Chapter 20
Decision Theory Without "Independence" or Without "Ordering"

What Is the Difference?

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Introduction

It is a familiar argument that advocates accommodating the so-called paradoxes of decision theory by abandoning the "independence" postulate. After all, if we grant that choice reveals preference, the anomalous choice patterns of the Allais and Ellsberg problems (reviewed in section "Review of the Allais and Ellsberg "Paradoxes"") violate postulate P2 ("sure thing") of Savage's (1954) system. The strategy of making room for new preference patterns by relaxing independence is adopted in each of the following works: Samuelson (1950), Kahneman and Tversky's "Prospect Theory" (1979), Allais (1979), Fishburn (1981), Chew and MacCrimmon (1979), McClennen (1983), and in closely argued essays by Machina (1982, 1983 [see the latter for an extensive bibliography]).

There is, however, a persistent underground movement that challenges instead the normative status of the "ordering" postulate for preference. Those whose theories evidence some misgivings about ordering include: Good (1952), C. A. B. Smith (1961), Levi (1974, 1980), Suppes (1974), Walley and Fine (1979), Wolfenson and Fine (1982), and Schick (1984). And abandoning ordering is a strategy that has been used to resolve group decision problems. For this see Savage (1954, section 7.2) and Kadane and Sedransk (1980), and see Kadane (1986) for an application to clinical trials. "Regret" models also involve a failure of ordering since choice-with-regret does not satisfy Sen's (1977) principle of "independence of irrelevant alternatives": Savage (1954, section 13.5), Bell and Raiffa (1979) and Bell (1982), Loomes and Sugden (1982), and Fishburn (1983) discuss regret.

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Expected Utility for Simple Lotteries – A Review

For ease of exposition, let us adopt an axiomatization similar to the von Neumann and Morgenstern (1947) theory, as condensed by Jensen (1967). Let R be a set of β-many rewards (or payoffs), $R = (r_\alpha\colon \alpha \leq \beta)$. In the spirit of Debreu's (1959, chapter 4) presentation, we can think of R as including (infinitely divisible) monetary rewards. A (simple) lottery over R is a probability measure P on R with the added requirement that $P(X) = 1$ for some *finite* subset of rewards.

Lotteries are individuated according to the following (0th) *reduction postulate*: Let L_1, L_2 be two lotteries with probabilities P_1, P_2 and let $R_n = (r_1, \ldots, r_n)$ be the finite set of the union of the payoffs under these two lotteries. A convex combination, $\alpha L_1 + (1-\alpha)L_2$ $(0 \leq \alpha \leq 1)$, of the two lotteries is again a lottery with probability measure $\alpha P_1 + (1-\alpha)P_2$ over R_n. Thus, the set of lotteries is a mixture set $M(R)$ in the sense of Herstein and Milnor (1953).

Three postulates comprise expected utility theory:

(1) An ordering requirement: preference, $\lesssim$, a relation over $M \times M$, is a weak-order. That is, $\lesssim$ is reflexive, transitive, and all pairs of lotteries are comparable under $\lesssim$. (Strict preference, $<$, and indifference, $\sim$, are defined relations.)

(2) An Archimedean requirement: If $L_1 < L_2$ and $L_2 < L_3$, there is a nontrivial convex combination of L_1 and L_3 strictly preferred (and another combination strictly dispreferred) to L_2. That is, there exist

$$0 < \alpha, \beta < 1 \text{ with } \alpha L_1 + (1-\alpha)\,L_3 < L_2 \text{ and } L_2 < \beta L_1 + (1-\beta)\,L_3.$$

The point in assuming that R includes (infinitely divisible) monetary payoffs is made clear by the additional stipulation that each lottery in M carries a sure-dollar equivalent (under $\lesssim$):

$$\forall L \in \mathrm{M}\ \ \exists \$x \in \mathrm{R}\quad (L \sim L_{\$x})\,, \qquad (*)$$

where $L_{\$x}$ is a degenerate lottery having only one prize, $\$x$. Then principle (2) deserves its title for, with (*) and the added stipulation that more is (strictly) better when it comes to money, (1) and (2) entail a real-valued utility representation for $\lesssim$ (continuous in \$).[1] To simplify still further, let us restrict attention to lotteries with none but monetary payoffs.

[1]By assuming (*), we fix it that $M/\sim$ (no longer assumed to be a mixture set) has a countable dense subset in the $<$-order on $M/\sim$, e.g., the rational-valued sure-dollar equivalents. Then our first two postulates ensure a real-valued utility u on M with the property that $L_1 < L_2$ if and only if $u(L_1) < u(L_2)$. The point is that, without "independence," the usual Archimedean axiom is neither necessary nor sufficient for a real valued utility. See Fishburn (1970, Section 3.1) for details, or Debreu (1959, Chapter 4), who discusses conditions for u to be continuous. Debreu uses a "continuity" postulate in place of (2) that, in our setting, requires that if the sequence $[L_i]$ converges (in distribution) to the lottery L_i, and $L_i < L_k$, then all but finitely many of the $L_i < L_k$. If we extend $\lesssim$ to general distributions over R, Debreu's continuity postulate entails countably

(3) The "independence" principle: For all L_i, L_j, and L_k, and for all α ($0 < \alpha \leq 1$), $L_i \precsim L_j \iff \alpha L_i + (1-\alpha)L_k \precsim \alpha L_i + (1-\alpha)L_k$.

Let us examine these postulates for the special case of lotteries on three rewards: $R = (r_1 < r_2 < r_3)$, where the reward r_i is identified with the degenerate lottery having point-mass $P(r_i) = 1$ $(i = 1,2,3)$. Following the excellent presentation by Machina (1982), we arrive at a simple geometric account of what is permitted by expected-utility theory. Figure 20.1 depicts the consequences of postulates (l)–(3).

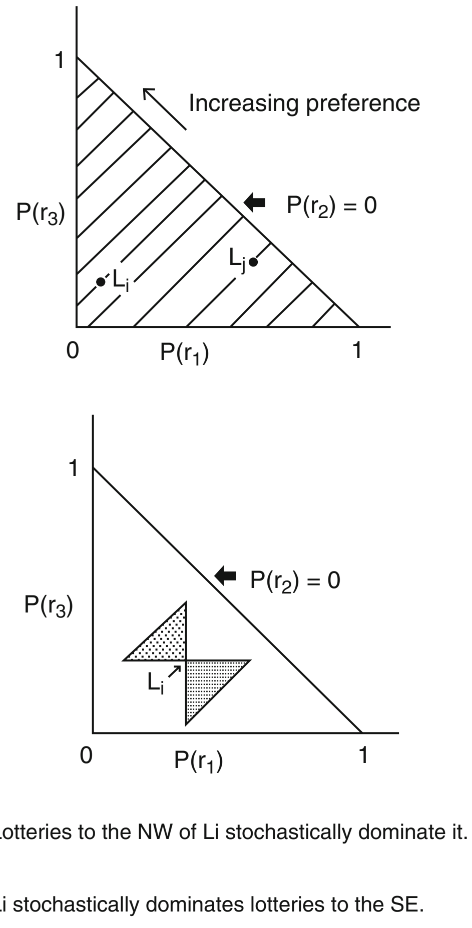

Fig. 20.1 Geometry of cardinal utility with three rewards

additive probability. See Seidenfeld and Schervish (1983) for some discussion of the decision-theoretic features of finitely additive probability.

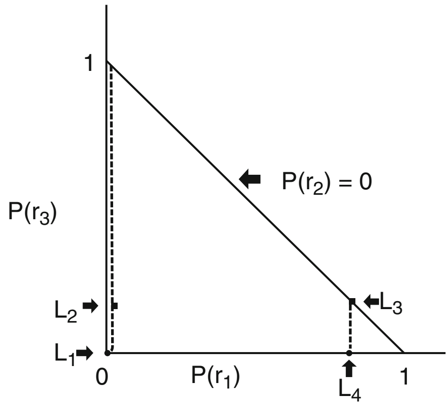

Fig. 20.2 Geometry of the Allais paradox

According to the postulates (l)–(3), indifference curves (~) over lotteries are parallel, straight lines of (finite) positive slope. L_i is (strictly) preferred to L_j, $L_j < L_i$, is just in case the indifference curve for L_i is to the left of the indifference curve for L_j.

Consider a lottery L_i, as in Fig. 20.2. Stochastic dominance provides a (strict) preference for lotteries to the NW of L_i, whereas L_i is (strictly) preferred to lotteries to its SE.[2] Thus, the indifference lines must have positive slope. Hence, in this setting with lotteries over three rewards, expected-utility theory permits one degree of freedom for preferences, corresponding to the choice of a slope for the lines of indifference.

In a collaborated effort, Seidenfeld et al. (1987, Section 1), we apply this analysis to the selection of "sizes" (α-levels) for statistical tests of a simple null hypothesis against a simple rival hypothesis. The conclusion we derive is the surprising "incoherence" (conflict with expected-utility theory) of the familiar convention to choose statistical tests with a size, e.g., $\alpha = .01$ or $\alpha = .05$, independent of the sample size. This reasoning generalizes that of Lindley (1972, p. 14, where he gives his argument for the special case of "0–1" losses). In a purely "inferential" (nondecision-theoretic) Bayesian treatment for the testing of a simple hypothesis versus a composite alternative, Jeffreys (1971, p. 248) argues for the same caveat about constant α-levels.

[2]Recall, lottery L_2 (first order) stochastically dominates lottery L_1 if L_2 can be obtained from L_1 by shifting probability mass from less to more desirable payoffs. More precisely, L_2 stochastically dominates L_1 if, as a function of increasingly preferred rewards, the cumulative probability distribution for L_2 is everywhere less than (or equal to) the cumulative probability distribution for L_1. Of course, whenever L_2 stochastically dominates L_1, there is a scheme for payoffs, in accord with the two probability measures, where L_2 weakly dominates L_1.

Review of the Allais and Ellsberg "Paradoxes"

3.1

Allais (1953) poses the following question. For the three rewards, $r_1 = \$0$, $r_2 = \$1$ million, and $r_3 = \$5$ million (so $r_1 < r_2 < r_3$), what are your preferences, in the choice between lotteries L_1 and L_2, and in the choice between lotteries L_3 and L_4, where:

$$L_1 - \text{with } P(r_2) = 1 \quad (\$1 \text{ million for certain}),$$
$$L_2 - \text{with } P(r_1) = .01, \; P(r_2) = .89, \text{ and } P(r_3) = .10,$$
$$L_3 - \text{with } P(r_1) = .90 \text{ and } P(r_3) = .10, \text{ and}$$
$$L_4 - \text{with } P(r_1) = .89 \text{ and } P(r_2) = .11?$$

The common response, choose L_1 over L_2, and L_3 over L_4, violates EU theory (under the assumption that the choices reveal $<$). This is made evident by an application, Fig. 20.2, of (Machina's) figure 1.

The lines connecting the pairs of lotteries in the two choices are parallel. Thus, regardless of the slope of the parallel, straight-line indifference curves (from Fig. 20.1) imposed on lotteries over the three rewards, either L_2 and L_3 are preferred to their rivals, or else L_1 and L_4 are preferred. EU precludes the common answer to Allais' question.

3.2

Ellsberg's (1961) paradox of preference for lotteries with known risk ($\in M$) over uncertain lotteries ($\notin M$), bearing unknown risk, does not fit the simple mixture-set model, M. We can accommodate Ellsberg-styled problems by generalizing our concept of acts so that an act is a function f from states, a (finite, exhaustive) partition, to distributions on the reward set R. These more general acts are called "horse lotteries" by Anscombe and Aumann (1963). Denote by M' ($\supset M$) the generalized mixture-set for the class of horse lotteries.[3] Then lotteries of known risk belong to this enlarged (mixture) set M' as a special case: they are the "constant" acts. That is, the acts of known risk are those for which f^{-1} is a determinate probability measure.

Let us see how an Ellsberg-styled paradoxical choice violates postulate 3, supposing (1) and (2) obtain, when the postulates are applied to M'. Imagine I

[3]To define the generalized mixture-set M', it suffices to define the operation of convex-combination of two (generalized) lotteries. This is done exactly as in Anscombe and Aumann's (1963) treatment of "horse lotteries." Horse lotteries, the generalized postulates (l)–(3) for horse lotteries, and, with the addition of two minor assumptions (precluding a preference-interaction between payoffs and states), the subjective expected-utility theory that results, are discussed by Fishburn (1970, Chapter 13) and briefly in section Sequential coherence of Levi's decision theory here.

have placed \$10 in one of two pockets, which are otherwise empty. Consider the following three lotteries:

L_{left} – take the contents of my left pocket,
L_{right} – take the contents of my right pocket, and
L_{mix} – take the contents of my left pocket if a "fair" coin lands tails up, and take the contents of my right pocket if the fair coin lands head up.

Lotteries L_{left} and L_{right} are uncertain prospects. Suppose you are indifferent ($\sim$) between these two, which you evaluate as having a sure-dollar equivalent of \$2.50. However, the third option, L_{mix}, is (under the "reduction" postulate) a lottery of known risk. That is, L_{mix} is a lottery with an equal (.5; .5) probability distribution on the two payoffs (\$0, \$10). In the spirit of the Ellsberg paradoxical choice, suppose you evaluate the fair gamble on these two payoffs as having, say, a sure-dollar equivalent of \$4.00. You (strictly) prefer the lottery of known risk, L_{mix}, to either of the two uncertain lotteries. Finally, as the coin flip gives you no relevant information about the location of the \$10, your conditional preferences over the two uncertain lotteries (and their \$2.50 equivalent) are unaffected by the outcome of the coin flip. Then, as L_{mix} is (under reduction) equivalent to the ($\alpha = .5$) convex combination of L_{left} and L_{right}, preference for "risk" over "uncertainty" violates the independence postulate 3, assuming (1) and (2) hold.[4]

In fact, given (1) and (2), this version of the Ellsberg paradox conflicts with a principle (4), (strictly) weaker than principle (3).

(4) Mixture dominance ("betweenness"): Of lotteries L_1 and L_2, if each is (weakly or strictly) preferred (or dispreferred) to a lottery L_3; so, too, each convex combination of L_1 and L_2 is (weakly or strictly) preferred (or dispreferred) to L_3.

[4]These preferences are in conflict with Savage's (1954) "sure-thing" postulate P2. P2 is inconsistent with the following two preferences:

(i) $L_{\text{right}} < L_{\text{mix}}$.
(ii) $L_{\text{left}} \sim L_{\text{right}}$, given the coin lands heads up.

Consider the four-event partition generated by whether the coin lands heads (H) or tails (T), and whether the \$10 is in the left (L) or right (R) pocket. Then, by (i), the first row (below) is preferred to the second. Savage's theory uses "called-off" acts to capture conditional preference. Thus, by (ii), the agent is indifferent between the third and fourth rows.

	HL	HR	TL	TR
L_{mix}	\$10	\$0	\$0	\$10
L_{right}	\$0	\$10	\$0	\$10
$L_{\text{left}} \mid H$	\$10	\$0	\$0	\$0
$L_{\text{right}} \mid H$	\$0	\$10	\$0	\$0

In terms of (Machine's) figure 1, mixture dominance entails linear indifference curves. (This follows directly with (4), as then indifference is preserved under convex combinations. In Fig. 20.1, the set of convex combinations of two lotteries graphs as a straight line.) But the conjunction of (1), (2), and (4) does not entail (3). Samuelson's (1950) "Ysidro" ranking, and the "weighted utility" theory of Chew (1981) satisfy (1), (2) and (4) but fail (3).[5] Chew (1983) shows that the Allais paradoxical choices are admitted by his theory. What we find here is that Ellsberg-styled preference for risk over uncertainty cannot be so easily absorbed. In order to admit the Ellsberg-styled paradoxical choices, mixture dominance, (4), too, must fail.

Objections to the Denial of "Independence"

On Failures of "Stochastic Dominance"

Kahneman and Tversky's (1979) intriguing alternative to EU, "Prospect Theory," gives a reconstruction of Allais' paradoxical choice behavior at the expense of the independence postulate. Call a simple lottery *regular* provided not all its payoffs are (strictly) preferred to "status quo." Recall, the ranking of a lottery by expected utility uses the formula:

$$\sum_i P\left(r_i\right) u\left(r_i\right).$$

For regular lotteries, the ranking of a lottery by prospect theory uses the formula:

$$\sum_i \pi\left[P\left(r_i\right)\right] \upsilon\left(r_i\right),$$

where υ is a value-function for rewards (akin to the utility u), and π is some monotone-increasing function with $\pi(0)=0$ and $\pi(1)=1$. Again, let us consider (regular) lotteries on three rewards $r_1 < r_2 < r_3$, where we may take r_1 as status quo. If (and only if) π is linear do we have agreement between prospect theory and EU (for then $\pi(x)=x$ with the scalar constant absorbed into the utility u, defined up to

[5] Let u be a utility on payoffs and assume u is positive. Denote by $E_u(L_1)$ the expected utility of lottery L_1 under utility function u. Denote by $L_1{}^{-1}$ the lottery that has payoffs with (multiplicative) inverse utility to L_1. Samuelson's (1950) "Ysidro" ranking, $\precsim_\gamma$, on lotteries is given by the function

$$Y\left(L_1\right) \;=\; \left[E_u\left(L_1\right)/E_u\left(L_1{}^{-1}\right)\right]^5.$$

Not only does $\precsim_\gamma$ satisfy the ordering, Archimedean, and mixture dominance postulates while failing independence but in addition $\precsim_\gamma$ respects stochastic dominance!

positive linear transformations). If π is not linear, so that prospect theory violates independence, stochastic dominance fails too.[6]

I do not know whether this aspect of prospect theory has been subjected to test for its descriptive accuracy. (I find it hard to believe that subjects would prefer a stochastically dominated lottery when the comparison involves just three rewards and the two lotteries involved assign identical probability to the status quo reward r_1.) In any event, it is normatively unacceptable to mandate a violation of stochastic dominance. What, after all, is left of the concern to avoid unnecessary losses (with respect to payoffs) when a theory *requires* a strict preference for an option dominated (on a set of positive probability)?

Thus, we shall examine only those violations of independence that induce preferences consistent with the partial order imposed by stochastic dominance. To that end, with an eye on the anticipated exchange between theories that abandon ordering versus those that abandon independence, I elevate respect for stochastic dominance to the status of a coherence condition.

Definition *A decision rule is* coherent *if* (*i*) *admissible choices under the rule are stochastically undominated, and* (*ii*) *admissibility is preserved under substitution* (*at choice points*) *of "indifferent" options.*

In section "Sequential coherence of Levi's decision theory", I summarize choice-based generalizations of "preference over rewards" (to explicate stochastic dominance) and "indifference over options" without assuming that choice induces a weak-order. However, in terminal decisions, when a choice rule induces an ordering $\lesssim$, condition (ii) adds nothing to (i). (See Sen's (1977) excellent discussion relating properties of choice rules to ordering.) That is, suppose lottery L_2 stochastically dominates L_1. By (i), L_1 is inadmissible when L_2 is available for choice, and $L_1 < L_2$. If, moreover, $L_1 \sim L_3$ and $L_2 \sim L_4$ then $L_3 < L_4$ by properties of $\lesssim$; so that inadmissibility of dominated options is preserved under substitution of indifferents, (ii). Therefore, in nonsequential decisions, and depending upon how indifference is defined without ordering, clause ii only serves as an added restriction on the coherence of choice rules that relax ordering. In sequential decisions the situation is rather different. As shown in section "Sequential incoherence – an example when mixture dominance fails, even though a choice rule induces a weak-order and respects stochastic dominance in nonsequential decisions, it may fail to be sequentially coherent.

The point of clause (ii) is to help identify a standard for evaluating decision rules predicated on the supposition that the agent's values for rewards (and for lotteries over those rewards) are stable over time. That is, this standard of coherence is

[6]The result is elementary and has been noted by many, including Kahneman and Tversky (1979, p. 283–284). Suppose π is not linear so that $\pi(p+q) > \pi(p) + \pi(q)$. Then by letting the value $\upsilon(r_2)$ approach the value $\upsilon(r_1)$, the agent is required (strictly) to prefer L_1: $P_1(r_1) = (1 - [p+q])$, $P_1(\mathrm{r}_2) = p + q$, and $P_1(r_1) = 0$ – over L_2: $P_2(r_1) = P_1(r_1)$, $P_2(r_2). = p$, $P_2(r_3) = q$, even though L_2 stochastically dominates L_1. The argument for the other case is similar: $\pi(p+q) < \pi(p) + \pi(q)$.

offered for assessing the performance of a choice rule in sequential decisions when basic values are unchanging. Clause (ii) is not cogent, I would argue, when basic values are subject to revision over time. Then there may be a current preference between two rewards that are (to be) judged indifferent relative to the future, changed values. Thus there is no reason to demand that substitution of "future" indifferents preserves the inadmissibility of what is, by current values, a dominated option.

Of course, the agent's knowledge of events, chance occurrences, and preceding choices inevitably changes in the course of a sequential decision. In fact, these changes in evidence are what makes valuable adaptive experimental designs.

Sequential Incoherence – An Example When Mixture Dominance Fails

Respect for stochastic dominance provides a safeguard that choice over lotteries attends to sure-gains in payoffs. Single stage (nonsequential) decisions are thereby protected from violations of weak dominance over rewards. That is not the case, however, when we attend to sequential decisions. Specifically, coherence in nonsequential decisions, in choices over lotteries, does not entail the sequential version of coherence in choices over plans. This is illustrated by an example.

Consider what happens when mixture dominance (4) fails: *Example*: Let lotteries L_1 and L_2 be indifferent with a sure-dollar equivalent of \$5.00. Suppose, contrary to (4), that an equal ($\alpha = .5$) convex combination of them is strictly preferred with a sure-dollar equivalent of, e.g., \$6.00. Denote this by $L_3 = .5\,L_1 + .5\,L_2 \sim \$6.00$. Then, by continuity of preference for monetary payoffs, there is some fee, $\$\in$, that can be attached to the *payoffs* of L_1 and L_2 (resulting in the lotteries denoted by "$L_1 - \in$" and "$L_2 - \in$") satisfying

$$L_4 = (L_3 - \in) = .5\,(L_1 - \in) + .5\,(L_2 - \in) \sim \$5.75.$$

Also, we can find some dollar prize strictly dispreferred to both of the $\$\in -$ modifications of L_1 and L_2, e.g., suppose

$$\$4.00 < (L_1 - \in)\,,(L_2 - \in)\,.$$

Thus, we have the sequence:

$$\$4.00 < (L_1 - \in)\,,(L_2 - \in) < L_1 \sim L_2 \sim \$5.00 < L_4 \sim \$5.75 < L_3 \sim \$6.00$$

and, by assumption, these preferences respect stochastic dominance in dollar payoffs. So nonsequential choices among these options, according to these preferences, result in no incoherence.

Imagine, however, that an agent with these same preferences faces the following, two-stage sequential decision. Initially (at choice point *A*), the agent has two (sequential) alternatives, plans 1 and 2. Under sequential plan 1, a fair coin is flipped; (a) if it lands heads up, the agent chooses between L_1 and a dollar prize of \$5.50; and (b) if it lands tails up, the choice is between L_2 and the dollar prize of \$5.50. Under sequential option 2, the fair coin is flipped; (c) if it lands heads up, the agent chooses between $L_1 - \in$ and \$4.00; and (d) if it lands tails up, the choice is between $L_2 - \in$ and \$4.00. (The problem is depicted by Fig. 20.3, where $L_1 \sim L_2 \sim \$5.00 < .5\,L_1 + .5\,L_2 \sim \$6.00$, and where one finds the \$∈ fee satisfying: $.5(L_1 - \in) + .5(L_2 - \in) \sim \$5.75$.)

How is the agent to choose between plans 1 and 2? It is clear, I think, that he should face up to what he knows his preferences are at choice nodes *B*, the choices he faces after the coin is flipped.[7] That is, the agent should assess the two (sequential) plans 1 and 2 in light of what he knows they lead to.

Under (1), if the coin lands heads up (a) he will choose \$5.50 over lottery L_1.[8] And if the coin lands tails up (b) again, he will choose the \$5.50 (over L_2). Thus, from the standpoint of (*A*), choosing plan 1 leads to a sure payoff of \$5.50.

Under (2), if the coin lands heads up (c) he will choose the lottery $L_1 - \in$ over the dollar reward of \$4.00. And if the coin lands tails up (d), the lottery $L_2 - \in$ is preferred to a sure \$4.00. Hence, from the standpoint of (*A*), choosing plan 2 leads

[7]Hammond's (1976, Section 3.3) felicitous phrase is that the agent uses "sophisticated" versus "myopic" choice.

[8]McClennen (1986, 1988, forthcoming) sketches a program of "resolute" choice to govern sequential decisions when independence fails. I am not very sure how resolute choice works. Part of my ignorance stems from my inability to find a satisfactory answer to several questions.

As I understand McClennen's notion of resolute choice, the agent's preferences for basic lotteries change across nodes in a sequential decision tree. (Then, the premises of the argument in Section 4 do not obtain.) In terms of the problem depicted in Fig. 4, at node *A* the agent resolves that he will choose L_1 at (a) of node *B*, and by so resolving increases its value at (a) of node *B* above the \$5.50 alternative.

There are several difficulties I find with this proposal. I suspect that the new value of L_1 at (a) will be fixed at \$6.00, and likewise for L_2 at (b). (The details of resolute choice are lacking on this point, but this suspicion is based on the observation that a minor variation in the sequential incoherence argument applies unless these two lotteries change their value from node *A* to node *B* as indicated. Just modify the construction so that the rejected cash alternative at *B* is \$6.00 – δ.) Then the assessed value of \$6.00 for L_3 (a mixture of the lotteries L_1 and L_2, now valued at \$6.00 each) is in accord with postulate (2). Such resolutions mandate that changes in preferences agree, sequentially, with the independence postulate. In terms of *sequential* decisions, is it not the case that resolute choice requires changes in values to agree with the independence postulate?

A second problem with resolute choice directs attention at the reasonableness of these mandatory changes in values. For example, consider the Ellsberg-styled choice problem described in Section 3.2. Cast in a sequential form, under this interpretation of resolute choice, if L_{mix} is most preferred, then the agent is required to increase the value for the option "take the contents of the right pocket," given that the coin lands heads up, over the value it has prior to the coin flip.

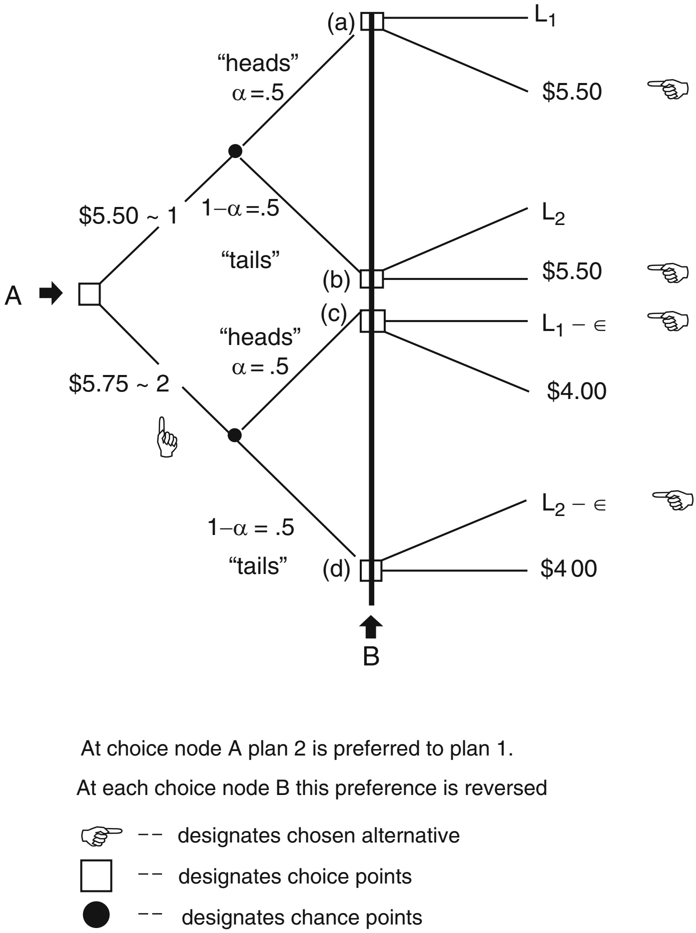

Fig. 20.3 An illustration of sequential incoherence for a failure of mixture of dominance ("betweenness")

to an equal ($\alpha = .5$) convex combination of the two lotteries $L_1 - \in$ and $L_2 - \in$. That is, from the perspective of choice node (A), sequential option 2 yields the lottery L_4, which is valued at \$5.75.

We make this reasoning precise with the following principle.

But the coin flip is irrelevant to a judgment of where the money is. However uncertain the agent is prior to the coin flip, is he not just as uncertain afterwards? Concern with uncertainty in the location of the money is the alleged justification for a failure of independence when comparing the three terminal options: L_{left}, L_{right}, L_{mix}, and declaring L_{mix} (strictly) better than the other two. What justifies the preference shift, given the outcome of the coin flip, when L_{right} becomes equivalued with an even-odds lottery over \$10 and \$0 despite the same state of uncertainty about the location of the money before and after the coin flip?

Dynamic Feasibility (DF)
To assess plan p *at a choice node* n_i, *anticipate how you will choose at its* (*potential*) *"future" choice nodes* n_j *and declare infeasible all future alternatives under* p *which are inadmissible at* n_j.

By this account, according to the principle DF, at (A) the agent prefers plan 2 over the rival plan 1. At (A) plan 2 is worth \$5.75 where plan 1 is worth only \$5.50. However, there is an embarrassment to these preferences. At choice nodes B, regardless of the fall of the coin, the agent prefers the choice he makes under plan 1 to what he chooses under plan 2.

If the coin lands heads up, the choice under (1), at (a), \$5.50 is preferred to the choice under (2), at (c), the lottery $L_1 - \in$. Likewise, if the coin lands tails up, the choice under (1), at (b), \$5.50 is preferred to the choice under (2), at (d), $L_2 - \in$. Therefore, though the agent prefers plan 2 to plan 1 initially [at (A)], he knows that this preference is reversed at (B), regardless of how the coin lands.

Under the indifferences of the ordering postulate and the preferences induced by stochastic dominance, at nodes B, the upshot is a contradiction in assessment of the sequential decision problem. The contradiction obtains as follows.

By postulate (1), the agent has a weak ordering of options at choice nodes B. There are some sure-dollar equivalents for the choices $L_1 - \in$ and $L_2 - \in$. By simple dominance, the sure-dollar equivalent for $L_1 - \in$ (or for $L_2 - \in$) is less than that for L_1. That is, each of $L_1 - \in$ and $L_2 - \in$ is worth less than \$5.00 and more than \$4.00. Since admissibility at choice nodes (such as nodes B) respects indifference, one's anticipation and knowledge [at (A)] of one's choices is only up to the level of indifferents. That is, the ordering postulate fixes choices only up to the equivalences of indifference, $\sim$. Thus, substitution of $\sim$−indifferents at choice nodes B leaves unchanged choices at those nodes. But plan 2 is dominated by plan 1 when, as dictated by postulate (1), the choices at nodes (c) and (d) [of (B)] are switched for their (indifferent) sure-dollar equivalents.

Of course, given the replacements of $L_1 - \in$ and $L_2 - \in$ by their sure-dollar equivalents at (A) plan 1 is preferred to plan 2 by dominance. (See Fig. 20.4, where, as in Fig. 20.3, $L_1 \sim L_2 \sim \$5.00 < .5\,L_1 + .5\,L_2 \sim \$6.00$, and where one finds the \$$\in$ fee satisfying: $.5(L_1 - \in) + .5(L_2 - \in) \sim \$5.75$.) Hence, subject to DF, applications of postulate (1) – ordering – at nodes B lead to contradictory conclusions with decisions made at node A.

Let us call this contradictory assessment an episode of *sequential incoherence*. That is, subject to Dynamic Feasibility, respect for stochastic dominance is not

Third, by what standards can the agent reassess the merits of a resolution made at an earlier time? In other words, how is the agent to determine whether or not to ignore an earlier judgment: the judgment to commit himself to a resolute future choice and thereby to change his future values. Once the future has arrived, why not instead frame the decision with the current choice node as the initial node without altering basic values? Unless this issue is addressed, the question of how to ratify a resolution is left unanswered, and the problem remains of how to make sense of the earlier resolution once the moment of choice is at hand.

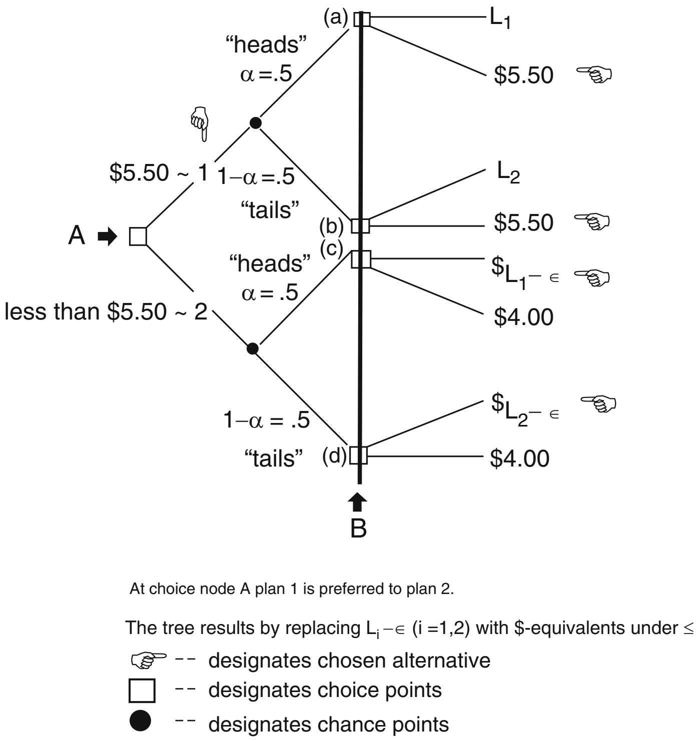

Fig. 20.4 An illustration of sequential incoherence for a failure of mixture of dominance (“betweenness”)

preserved under substitution of indifferent alternatives at choice nodes. That is, clause ii of coherence fails with this choice rule.

It is important to understand that, at (*A*) (in the problem depicted in Fig. 20.3), the agent has *no* “terminal” options, no choices of lotteries. In particular, at choice node *A*, the agent does not have the terminal option L_3. Nor does he have any of the terminal options corresponding to the other seven lotteries that arise from an equal ($\alpha = .5$) convex combination of the options available to him at nodes *B*. Thus, at (*A*), he does not have the choice of $5.50 outright. What he does have as a choice at (*A*) is plan 1, which, by DF, he equates with a certain $5.50. But plan 1 calls for decisions at nodes *B*, depending upon how the coin lands, and these subsequent choices are not to be ignored at (*A*). The principle of Dynamic Feasibility achieves a limited reduction of plans to terminal options.

In the language of game theory (Luce and Raiffa 1957, chapter 3), the sequential decision problem (above) is in *extensive* form. What we learn from this problem is that, when mixture dominance fails, even with DF, sequential decisions in extensive form are not equivalent to the normal form one-stage (nonsequential) decisions that result by ignoring subsequent choice nodes like (*B*). In a normal form version of this sequential problem each of the two plans is represented by a set of four lotteries. In normal form, the decision is among the eight lotteries:

$$[L_{3,}\,(.5\cdot\ \$5.50+.5L_2)\,,(.5L_1+.5\cdot\ \$5.50)\,,\$5.50,\ldots,\ \$4.00]\,.$$

Of these L_3 is most preferred, say. In normal form, then, plan 1 is chosen and is valued at \$6.00 ($\sim L_3$). However, the argument offered in this section (establishing sequential incoherence) does not presume the equivalence of extensive and normal forms.[9]

In the sequential problem, at node A, the agent knows L_3 is not available to him under plan 1. This is because he knows that (at nodes B) the dollar prize (\$5.50) is preferred to each of the lotteries L_1 and L_2. Under these preferences, the choice of plan 1 at (A) in the hope that L_1 will be chosen if heads and L_2 if tails is a pipe dream – mere wishful thinking that is brought up short by Dynamic Feasibility.[10]

Sequential Incoherence from Failures of Independence

What is the relation between failures of independence and episodes of sequential incoherence? An answer is given by the central critical result of this essay:

Theorem *If $\lesssim$ is a weak-order* (*1*) *of simple lotteries satisfying the Archimedean postulate* (2) *with sure-dollar equivalents for lotteries, and if $\lesssim$ respects stochastic dominance in payoffs* (*"a greater chance at more is better"*), *then a failure of independence*, (*3*), *entails an episode of sequential incoherence*.

[9]By contrast, Raiffa's (1968, pp. 83–85) classic objection to the failure of independence in the Allais paradox depends upon a reduction of extensive to normal forms. Also, in his interesting discussion, Hammond (1984) requires the equivalence of extensive and normal forms through his postulate of "consequentialism" in decision trees. These authors defend a strict expected-utility theory, in which the equivalence obtains. Likewise, LaValle and Wapman (1986) argue for the independence postulate with the aid of the assumption that extensive and normal forms are equivalent.

The analysis offered here does not presume this equivalence, nor does avoidance of sequential incoherence entail this equivalence, since, e.g., it is not satisfied in Levi's theory either – though his theory avoids such incoherence. Hence, for the purpose of separating decision theories without independence from those without ordering, it is critical to avoid equating extensive and normal forms of decision problems.

[10]One may propose that, by force of will, an agent can introduce new terminal options at an initial choice node, corresponding to the "normal" form version of a sequential decision given in "extensive" form. Thus, for the problem depicted in Fig. 20.3, the assumption is that the agent can create the terminal option L_3 at node A by opting for plan 1 at A and then choosing L_1 at (a) and L_2 at (b).

Whatever the merit of this proposal, it does not apply to the sequential decisions discussed here, since, by stipulation, the agent cannot avoid reconsideration at nodes B. There may be some problems in which agents can create new terminal options at will, but that is not a luxury we freely enjoy. Sometimes we have desirable terminal options and sometimes we can only plan. (See Levi's [1980, Chapter 17] interesting account of "using data as input" for more on this subject.)

Proof The proof is given in two cases. (The argument for the second case uses the full assumption of stochastic dominance rather than the weaker assumption [used in Case 20.1] that < respects simple dominance in dollar payoffs.)

Case 20.1 Let $L_1 \lesssim L_2$, yet for some L_3 and α, $\alpha L_2 + (1-\alpha)L_3 < \alpha L_1 + (1-\alpha)L_3$. Let $\$X \sim \alpha L_2 + (1-\alpha)L_3$, $\$Z \sim \alpha L_1 + (1-\alpha)L_{3,}$ and $\$\mathcal{U} \sim L_3$, with $X < Z$. By our assumptions of a weak order for preference, continuity in dollar payoffs, and the strict preference for more (over less) money, there is some \$2∈ fee and amount \$*Y* for which:

$$\$X < \alpha\,(L_1 - 2\in) + (1-\alpha)\,(L_3 - 2\in) \sim \$Y < \$Z.$$

Next, choose fees \$γ and \$δ so that:

$$L_2 - \gamma < L_1 - 2\in \text{ and } \$\,(\mathcal{U} - 2\delta) < \mathrm{L}_3 - 2\in.$$

If we consider the sequential decision problem whose "tree" is depicted in Fig. 20.5 for Case 20.1, we discover by the same reasoning we used in the example above:

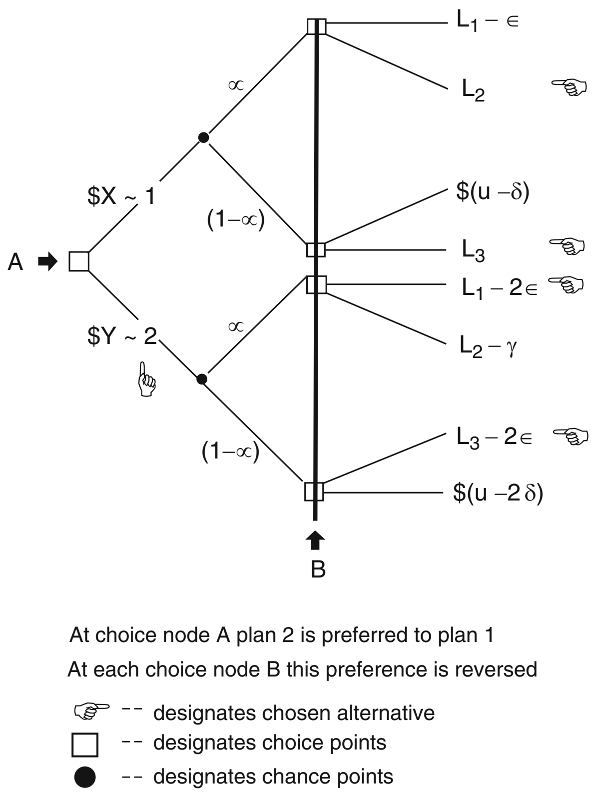

Fig. 20.5 Sequential incoherence for failures of "independence": Case 20.1

At node A, plan 1 is valued at $\$X$, whereas plan 2 is valued at $\$Y$. Thus, at node A, plan 2 is the preferred choice.

But at nodes B, regardless of which "chance event" occurs, the favored option under plan 1 is preferred to the favored option under plan 2. Thus, the preferences leading to a failure of independence in Case 20.1 succumb to sequential incoherence. An application of indifferences (from the ordering postulate 1 at nodes B leads to an inconsistent evaluation, at (A), of the sequential plans 1 and 2.

Case 20.2 $L_1 < L_2$, yet there are L_3 and $\alpha > 0$ with $\alpha L_1 + (1-\alpha)L_3 \sim \alpha L_2 + (1-\alpha) L_3$. Let $\$\mathcal{U} \sim L_3$ and $\$Z \sim \alpha L_1 + (1-\alpha)L_3$. Then choose an $\$\in$ fee so that $L_1 < L_2 - \in$. Let $\$X \sim \alpha(L_2 - \in) + (1-\alpha)L_3$. Then by stochastic dominance, $X < Z$. Next, by continuity, choose a $\$\delta$ fee to satisfy $\$Y \sim \alpha(L_1 - \delta) + (1-\alpha)(L_3 - \delta)$, where $X < Y < Z$. Choose an integer n so that $L_2 - n\in < L_1 - \delta$. Finally, find any $\$\gamma$ fee and choose an integer m so that $\$(u - m\gamma) < L_3 - \delta$.

Consider a sequential decision problem whose "tree" is depicted in Fig. 20.6 for Case 20.2. Once again we find an episode of sequential incoherence as:

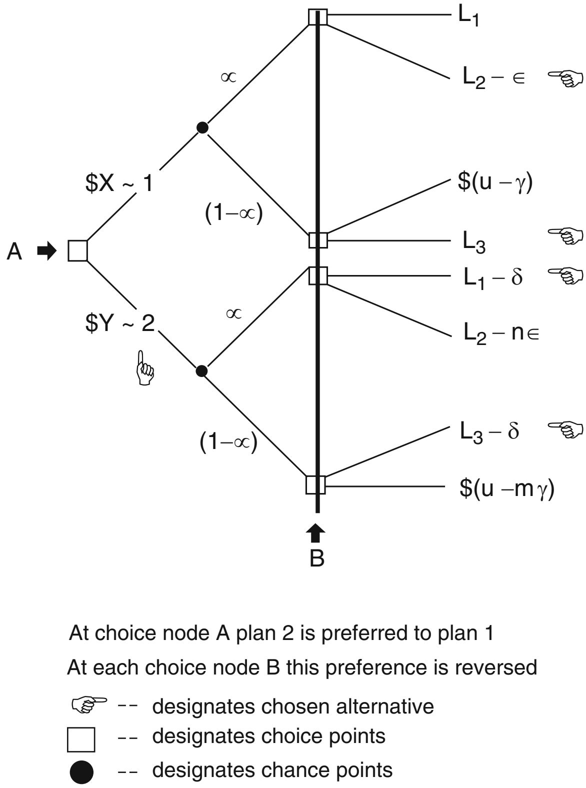

Fig. 20.6 Sequential incoherence for failures of "independence": Case 20.2

At (*A*), plan 1 is valued at \$*X*, whereas plan 2 is valued at \$*Y*. Thus, at node *A* plan 2 is the preferred choice.
At node *B*, regardless of which chance event occurs, the favored option under plan 1 is preferred to the favored option under plan 2. Thus, the preferences leading to a failure of independence in Case 20.2 succumb to sequential incoherence.

Concluding Remark

Can familiar "Dutch Book" arguments (de Finetti 1974; Shimony 1955) be used to duplicate the results obtained here? Do the considerations of book establish sequential incoherence when independence fails? I think they do not.

The book arguments require an assumption that the concatenation (conjunction) of favorable or indifferent or unfavorable gambles is, again, a favorable or indifferent or unfavorable gamble. That is, the book arguments presume payoffs with a simple, additive, utility-like structure. The existence of such commodities does not follow from a (subjective) expected-utility theory, like Savage's. And rivals to EU, such as Samuelson's (1950) "Ysidro" ranking, can fail to satisfy this assumption though they respect stochastic dominance in lotteries. Thus, in light of this assumption about combinations of bets, the Dutch Book argument is not neutral to the dispute over coherence of preference when independence fails. (Of course, that debate is not what Dutch Book arguments are designed for.)[11]

This objection to the use of a book argument does not apply to the analysis presented here. The argument for sequential incoherence is not predicated on the Dutch Book premise about concatenations of favorable gambles. That assumption is replaced by a weaker one, to wit: $\lesssim$ respects stochastic dominance in \$-rewards. There is no mystery why the weakening is possible. Here, we avoid the central question addressed by the Dutch Book argument: When are betting odds subjective probabilities? The book arguments pursue the representation of coherent belief as probabilities, given a particular valuation for combinations of payoffs. Instead, the spotlight here is placed on the notion of coherent sequential preference, given a preference (a weak ordering) of lotteries with canonical probabilities.

[11]See Frederic Schick's "Dutch Bookies and Money Pumps" (1986) for discussion of the import of this concatenation assumption in the Dutch Book argument. Its abuse in certain "intertemporal" versions of Dutch Book is discussed in Levi (1987).

Summary

Attempts to generalize EU by denying independence, while retaining the ordering and Archimedean postulates, fail the test of coherence in simple sequential choices over lotteries with dollar rewards.

Sequential Coherence of Levi's Decision Theory

A detailed analysis of Levi's Decision Theory (LDT), a theory without the ordering postulate, is beyond the scope of this essay. Here, instead, I shall merely report the central results that establish coherence of LDT in sequential choices over horse lotteries, a setting where both values (utility/security) and beliefs (probability) may be indeterminate. (I give proofs of these results in a technical report Seidenfeld (1987)).

To begin with, consider the following choice-based generalizations of the concepts: indifference, preference, and stochastic dominance. These generalizations are intended to apply in the domain of horse-lottery options, regardless of whether or not a decision rule induces a weak-ordering of acts.

The notational abbreviations I use are these. An option is denoted by o_i and, since acts are functions from states to outcomes, also by the function on states $o_i(s)$. Sets of feasible options are denoted by O, and the admissible options (according to a decision rule) from a feasible set O are denoted by the function C[O].

Call two options $\approx -$*indifferent*, if and only if, whenever both are available either both are admissible or neither is.

Definition

$$\mathrm{o}_1 \approx \mathrm{o}_2 \iff \forall\,(\mathrm{O})\ (\{\mathrm{o}_1, \mathrm{o}_2\} \subset \mathrm{O} \Rightarrow (\mathrm{o}_1 \in \mathrm{C}\,[\mathrm{O}] \iff \mathrm{o}_2 \in \mathrm{C}\,[\mathrm{O}])).$$

When a choice rule induces a weak-order, denoted by $\lesssim$, then $\approx$ is just the symmetrized $\sim$ relation: $(\mathrm{o}_1 \sim \mathrm{o}_2) \iff (\mathrm{o}_1 \lesssim \mathrm{o}_2)$ *and* $(\mathrm{o}_2 \lesssim \mathrm{o}_1)$.

Next, define a choice-based relation of categorical preference over rewards using a restricted version of Anscombe and Aumann's (1963, p. 201) "Assumption 1," modified to avoid the ordering postulate. [This assumption is part of the value-neutrality of states. See Drèze's (1985) monograph for a helpful discussion of related issues.] Let o_1 and o_2 be two horse lotteries that differ only in that o_2 awards reward r_2 on states where o_1 awards reward r_1. Then reward r_2 is categorically preferred to reward r_1 just in case o_1 is inadmissible whenever o_2 is available. In symbols,

Definition *Reward* r_2 *is* categorically preferred *to reward* $\mathrm{r}_1 \iff \forall(\mathrm{O})\ \forall(\mathrm{o}_1 \neq \mathrm{o}_2)\ (\forall(\mathrm{s})[\mathrm{o}_1(\mathrm{s}) = \mathrm{o}_2(\mathrm{s}) \vee (\mathrm{o}_1(\mathrm{s}) = \mathrm{r}_1\ \&\ \mathrm{o}_2(\mathrm{s}) = \mathrm{r}_2)]\ \&\ \mathrm{o}_2 \in O \Rightarrow \mathrm{o}_1 \notin \mathrm{C}[\mathrm{O}])$.

Third, we need to make precise a suitable generalization of stochastic dominance among lotteries. Recall, when there is an ordering ($\lesssim$) of rewards, lottery L_2 stochastically dominates lottery L_1 if L_2 can be obtained from L_1 by shifting some distribution mass from less to more preferred rewards. Thus, L_2 stochastically dominates L_1 just in case $\alpha L_2 + (1-\alpha)L_3$ stochastically dominates $\alpha L_1 + (1-\alpha)L_3$ $(0 < \alpha \leq 1)$. Rely on this biconditional to formulate the following $\prec -$*dominance* relation over horse lotteries, defined for an arbitrary choice function C.

Definition $o_1 \prec o_2 \iff \forall(O)\forall(o)\forall(\alpha > 0)\ ((\alpha o_2 + (1-\alpha)o) \in O \Rightarrow (\alpha o_1 + (1-\alpha)o) \notin C[O])$. Trivially, if $o_2 \prec -$*dominates* o_1, *then* (*let* $\alpha = 1$) o_1 *is inadmissible whenever* o_2 *is available.*

Sequential coherence for a decision rule requires, then, that

(i) shifting distributional mass from categorically less to categorically more preferred rewards produces an $\prec -$dominating option, and
(ii) the inadmissibility of $\prec -$dominated options is preserved under substitution (at choice nodes) of $\approx -$indifferents.

To see why LDT is sequentially coherent, recall that admissibility in Levi's (1974, 1980) decision theory is determined by a two-tiered lexicographic rule. The first tier is "*E*-admissibility." An option is *E*-admissible in the set O of feasible options, provided it maximizes expected utility (over O) for some pair $(P, \mathcal{U})$ – where P is a personal probability in the (convex) set **P** that represents the agent's beliefs about the states, and where $\mathcal{U}$ is a (cardinal) utility in the (convex) set **U** that represents one aspect of the agent's values over the rewards.

Definition o *is* E *-admissible* $\iff \exists(P,U)\ \forall(o' \in O)\ E_{PU}[o] \geq E_{PU}[o']$.[12]

The second tier in admissibility requires that an option be "S-admissible." This condition demands of an option that it be *E*-admissible and maximize a "security" index among the *E*-admissible options. The security of an option reflects yet another aspect of the agent's value structure. For purposes of this section, the notion of security is illustrated with three (real-valued) varieties:

$\mathrm{Sec}_o[o] = O$ – a vacuous standard, where all options have equal security;
$\sec_1[o] = \mathrm{infemum}_{U,\,R_o}\, \mathcal{U}[r]$, where R_o is the set of possible outcomes (rewards) for option o and $r \in R_o$. Thus, $\sec_1$ is security indexed by the "worst" possible reward, a maximin consideration;

[12]Levi (1980, Section 5.6) offers a novel rule, here called rule', for determining expectation-inequalities when the partition of states, π, is finite but when events may have subjective probability 0. The motive for this emendation is to extend the applicability of "called-off" bets (Shimony 1955) to include a definition of conditional probability given an event of (unconditional) probability 0. Also, it extends the range of cases where a weak-dominance relation determines a strict preference.

Given a probability/utility pair $(P, \mathcal{U})$, maximizing †-expected utility (with rule') includes a weak-order that satisfies the independence axiom, though, †-expectations may fail to admit a real-valued representation, i.e., the "Archimedean" axiom is not then valid. Under rule†, given a pair $(P, \mathcal{U})$, †-expectations are represented by a lexicographic ordering of a vector-valued quantity.

$\text{sec}_2[o] = \text{infemum}_{\text{P}\times\text{U}}\, E_{P,U}[o]$ – security indexed by the least expected utility, also called the "Γ-minimax" level for option o. (I avoid the minor details of defining sec_2 when events have probability O and expectations are determined by rule†, as reported in note 12.)

Thus, an option is admissible in LDT just in case it is both E-admissible and maximizes security among those that likewise are E-admissible. As a special case, when security is vacuous or is indexed by sec_2, and when both **P** and **U** are unit sets, C[O] is the subset of options that maximize subjective expected utility: the strict Bayesian framework. Then admissibility satisfies the ordering and independence postulates (and the Archimedean axiom, too, provided events have positive probability or rule† is not used).

The next few results, stated without proof, provide the core of the argument for sequential coherence of LDT. Condition i of coherence in nonsequential decisions, and more, is shown by the following.

Theorem 20.1 *If* o_2 *can be obtained from* o_1 *by shifting distribution masses from categorically less to more preferred rewards, then* $\text{o}_1 < \text{o}_2$*, and thus* o_1 *is inadmissible whenever* o_2 *is available.*

The theorem follows directly from a useful lemma about $<$ and categorical preference in LDT.

Lemma $\text{o}_1 \prec \text{o}_2 \iff \forall(\text{P, U})\text{E}_{\text{PU}}(\text{o}_1) < \text{E}_{\text{PU}}(\text{o}_2)$. *Thus,* r_2 *is categorically preferred to* $\text{r}_1 \iff \forall(\text{U})\ \text{U}(\text{r}_1) < \text{U}(\text{r}_2)$.

Thus, both $\prec$-dominance and categorical preference are strict partial orders, being irreflexive and transitive. In addition, following obviously from its definition, $\prec$-dominance satisfies the independence axiom. [Note the term "categorical preference" is borrowed from Levi (1986b, p. 91). The lemma provides the collateral for this conceptual loan.]

Condition ii of coherence for LDT in nonsequential decisions is shown by a result that two options are $\approx$ –related exactly when they have the same security index and are indiscernible by expectations:

Theorem 20.2 $\text{o}_1 \approx \text{o}_2 \iff (sec[\text{o}_1] = sec[\text{o}_2]\ \&\ \forall(\text{P,U})(\text{E}_{\text{PU}}[\text{o}_1] = \text{E}_{\text{PU}}[\text{o}_2])$. *Thus,* $\approx$ *is an equivalence relation and, in nonsequential decisions, admissibility is preserved under substitution of* $\approx$ *–indifferent options.*

In order to extend the analysis to sequential decisions, the notion of $\approx$ – indifference is generalized to include conditional assessments, conditional upon the occurrence of either "chance" or "event" outcomes. The next corollary is elementary and indicates how conditional $\approx$ –indifference applies when for instance, choice nodes follow chance nodes.

Corollary 20.1 $\approx$ *is preserved under chance mixtures,*

$$\forall\ (i, j, k \text{ and } \alpha)\ o_{\text{i}} \approx o_{\text{i}} \Rightarrow (\alpha o_{\text{i}} + (1-\alpha)\, o_{\text{k}} \approx \alpha o_i + (1-\alpha)\, o_{\text{k}})\,.$$

Under the principle of Dynamic Feasibility (which provides a limited reduction of sequential plans to nonsequential options), these findings combine to support the desired conclusion.

Sequential coherence for LDT

(i) Admissible options are $\prec$ −undominated among the dynamically feasible alternatives, and
(ii) Provided the agent updates his beliefs by Bayes’ rule and does not change his value structure, admissibility is preserved under the substitution of $\approx$ −indifferent alternatives at choice nodes.

Conclusions About Relaxing Independence or Ordering

I have argued that decision theories that relax only the independence postulate succumb to sequential incoherence. That is, such programs face the embarrassment of choosing stochastically dominated options when, in simple two-stage sequential decisions, dollar equivalents are substituted for their indifferent options at terminal choice nodes. Moreover, the criticism does *not* presume an equivalence between sequential decisions (in extensive form) and their normal form reductions; instead, all decisions are subject to a principle of Dynamic Feasibility.

In section “Sequential coherence of Levi’s decision theory”, I generalize sequential coherence to choice rules that may not induce an ordering of options by preference. Also, I outline reasons for the claim that Levi’s Decision Theory is sequentially coherent (in a setting where both belief and value are subject to indeterminacy). Since Levi’s theory is one that fails the ordering postulate, the combined results establish a demarcation between these two strategies for relaxing traditional (subjective) expected-utility theory. The difference is that only one of the two approaches is sequentially coherent.

Acknowledgments I have benefitted from discussions with each of the following about the problems addressed in this essay: W. Harper. J. Kadane, M. Machina, P. Maher, E. F. McClennen, M. Schervish; and I am especially grateful to I. Levi.

Discussions

Editors’ Note

Subjective expected-utility theory provides simple and powerful guidance concerning how to make rational decisions in circumstances involving risk. Yet actual decision making often fails, as has been well known for decades, to conform to the theory’s recommendations. If subjective expected-utility theory represents the ideal of rational behavior, these failures may simply show that people often behave irrationally. Yet if the gap between ideal and actual behavior is too wide, or if behavior that on the best analysis we can make is rational but not consistent with subjective expected-utility theory, then we may come to doubt some of the axioms of the theory. Two main lines of revision have been suggested: either weakening the "ordering" axiom that requires preferences to be complete or surrendering the so-called independence principle. Although the issues are highly abstract and somewhat technical, the stakes are high; subjective expected-utility theory is critical to contemporary economic thought concerning rational conduct in public as well as private affairs.

In the preceding article, "Decision Theory without 'Independence' or without 'Ordering': What Is the Difference?" Teddy Seidenfeld argued for the sacrifice of ordering rather than independence by attempting to show that abandoning the latter leads to a kind of sequential incoherence in decision making that will not result from one specific proposal (Isaac Levi's) for abandoning ordering. In their comments in this section, Edward McClennen, who supports surrendering the independence postulate rather than ordering, and Peter Hammond, who argues against any weakening of subjective expected-utility theory, discuss Seidenfeld's argument from their quite different theoretical perspectives.

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