where p entails q. Then the probability of q given p, or the *conditional probability* of q on p, is equal to unity; $\mathrm{P}(q, p) = 1$. And similarly, if p and q are logically inconsistent then $\mathrm{P}(p, q) = 0$ and $\mathrm{P}(q, p) = 0$. Now if we allow for other kinds of logical links, of varying strengths, we can imagine that, in general, $\mathrm{P}(p, q) = r$ where r is any real number between zero and one. It is worth noting that Popper explained the logical interpretation—or more accurately his variant thereof—rather differently, by appealing to the notion of logical content. Specifically, he claimed that $\mathrm{P}(a, b)$, interpreted logically, measures: 'the degree to which the statement a contains information which is contained by b' ([26], p. 292). (A hint that this is problematic may be gleaned by thinking of the rule of 'or introduction' in natural deduction. If a is not entailed by b, then a will have infinitely many consequences that b does not. And *vice versa*.)

How about *unconditional* probabilities? Keynes ([21], pp. 6–7) declared that: 'No proposition is in itself either probable or improbable, just as no place can be intrinsically distant; and the probability of the same statement varies with the evidence presented, which is, as it were, its origin of reference.' So in general, on the logical view, talk of unconditional probabilities is understood as elliptical. When I talk of the probability of the next president of the USA being a Democrat, for instance, I assume some things are true; that the Democratic Party is not a figment of my imagination, that the USA is a real country, and so forth. But we may nevertheless define unconditional probabilities in terms of a special class of conditional probabilities, following the suggestion of Popper ([26], pp. 284–285). The idea is that $\mathrm{P}(p)$ may be understood to represent $\mathrm{P}(p, T)$, where T represents any tautology. This gives a probability that doesn't depend on anything other than the laws (or axioms) of logic being true.

So far I have mentioned only logical relations. But as this is an *epistemic* interpretation of probability, the reader may be wondering how *we* come into the picture. The short answer is that most advocates of the logical interpretation have held that our personal degrees of belief—this is a technical term, about which I will say more shortly—should map on to the equivalent logical relations in order to be rational. Again, an analogy with entailment helps. If p entails q and I'm certain that p, then I ought to be certain that q (subject to some other appropriate conditions

[3]There is also a 'classical interpretation', which predates the logical one. On the classical view, probabilities are defined (roughly) in terms of the ratio of favourable outcomes to possible outcomes. The problem with this view is that it appears to require that each outcome be *equipossible*. So it could not handle a biased coin; e.g. in calculating the probability of heads on one flip, we'd always arrive at one half. (And someone might very well take the coin landing on its edge to be possible, and thereby be forced to conclude that the probability of heads was a third.) For more on the classical interpretation, see Gillies [13] and Rowbottom [35].

obtaining, e.g. that I desire to know whether q and recognise that p entails q).[4] More generally, let p entail q to a specific degree: $\mathrm{P}(q, p) = r$. My degree of belief in q given p—$\mathrm{D}(q, p)$—will be rational only if it maps on to the appropriate logical relation, i.e. only if $\mathrm{D}(q, p) = \mathrm{P}(q, p) = r$. This was the view of Keynes ([21], p.4):

> What particular propositions we select as the premises of our argument naturally depends on subjective factors peculiar to ourselves; but the relations, in which other propositions stand to these, and which entitle us to probable beliefs, are objective and logical.

I must reiterate, because it is a common error to think otherwise, *that the logical view does not say that probabilities are rational degrees of belief*. As Keynes ([21], p. 11) clearly stated, 'probability':

> In its most fundamental sense . . . refers to the logical relation between two sets of propositions . . . Derivative from this sense, we have the sense in which . . . the term *probable* is applied to the degrees of rational belief.

I emphasise this issue because accepting a logical interpretation of probability does not entail accepting that a degree of belief is rational if and only if it maps on to the appropriate logical relation (in the way described above). Take Popper as a case in point. One of his most striking theses was that the logical probability of any synthetic universal statement is zero relative to any finite evidence, and plausibly any evidence that we might possess; see Popper ([25], Appendix *vii) and Rowbottom ([31], Section 2.3; [32]). But he did not conclude that it is irrational to believe in such laws.

The main problem with the logical interpretation is that it is hard to see how such logical probabilities may be measured (or how to define them *operationally*).[5] And if they cannot be measured, then it seems reasonable to doubt that they exist. Keynes [21] had a rather complicated position on this issue.[6] Roughly—see O'Donnell [24] and Rowbottom [30] for more—he thought that some relations can be grasped by intuition, but that others can only be calculated by employing an *a priori* synthetic principle, namely the principle of indifference.[7] In essence, his idea was that this principle is applicable when our intuition fails, as an extension of logical proof to non-demonstrative cases:

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[4]How to connect reasons for belief and entailment is much more complicated than it may first appear. See, for example, Streumer [45].

[5]In the words of De Finetti ([7], p. 23): 'For any proposed interpretation of Probability, a proper operational definition must be worked out: that is, a device apt to measure it must be constructed.' One could argue with this, of course, but it seems odd to want to posit a kind of probability that isn't generally measurable! What purpose would it serve?

[6]Keynes also believed in non-numerical probabilities, which complicates matters further, but we can put this to one side for present purposes.

[7]This was earlier called 'the principle of non-sufficient reason', and goes back (although not under the same name) to Bernoulli [2].

> If the truth of some propositions, and the validity of some arguments, could not be recognised directly, we could make no progress... [T]he method of logical proof... enables us to know propositions to be true, which are altogether beyond the reach of our direct insight... ([21], p. 53, f.1).

So just arriving at the correct degree of belief in p given q, say as a result of brainwashing, is not sufficient to have a rational degree of belief in p given q. (It is only *necessary*, for any rational degree of belief, that $D(q, p) = P(q, p) = r$.) One must have some suitable procedure for arriving at $P(p, q)$. And this brings us to the principle of indifference:

> [T]hat equal probabilities must be assigned to each of several arguments, if there is an absence of positive ground for assigning *unequal* ones ([21], p. 42).

The problem with the application of the principle is that it leads to paradoxical results when there is more than one way to 'carve up' the space of possibilities. The most famous and widely discussed of these are the water-wine paradox (see van Fraassen [46], pp. 304–305 and Mikkelsson [23]) and Bertrand's ([3], p. 4) geometrical paradox (see Rowbottom [34]). But an even simpler example will suffice.

"If I flip a coin twice, what is the probability of getting two heads?" One respondent might take there to be three different possibilities—no heads, one head, and two heads—and conclude, assigning each an equal probability by the principle of indifference, that the answer is one third. Another might take there to be four different possibilities—no heads, one head and one tail (in order), one tail and one head (in order), and two heads—and instead conclude that the answer is a quarter. Which is right?

Several readers may think that the latter is clearly correct, and indeed Keynes ([21], p. 60) introduced an indivisibility criterion in order to argue for this; in short, the possibility of 'one head' is divisible into the possibilities of 'one head and one tail (in order)' and 'one tail and one head (in order)'. It might surprise those readers to learn that Carnap ([5], pp. 562–565) instead argued that one should think of possibilities in the first way in the coin flip example. His motivation was that learning by Bayesian conditionalisation, on the basis of the behaviour of the coin in repeats of the experiment, would otherwise not be possible. See Gillies ([13], p. 45–46) for more discussion of this.

In any event, the whole point of paradoxes such as Bertrand's is that they concern continuous cases and hold even if one introduces an indivisibility criterion. So despite occasional attempts to defend the principle of indifference or similar strategies—see Jaynes [18], Marinoff [22]—the more widespread view—see van Fraassen [46], Gillies ([13], p. 49), Shackel [43], and Rowbottom [34]—is that the paradox is insoluble.[8]

[8]This said, some putative solutions have recently appeared. See Aerts and Sassoli de Bianchi [1], and Gyenis and Rédei [14]. Both papers are rather technical in character.

21.4 The Subjective Interpretation

The subjective interpretation eschews talk of logical relations, to focus instead on personal degrees of belief.[9] (We'll come to the ontology of 'degrees of belief' a little later.) Sometimes it is said that in the subjective interpretation, probabilities *just are* degrees of belief. But this is a mistake, albeit one supported by a casual reading of the likes of De Finetti.[10] The reason is that not all degrees of belief need satisfy the probability calculus; and as a matter of fact, even clever people like logicians and mathematicians can have inconsistent beliefs. Thus the subjective interpretation relies on the idea that it is *necessary* for a person's degrees of belief to satisfy the probability calculus in order for those degrees of belief to be rational. So following Keynes ([21], p. 20), advocates of the subjective view hold that: 'Belief, whether rational or not, is capable of degree'. In addition, they hold that *rational* degrees of belief do not violate the axioms of probability; they range between 0 and 1, and so on.

In fact, Ramsey and De Finetti hailed it as a virtue of the subjective interpretation that the axioms of probability can be *derived* from a consideration of the rules that degrees of belief ought to obey, via a consideration of rational betting behaviour. This is usually known as the Dutch Book Argument, and sometimes as the Ramsey-De Finetti theorem. The basic idea is simple. Put someone in a betting scenario. If they bet in such a way as to be susceptible to losing whatever happens, then they must have accepted bets with betting quotients—these will be explained below—that fail to satisfy the axioms of probability. Now assume that those betting quotients reflect the person's degrees of belief and that being susceptible to losing whatever happens is irrational. The conclusion is that rational degrees of belief satisfy the axioms of probability.

One way to set up such a scenario is as follows. T (a tester) asks B (a bettor) to choose a number q, her betting quotient on E, on the understanding that T will then choose a stake S, that B will pay T the sum of qS, and that B will receive S if E occurs. If E does not occur, T will keep the sum of qS. Note that T may choose a positive of negative value for S. (Giving a sum of qS, in the event that S is positive, is equivalent to receiving a sum of $q|S|$ in the event that S is negative, and so forth. So if S is negative, then T will pay B the sum of $q|S|$, in return for $|S|$ if E occurs; else, B will keep $q|S|$.) A final stipulation is that B does not know which way she will be betting (i.e., whether T will choose a positive or negative value for S).

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[9]In the words of Ramsey ([27], pp. 72–73): 'the kind of measurement of belief with which probability is concerned is . . . of belief *qua* basis of action . . . with beliefs like my belief that the earth is round . . . which would guide my action in any case to which it was relevant.' Ramsey's example seems somewhat odd, however, because it seems that all conceivable beliefs can be relevant to action in appropriate cases. For example, I could be asked what I believe about some obscure philosophical issue and desire to express the truth. An asseveration I made in response would be guided by that belief.

[10]The following statement, for example, is misleading: 'only subjective probabilities exist—i.e. the degree of belief in the occurrence of an event attributed by a given person at a given instant with a given set of information.' (De Finetti [8], pp. 3–4)

You may already have thought of several problems with this scenario. If the stake is too small, such as $0.01, then B might give any old answer. Or B may fear paying T the sum of qS in the event that T selects S positive if E concerns the far future because she'll then be out of pocket for a long time—and select a value of q with the intent to entice B to select a negative value for S. These problems can solved by refining the scenario; the magnitude of the stake can be set so that it's significant, the betting procedure can be altered such that money is always given to the tester (or the bets can be on utilities rather than cash), and so forth. But there are other problems that appear insoluble. For example, the fact that B does not know which way T is going to bet does not mean that she should not have an opinion about how T will bet. And recognising this leads to the rather remarkable result that a bettor might rationally select a betting quotient of zero for an event that she is sure will occur; see Rowbottom [28]. According to the axioms of probability, though, the probability of such an event should be unity! Similar problems have been discussed in considerable depth elsewhere, e.g. by Seidenfeld et al. [42] and Hájek [16].[11]

Given the problems with the Dutch book argument, it is surprising that a much more promising alternative, proposed by De Finetti [7], is less well known. This is to use a scoring rule. The idea is to put the bettor—or the person whose degrees of belief you want to elicit—into a situation in which she believes that she will be penalised by a particular loss, dependent on her forecast (which is a number similar to a betting quotient in the gambling case above) and what happens. Now we can proceed only by assuming that she wishes to minimise her expected loss, and need not worry about how she anticipates that someone else—a tester (or bookie)—will behave. For more on this strategy, see Schervish et al. [37].

It is crucial to note that there are several different understandings of degrees of belief. De Finetti, for instance, thought of these in an *operational* and *behavioural* manner, such that they are *identical* to betting quotients or forecasts. Most plausibly, degrees of belief might be understood, in such a vein, as dispositions to bet or forecast in particular ways. By contrast, the more popular view at present is that degrees of belief are *credences* or degrees of confidence. Such credences may not be reflected in betting behaviour, which is an advantage. One worry, however, is how to measure them (as something above and beyond, say, forecast dispositions). Perhaps the most obvious route is to appeal to some kind of personal awareness, like strength of feeling. But this is a dubious move. As Ramsey ([27], p. 71) pointed out:

> This view would be very inconvenient, for it is not easy to ascribe numbers to the intensities of feelings; but apart from this it seems to me observably false, for the beliefs which we hold most strongly are often accompanied by practically no feeling at all; no one feels strongly about things he takes for granted.[12]

[11] A good place to read more about Dutch Books is Hájek [17].

[12] Note, in particular, the comment about ascribing numbers to intensities of feeling. The idea that people can have precise degrees of belief corresponding to any rational number between 0 and 1—or perhaps beyond, if we're discussing degrees of belief that don't satisfy the probability

So just as there are different accounts of belief, in the philosophy of mind, there are different accounts of degrees of belief in the philosophy of probability (or formal epistemology). In fact, here is an area in which there is the potential for a valuable exchange between these two areas of philosophy. This is suggested by my recent debate with Eric Schwitzgebel—see Schwitzgebel [38–40] and Rowbottom [29, 36]—on whether degrees of belief can explain cases of apparent 'in-between believing'. De Finetti's behavioural approach seems to have an analogue in the dispositional approach to belief (although De Finetti was concerned with only a narrow range of dispositions, at best). So are credences typically construed as mental representations, i.e. as beliefs are on a representational account in the philosophy of mind? (And might they not instead be construed as dispositional but not limited to betting and/or forecast scenarios?) For more on the nature of degrees of belief, see the discussion of Eriksson and Hájek [10]. They suggest that degrees of belief are primitive, and point to the success that decision theory has had without providing an analysis of the notion. But they do not engage with contemporary philosophy of mind.

21.5 Objective Bayesianism

The basic idea behind objective Bayesianism—which is championed by Jaynes [20] and Williamson [47, 48], for example—is that one can start in the same way as an advocate of the subjective view does, e.g. with a betting or forecast scenario, and then show that there are constraints on rational degree of belief which subjectivists do not consider. It is therefore unsurprising to find that Gillies ([12], Sect. 2) characterises 'objective Bayesianism' as follows:

> [The] approach could be called the 'topping-up' version of the logical interpretation of probability. The idea is to start with purely subjective degrees of belief. We then add one rationality constraint (coherence) to obtain the axioms of probability. However, this might be 'topped-up' by further rationality constraints derived from logical or inductive intuition. Thus the choice of different probabilities allowed by the subjective theory would be narrowed down, and eventually it might be possible to get back to a single rational degree of belief as in the original logical theory.

To be more specific, objective Bayesians think that degrees of belief should (a) be probabilities (in the sense of satisfying the calculus), (b) reflect the evidence of their possessors, especially in so far as this concerns observed frequencies and/or estimates of aleatory probabilities, and (c) otherwise be maximally non-committal. Williamson [48] calls these the (a) *probability*, (b) *calibration*, and (c) *equivocation*

calculus—is clearly an idealisation. It's more realistic to think that they lie in particular intervals. There is a related literature on imprecise probabilities, and in fact the idea of working with intervals was discussed at considerable length by Keynes [21]. For more on the notion, which is growing in popularity, see http://www.sipta.org and Bradley [4].

norms. With regard to (c), a principle called ‘the maximum entropy principle’ is used in place of the principle of indifference. Jaynes ([19], p. 623) described this as ‘an extension of the principle of insufficient reason [i.e. indifference]’, and declared ‘that it can be asserted for the positive reason that it is uniquely determined as the one which is maximally noncommittal with regard to missing information’. It is superior to the principle of indifference at least in so far as it has greater generality.

How close the objective Bayesian view *really* is to the logical one is contested, and the disagreement rests to some extent on how one reads Keynes; see Rowbottom [30] and Williamson ([48], p. 23).[13] One potential advantage is the avoidance of the posit of logical relations ‘out there’; instead rational degrees of belief may be *defined* in terms of degrees of belief that satisfy the aforementioned norms. But it is fair to say that some of the strongest criticisms of objective Bayesianism have a similar flavour to the strongest criticisms of the logical view. In particular, they focus on the successor to the principle of indifference, namely the maximum entropy principle. One suggestion is this is just as paradoxical as its forerunner; see Seidenfeld [41].

The phrase ‘objective Bayesian’ might also give the false impression that (learning by) Bayesian conditionalisation plays a central role in the interpretation of probability being proposed. But as Williamson [47, 48] makes clear, objective Bayesians may hold that individuals *should not* update their degrees of belief by Bayesian conditionalisation. This brings us on to the next section.

21.6 ‘Degree of Belief’ Interpretations

It might be preferable to oppose the logical interpretation of probability to *degree of belief interpretations of probability*. (This is new terminology, but it seems fitting.) Compare the subjective and objective Bayesian interpretations, or what we might now call ‘subjectivism’ and ‘objectivism’. On a *pure* subjective view, having degrees of belief that satisfy the axioms of probability is sufficient for rationality. But some introduce further rationality requirements, e.g. that degrees of belief should reflect observed frequencies where appropriate, as ‘top ups’. Ultimately, it is best to understand this in terms of a spectrum; strong objectivists (like some ‘objective Bayesians’) hold that personal probabilities should always have special unique values, whereas pure subjectivists only require that they lie within a particular range. Those in the middle of the spectrum think that personal probabilities should have unique values in some contexts, but not in others, or think the range is narrower than pure subjectivists do.

Williamson [48], for instance, holds that sometimes there are different permissible ways to *equivocate*, or to be non-committal. And when this is the case, he

[13] For example, I argue that Keynes did hold that observed frequencies should constrain our degrees of belief, or at least that his interpretation could easily accommodate this idea. I also dispute the view that the principle of indifference is not as well motivated as the maximum entropy principle.

thinks there's a free choice about how to equivocate. As such, he holds that in some circumstances there is no particular personal probability distribution that one ought to adopt, although some distributions will nevertheless be wrong (in so far as they don't equivocate). Think back to the earlier example of the two flips of the coin. One can opt to use a sample space of {no heads, one head, two heads} or instead of {no heads, one head and one tail, one tail and one head, two heads}. If one equivocates on the first space, the probability of no heads will be one third. If one equivocates on the second space, the probability of no heads will be one quarter. So Williamson might suggest that the probability one has for 'no heads' ought to be one of these two values, other than anything else.[14]

21.7 Group Level Interpretations

One final kind of interpretative strategy is to consider *group*, rather than personal, degrees of belief. The idea behind this approach, which was first proposed by Gillies [11], is that a group can have a Dutch Book made against it if its members do not have the same degree of belief assignments, i.e. reach consensus, and those assignments do not satisfy the probability calculus. Imagine a married couple, with pooled financial resources. Romeo bets £100 that it will rain tomorrow, at even odds. Juliet simultaneously bets £150, at three to one on, that it will not rain. No matter what happens, they will lose £50. (Note that this depends on the individuals having degrees of belief too; so a group level interpretation is *supplemental* to a personal level one.)

This idea has not really caught on, but it can be developed in several ways as shown in Rowbottom ([33]; [35], Chapter 6). For example, group degrees of belief may be interpreted differently from personal degrees of belief; the latter might be understood as credences, while the former might be understood as agreed betting quotients (along with the appropriate dispositions). So a group may be understood to reach consensus about how to bet without sharing individual credences.

Furthermore, it is possible to consider a spectrum of group interpretations, ranging from purely intersubjective to 'interobjective'. On a pure intersubjective account, it does not matter how consensus is reached; it is enough that it is present. On an interobjective account, by contrast, particular procedures are also required in order to form consensus, e.g. critical discussion with input from all members of group who have relevant degrees of belief, and/or relevant expertise. There will therefore be some scenarios, at least, in which group probabilities have unique values.

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[14] Williamson would presumably insist that the sample space in *this* case should be the latter. However, when the sample spaces are continuous, e.g. in the paradox of Bertrand [3], he thinks that it is allowable to equivocate on the basis of different sample spaces. In short, the idea is that the sample space to use is not clearly specified in the way the problem is set up.

Whatever else might be said about the merits of such approaches, it is sometimes the case that group decisions are better than individual ones; and talk of probabilities at the group level may therefore prove useful in the context of confirmation theory. (Often we're interested in the recommendation of a group, on the basis of the union of the background knowledge of the members. And it seems natural to talk about what the groups thinks, its degrees of confidence, and so forth.) But this research programme is still in its infancy.

Recommended Further Reading

Rowbottom [35] is the most accessible introduction to the interpretation of probability, and requires no mathematical background. It also covers the significance of the interpretation of probability in several contexts: philosophical, social scientific, and natural scientific.

Childers [6] is an intermediate-level introduction. It is especially noteworthy for its extended discussion of the maximum entropy principle, which lies at the heart of objective Bayesianism.

Gillies [13] is the classic textbook on the interpretation of probability. It is more advanced in character than the aforementioned books, and is a very rewarding read for those with solid mathematical backgrounds.

Eagle [9] is a useful collection of classic work on the philosophy of probability (rather than only 'contemporary readings', as its title unfortunately suggests). It is at an advanced, research, level.

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