

Optimists enrich the present, enhance the future, challenge the improbable and attain the impossible.

William Arthur Ward

He that would have the fruit must climb the tree.

Thomas Fuller

Imagination is more important than knowledge.

Albert Einstein

7.1 Chapter Purpose and Strategy

With optimism and effort we will climb together the tree of material balance.

This chapter is about one of the most important and fundamental topics for a process engineer: material balance. The significance and importance of this chapter lies in the fact that we expect to accomplish two objectives simultaneously: delight students with the prospect of a satisfying process engineering career and train and prepare them for one of the most relevant topics: material balance. We will use many examples to teach you, step by step, how to approach, formulate, and solve material balance problems and, at the same time, through diverse examples, reveal the breadth of applications of process and bioprocess engineering. You will be exposed to classical problems of chemical engineering, environmental engineering, food engineering, biochemical engineering, biotechnology, and others.

How difficult is material balance? At this stage, this is a valid question because, as mentioned earlier, and as we will discover, material balance forms the core of this book and is a vital tool for process engineers. Soon enough, certain basic engineering courses will be easier for you because of your knowledge of material balance. In addition, you will discover many applications that are relevant to everyday life.

That is why we strongly suggest that you follow, step by step, all the advice in the first sections of this chapter.

Are material balance problems difficult? The definitive answer is no, and actually, they are easy. Although they might appear difficult, in reality, with adequate training and strategy they can be made easy, fun, and, most important, heartwarming. Some problems, and probably most real-world

problems, can be nightmarish if not approached following an adequate procedure. Perhaps the key word in this chapter is PROCEDURE!

We base this approach on experience with several generations of freshmen who, after following the appropriate steps, became knowledgeable about material balance concepts and their applications.

We now invite you on this important and pleasurable journey to learn, face, formulate, solve, and apply material balance problems in process and bioprocess engineering.

7.2 What is a process?

What is a process? A process is a series of operations (normally called unit operations, like dehydration, evaporation, crystallization, or fermentation, for example) accomplished in the manufacture of an intermediate or end product (Fig. 7.1).

What is a system? A system is specific part of a process specifically chosen by the process engineer to carry out a detailed analysis, in this case, a material balance analysis (Fig. 7.1).

The dotted lines in Fig. 7.1 show the different possibilities for choosing a system for analysis. As we will explain later, in order to analyze the whole process, it is advisable to analyze it unit by unit.

How processes are classified? We will identify two classifications: (a) how processes are operated as a function of time and (b) the mode of the process operation.

(a) How processes are operated as a function of time.

A **steady-state process** is one in which the system variables do not change over time. Imagine a system where we are measuring its variables (e.g., temperature, pressure, inflow rates, and outflow rates). We repeat the measurements at various times. If the system is in steady state, every time we take a measurement, all of the variables will have the same value. For example, if an equipment is fed with a stream of 10 kg/h, has a retention mass in the interior of 600 kg, and at the output has two outputs, one with a stream of 8 kg/h and the other with a stream of 2 kg/h, then the question is: is the equipment (system) under steady state? Yes. Why? Because the total mass flow rate at the input is

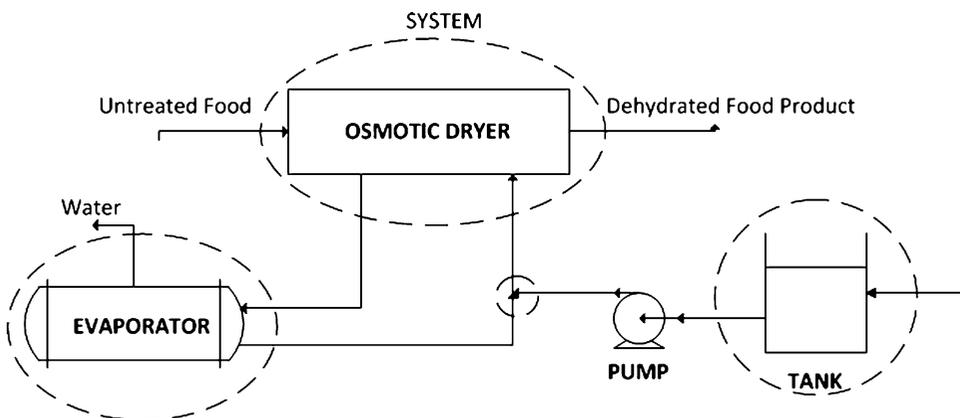


Fig. 7.1 Osmotic dehydration process

Fig. 7.2 Material balance in a steady-state process

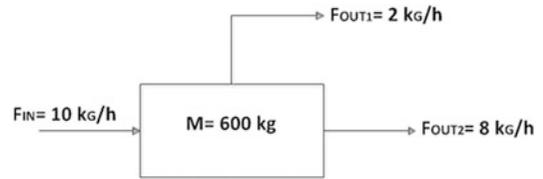
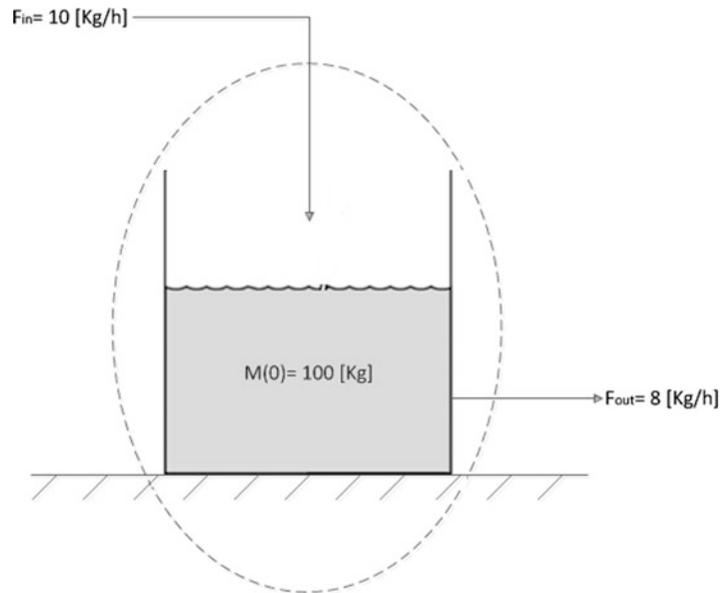


Fig. 7.3 Transient or unsteady-state process



constant and equal to the total mass flow rate at the output; therefore, the retention on the equipment will remain invariable (Fig. 7.2). As expressed in (7.1):

$$\text{Total input mass} - \text{Total output mass} = 0(\text{steady state}). \quad (7.1)$$

The dotted lines in Fig. 7.2 show the chosen system for analysis. Thus,

$$10 \text{ kg/h} - 8 \text{ kg/h} - 2 \text{ kg/h} = 0. \quad (7.2)$$

A **transient or unsteady-state process** is one in which the system variables change over time. Imagine the same system discussed previously under steady-state process. We take measurements of the system variables at some time t . If we return later to take the same measurements and find that the variables have changed, then the system is transient or in an unsteady state. An example would be a system in which the input mass flow rate and output mass flow rate differ. For example, if a tank is fed with a stream of 10 kg/h (Fig. 7.3), has an initial retention mass at the interior of $M(0) = 100 \text{ kg}$, and at the output has one stream of 8 kg/h, then the question is: is the tank (system) under steady state? No. Why? Because the mass flow rate at the input is greater than the output mass flow rate; therefore, the

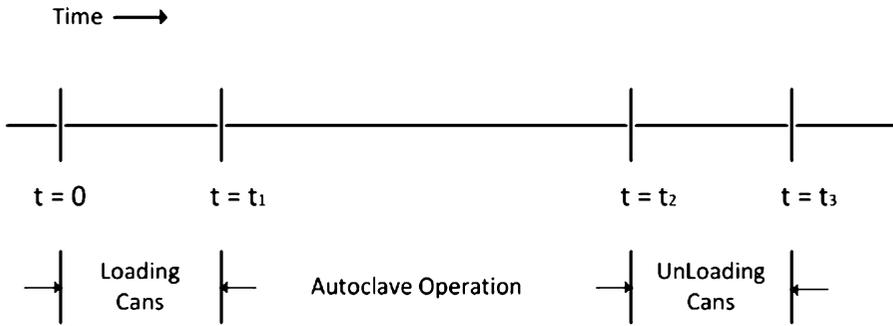


Fig. 7.4 Batch operation of an autoclave in a canning plant

retention on the tank will vary over time [$M(t)$ will accumulate over time]. The accumulation rate will be 2 kg/h. As expressed in (7.3):

As in Fig. 7.3 the dotted lines show the system under study. Thus,

$$10 \text{ kg/h} - 8 \text{ kg/h} = 2 \text{ kg/h.} \quad (7.3)$$

(b) What is the mode of process operation?

A **continuous process** is one in which the input and output streams operate uninterrupted. The product is continuously generated. Some examples are tomato concentration (food process), oil refinery (chemical process), and the brewing of beer (bioprocess).

A **batch process** is one in which the system is fed and closed, and then after a specified amount of time, product is obtained. Examples include soups, jams, specialty chemicals, canned foods, and wine fermentation. Briefly, in the case of canned foods, untreated cans are charged to the autoclave (retort); then the system is closed and operated at approximately 120 °C to sterilize the cans. After a certain amount of time (normally 60–90 min) the system is opened when the cans are commercially sterile and ready for the supermarket.

As shown in Fig. 7.4 the operation is stopped to load cans into the autoclave and then stopped after operation to unload cans.

A **semibatch (also called semicontinuous) operation** is an operation that has some continuous-process features combined with batch processes. Some process units are operated in a batch mode and other process units are operated in a continuous mode. An example of this type of process is also seen in retort processing. Here, individual retorts operate in batch mode, as shown earlier in Fig. 7.4. However, if we imagine a bank of three retorts operating in a plant, one of which is being loaded, another unloaded, and the third being operated (Fig. 7.5), it becomes apparent that from the perspective of the overall system, the process is continuous, even if the individual retorts are batch operated.

7.3 What Is Material Balance?

Material balance involves making an inventory of the quantities going in and out of a system. These quantities could be as simple as the total mass. The principles of material balances can be expanded to the balance of energy, electrical charge, and virtually any quantity that is conserved.

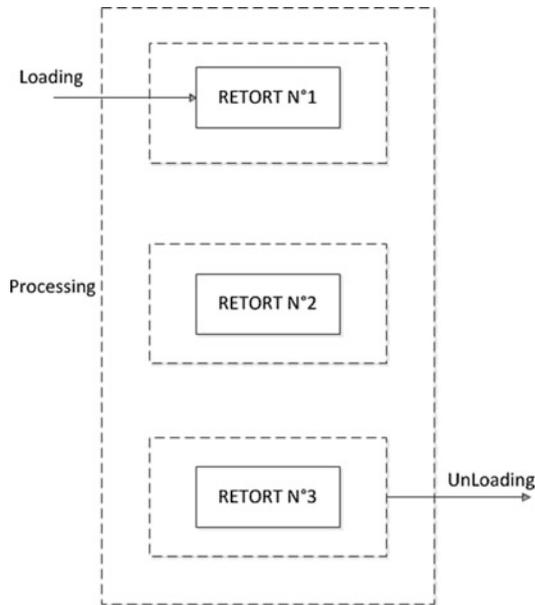


Fig. 7.5 Battery of three batch retorts operated in a continuous mode

Material balance is based on the law of conservation of mass that was established by Antoine Lavoisier (1743–1794) and states that *mass can neither be created nor destroyed, but is conserved (in any ordinary chemical reaction)*.

In general and simple terms (Fig. 7.6) we can express a total mass balance for a given unit or system as follows:

$$\begin{aligned} \text{Total input mass flow rate} - \text{Total output mass flow rate} \\ = \text{Rate of accumulation [unsteady-state condition]}. \end{aligned} \quad (7.4)$$

If input and output mass flow rates are equal, then we can say that the unit or system is under steady-state conditions because all mass flow rates and the retention mass of the system remain constant (further analysis and more explanations are given in Sect. 7.4). Therefore:

$$\text{Total input mass} - \text{Total output mass} = 0 \quad [\text{steady-state condition}]. \quad (7.5)$$

For example, Fig. 7.6 shows a tank (system) with two streams, F_{IN} and F_{OUT} . If $F_{\text{IN}} = F_{\text{OUT}}$, then we have a steady-state condition (7.5), where M is constant, i.e., it does not change with time. On the other hand, if $F_{\text{IN}} \neq F_{\text{OUT}}$, then the system is in an unsteady-state condition (7.4) and $M(t)$ changes with time, where $M(t)$ will increase versus time if $F_{\text{IN}} > F_{\text{OUT}}$ or decrease if $F_{\text{IN}} < F_{\text{OUT}}$.

Numerical examples

Example 1. Steady-state condition. A tank holding 600 kg H₂O (Fig. 7.7) is fed with 1,000 kg/h H₂O and has two streams going out, one with 800 kg/h. What must the mass flow rate of the other stream ($F_{\text{OUT}2}$) be to maintain the tank in steady-state condition?

Fig. 7.6 Materials balance in a system

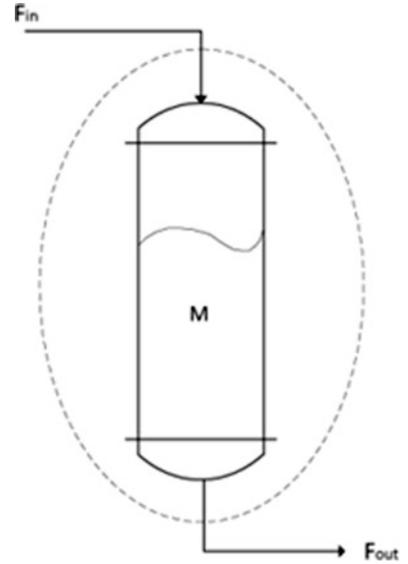
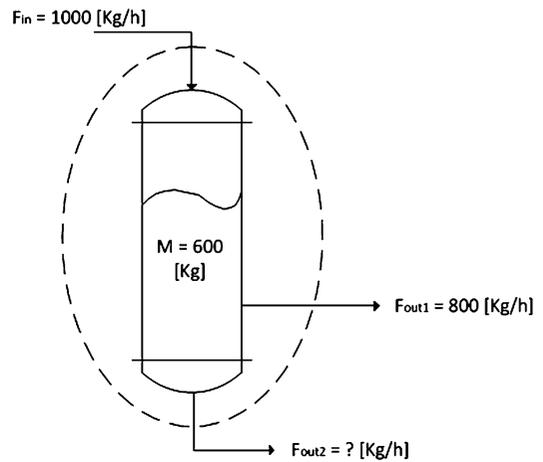


Fig. 7.7 Tank in a steady-state condition



According to (7.1) (steady state), if you sum up all input streams (in this case 1,000 kg/h) and subtract all the streams going out, the result should be 0.

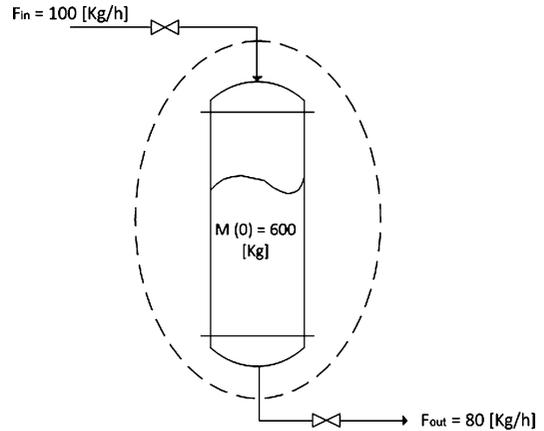
Where F_{OUT2} is the unknown stream, then

$$1,000 - 800 - F_{OUT2} = 0. \quad (7.6)$$

Therefore, $F_{OUT2} = 200$ kg/h, where 0 in (7.6) means that the retention mass within the tank (600 kg) does not change with time (steady state); all the mass that enters the tank (1,000 kg/h) leaves (800 kg/h plus 200 kg/h). Although not explicitly mentioned, the tank in this analysis is the system under study.

One of the lessons here is that to carry out a material balance you must always specify the system, then it will be clear how to identify all the streams going in and out of the system; in this case, the

Fig. 7.8 Tank in an unsteady-state condition



system was the tank. In addition, it is very important not only to define variables but also the way they are codified. Real material balance problems have several variables and sometimes tens of variables. Therefore, you need to carefully define each variable and codify them in such a way that it will be easy for you to identify and associate each variable with the flow diagram.

Example 2. Unsteady-state condition. A tank initially contains 600 kg H₂O and is fed with 100 kg/h H₂O and has one stream going out at 80 kg/h. What is the mass balance for this system? Is the system under steady state?

Learning the lesson of the first example, we will first define the system under analysis (dotted lines).

System: tank. Therefore, the mass balance in this case is

$$100 - 80 = 20 = \text{Accumulation.} \quad (7.7)$$

Given that the right-hand-side term in (7.7) is equal to 20 (different from 0), we can state that the system (tank) is in an unsteady-state condition and, in addition, because the term is positive, the system is accumulating water at a ratio of 20 kg/h. As seen in Fig. 7.8, $M(0) = 600$, indicating that the mass in the tank at time 0 is 600 but, as was shown, is increasing at a ratio of 20 kg/h.

Summary

Although simple, some steps are common and necessary in all material balance problems. (a) Graphically represent your process (tank and streams). (b) Define the system under analysis (here, the tank). (c) Define all the variables (input and output streams). (d) Formulate the material balance problem and solve it.

As we progress through the chapter we will be discovering and structuring a general procedure to facilitate the formulation and solution of real material balance problems.

7.4 General Concepts on Material Balance

Mass is conservative. In simple terms, if you feed 1,000 kg of materials to a given process, you will get the same amount at the end; you cannot get 1,500 kg! Although reactions can occur, the total amount of matter will remain the same. Matter can be transformed but neither created nor destroyed. Exceptions are nuclear reactions, but these reactions are beyond the scope of this book.

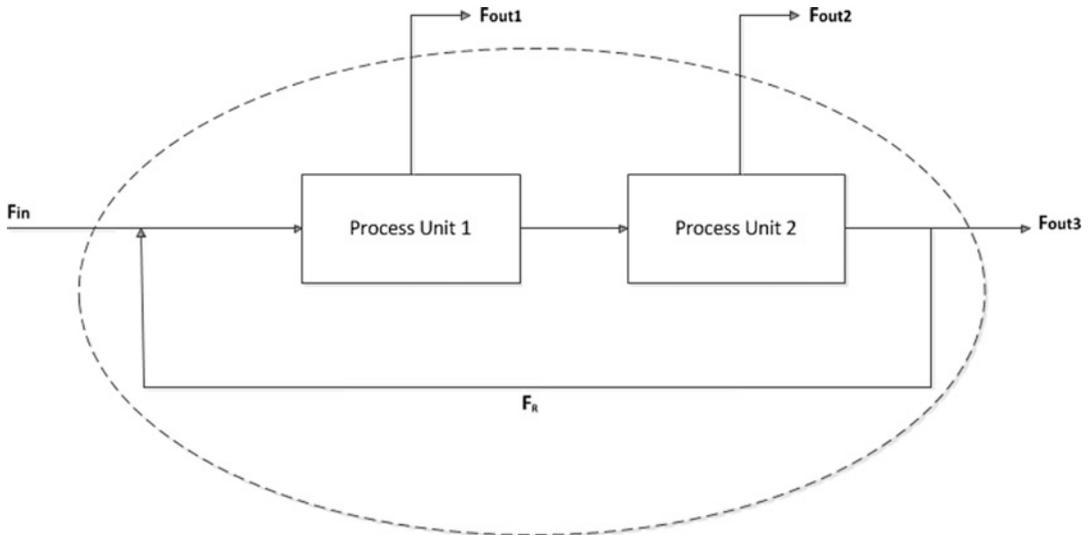


Fig. 7.9 Process with two units and a recycle

Mass is additive. Can balances be done based on the volume rate of the streams in a given process? No, because volume is not always additive. When you add X kg of material A to Y kg of material B, you will always obtain $X + Y$ kg of the A and B mixture. We cannot say the same of volume. For example, 1 L H_2O plus 1 L ethanol (at ambient conditions) will give you less than 2 L of the mixture. On the other hand, 1 kg H_2O plus 1 kg ethanol will always give you 2 kg of the mixture. Students are encouraged to investigate why sometimes (as in the example of H_2O plus ethanol) the volume is not necessarily additive and why the resulting volume, as in this example, is different than expected (see solved problem 12 in Sect. 5.3.2, Chap. 5).

Mass units. Before formulating and carrying out a material balance analysis and calculations (especially for nonreactive systems), it is strongly advised that all streams be expressed in mass units. As we will see in the next chapter, for reactive systems, it might be better to express streams in molar units.

Graphical representation and total mass balance. Figure 7.9 shows a typical scheme or diagram for a mass balance including two units and a recycle stream (F_R). The dotted line shows that the whole system has been selected as a system. Assuming steady state,

$$F_{IN} - F_{OUT1} - F_{OUT2} - F_{OUT3} = 0. \quad (7.8)$$

7.5 Why Material Balance Is Relevant for Chemical and Bioprocess Engineers

Material balance takes an exact account of all materials entering, leaving, accumulating, or being depleted in a given process unit for a specific time interval. The practical use of material balance lies in the fact that in reality it is very difficult for a process engineer to make direct measurements of all the masses of each process stream.

As you will discover in your development and education as an engineer, and later on as a professional, material balance is vital for process engineering. Material balance has broad applications that are even beyond the fields of process and bioprocess engineering (e.g., economics).

Making the inventory of material that enters, leaves, or is generated within a system allows one to know whether the system will be enriched or depleted with the material. In that way it is possible to determine how the system will change, and even the rate of change. This is relevant in equipment sizing, where a decision of how long it will take for the equipment to fill to capacity (e.g., with the tank in a toilet, the time for filling it up is given by the flow of water and the size of the tank; the size of the tank is associated with the flow required to clean it up. At every step in the process, volumes and times are determined by the material balance).

Several applications in chemical and bioprocess engineering work in such a way that the system is designed not to gain or be depleted of materials (or molecules or electrical carriers). This is known as a system operating in steady-state condition. In that case, the material balance changes to one of considering what is required to enter the system (after the generation or transfer of materials) to obtain a desired outflow.

Furthermore, the main focus of many material balance problems is to determine how much the processes operating inside the system must transform materials to obtain a desired product. This is how chemical and bioprocesses like chemical reactors, distillation columns, and fermenters (to mention a few) are designed. It is after this material balance step that the knowledge to design those operations merges (beyond the scope of this book). The sizing and design of this equipment will require knowledge of fluid mechanics, heat transfer, and mass transfer (in Chap. 6 we provided a glimpse of these topics).

In processes consisting of several units (e.g., chemical reactors, equipment for mixing/purification, autoclaves, fermenters) the analysis gets more complex, but the material balance is still a must and, as we will explain later, conducted on each unit, thus enabling the following steps in the design and specification of equipment.

Usually the material balance is the first step in process design and engineering projects. It is present in the day-to-day monitoring and supervision of chemical and bioprocessing plants. It is by far the most relevant tool for chemical and bioprocess engineers.

7.6 Formulating Material Balance Equations (Steady-State and Continuous Operation)

As mentioned at the beginning of Sect. 7.3, Antoine Laurent Lavoisier (1743–1794) was the first scientist to suggest that matter is neither created nor destroyed but conserved. The law of conservation of matter postulates that the amount of material before and after a process is strictly the same. What the law of conservation of matter implies is that, beyond transformations, matter continues to exist. In other words, atoms react with each other as substances, but atoms are neither created nor destroyed.

Formulation of a general mass balance for nonreactive and open systems under steady-state conditions (process unit):

According to (7.5) and Fig. 7.10 we can write a **TOTAL MASS BALANCE** as

$$\text{Total input mass flow rate} - \text{Total output mass flow rate} = 0 \quad [\text{steady-state condition}]. \quad (7.9)$$

Then

$$\left(\sum_{i=1}^{i=n} \dot{m}_i \right)_{\text{in}} - \left(\sum_{j=1}^{j=k} \dot{m}_j \right)_{\text{out}} = 0, \quad (7.10)$$

Fig. 7.10 General material balance for an open system

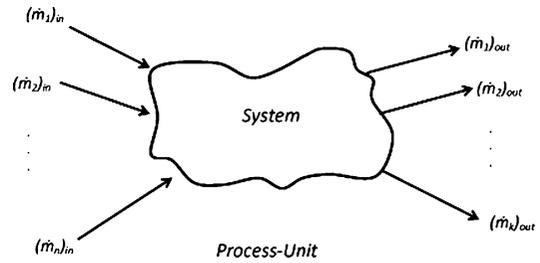
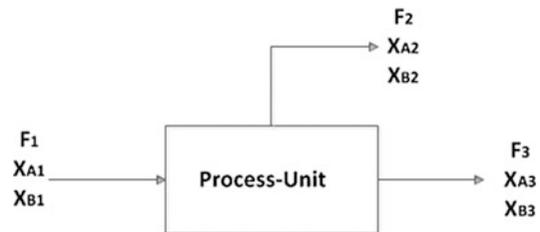


Fig. 7.11 Simple process-unit



where $(\dot{m}_i)_{in}$ is the mass flow rate of stream i entering the system (mass/time) and $(\dot{m}_j)_{out}$ is the mass flow rate of stream j leaving the system (mass/time).

In addition, given that it is a nonreactive system, the mass of each component is conserved. Then we can write one mass balance for each component. If we have p components, then we can formulate p additional equations. Writing the mass balance for some specific component r ,

$$\left(\sum_{i=1}^n x_{ri} \dot{m}_i \right)_{in} - \left(\sum_{j=1}^m x_{rj} \dot{m}_j \right)_{out} = 0, \quad (7.11)$$

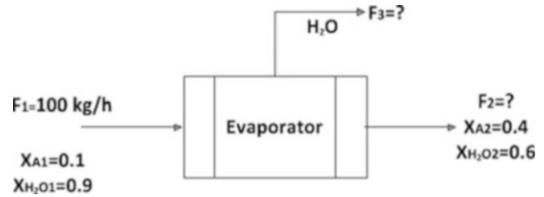
where x_{ri} is the mass fraction of component r in stream i , and x_{rj} is the mass fraction of component r in stream j .

The first term in (7.11) represents the addition of masses of component r in all input streams. Thus, the first term represents all masses of component r entering the system. In the same way, the second term represents all masses of component r leaving the system.

Therefore, we can write one total mass balance and, in addition, one mass balance for each component (1, 2, ..., p). Since we have p components, we are able to write, in total, $p + 1$ equations, but **ONLY p of them are independent!** Why? For example, if we sum up all the material balance equations formulated for each component, then we discover that the result is equal to the total mass balance. **Remember, the number of independent equations is equal to the number of components in the process unit (system).**

Example 3. Independent material balance equations. In this example, we will prove quantitatively that the number of independent material balance equations is equal to the number of components. In Fig. 7.11 is depicted a simple process unit with one input stream with components A and B and two output streams, also with components A and B.

Fig. 7.12 Material balance in a single effect evaporator



Material balance

Total mass balance:

$$F_1 - F_2 - F_3 = 0. \quad (7.12)$$

Mass balance for component A:

$$x_{A1} \times F_1 - x_{A2} \times F_2 - x_{A3} \times F_3 = 0. \quad (7.13)$$

Mass balance for component B:

$$x_{B1} \times F_1 - x_{B2} \times F_2 - x_{B3} \times F_3 = 0. \quad (7.14)$$

As mentioned earlier, in this case we have two components, so we should have only two independent equations. For example, if we sum up (7.13) and (7.14), we obtain

$$(x_{A1} + x_{B1}) \times F_1 - (x_{A2} + x_{B2}) \times F_2 - (x_{A3} + x_{B3}) \times F_3 = 0, \quad (7.15)$$

where $(x_{A1} + x_{B1}) = (x_{A2} + x_{B2}) = (x_{A3} + x_{B3}) = 1$, meaning that (7.15) is exactly the same as (7.12). Therefore, if you use (7.13) and (7.14), then (7.15) is not independent. The same happens if you choose to use, for example, (7.12) and (7.13); then (7.14) will not be independent. The corollary is that if you have n components, then you have n independent material balance equations. However, you have the freedom to select which ones to use.

Example 4. Product concentration. A single-effect evaporator is fed with 100 kg/h of a stream that contains components A and H₂O. At the output, one stream contains just H₂O (as a vapor) and the other, the stream concentrated in A, contains H₂O and component A. If the concentration of component A is 10 % w/w in the feed stream and 40 % w/w at the output stream, then (a) how much H₂O was removed in the evaporator (evaporated) and (b) what is the mass flow rate of the concentrated product?

According to the earlier examples in this chapter we will follow an intuitive procedure. Thus,

- (I) Draw a flow diagram of the problem statement (Fig. 7.12):
- (II) Variable definitions and codification

We would like to emphasize the importance of defining all variables and codifying them accordingly. Do not use generic terms like X, Y, and Z as variables because it is much better to codify variables in such a way that you can always associate them with your problem (flow diagram). In addition, it will be easier to have one codification for each variable.

F_1 : Feed stream (kg/h) (100 kg/h)

F_2 : Stream with concentrated product (component A) (kg/h)

F_3 : Stream with evaporated H₂O

x_{A1} : Mass fraction of component A in feed stream (0.1 w/w)

x_{A2} : Mass fraction of component A in output stream (0.4 w/w)

x_{H_2O1} : Mass fraction of H₂O in feed stream (0.9 w/w)

x_{H_2O2} : Mass fraction of component H₂O in output (0.6 w/w)

Although the mass fraction of H₂O it is not given directly in the problem statement, because there are two components, the addition of mass fraction of component A and mass fraction of H₂O must be 1 in both streams.

- (III) Mass balance formulation and solution. In this case we have two components (A and H₂O), so we can formulate two independent equations, as follows:

We will choose, first, the mass balance for component A because it has the advantage that it appears only in two of the three streams.

Mass balance for component A.

Mass of component A entering system – Mass of component A leaving system = 0 (steady state); thus, $0.1 \times 100 - 0.4 \times F_2 = 0$, then $F_2 = 25$ kg/h.

Total mass balance

Total mass entering system – Total mass leaving system = 0 (steady state). Therefore,

$100 - F_2 - F_3 = 0$, then substituting for F_2 , we obtain $F_3 = 75$ kg/h. Therefore, (a) 75 kg/h and (b) 25 kg/h.

This easy example helps in understanding some basic concepts of material balance and demonstrates a procedure to be considered when solving complex material balance problems. In the following sections, we will further analyze the specifics of material balance and then design and develop a general procedure for solving material balance problems, whether basic or complex.

7.7 Material Balance Basics

As described here, it is important to consider the process of approaching, formulating, and solving material balance problems.

1. If the flow diagram is not provided with the problem statement, then draw a complete one on a whole page. Clarity at this point is critical and will help you get a feel for the problem. If a flow diagram is provided, sometimes it will be better to redo and complete it with all the available information.
2. Define all variables (known or unknown). Some variables will be specified, but it is advisable to have a definition for each variable and codify it in such a way that it will be easy for you to match each variable with your flow diagram. As was just mentioned in example 3, in general, avoid using generic variable names such as X , Y , and Z . It is much clearer to codify the variable according its role in the problem. An exception is the use of x for mass fraction composition.
3. To formulate a specific material balance, it is vital to first define the system under analysis. But who defines the system? You, the person formulating it! As shown in example 3, before writing a material balance it is strictly necessary to define the system and its boundaries. Only then can we write a material balance equation and determine which flows are entering and leaving the system. For an example, see Fig. 7.13.
4. For nonreactive systems (this chapter) it is better to use mass units for all streams (Fig. 7.13). As was pointed out earlier, mass is conservative. Recall the example of ethanol and water that showed that volume was not necessarily additive. As we will see in the next chapter (material

Fig. 7.13 Distillation column

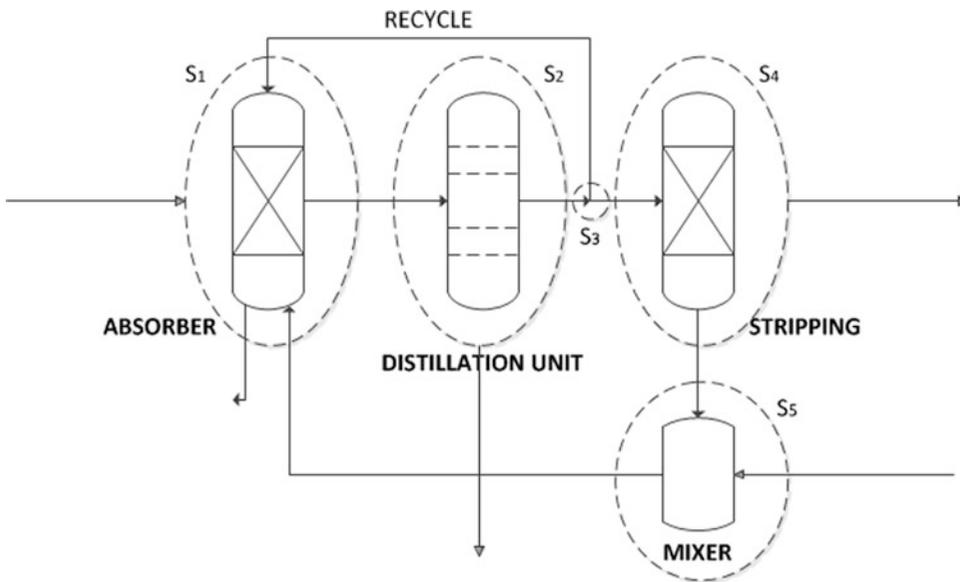
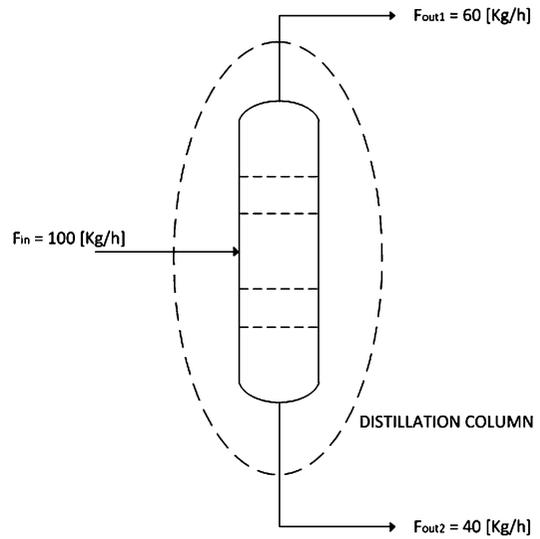


Fig. 7.14 Complex process including different units and a recycle

balance for reactive systems), moles should be used when the material balance includes reactions (due to stoichiometry).

- Subdivide the whole process into its different units and analyze each unit step by step (Fig. 7.14). In complex processes that includes several units it is advisable to analyze one unit at a time. Normally you cannot solve the material balance problem for each unit independently, but to formulate the whole mathematical problem, it is better to go step by step (unit by unit).

As depicted in Fig. 7.14, the whole process is divided into five systems ($S_1, S_2, \dots,$ and S_5) for further study. In addition, we can consider the total system (S_T), but of these six systems only five are independent.

6. In each unit of the processes, identify all the independent material balance equations. If you have n components, then you can write n independent material balance equations. In example 3, the feed and output streams had two components and then we were able to write two independent equations.
7. Minimize as much as possible the number of variables. For example, if you have two components in a stream, say H_2O and solids, you can use two variables, one for the mass fraction w/w of H_2O (X_{H_2O}) and one for the solids (X_{SOLIDS}), and add one relationship that says $X_{H_2O} + X_{SOLIDS} = 1$, or just use one variable (better), say X_{H_2O} , knowing that you can calculate “mentally” X_{SOLIDS} from $X_{SOLIDS} = 1 - X_{H_2O}$. As in example 3, we obtained the mass fraction of H_2O from $X_{H_2O1} = 1 - X_{A1}$. An example is given in Warm-up Example 2 in Sect. 7.8.3.
8. If a number of substances is being maintained in fixed proportions in the process, it is better to consider them all together, enclosing them in one variable (see proposed problem 13 in Sect. 7.10). For example, if a food material (e.g., tomato juice) is to be concentrated by removing water, then all components (e.g., carbohydrates, proteins, fats) will remain together in the concentrated stream; then it would be most efficient to call all of them solids and use just one variable for it all.
9. A substance that enters the process and leaves without transformation is very important as a reference substance (or tie substance). Again, in example 4, we chose component A to formulate one of the material balance equations. The advantage in doing that was that component A was involved in just two streams and facilitated calculations.

First, for now, these nine points are simply a list of recommendations, but we will analyze these recommendations later when we focus on developing an integral and general procedure for approaching, formulating, and solving material balance problems. Second, we will exemplify the relevance of these tips on warm-up examples and when solving problems in Sect. 7.9 (solved exercises).

7.8 Designing and Structuring a General Procedure to Formulate and Solve Material Balance Problems

In this and the following chapter, we will dedicate all our efforts to teaching students mainly material balance under steady-state conditions in continuous mode. This option has the advantage of simplifying the mathematics involved and, in addition, at the same time does not limit us in our attempt to cover a very broad range of interesting examples in chemical and bioprocess engineering.

7.8.1 Developing a General Procedure for Material Balance Problems

As shown in example 4, following a simple and intuitive procedure the solution of a material balance problem seems straightforward. We need to acknowledge that example 4 is simple and not representative of real material balance problems, but on the other hand, it showed us that a sound fundamental strategy could help to simplify the solution. Systematically, our experience with several generations of freshmen has taught us that the following procedure (Fig. 7.15) helps students and professionals have clarity when facing flow diagrams, from the simple to the intricate.

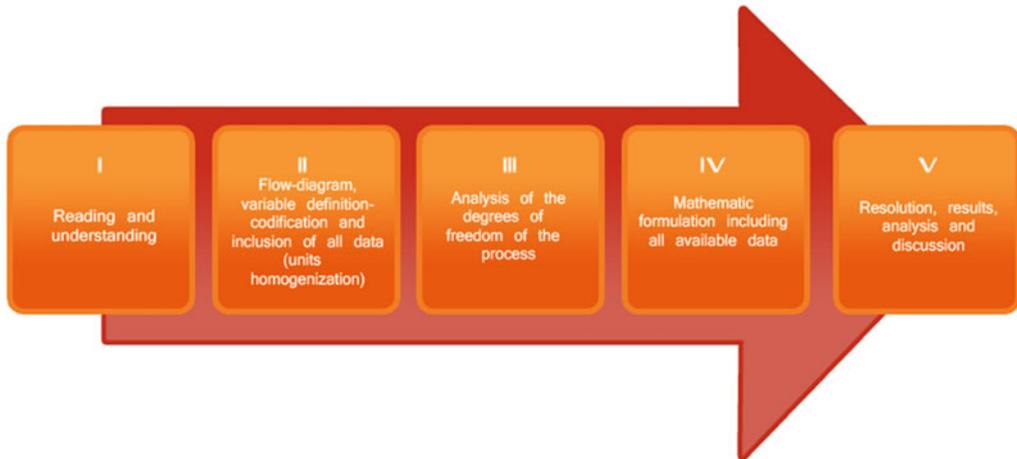


Fig. 7.15 General strategy and procedure to solve materials balance problems

This scheme may seem very detailed and often unnecessary for simple material balance problems. However, as will be seen again and again throughout the examples, this procedure is vital in solving most real plant material balance problems. In some problems, you might feel very confused and possibly a bit tormented by the intricate flow diagrams, but if you follow the proposed procedure step by step, most, if not all, problems can be correctly formulated and solved. We will be very emphatic that for you, as a freshman, but probably for most engineering students as well, all material balance problems should be tackled with a clear strategy, e.g., the strategy proposed here. This will be further shown in warm-up examples and in Sect. 7.9 (solved exercises).

7.8.2 Understanding the Procedure to Approach, Formulate, and Solve Material Balance Problems

In this section we present and describe, step by step, this schematic procedure to approach material balance problems.

The content presented below is essential and indispensable for your training as a future process engineer. A clear understanding on how to approach, formulate, and solve problems of material balance is extremely important and vital for a process engineer. If, after this chapter you develop the capacity and methodology to solve most problems of material balance, then the purpose of the chapter will have been fulfilled and you can consider yourself halfway to your goal of becoming an engineer!

As was emphasized in Chap. 5, the most important thing in solving a problem is to have a strategy and a method, i.e., a set of rules that guarantees the optimal decision and procedure. In addition, here again, at this stage, the most critical aspect is not to obtain a final solution but to be able to mathematically express and formulate the verbal statement of a material balance problem. Often we have been surprised when freshmen possessing a very basic background can approach, formulate, and solve challenging and complex material balance problems that include multiple units (e.g., Sect. 7.10, exercise 28). Our own challenge is to provide you with a method and empower you to solve real-world problems.

7.8.2.1 Procedure Description and Analysis

As depicted in Fig. 7.15 the proposed procedure consists of five steps in series that are described and analyzed below. Later, in the warm-up examples, we will apply it to problems.

STEP I

Reading and understanding

This first step is critical. Sometimes with long statements and when complex flow diagrams are included it is easy to get confused. If the problem statement includes the flow-diagram then a good strategy will be to read the statement and at the same time try to match it with the flow-diagram. Most of the time, it will be easier to follow the statement with the diagram and then you will start to understand and feel the problem. If the diagram is not provided, then you need to draw one right away: do not wait until Step II of this procedure. An idea is to start constructing the flow-diagram step by step reading each phrase of the problem at a time. You will have a good understanding if you match the problem statement with the flow-diagram. In addition, at this stage of your engineering career we strongly recommend you to familiarize yourself (just to gain a general understanding) with the main function of some common equipment used in process engineering. As we will see in the following examples and sections 7.9 and 7.10 some of the equipment items are: Distillation columns, Crystallizers, Evaporators, Presses, Settlers, and Humidifiers. In chapter 8 (materials balance for reactive systems) you will be exposed to reactors and fermenters.

STEP II

Flow-diagram, variable definition-codification and inclusion of all available data

Possibly most problems will include a flow-diagram but at this stage we recommend not only drawing a flow-diagram but also include all the available data and, in addition, include (or define) all the variables. All available means, all, not only the information related to mass flow rate of the streams and compositions, but also any given relationship (e.g. $F_1/F_2 = 2$, the ratio of two flow streams). Because we will be adding codified variables in the flow-diagram, then it is strictly necessary to define each variable of the problem. As mentioned the right codification is very important. Then, we will have a clear understanding of all variables involved at its role in the problem and which ones are known and unknown, in addition, all given relationships. This step will be done with a complete flow-diagram and a list with well-defined and codified variables. If the flow-diagram is not attached to the problem, then we must devote ourselves to "decipher" the problem verbal-statement and construct (draw) the flow-diagram, and if the flow-diagram is attached, in some cases we might re-draw the flow-diagram to have breadth and clarity of each of the units and processing lines. In some specific cases, it is necessary to add "units" to the flow-diagram, for example, when a recycle stream, a by-pass or a side-stream join another stream is better to add a mixer for clarity (see examples and definitions of recycle and side-stream in Step III).

STEP III

Analysis of the degrees of freedom of the process

Firstly, what are the degrees of freedom? For example if you are solving the equation $X + Y = 5$, we can say that it has infinite solutions because for a specific value for X you get a specific value for Y , but for a

different value of X you get a new value for Y . This situation occurs because you have 2 variables (X and Y) and just 1 equation, then you have 1 degree of freedom, meaning that you have the freedom to fix the value of one variable and then solve the problem for the unknown value of the other variable. In this simple case we can state that the degrees of freedom (DF) are:

$$DF = \text{Number of variables (unknowns)} - \text{Number of equations} \quad 7.16$$

Where the problem is set and ready to be solved when $DF = 0$ (e.g. 2 variables and 2 equations).

In materials balance problems we face a similar situation. We will be expressing our materials balance with equations. Although, in materials balance we will face a slightly different situation because equations are not given to us: we need to formulate them. In addition, some variables will be specified and also some relations will be given. Although conceptually, it is the same the situation, the definition of DF in materials balance problems is slightly different and will include additional terms.

Why is the analysis of degrees of freedom important? To mathematically solve the materials balance problem we need to solve a set of equations and as mentioned to set the problem we need to reach the state of $DF = 0$. For example if the degrees of freedom are 2, then, we have the freedom to fix two variables in the process. Even though, when $DF > 0$ you have the freedom to fix some variables (e.g. if $DF = 4$ you can fix four variables) your freedom is limited. Yes, you will be able to fix four variables but with limitations. Why? For example all mass flow rates should be positive numbers, all mass fractions should be positive, the sum of all mass fractions in a stream should be 1, etc. We will further analyze these limitations.

Definition of degrees of freedom in a materials balance problem

As we have seen through examples 1, 2, and 3 in materials balance problems we have variables and equations but also we could have variables that are previously specified and some relationships could be given. Then to analyze the degrees of freedom in materials balance problems we will define:

DF : Degrees of freedom of the process.

NV : Total number of variables in the process (mass flow rates and mass fractions).

NMB : Total number of independent materials balance equations in the whole process.

NSV : Total number of independent specified variables (e.g. Feed flow = 100 kg/h)

NR : Total number of given relationships among the variables (e.g. $F_1 + F_2 = 150$ kg/h)

Therefore the degrees of freedom in a materials balance problem can be generalized by the following equation:

$$DF = NV - (NMB + NSV + NR) = (NV - NSV) - (NMB + NR) \quad 7.17$$

The right hand side of equation 7.17 could be interpreted as the number of unknowns ($NV - NSV$) less number of independent equations ($NMB + NR$).

Examining equation 7.17 we can identify three different cases as follows:

- a) $DF > 0$, the problem is under-specified. To solve it you need to provide some extra information (e.g. if $DF = 1$, then you have the freedom to fix one of the process variables, for example if the feed flow rate is unknown you can assign a value, say 100 kg/h and then DF will be 0). When you assign a value to a variable, means that you are adding an equation e.g. $F_1 = 100$.
- b) $DF = 0$, the problem is correctly specified, and you can proceed to solve it.
- c) $DF < 0$, the problem is over-specified, and then some of the information is redundant and must be discarded. In addition, it is necessary to check if the redundant information is consistent and check which information will be discarded.

It is not necessarily trivial or straightforward to determine the respective values for NV , NMB , NSV and NR , thus we will devote the next section to explain how to determine each value in a given process.

- a) **Total number of variables (NV), Total number of specified variables (NSV) and Total number of relationships (NR).** As expressed in the previous step (step II) all variables (known or unknown) will be included in the flow-diagram. Then from the complete flow-diagram developed in step II we can get NV , NSV and NR .
- b) **Total number of independent materials balance equations (NMB).** It is necessary to determine which are all the equations (without writing them) that can be formulated. Frequently a process is composed of multiple units, and sometimes including by-passes, side-streams and recycles. Inferring from section 7.6 if a process-unit has a total of n_i components entering and going out of the unit then the number of independent materials balance on the unit is n_i . Strictly, you can formulate $n_i + 1$ equations, one per each component and, in addition, a total mass balance, but only n_i of them are truly independent. It is important is to recall that you have the freedom to select which n_i equations you will be using from the $n_i + 1$ total equations. For example you can include or not within the n_i equations the total mass balance.

In the same manner, if the complete process has P units and each unit has n_i components (where i represents the number of the unit, from 1 to P), then the total number of independent materials balance equations is:

$$NMB = \sum_{i=1}^{i=P} n_i = n_1 + n_2 + \dots + n_p \quad 7.18$$

Where:

NMB : Number of independent materials balance equations.

P : Number of process units.

n_i : Number of components in unit i ($i = 1, 2, 3, \dots, P$)

As will be shown in warm-up example 2 if the process includes by-passes, side-streams, and/or recycle streams (see Figure 7.16) it is advisable to include an additional process-unit (mixer) in your flow-diagram showing the mixture of one stream and, for example, a recycle stream. This advice is critical because you need to consider this "additional unit" when determining the number of independent materials balance equations (*NMB*). In addition, if a stream is divided (e.g. side-stream), all new streams will have the same composition but you need to add one total material balance equation. When the side-stream joins the process again, it is necessary to add a new process-unit (mixer).

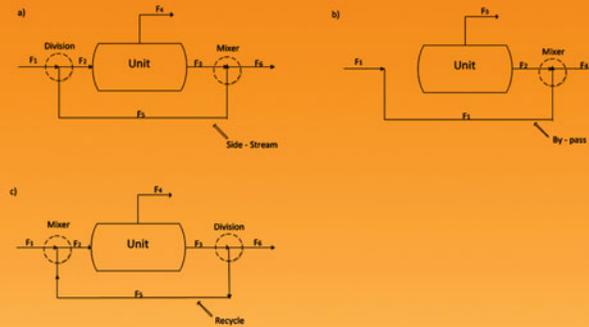


Fig. 7.16 A process depicting side-stream (a), by-pass (b) and recycle (c)

As depicted in Figures 7.16a, b and c we have added dotted line where two streams are mixed. When writing the materials balance equations it is necessary to consider this situation as a new unit: mixer. As an example Figure 7.16a is better drawn as shown in Figure 7.17.

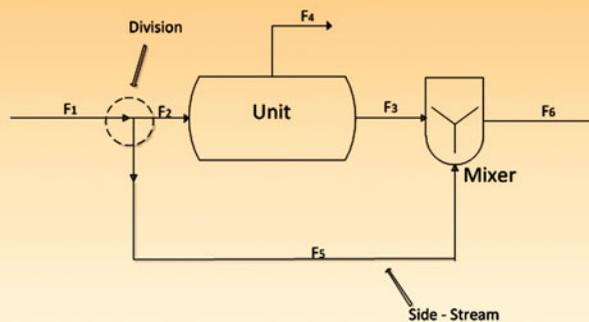


Fig. 7.17 Process with a side-stream

As you can comprehend from Figures 7.16 and Figure 7.17 we can state that:

Each side-stream and a recycle stream in the process will add a division (one total mass balance) and a mixer. The mixer will add a number of material balance equations according to the total number of components involved in it.

Each by-pass will add a mixer (as shown in Figure 7.17) where the additional number materials balance equations will be directly related with the total number of components involved in it.

Additional considerations on the degrees of freedom. We already have all the tools to determine the values for NV , NMB , NSV and NR , then we can determine DF . Now we need to understand that its use has some limitations. For example if we have some degrees on freedom in your process, then, certainly you will be able to fix some unknown variables in your process but with some limitations. For example if a process-unit has three degrees of freedom and with one stream at the input and two streams at the output you cannot fix all three streams arbitrarily, there is a mass balance equation that relates these three streams, then, in this case, you can fix two of these three streams. We are not implying that you cannot use all the DF , just to state that some constraints limit your freedom in the choices of values and variables used as DF . This concept will be further analyzed and exemplified in the warm-up examples and reinforced in section 7.9. For example in warm-up example 2 we will show how to analyze the degrees of freedom in a multiple unit process including a side-stream.

As a teaching and learning-process strategy, in this chapter, most of the problems will be correctly specified or under specification (i.e. $DF \geq 0$). As mentioned before if $DF < 0$, not only is it required to discard the redundant information but also is necessary to check if the redundant information is consistent within the problem.

STEP IV

Mathematical formulation including all available data

Firstly, we already have a complete flow-diagram (step II) with all variables and available information. Secondly, with the analysis of degrees of freedom (step III) we have determined how many independent materials balance equations (NMB) there are in the process. Thirdly, given that we already know what is the number of independent materials balance in the process, then we need to determine which of the equations will be used. For example in each unit we can choose whether or not to use the total mass balance. In addition, we need to decide whether or not we will use the global mass balance. Given that normally we have information on the input and output streams and its composition appears reasonable, first, to explore the possibility to use the global mass balance. Therefore we need to write all the selected equations and if $DF > 0$ then specify some variables to get $DF = 0$ to set and solve the problem.

STEP V**Solution, results, analysis and discussion**

Normally, in mathematical terms the problem to be solved will be a system of equations. Although sometimes, it might be hard to solve, our main goal is to empower you until the point that you can correctly formulate the whole materials balance problem. In addition, if you are able to solve it, great! You will eventually learn the procedures and tools for solution of large systems of equations.

Once you have the results is necessary to check if they are according to your expectations and more than that, hopefully test if they are correct. In warm-up examples 1 and 2 we will show you some cases were results are properly tested. In addition, this concept will be further analyzed in section 7.9 (solved problems).

Example 5. Analyzing the degrees of freedom. A separator is fed with a stream with solids and H₂O (Fig. 7.18), and at the output there are two streams, both containing solids and H₂O.

One of your best friends and also your classmate in Introduction to Chemical and Bioprocess Engineering states that in the system (separator) you can identify four degrees of freedom. He argues that there are six variables (three streams and, e.g., three solid mass fractions). Thus, $NV = 6$, and because the streams contain two components, he argues that you can formulate two independent material balances ($NMB = 2$). In addition, there are no specified variables and no relationships are given. Therefore, $NSV = 0$ and $NR = 0$. At this point, thanks to your knowledge of material balance, you fully agree with your classmate because applying (7.17) you get

$$DF = NV - NMB - NSV - NR = 6 - 2 - 0 - 0 = 4.$$

Then your friend says that he will specify four variables ($NSV = 4$) to solve the problem. He chooses the solid mass fraction at the feed stream ($x_{S1} = 0.1$) and the flow rate of the three streams ($F_1 = 100$ kg/h, $F_2 = 80$ kg/h, and $F_3 = 20$ kg/g) and tells you to quantitatively solve the problem. Then, first, you dutifully follow the procedure presented in this book, reading and rereading the problem and drawing a flow diagram (steps I–III) including all data, defining all variables (Fig. 7.19), and determining the degrees of freedom.

Step I–III

F_1 : Mass flow rate of feed stream (kg/h)

x_{S1} : Solid mass fraction composition in feed stream

F_2 : Mass flow rate of first output stream (kg/h)

x_{S2} : Solid mass fraction composition in first output stream

F_3 : Mass flow rate of second output stream (kg/h)

x_{S3} : Solid mass fraction composition in second output stream

According to your friend you have $DF = NV - NMB - NSV - NR = 6 - 2 - 4 - 0 = 0$. Therefore, the problem is set and ready to be solved.

Step IV**Mathematical formulation including all available data**

First, we will write the material balance in generic terms without any data as follows:

Fig. 7.18 Analysis of the degrees of freedom in a process

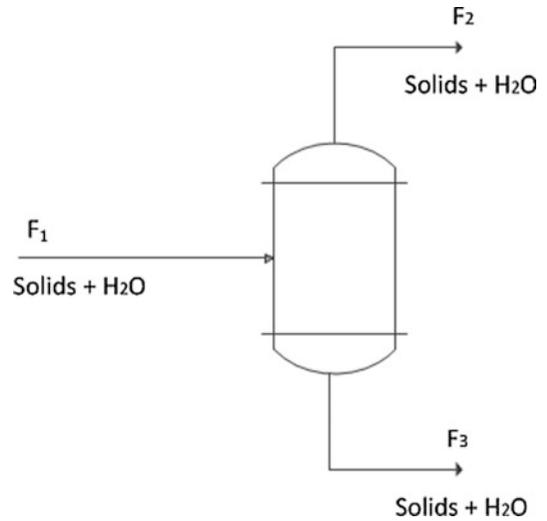
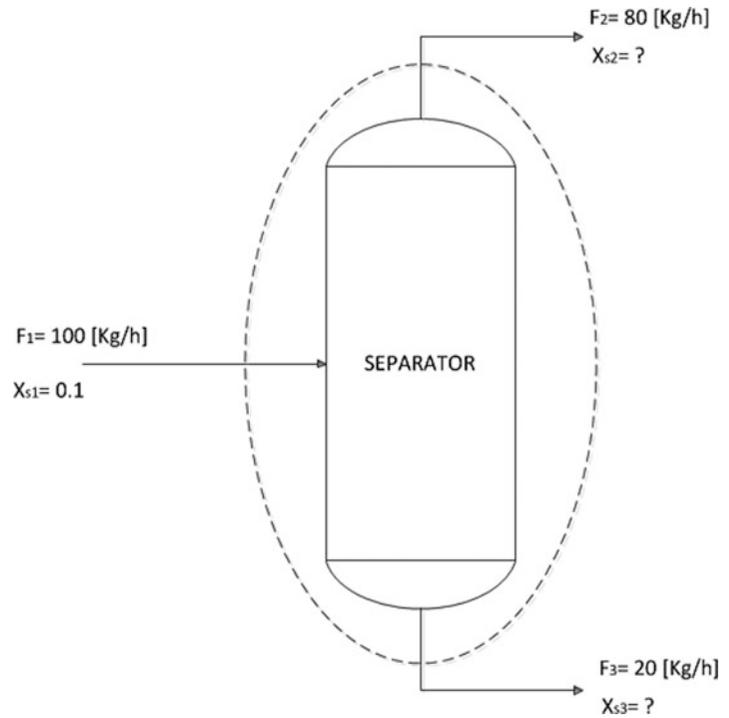


Fig. 7.19 Flow diagram including all variables and available data



Mass balance for solids

$$F_1 x_{S1} - F_2 x_{S2} - F_3 x_{S3} = 0. \quad (7.19)$$

Total mass balance

$$F_1 - F_2 - F_3 = 0. \quad (7.20)$$

Therefore, replacing the specified variables given by your classmate in (7.19) and (7.20) we get

$$100 \times 0.1 - 80 \times x_{S2} - 20 \times x_{S3} = 0, \quad (7.21)$$

$$100 - 80 - 20 = 0. \quad (7.22)$$

We did follow the advice given by our friend, and it seemed correct, but we have one equation (7.21) and two variables (x_{S2} and x_{S3}). Clearly 7.22 is not an equation. What went wrong? Strictly speaking, we have one degree of freedom, and we cannot solve the problem!

Our friend told you that he will be specifying four variables, and we believed him. The question is whether or not he really specified four variables. The definitive answer is no. Why not? He specified x_{S1} , F_1 , F_2 , and F_3 . It seems that he is specifying four variables but really he is only specifying three. If he specifies F_1 and F_2 , he does not have the freedom to fix F_3 because F_3 is already fixed by the total mass balance ($F_1 - F_2 - F_3 = 0$). As was mentioned when we were analyzing step III of the proposed procedure, there are some limitations in assigning the degrees of freedom. Yes, if we have four degrees of freedom, then we will be able to specify four variables, but with limitations, as shown in this example. The lesson here is to be cautious and select the variables to be specified in such a way that they are independent of each other. In this example F_1 , F_2 , and F_3 are not all independent because there is a relationship among them (total mass balance). We will return to this important aspect in Sect. 7.9.

7.8.3 Solving Material Balance Problems with the Proposed Procedure

As Einstein said, “*Example is not another way to teach, it is the only way to teach.*” Because we agree with this statement, we are fully committed to giving you a large number of examples, either solved or to be solved. In this section we will provide some warm-up examples to start practicing the proposed procedure.

It is definitely advisable to follow these two warm-up examples step by step. They are simple but will guide you in the first steps of the proposed procedure.

Warm-Up Example 1

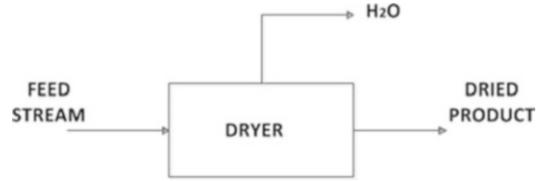
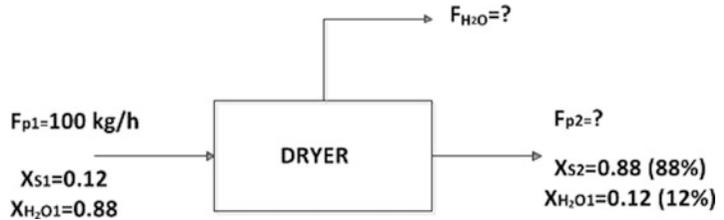
Dryer [3]. 100 kg/h of a food are fed to a continuous dryer (H_2O removal) operated under steady-state conditions to reduce its moisture content from 88 to 12 %. Assume that for the purpose of the problem the food has two components: solids and H_2O (Fig. 7.20). (a) What is the amount of dried food obtained (kg/h)? (b) How much H_2O (kg/h) is removed in the continuous dryer?

Solution

Step I

Reading and Understanding

It might appear to be an exaggeration to follow the proposed procedure (Fig. 7.15) for such a simple problem that an advanced student could probably solve without any strategy. Experience has repeatedly shown us that, although it is simple, it is better and advisable to always follow the proposed procedure. It is a good idea to remember our experience with the shoe problem in Chap. 5. Maybe you solved it mentally and got the wrong answer. Our purpose here is to familiarize you with the proposed procedure and demonstrate its use.

Fig. 7.20 Drying process**Fig. 7.21** Drying process including all variables and available data

In this case we have a simple unit operation for water removal to significantly reduce the initial product moisture content. A dried food product is much more stable than a high moisture product and has an extended shelf life.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

Variable codification is very important and crucial for your own clarity when formulating and solving problems, so we propose the following:

F_{P1} : Mass flow rate of feed stream (kg/h)

x_{S1} : Solid mass fraction composition in feed stream

x_{H_2O1} : H_2O mass fraction composition in feed stream

F_{P2} : Mass flow rate of first output stream (kg/h)

x_{S2} : Solid mass fraction composition in first output stream

x_{H_2O2} : H_2O mass fraction composition in first output stream

F_{H_2O} : Mass flow rate of H_2O the second output stream (kg/h)

Then the total number of variables is seven ($NV = 7$).

NOTE: As we will see in warm-up example 2, it is not necessary here to use two variables for mass fraction composition in each stream. For example, we know that $x_{H_2O1} = 1 - x_{S1}$, so we can reduce the total number of variables used here to just one mass fraction as a variable. In this simple example, it is not really critical, but in general we will try to minimize the number of variables!

Step III

Analysis of degrees of freedom in process

In this case, we have two components, solids and H_2O , and one process unit (dryer). Therefore, we can formulate two independent material balances (later we will decide which ones and then write its equations).

Thus, $NMB = 2$.

Observing Fig. 7.21 we notice that we have five specified variables and no relationships are given. Thus, $NSV = 5$ and $NR = 0$.

Finally, the number of degrees of freedom (DF) in this example is

$$DF = NV - NMB - NSV - NR = 7 - 2 - 5 - 0 = 0; \quad DF = 0. \quad (7.23)$$

Thus, the problem is set and ready to be quantitatively formulated and solved.

Step IV

Mathematical formulation including all available data

As mentioned, we can write two independent material balance equations. Given that solids are just in two of the three streams, it is a good option to write a mass balance for solids. The other good option in this case is the total mass balance. Thus:

Mass balance for solids

$$x_{S1}F_{P1} - x_{S2}F_{P2} = 0. \quad (7.24)$$

Total mass balance

$$F_{P1} - F_{P2} - F_{H2O} = 0. \quad (7.25)$$

Therefore, the complete mathematical formulation for the whole process including all available data will be.

Specified variables

$x_{S1} = 0.12$; $x_{H2O1} = 0.88$; $x_{S2} = 0.88$; $x_{H2O2} = 0.12$ and $F_{P1} = 100$ kg/h.

Therefore, substituting into (7.24) and (7.25) we get

Mass balance for solids

$$0.12 \times 100 - 0.88 \times F_{P2} = 0. \quad (7.26)$$

Total mass balance

$$100 - F_{P2} - F_{H2O} = 0. \quad (7.27)$$

Step V

Solution, results, analysis, and discussion

The solution to these equations is straightforward. From (7.26) we get $F_{P2} = 13.6$ kg/h, and then, substituting F_{P2} in (7.27), we get $F_{H2O} = 86.4$ kg/h.

Clearly to reduce the humidity from 88 to 12%, a large amount of H₂O was removed (86.4 of 100 kg/h in the feed stream). Although the results look reasonable, it is much better to try to test whether the results are correct. For example, the feed stream (100 kg/h) contained 12% solids. Thus, we have 12 kg/h of solids. In the output stream ($F_{P2} = 13.6$ kg/h) we have the same amount of solids (12 kg/h), and so the mass fraction of solids at the output should be 12/13.6, which is 0.88, or 88%, solids!

Warm-up Example 2

Evaporation unit and a dryer [5]. An unusual and extremely delicate pharmaceutical product should be dried to reach a very low moisture content (3% w/w). The process (Fig. 7.22) has two

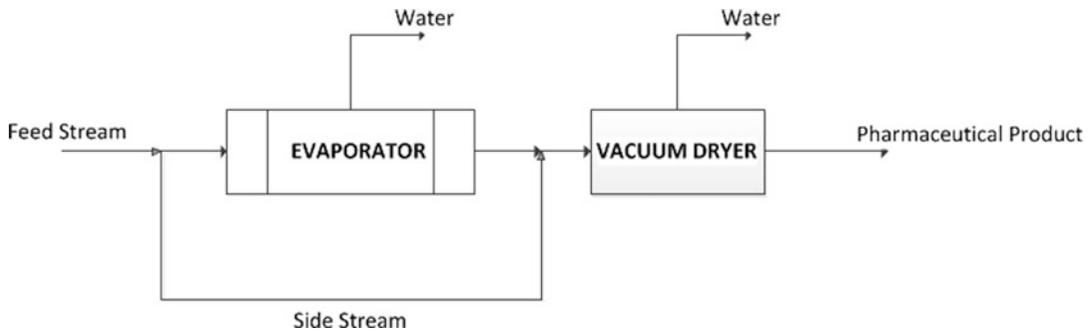


Fig. 7.22 Three step process: Evaporation, mixing and drying

units, first a single-effect evaporator and then a vacuum dryer. The feed stream has a mass flow rate of 1,000 kg/h with 12 % w/w solids (assume that the feed stream has two components: solids and H₂O). The feed stream is divided into two streams, one going directly to the single-effect evaporator and the other (side stream) joining the output stream of the evaporator that contains the solids. The output of the single-effect evaporator has two streams, one stream with just H₂O and a second stream with solids and H₂O (the one that receives and is mixed with is the side stream). This stream is fed to the vacuum dryer to remove more H₂O and reach the desired final 3 % w/w moisture content. If the side stream has a mass flow rate of 200 kg/h and the single-effect evaporator removes H₂O at a rate of 500 kg/h, then: (a) What is the mass flow rate of the pharmaceutical product? (b) What is the mass fraction of solids at the output stream of the single-effect evaporator?

Step I

Reading and understanding

This problem includes two units (single-effect evaporator and a vacuum dryer) and, in addition, a side stream. Considering that our first object in the material balance is to minimize the number of variables, we will consider one unknown mass fraction for each stream (because, as mentioned, the sum of the two mass fractions is 1). In addition, the side stream adds a division (one material balance) and a mixer (two material balances).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

As depicted in the flow diagram (Fig. 7.23), we have included a mixer and all data. Thus, we can identify 14 variables. In addition, as shown in the flow diagram, the variables are defined as follows:

F_1 : Mass flow rate of feed stream (kg/h)

x_1 : Solid mass fraction composition in feed stream

F_2 : Mass flow rate of side stream (kg/h)

x_2 : Solid mass fraction composition in side stream ($x_1 = x_2 = x_3$)

F_3 : Mass flow rate entering single-effect evaporator (kg/h)

x_3 : Solid mass fraction composition in stream entering single-effect evaporator

F_4 : Mass flow rate of H₂O out of single-effect evaporator (kg/h)

F_5 : Mass flow rate out of single-effect evaporator (kg/h)

x_5 : Solid mass fraction composition in stream out of single-effect evaporator

F_6 : Mass flow rate entering the vacuum dryer (kg/h)

x_6 : Solid mass fraction composition in stream entering the vacuum dryer

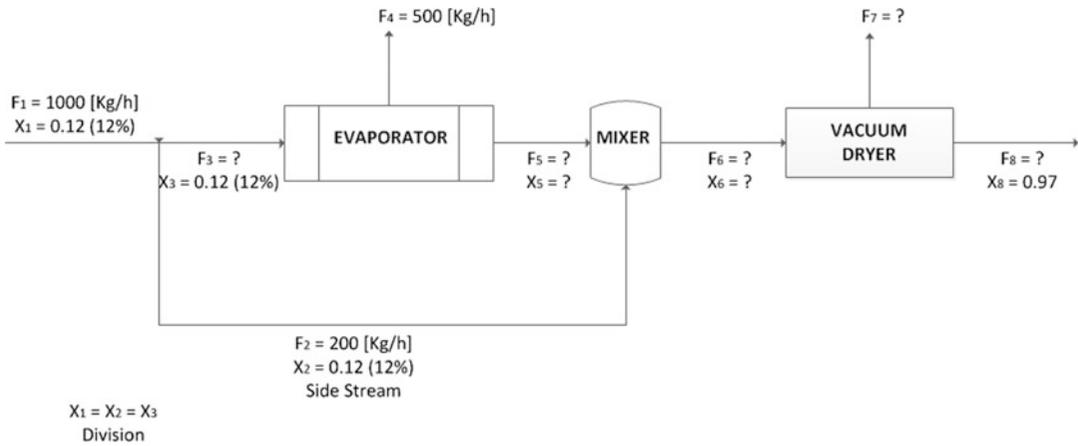


Fig. 7.23 Three step process including all variables

F_7 : Mass flow rate of H_2O out of vacuum dryer (kg/h)

F_8 : Mass flow rate of delicate product (3 % humidity) (kg/h)

x_8 : Solid mass fraction composition at stream with delicate product

Thus, the total number of variables is 14 (NV = 14).

Step III

Analysis of degrees of freedom in process

In this process we have two units, but because the side stream is mixed with the output stream of the single-effect evaporator, we added a third unit—a mixer (Fig. 7.23). Therefore, given that we have two components in each unit, the total independent material balance is 6 ($n_1 + n_2 + n_3 = 2 + 2 + 2 = 6$). In addition, the feed stream is divided in two, and here we can add one more independent material balance equation, so NMB = 7.

As depicted in Fig. 7.23, the number of specified variables is 7 (NSV = 7). Given that there is no relationship information, we have

$$DF = NV - NMB - NSV - NR = 14 - 7 - 7 - 0 = 0; \quad DF = 0. \quad (7.28)$$

Thus, the problem is set and we can proceed to formulate the equations and solve the problem.

Step IV

Mathematical formulation including all available data (Fig. 7.23)

System S_1

Total mass balance at division (side stream):

$$1,000 - 200 - F_3 = 0. \quad (7.29)$$

System S_2

Total mass balance at evaporator:

$$F_3 - 500 - F_5 = 0. \quad (7.30)$$

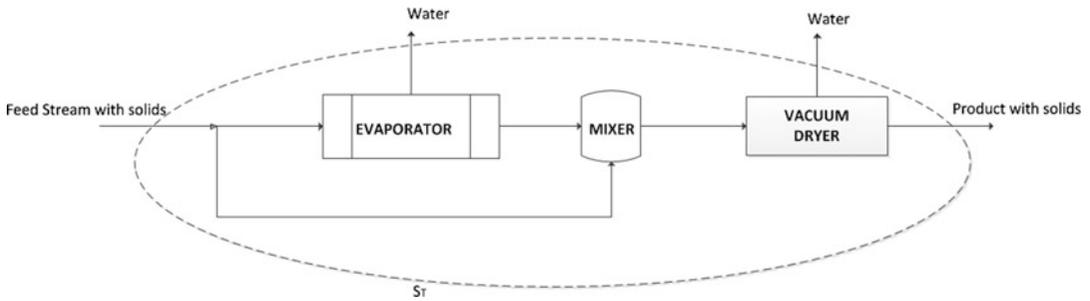


Fig. 7.24 Global material balance in the three step process

Solid mass balance at evaporator:

$$0.12 \times F_3 - x_5 \times F_5 = 0. \quad (7.31)$$

System S_3

Solid mass balance at mixer (side stream):

$$0.12 \times 200 + x_5 \times F_5 - x_6 \times F_6 = 0. \quad (7.32)$$

Total mass balance at mixer (side stream):

$$200 + F_5 - F_6 = 0. \quad (7.33)$$

System S_4

Total mass balance at vacuum dryer:

$$F_6 - F_7 - F_8 = 0. \quad (7.34)$$

Solid mass balance at vacuum dryer:

$$x_6 \times F_6 - 0.97 \times F_8 = 0. \quad (7.35)$$

Step V

Solution, results, analysis, and discussion

As shown in (7.29)–(7.35), we have formulated seven equations and we have seven unknowns. As is normal, in this material balance problem (steady state and continuous operation) the equations are simple and not necessarily hard to solve. In this case, it is relatively straightforward, starting with (7.29), and then we finally get

$F_3 = 800$ kg/h; $F_5 = 300$ kg/h; $x_5 = 0.32$ kg solids/kg solution; $F_6 = 500$ kg/h; $x_6 = 0.24$ kg solids/kg solution; $F_7 = 3.76 \times 10^2$ kg/h; $F_8 = 1.23 \times 10^2$ kg/h.

Answering the questions of the problem, (a) 1.23×10^2 kg/h of the delicate pharmaceutical product (F_8), and (b) 0.32 kg solids/kg solution (x_5).

Testing the results. If you formulate a total mass balance for solids in the whole process, then you can write (Fig. 7.24)

$$0.12 \times 1,000 - 0.97 \times F_8 = 0 \quad (S_T \text{ in Fig. 7.24}), \text{ and thus } F_8 = 1.23 \times 10^2 \text{ kg/h.}$$

Recall that it is not always simple to test the results, but at the very least you need to analyze the results and try to figure out if they make sense in your problem.

Although, strictly speaking, this is not a very difficult problem, it is a good example of how easy it is to manage a problem with several units and large numbers of variables and equations.



Please stop and take your time to quietly read the following message.

ATTENTION

As was the similar message in Chap.5, here again, we consider it crucial for you to read the statement of each solved problem and then try to solve it without looking at the proposed solution. On the first few problems, it might be a little boring to follow the proposed procedure, but we are convinced that by the end you will feel that it was worth it. Follow the proposed procedure step by step, and for all solved-problems presented here, do not skip any problems. Only once you have done this, having resolved all issues, should you proceed, with firm resolve, to all the proposed problems. And never forget the methodology!

“I hear, and I forget. I see, and I remember. I do, and I understand”

CHINESE PROVERB

7.9 Solved Problems

The aim of this section is to familiarize you with the proposed procedure and to reveal that most real-world problems will be hard to solve without some sort of procedure. Initially, with simple problems (simple units), some steps will be developed together, but as problems become more complex, it will be necessary to follow rigorously the proposed strategy step by step.

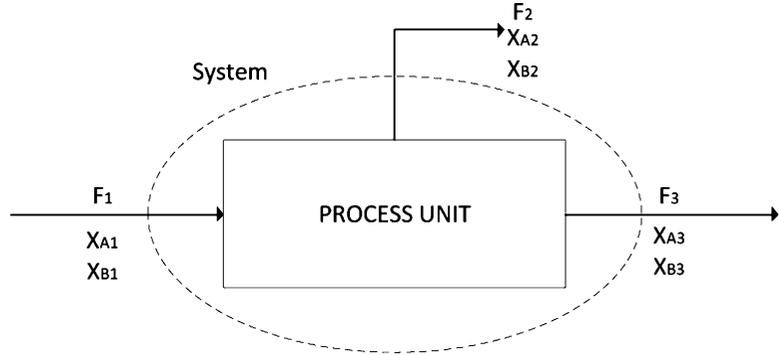
As we tell our students, just dutifully follow our recommendations, and, as we have stated from the beginning, the key word is PROCEDURE. A good example is problem 15. It involves 31 variables and several “hidden” relationships, and we categorized its difficulty as 10^+ . But not to worry! If you start from problem one and advance step by step, you will be prepared for problem 15.

As stated in Sect. 7.1, one of our goals is to familiarize students with the broad application of chemical and bioprocess engineering. Although the main goal is to enable you to solve material balance problems, in addition, you will discover, through the examples, why some processes include recycle streams (e.g., problems 10 and 11) and purges (e.g., problem 15).

SIMPLE UNITS

1. **Separation process [3].** A stream with three components, A, B, and C, is fed to a process unit. At the outlet of the equipment are two streams, each containing components A, B, and C. (a) Draw a schematic representation of this process. (b) How many variables are in this process? (c) Determine the degrees of freedom of the process.

Fig. 7.25 Separation process



Step I and II

Reading and understanding a flow diagram, variable definition and codification, and inclusion of all available data

(a) Draw a schematic representation of this process (Fig. 7.25)

F_1 : Mass flow rate of feed stream (kg/h)

x_{A1} : Mass fraction of component A in feed stream

x_{B1} : Mass fraction of component B in feed stream

F_2 : Mass flow rate of second stream (kg/h)

x_{A2} : Mass fraction of component A in second stream

x_{B2} : Mass fraction of component B in second stream

F_3 : Mass flow rate of third stream (kg/h)

x_{A3} : Mass fraction of component A in third stream

x_{B3} : Mass fraction of component B in third stream

(b) How many variables are in this process? $NV = 9$, and, given that there are no specified variables or any given relationship, $NSV = 0$ and $NR = 0$.

Step III

Analysis of degrees of freedom in process

Because the streams have three components, we can formulate three independent material balances. Recalling (7.13) we obtain:

(c) Determine the degrees of freedom of the process:

$$DF = NV - NMB - NSV - NR = 9 - 3 - 0 - 0 = 6; \quad DF = 6.$$

We have already answered all the questions and will stop here. In addition, we have no data to do any further calculations. But remember, you can specify six variables, but they must be independent of each other. We learned from example 5 that you cannot arbitrarily fix the three flow streams.

2. Separation process with a side stream [4]. Although it normally causes confusion, the concepts of side stream and bypass are clearly different. A bypass stream does not pass through a process, i.e., the stream “dodges” the process and is passed by the following process. A side stream, on the other hand, is divided into two streams, one part going through the process and the other being “bypassed” and then joining with the stream that was processed. A generic example of a side stream is presented in this exercise.

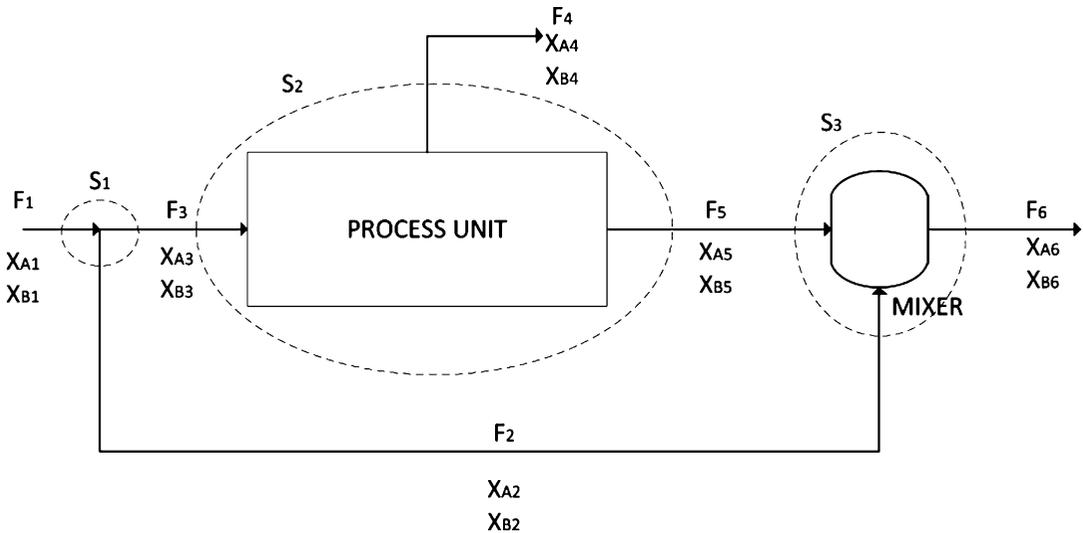


Fig. 7.26 Separation process with a side stream

Like the previous exercise (three streams and three components per stream), this one includes a side stream, where part of the feed stream is “bypassed” to join one of the streams as it leaves the equipment. (a) Draw a schematic representation of this process. (b) How many variables are in this process? (c) Determine the degrees of freedom in this process.

Step I and II

Reading and understanding a flow diagram, variable definition and codification, and inclusion of all available data

(a) Draw a schematic representation of this process.

As depicted in Fig. 7.26, we have included a mixer, as recommended in Sect. 7.8.2.1. Another interesting point for discussion is the relevance of including or not x_{A2} , x_{B2} , x_{A3} , and x_{B3} as variables. When the feed stream is separated, the flow rate of each new stream is unknown, but their concentrations are exactly the same as that of the original stream; it is simply a division. Therefore, $x_{A1} = x_{A2}$, $x_{A1} = x_{A3}$, $x_{B1} = x_{B2}$, and $x_{B1} = x_{B3}$. If we consider them variables, then we need to include these four equations. Our advice is to not consider them variables and assume that they are already considered in x_{A1} and x_{B1} .

F_1 : Mass flow rate of feed stream (kg/h)

x_{A1} : Mass fraction of component A in feed stream

x_{B1} : Mass fraction of component B in feed stream

F_2 : Mass flow rate of second stream (kg/h)

~~x_{A2} : Mass fraction of component A in the second stream.~~

~~x_{B2} : Mass fraction of component B in the second stream.~~

F_3 : Mass flow rate of third stream (kg/h)

~~x_{A3} : Mass fraction of component A in the third stream.~~

~~x_{B3} : Mass fraction of component B in the third stream.~~

F_4 : Mass flow rate of third stream (kg/h)

x_{A4} : Mass fraction of component A in fourth stream

x_{B4} : Mass fraction of component B in fourth stream

- F_5 : Mass flow rate of third stream (kg/h)
 x_{A5} : Mass fraction of component A in fifth stream
 x_{B5} : Mass fraction of component B in fifth stream
 F_6 : Mass flow rate of third stream (kg/h)
 x_{A6} : Mass fraction of component A in sixth stream
 x_{B6} : Mass fraction of component B in sixth stream

(b) How many variables are in this process? $NV = 14$, and given that there are neither specified variables nor any relationship given, $NSV = 0$ and $NR = 0$.

Step III

Analysis of degrees of freedom in process

Because the streams have three components, we can formulate three independent material balance equations for each process unit (two process units, systems S_2 and S_3 , and six equations). In addition, we have a division (system S_1), adding one more total mass balance. Therefore, $NMB = 7$.

(c) Determine the degrees of freedom in this process:

$$DF = NV - NMB - NSV - NR = 14 - 7 - 0 - 0 = 7; \quad DF = 7.$$

3. **Dryer [6].** A wet material is passed through a dryer unit to attain some specific moisture content. Hot fresh air is fed to the dryer and at the outlet, part of the humid air is recycled. (a) How many variables can you identify? (b) Determine the degrees of freedom in this process unit.

Step I

Reading and understanding

Remember, familiarize yourself with the equipment used in process and bioprocess engineering. In this specific case it is not necessary to learn and become familiar with all classes of dryers, but at least familiarize yourself with its main function. Given that the flow diagram of the process is not included, it is important to read carefully and put the statement of the problem in a clear and schematic figure. Particularly in this problem a good schematic representation of the drying process is critical. As we will see, this type of problem presents its own special features, and the way one defines systems is important. Do not be too concerned about its difficulty because we have intentionally included this example for its difficulty to characterize the process and its systems.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

- F_1 : Mass flow rate of wet material (kg/h)
 x_{S1} : Mass fraction of solids on wet material stream
 F_2 : Mass flow rate of dried material stream (kg/h)
 x_{S2} : Mass fraction of solids on dried material stream
 F_3 : Mass flow rate of hot fresh air (kg/h)
 x_{H_2O3} : Mass fraction of H_2O in hot fresh air
 F_4 : Mass flow rate of air entering dryer (kg/h)
 x_{H_2O4} : Mass fraction of H_2O in air entering dryer
 F_5 : Mass flow rate of air leaving dryer (kg/h)
 x_{H_2O5} : Mass fraction of H_2O in air leaving dryer
 F_6 : Mass flow rate out of process (kg/h)
 F_7 : Mass flow rate of recycle stream (kg/h)
 F_8 : Mass flow rate of H_2O evaporated from wet material (kg/h)

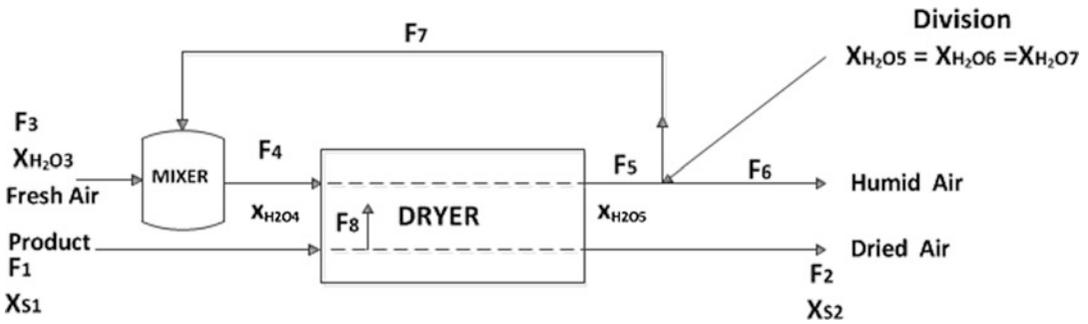


Fig. 7.27 Drying process

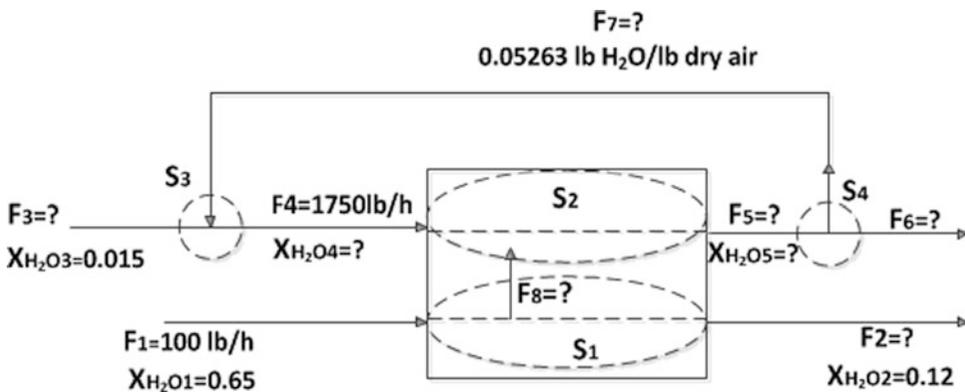


Fig. 7.28 Drying process including all variables and available data

One key feature in this example is that we have conveniently separated the drying process into two systems. Whenever you have data on the humidity of air, it is convenient to separate it into two systems, as depicted in Fig. 7.27. In addition, as explained in the previous problems, we are not considering x_{H_2O6} and x_{H_2O7} as variables because they are equal to x_{H_2O5} (due to the division).

(a) How many variables can you identify? $NV = 13$, and given that there are no specified variables or any given relationship, $NSV = 0$ and $NR = 0$.

Step III

Analysis of degrees of freedom in process

As shown in the flow chart (Fig. 7.28), the whole process has been divided into four systems, where in systems S_1, S_2 , and S_3 we can formulate two material balances in each one. System S_4 is a division that adds one more material balance. Therefore, $NMB = 7$.

(b) Determine the degrees of freedom in this process unit:

$$DF = NV - NMB - NSV - NR = 13 - 7 - 0 - 0 = 6; DF = 6.$$

4. **Dryer [5].** 100 lb/h of a wet material with 65 % humidity w/w, and the rest can be considered a solid that is passed through a dryer where the end product reaches 12 % humidity w/w. The hot fresh air has 1.5 % humidity w/w and before entering the dryer is mixed with recycled air with 0.05263 lb

H₂O/lb of dry air. If 1,750 lb/h of humid air are fed to the dryer, then: (a) What is the mass flow rate of hot fresh air? (b) What is the mass flow rate of the recycled air?

Step I

Reading and understanding

This problem is similar to the previous problem but with quantitative data. Then the number of variables should be the same ($NV = 13$) and the degrees of freedom will change because here we have some specified variables.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1 : Mass flow rate of wet material in pounds per hour (lb/h)

x_{H_2O1} : Mass fraction of water in wet material stream (w/w)

F_2 : Mass flow rate of dried stream (lb/h)

x_{H_2O2} : Mass fraction of water in dried material stream (w/w)

F_3 : Mass flow rate of hot fresh air (lb/h)

x_{H_2O3} : Mass fraction of H₂O in hot fresh air (w/w)

F_4 : Mass flow rate of air entering dryer (lb/h)

x_{H_2O4} : Mass fraction of H₂O in air entering dryer (w/w)

F_5 : Mass flow rate of air out of dryer (lb/h)

x_{H_2O5} : Mass fraction of H₂O in air out of dryer (w/w)

F_6 : Mass flow rate out of process (lb/h)

F_7 : Mass flow rate of recycle stream (lb/h)

F_8 : Mass flow rate of H₂O evaporated from wet material (lb/h)

Thus, $NV = 13$, and there are six specified variables and no relationships are given. Therefore, $NSV = 6$ and $NR = 0$.

Step III

Analysis of degrees of freedom in process

As shown in the flow chart (Fig. 7.28), the whole process, as in the previous problem, has been divided into four systems. In each of systems S_1 , S_2 , and S_3 we can formulate two material balances. System S_4 is a division that adds one more material balance. Therefore, $NMB = 7$. Replacing NV , NMB , NSV , and NR in (7.13), we get

$$DF = NV - NMB - NSV - NR = 13 - 7 - 6 - 0 = 0; \quad DF = 0.$$

meaning that the problem is set and can be solved.

Step IV

Mathematical formulation including all available data

As depicted in Fig. 7.28, the process is divided into four systems (S_1 , S_2 , S_3 , and S_4). In addition, as mentioned in systems S_1 , S_2 , and S_3 , we can formulate two material balances in each one and write one more material balance (total mass balance) in system S_4 . In addition, as shown in the flow diagram, there are seven unknowns, and we can formulate seven equations. Before formulating the equations of systems S_1 , S_2 , S_3 , and S_4 it is necessary to have consistent units in all variables. For example, the humidity of the recycled air stream is given in pounds of water per pound of dry air (lb H₂O/lb dry air). To be consistent, it is necessary to express this as pounds of water per pound

of humid air (lb H₂O/lb humid air), where 0.05263 lb H₂O/lb dry air is equivalent to 0.05263/(1 + 0.05263) lb H₂O/lb humid air $\sim 5.000 \times 10^{-2}$ lb H₂O/lb humid air. Therefore, $x_{\text{H}_2\text{O}_7} = 5.000 \times 10^{-2} = x_{\text{H}_2\text{O}_5}$. The hot fresh air has 1.5 % humidity w/w. Therefore, $x_{\text{H}_2\text{O}_3} = 0.015$ lb H₂O/lb humid air.

System S₁

Total mass balance:

$$100 - F_2 - F_8 = 0. \quad (7.36)$$

Mass balance for H₂O:

$$0.65 \times 100 - 0.12 \times F_2 - F_8 = 0. \quad (7.37)$$

System S₂

Total mass balance:

$$1,750 + F_8 - F_5 = 0. \quad (7.38)$$

Mass balance for H₂O:

$$X_{\text{H}_2\text{O}_4} \times 1,750 + F_8 - 5.000 \times 10^{-2} \times F_5 = 0. \quad (7.39)$$

System S₃

Total mass balance:

$$F_3 + F_7 - 1,750 = 0. \quad (7.40)$$

Mass balance for H₂O:

$$0.015 \times F_3 + 5.000 \times 10^{-2} \times F_7 - 1,750 \times X_{\text{H}_2\text{O}_4} = 0. \quad (7.41)$$

System S₄

Total mass balance:

$$F_5 - F_6 - F_7 = 0. \quad (7.42)$$

Step V

Solution, results, analysis, and discussion

First, we have seven equations and seven unknowns. Sometimes, this might be a bit difficult, but it is worth trying. Solving (7.36) and (7.37) we obtain $F_2 \sim 39.773$ lb/h and $F_8 \sim 6.2 \times 10$ lb/h. Then from (7.38) and (7.39) we get $F_5 \sim 1.81 \times 10^3$ and $X_{\text{H}_2\text{O}_4} \sim 0.0173$. Finally, from (7.40)–(7.42) we obtain $F_7 \sim 1.15 \times 10^2$ lb/h, $F_3 \sim 1.63 \times 10^3$ lb/h, and $F_6 \sim 1.69 \times 10^3$ lb/h.

(a) What is the mass flow rate of hot fresh air? $F_3 \sim 1.63 \times 10^3$ lb/h.

(b) What is the mass flow rate of recycled air? $F_7 \sim 1.15 \times 10^2$ lb/h.

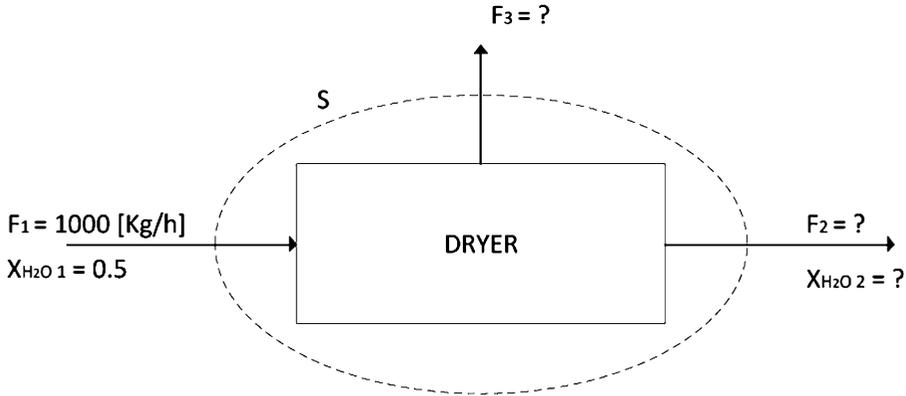


Fig. 7.29 Drying process including all variables and available data

One way to test the results is to carry a global mass balance of the whole process (not included in (7.36)–(7.42). In Fig. 7.28 the total mass balance corresponds to $F_1 + F_3 - F_2 - F_6 = 0$. Replacing each value we confirm that this equation is correctly accomplished with the obtained values for the streams. In addition, $F_3 + F_7$ should add up to 1,750 lb/h, and it does.

There are several ways to see whether at least the results are in the expected range. For example, $x_{\text{H}_2\text{O}3} < x_{\text{H}_2\text{O}4} < x_{\text{H}_2\text{O}7}$ (recall that $x_{\text{H}_2\text{O}5} = x_{\text{H}_2\text{O}7}$). As you can verify, $x_{\text{H}_2\text{O}4}$ is greater than $x_{\text{H}_2\text{O}3}$ and less than $x_{\text{H}_2\text{O}7}$.

5. Dryer analysis [6]. 1,000 kg/h of a wet material with 50 % moisture content will be dehydrated. The manager wants a very dry product and obtains 400 kg/h end product. You do some thought and conclude that it would be impossible to obtain less than 500 kg/h dried product (just removing H_2O). The manager protests your claim that he occupied one degree of freedom, which is exactly the degrees of freedom of the process. Is he right? Why not?

Step I

Reading and understanding

As you indicate, if the material has 50 % moisture content, then if you remove all the H_2O , your dried product will have a mass flow rate of 500 kg/h. On the other hand, the manager might be right when he says that the process has 1 degree of freedom.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data (Fig. 7.29)

F_1 : Mass flow rate of wet material (kg/h)

$x_{\text{H}_2\text{O}1}$: Mass fraction of water in wet material stream (w/w)

F_2 : Mass flow rate of the dried stream (lb/h)

$x_{\text{H}_2\text{O}2}$: Mass fraction of water in dried material stream (w/w)

F_3 : Mass flow rate of H_2O (kg/h)

Therefore, $\text{NV} = 5$, and there are two specified variables (F_1 and $x_{\text{H}_2\text{O}1}$), and no relationships are given. Thus, $\text{NSV} = 2$ and $\text{NR} = 0$.

Step III**Analysis of degrees of freedom in process**

The wet material has two components (solids + H₂O), and the process is carried out in one process unit (dryer). Therefore, we are able to formulate two independent material balances:

$$\text{NMB} = 2.$$

Thus,

$$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = 5 - 2 - 2 - 0 = 1:$$

$$\text{DF} = 1.$$

Yes, the manager is right when he says that the process has one degree of freedom. Yes, we can fix one variable, but, remember, with limitations. We have three unknowns, and we can fix one variable (DF = 1).

What are the limitations in this case?

$F_3 \leq 500$ kg/h, the mass flow rate of the wet material is 1,000 kg/h with 50 % humidity, and so the amount of H₂O in F_1 is 500 kg/h. Of course, you cannot remove more than 500 kg/h.

$F_2 \geq 500$ kg/h if the amount of H₂O removed is ≤ 500 kg/h. Thus, F_2 must be ≥ 500 kg/h. Why? The total mass balance in this process is $1,000 - F_2 - F_3 = 0$. Then $F_2 = 1,000 - F_3$. If $F_3 \leq 500$ kg/h, then $F_2 \geq 500$ kg/h.

Therefore, the manager was right that the degree of freedom in this process was one but wrong when he set $F_2 = 400$ kg/h. In addition, we suggest that you analyze the limitations for $X_{\text{H}_2\text{O}}$.

MULTIPLE UNITS

6. **H₂O removed from two processes in series including a recycle [6].** In generic terms, a recycle stream is an outlet stream that is divided into two streams, one stream returning to the process (e.g., to the feed stream) and the other stream going on to the following stage of the process, if any.

In the following generic process there are two process units in series including a recycle stream. The feed stream contains A, B, and H₂O. In each process unit, part of the H₂O is removed. At the outlet of the second process unit, the stream is divided into two streams; one of the streams is recycled to join the feed stream. (a) Draw a schematic representation of the complete process. (b) How many variables are in this process (NV)? (c) Determine the degrees of freedom in this process (DF).

Step I**Reading and understanding**

This process has two units and a recycle stream. As was stated earlier, the addition of a recycle stream means that we need to consider a division (add one total mass balance) and add a mixer (in this case adding three material balances because the stream has three components).

Step II**Flow diagram, variable definition and codification, and inclusion of all available data**

(a) Draw a schematic representation of the complete process

F_1 : Mass flow rate of feed stream (kg/h)

x_{A1} : Mass fraction of component A in feed stream

x_{B1} : Mass fraction of component B in feed stream

F_2 : Mass flow rate of the input stream in first process unit (kg/h)

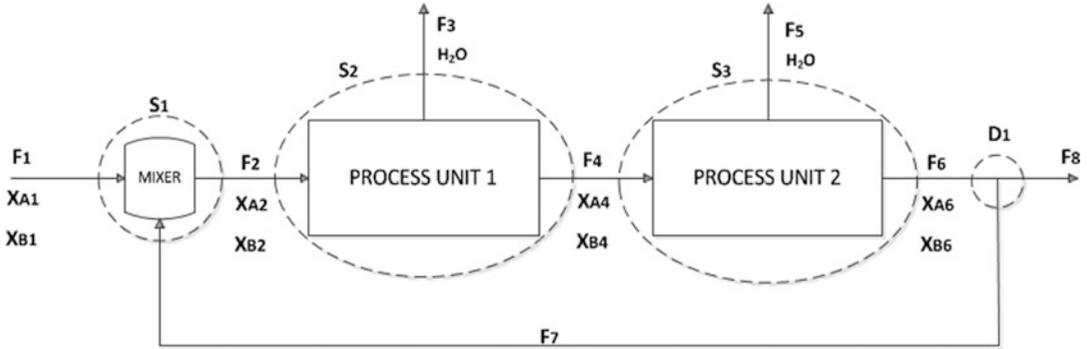


Fig. 7.30 Process in series to remove water

x_{A2} : Mass fraction of component A in input stream in first process unit

x_{B2} : Mass fraction of component B in input stream in first process unit

F_3 : Mass flow rate of H_2O at output in first process unit (kg/h)

F_4 : Mass flow rate at outlet of first process unit (kg/h)

x_{A4} : Mass fraction of component A at outlet of first process unit

x_{B4} : Mass fraction of component B at outlet of first process unit

F_5 : Mass flow rate of H_2O at output in second process unit (kg/h)

F_6 : Mass flow rate at outlet of first process unit (kg/h)

x_{A6} : Mass fraction of component A at outlet of second process unit

x_{B6} : Mass fraction of component B at outlet of second process unit

F_7 : Mass flow rate of recycle stream (kg/h)

F_8 : Mass flow rate at outlet of whole process (kg/h)

As shown in the division, when a stream is separated, the flow rate of each new stream is unknown, but their concentrations are exactly the same as the original stream; it is simply a division ($x_{B6} = x_{B7} = x_{B8}$). Then the outlet stream of the process and the recycle stream have already assigned variables for their concentrations of A and B (x_{A6} and x_{B6}). In addition, the water removed in each stage is pure (100 % H_2O).

(b) How many variables are in this process?

As depicted in Fig. 7.30, we have 16 variables ($NV = 16$), and no specified variable or relationships are given. Thus, $NSV = 0$ and $NR = 0$.

Step III

Analysis of degrees of freedom in process

With the inclusion of the mixer we now have three process units and one division. Therefore, we can formulate ten independent material balances (three per unit and one at the division), so $NMB = 10$.

(c) Determine the degrees of freedom in this process (DF).

$$DF = NV - NMB - NSV - NR = 16 - 10 - 0 - 0 = 6; DF = 6.$$

7. Multieffect evaporator [8]. Multieffect evaporators are simple evaporators connected in series in order to reduce the consumption of energy (steam) per kilogram of water evaporated from the fluid under concentration. This is achieved by reusing the vapors coming out of each effect in the following or preceding effect (depending on whether it is a cocurrent or countercurrent). Figure 7.31 is a generic schematic representation of a multieffect evaporator operated under a countercurrent, i.e., the diluted

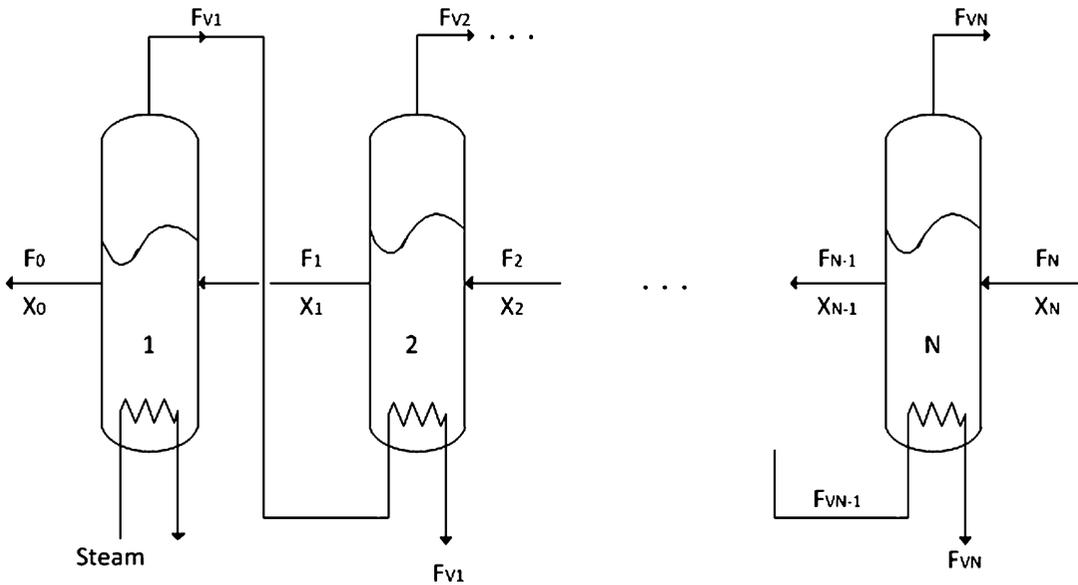


Fig. 7.31 Multi-effect evaporator

fluid is fed to the last effect (the lowest temperature). Assuming that the feed fluid is composed of solids and water: (a) How many variables are in this process? (b) How many independent material balances can be formulated? (c) Determine the degrees of freedom of the multieffect evaporator process (DF).

Step I

Reading and understanding

According to the problem statement and as shown in Fig. 7.31, the vapor generated in the following effect is used in the preceding effect as energy. In this arrangement, just the first effect uses live steam from the boiler. Then in each effect we have three streams, two for the product and the third of vapor. The stream shown below in each effect is used just as a medium to supply energy. Strictly speaking, it will not be part of the system in terms of mass balance.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

As depicted in Fig. 7.32, S_1, S_2, \dots, S_N are the systems chosen for material balance analysis (similar to the one shown in Fig. 7.33).

According to the flow diagram provided in the problem and with our additions (systems) we have

$F_0, F_1, F_2, \dots, F_N$: Mass flow rate of each stream of product (kg/h);

$F_{V1}, F_{V2}, \dots, F_{VN}$: Mass flow rate of vapor per each effect;

$x_0, x_1, x_2, \dots, x_N$: Mass fraction of solids in each stream of product.

Then we have $N + 1$ streams of product, N streams of vapor, and $N + 1$ concentrations, in total $3N + 2$.

(a) How many variables are in this process?

$NV = 3N + 2$. In addition, $NSV = 0$ and $NR = 0$.

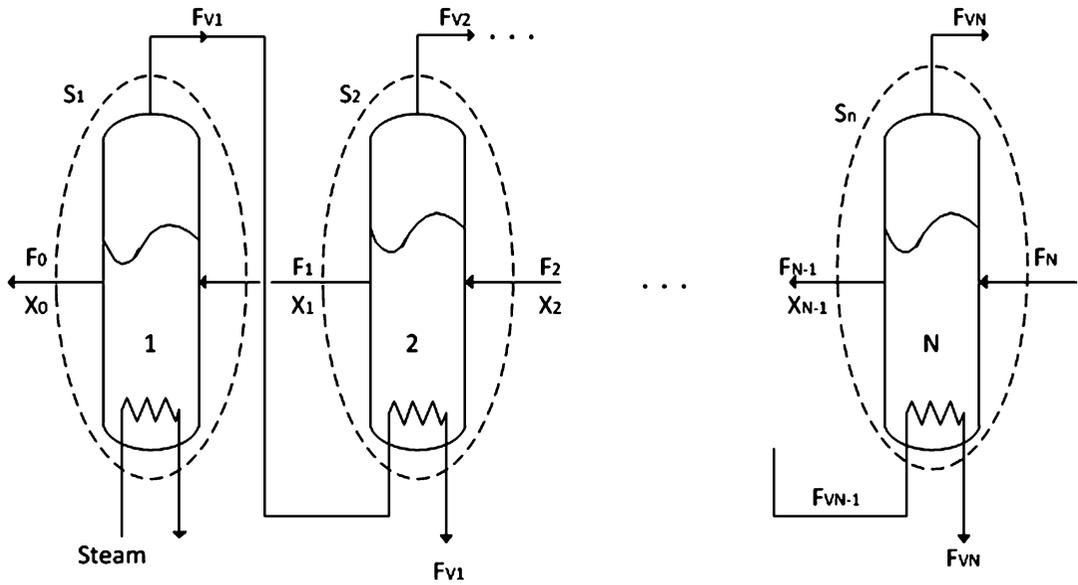


Fig. 7.32 Multi-effect evaporator showing systems S_1, S_2, \dots, S_N

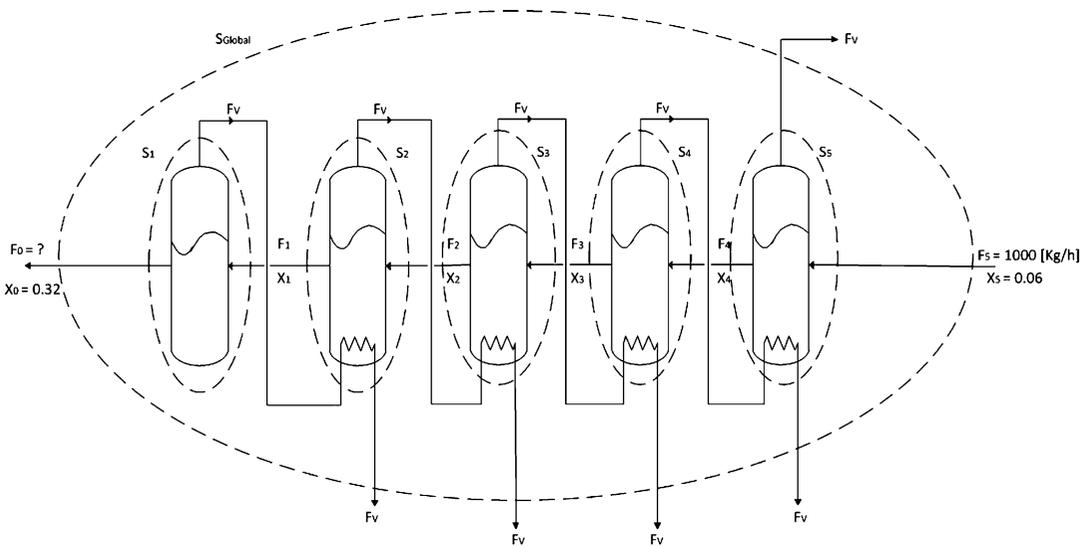


Fig. 7.33 Multi-effect evaporator for tomato paste

Step III

Analysis of degrees of freedom in process

(b) How many independent material balances can be formulated?

In each effect we can formulate two independent material balances (two components). Thus, in total we can formulate $2N$ equations. Therefore,

$$\text{NMB} = 2N.$$

(c) Determine the degrees of freedom of the multieffect evaporator process (DF).

$$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = (3N + 2) - 2N - 0 - 0 = N + 2; \text{DF} = N + 2.$$

8. **Tomato concentrates [6].** 1,000 kg/h tomato juice (6 % solids and H₂O) are fed to a five-effect evaporator operated under a countercurrent to reach 32 % solids at the outlet. The amount of H₂O removed in each effect is the same. (a) Determine the degrees of freedom of the process. (b) Determine the solid concentration at the outlet of each of the following effects (2, 3, 4, and 5). (c) What is the flow rate at the outlet of the first effect?

Step I

Reading and understanding

This problem is a practical application of the preceding one. Therefore, we will use a similar nomenclature to define all the variables but assume that the amount of H₂O evaporated, in each effect, is the same. Thus,

$$F_{V1} = F_{V2} = F_{V3} = F_{V4} = F_{V5} = F_V.$$

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

$F_0, F_1, F_2, F_3, F_4, F_5$: Mass flow rate of each stream of product (kg/h)

F_V : Mass flow rate of vapor per each effect (kg/h)

$x_0, x_1, x_2, x_3, x_4, x_5$: Mass fraction of solids in each stream of product

Therefore, $\text{NV} = 13$, $\text{NSV} = 3$ (F_5, x_5 , and x_0), and $\text{NR} = 0$.

Step III

Analysis of degrees of freedom in process

In each effect we can formulate two independent material balances. Thus, $\text{NMB} = 10$.

(a) Determine the degrees of freedom of the process (DF).

$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = 13 - 10 - 3 - 0 = 0$; $\text{DF} = 0$. Therefore, the problem is ready to be solved.

Step IV

Mathematical formulation including all available data

As we have learned from previous problems, a good choice for material balance in this case is the solid and total mass balance for each effect. In addition, we can formulate ten independent material balances. We can do that by formulating two equations per system (S_1, S_2, S_3, S_4 , and S_5) and not using the global mass balance (S_T). In this situation, given that we have information at the input and output of the whole process, it appears convenient to use the global mass balance (total and solids) as two of the ten equations.

System S_T (global)

Total mass balance:

$$1,000 - F_0 - 5 \times F_V = 0. \quad (7.43)$$

Solid mass balance:

$$0.06 \times 1,000 - 0.32 \times F_0 = 0. \quad (7.44)$$

System S_5

Total mass balance:

$$1,000 - F_4 - F_V = 0. \quad (7.45)$$

Solid mass balance:

$$0.06 \times 1,000 - x_4 \times F_4 = 0. \quad (7.46)$$

System S_4

Total mass balance:

$$F_4 - F_3 - F_V = 0. \quad (7.47)$$

Solid mass balance:

$$x_4 \times F_4 - x_3 \times F_3 = 0. \quad (7.48)$$

System S_3

Total mass balance:

$$F_3 - F_2 - F_V = 0. \quad (7.49)$$

Solid mass balance:

$$x_3 \times F_3 - x_2 \times F_2 = 0. \quad (7.50)$$

System S_2

Total mass balance:

$$F_2 - F_1 - F_V = 0. \quad (7.51)$$

Solid mass balance:

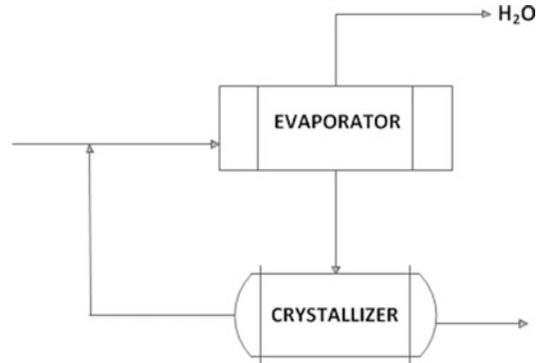
$$x_2 \times F_2 - x_1 \times F_1 = 0. \quad (7.52)$$

Step V

Solution, results, analysis, and discussion

Then, from step IV, we have ten equations and ten variables. Solving first (7.44) we get $F_0 = 187.5$ kg/h, then from (7.43) we obtain $F_V = 162.5$ kg/h. After these calculations the rest is straightforward; just continue with (7.45) and so on.

Fig. 7.34 Evaporation-Crystallization system including recycle



(b) Determine the solid concentration at the outlet of each of the following effects (2, 3, 4, and 5).

$$x_1 = 0.1714, x_2 = 0.1171, x_3 = 0.0889, x_4 = 0.0716$$

(c) What is the flow rate at the outlet of the first effect?

$$F_0 = 187.5 \text{ kg/h}$$

One important lesson is that, sometimes, it is not clear what the right decision is as to which equations to formulate to facilitate the mathematical solution. Several times, as in this example, the total and solid mass balance for the whole process (global) could be the trigger for a quick solution. There is no general rule on how to select the right equations in such a way as to minimize your potential struggle with the mathematical solution. On the other hand, today you have many tools, and so the mathematical solution will not necessarily pose a problem. In addition, as has been expressed in different parts of the text, the most important thing at this stage is to learn material balance.

9. Evaporation-crystallization including recycle [8]. As depicted in Fig. 7.34, the process is composed of an evaporation unit and a crystallizer.

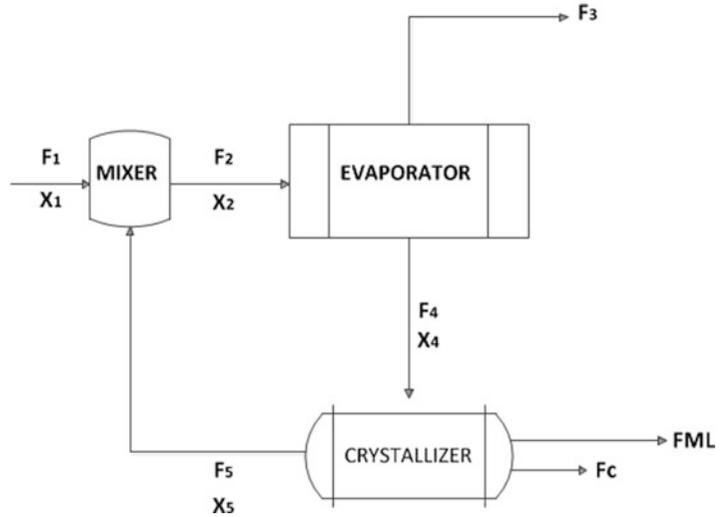
The process is operated in a continuous mode under steady state. A solution with component A and H₂O will pass first to an evaporator unit to remove some H₂O and then through a crystallizer. The mother liquor (water and component A) will be recycled; and the crystals leaving the process will carry, on their surfaces, a small amount of the mother liquor. (a) How many variables you can identify in this process? (b) Determine the degrees of freedom of the process.

Step I

Reading and understanding

In this case, as discussed earlier, it is better to improve or redo the schematic diagram of the process provided in this problem. For example, given that the process includes a recycle stream, it is advisable, for clarity, to include a mixer in the flow diagram. In addition, remember to familiarize yourself with the unit operations covered in this process, evaporation and crystallization. One strategy that we suggest in this problem is to treat the stream with crystal and mother liquor as two different streams.

Fig. 7.35 Evaporation-Crystallization system including all variables



Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1, F_2, F_3, F_4, F_5 (recycled mother liquor), F_C, F_{ML} (mother liquor (F_{ML}) and crystals (F_C)): mass flow rate of streams kg/h

x_1, x_2, x_4, x_5 : Concentrations w/w (where $x_5 = x_{ML}$).

We have decided to separate the output mass flow rate of the crystallizer into two streams for clarity. In the next solved exercise we will show why it is better to separate this output flow into two streams.

(a) How many variables can you identify in this process?

From Fig. 7.35, $NV = 11$, $NSV = 0$, and $NR = 0$.

Step III

Analysis of degrees of freedom in process

As depicted in Fig. 7.35, now the process includes three process units: a mixer, evaporator, and crystallizer. The fluid has two components, so we can formulate two equations per unit. Therefore,

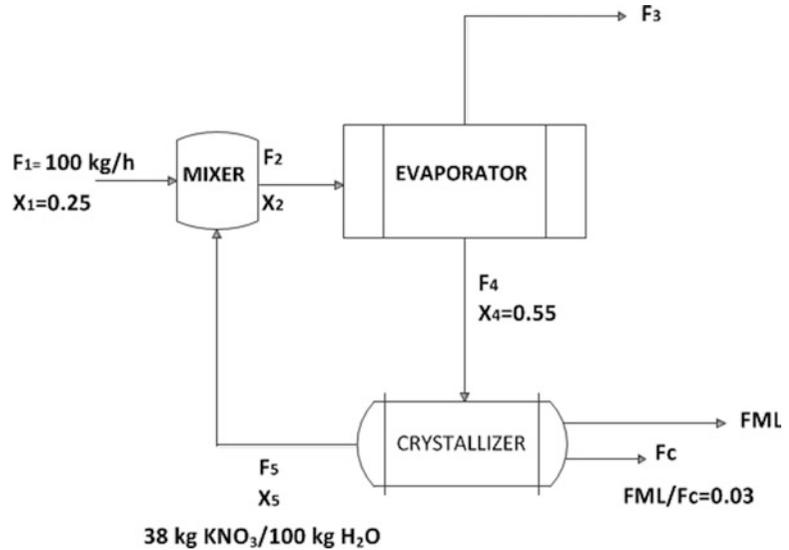
$$NMB = 6.$$

(b) Determine the degrees of freedom in the process.

$$DF = NV - NMB - NSV - NR = 11 - 6 - 0 - 0 = 5; \quad DF = 5.$$

10. KNO_3 crystallization with recycle [8]. 100 kg/h of a solution with 25 % KNO_3 will be crystallized in an evaporation-crystallization unit, as shown in the previous exercise. Upon leaving the evaporation unit, the solution has a concentration of 55 % KNO_3 . At the crystallizer the temperature is dropped and the mother liquor has 38 kg of KNO_3 /100 kg of H_2O . The mother liquor is recycled and the crystals are removed, carrying on the surface a small amount of mother liquor (mass of mother liquor/mass crystals = 0.03 w/w). (a) How much H_2O is evaporated? (b) What is the flow rate (kg/h) of the recycle stream? (c) What is the flow rate of pure KNO_3 crystals? (d) What is the efficiency of the process (mass of KNO_3 crystals/mass of KNO_3 fed)?

Fig. 7.36 KNO_3 crystallization with recycle including all variables and available data



Step I

Reading and understanding

First, we need to follow the advice from the previous problem and draw a good flow diagram (Fig. 7.36). In addition, consider separating the flow rate at the output of the crystallizer into two streams, crystals (F_C) and mother liquor (F_{ML}).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

$F_1, F_2, F_3, F_4, F_5, F_C, F_{ML}$: Mass flow rate of streams (kg/h)

x_1, x_2, x_4, x_5 : KNO_3 concentrations w/w (kg KNO_3 /kg solution)

(a) How many variables can you identify in this process?

From Fig. 7.36, $NV = 11$, $NSV = 4$ (F_1, x_1, x_4 , and x_5), and $NR = 1$ ($F_{ML}/F_C = 0.03$). We need to consider that x_5 has been given, but indirectly.

Step III

Analysis of degrees of freedom in process

Now (Fig. 7.36) the process consists of three process units, and the fluid in each has two components. Therefore, $NMB = 6$, and thus $DF = 11 - 6 - 4 - 1 = 0$. The problem can now be solved.

Step IV and V

Mathematical formulation including all available data, solution, results, analysis, and discussion

First, as mentioned, x_5 has been given, but only indirectly. We need to express the concentration as kg KNO_3 /kg solution. The concentration of the mother liquor is given as 38 kg KNO_3 /100 kg of H_2O and, expressed as kg KNO_3 /kg, the solution will be

$$x_5 = 38 \text{ KNO}_3 / (38 \text{ kg KNO}_3 + 100 \text{ kg H}_2\text{O}) \sim 0.28 \text{ kg KNO}_3 / \text{kg solution.}$$

From Fig. 7.36 we know that we have seven unknowns, and given that $NMB = 6$ and $NR = 1$, we can formulate seven independent equations.

As in the previous problem, we need to analyze and try to discover the best equations to use. Because we have complete information on the feed stream and relations at the output of the crystallizer we will use the global mass balance (S_T). Normally, it is a good idea to explore the global mass balance because usually we have information on the feed stream and of the end product.

System S_T (global)

Total mass balance:

$$100 - F_3 - F_C - F_{ML} = 0. \quad (7.53)$$

Mass balance for KNO_3 :

$$0.25 \times 100 - F_C - 0.28 \times F_{ML} = 0 \quad (7.54)$$

and

$$F_{ML}/F_C = 0.03. \quad (7.55)$$

Before continuing, here we have three equations (7.53)–(7.55) and three unknowns (F_3 , F_C , and F_{ML}). Therefore,

$$F_C = 24.8 \text{ kg/h}, F_{ML} = 0.744 \text{ kg/h}, F_3 = 74.5 \text{ kg/h}.$$

At this point we can answer the following questions: (a) How much H_2O is evaporated? $F_3 = 74.5$ kg/h. (c) What is the flow rate of pure KNO_3 crystals? $F_C = 24.8$ kg/h. (d) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)? We can answer this question as follows:

η : Efficiency (% mass of KNO_3 crystals/mass of KNO_3 fed). Therefore,

$$\eta = 100 \times F_C / (0.25 \times 100) = 100 \times 24.8 / 25 = 99.2 \text{ \%}.$$

To answer question (b) regarding the flow rate (kg/h) of the recycle stream, we will formulate all mass balances for systems S_1 , S_2 , and S_3 ; then, using the values obtained for F_C , F_{ML} , and F_3 , and selecting some specific equations, we will calculate F_5 .

System S_1 (Mixer)

Mass balance for KNO_3 :

$$0.25 \times 100 + 0.28 \times F_5 - x_2 \times F_2 = 0. \quad (7.56)$$

Total mass balance:

$$100 + F_5 - F_2 = 0. \quad (7.57)$$

System S_2 (Evaporator)

Mass balance for KNO_3 :

$$x_2 \times F_2 - 0.55 \times F_4 = 0. \quad (7.58)$$

Total mass balance:

$$F_2 - F_4 - F_3 = 0. \quad (7.59)$$

System S_2 (Crystallizer)

Mass balance for KNO_3 :

$$0.55 \times F_4 - 0.28 \times F_5 - F_C - 0.28 \times F_{\text{ML}} = 0. \quad (7.60)$$

Total mass balance:

$$F_4 - F_5 - F_C - F_{\text{ML}} = 0. \quad (7.61)$$

Notice that in total we have written nine equations (7.53)–(7.61); clearly they are not all independent. As stated, we have seven independent equations. Observing (7.56)–(7.61) we will select (7.56), (7.58), and (7.61) to calculate F_5 . Inserting values and working with (7.56) and (7.58) we get

$$x_2 \times F_2 = 0.25 \times 100 + 0.28 \times F_5, \text{ and } x_2 \times F_2 = 0.55 \times F_4.$$

Therefore,

$$0.25 \times 100 + 0.28 \times F_5 = 0.55 \times F_4. \quad (7.62)$$

And (7.61) states

$$F_4 - F_5 - 24.8 - 0.744 = 0. \quad (7.63)$$

Thus, $F_5 = 39.9$ kg/h and $F_4 = 65.4$ kg/h.

(b) What is the flow rate (kg/h) of the recycle stream? $F_5 = 39.9$ kg/h.

The lesson here is that from step III we already know that we can solve the problem, but we are dealing with seven equations and seven unknowns. As shown, analyzing system S_T first was a good choice. In addition, we would like to emphasize that it would be very good if you could solve the problem, but remember that formulating a problem well and understanding the problem comprise 99 % of our goal.

Now we invite you to test the results. For example, given that the composition of KNO_3 in stream 2 comes from the mixture of the feed stream and the recycle stream, we must expect that

$$0.25 \leq x_2 \leq 0.28.$$

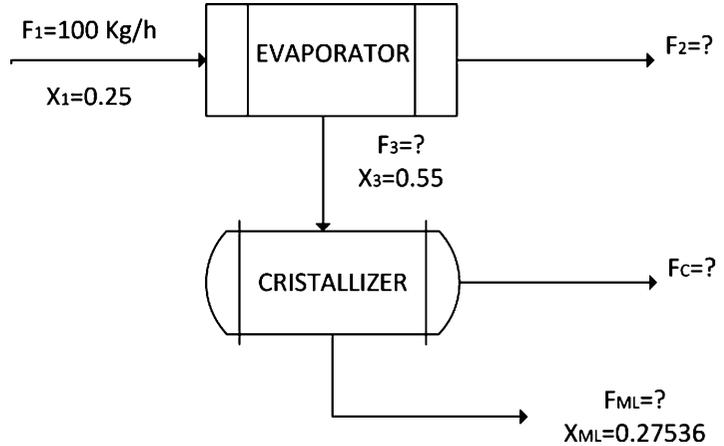
First we will obtain F_2 from (7.57) and then we will calculate x_2 from (7.56):

$$100 + F_5 - F_2 = 0; F_2 = 100 + F_5 = 1.40 \times 10^2 \text{ kg/h.}$$

$0.25 \times 100 + 0.28 \times F_5 - x_2 \times F_2 = 0$; replacing and rearranging, we get $x_2 = 0.257$ kg KNO_3 /kg solution, which, as expected, is in the right range.

11. KNO_3 crystallization without recycle [6]. 100 kg/h of a solution with 25 % KNO_3 will be crystallized in an evaporation–crystallization unit. After the evaporation unit the solution has a

Fig. 7.37 KNO_3 crystallization without recycle including all variables and available data



concentration of 55 % KNO_3 . At the crystallizer the temperature is dropped and the mother liquor has 38 kg of KNO_3 /100 kg of H_2O . (a) How much H_2O is evaporated? (b) What is the flow rate of pure KNO_3 crystals? (c) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)? (d) Compare and discuss these results with those of the previous example.

Step I

Reading and understanding

First, this problem is similar to the previous one but without recycle. Again we will consider separating the flow rate at the output of the crystallizer into two streams, crystals and mother liquor.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

$F_1, F_2, F_3, F_C, F_{ML}$: mass flow rate of streams kg/h

x_1, x_3, x_{ML} : KNO_3 concentrations w/w (kg KNO_3 /kg solution)

(a) How many variables you can identify in this process?

From Fig. 7.37, $NV = 8$, $NSV = 4$ (F_1, x_1, x_3 , and x_{ML}), and $NR = 0$. Assume x_{ML} is given indirectly (see previous problem with variable x_5).

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.37, the process consists of two process units in each of which the fluid has two components. Thus, $NMB = 4$, and then $DF = 8 - 4 - 4 - 0 = 0$. The problem can now be solved.

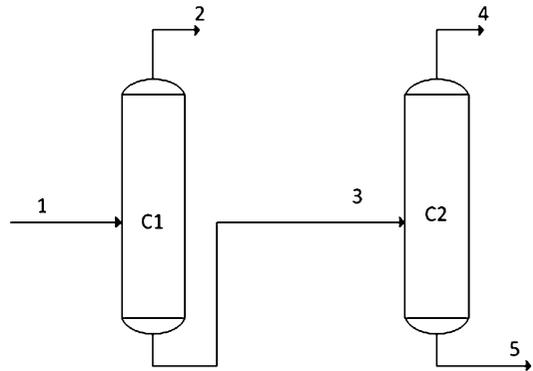
Step IV and V

Mathematical formulation including all available data, resolution, results, analysis, and discussion

As mentioned, x_{ML} is given indirectly and, similarly to the previous problem, we get

$$x_{ML} = 38 \text{ KNO}_3 / (38 \text{ kg KNO}_3 + 100 \text{ kg H}_2\text{O}) \sim 0.28 \text{ kg KNO}_3 / \text{kg solution.}$$

Fig. 7.38 Two distillation columns in series



System S_1 (evaporator)

Total mass balance:

$$100 - F_2 - F_3 = 0. \quad (7.64)$$

Mass balance for KNO_3 :

$$0.25 \times 100 - 0.55 \times F_3 = 0. \quad (7.65)$$

From (7.65) we get $F_3 = 45.5$ kg/h, and thus, replacing F_3 in (7.64), we obtain $F_2 = 54.5$ kg/h.

System S_2 (crystallizer)

Total mass balance:

$$45.5 - F_C - F_{ML} = 0. \quad (7.66)$$

Mass balance for KNO_3 :

$$0.55 \times 45.5 - F_C - 0.28 \times F_{ML} = 0. \quad (7.67)$$

Then from (7.66) and (7.67) we obtain $F_C = 17.2$ kg/h and $F_{ML} = 28.2$ kg/h.

(a) How much H_2O is evaporated? $F_2 = 54.5$ kg/h.

(b) What is the flow rate of pure KNO_3 crystals? $F_C = 17.2$ kg/h.

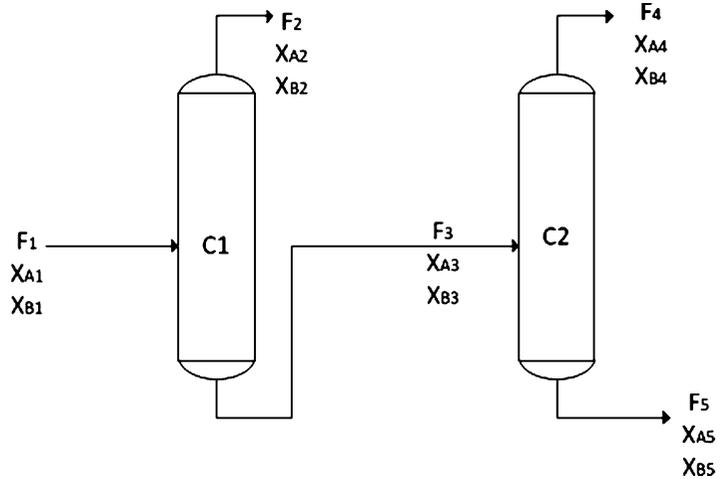
(c) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)?

$$\eta = 100 \times F_C / (0.25 \times 100) = 100 \times 17.2 / 25 = 68.9 \%$$

(d) Compare and discuss these results with those of the previous example. First, the efficiency of the process fell from a high of 99.2 % (with recycle) to a low of 68.9 % (without recycle). Second, the decision to use or not recycle should be analyzed in economic terms because the inclusion of recycle will imply some additional equipment, like, for example, a mixer, pumps, or fittings. However, the increase in efficiency is so high that it would most likely be worth including recycle stream.

12. Distillation [6]. A distillation process composed of two units operating in series under steady-state conditions is fed by a solution containing components A, B, and C (in all streams) (Fig. 7.38).

Fig. 7.39 Two distillation columns in series including all variables



(a) How many variables are in this process? (b) Determine the degrees of freedom of the process.

Step I

Reading and understanding

In simple terms, distillation is a classical unit operation in chemical and bioprocessing companies with the aim of separating mixtures based on the difference volatilities of the components. There are several applications of distillation, for example, in crude oil, water (to remove impurities), and fermented solutions.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data (Fig. 7.39)

F_1 : Mass flow rate of feed stream (kg/h)

x_{A1} : Mass fraction of component A at feed stream

x_{B1} : Mass fraction of component B at feed stream

F_2 : Mass flow rate of top stream of first column (kg/h)

x_{A2} : Mass fraction of component A at top of first column

x_{B2} : Mass fraction of component B at top of first column

F_3 : Mass flow rate of bottom stream of first column (kg/h)

x_{A3} : Mass fraction of component A at bottom of first column

x_{B3} : Mass fraction of component B at bottom of first column

F_4 : Mass flow rate of top stream of second column (kg/h)

x_{A4} : Mass fraction of component A at top of second column

x_{B4} : Mass fraction of component B at top of second column

F_5 : Mass flow rate of bottom stream of second column (kg/h)

x_{A5} : Mass fraction of component A at bottom of second column

x_{B5} : Mass fraction of component B at bottom of second column

(a) How many variables are in this process? $NV = 15$, $NSV = 0$, and $NR = 0$.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.39, the process consists of two process units, and each stream has three components. Thus, $NMB = 6$, and then $DF = 15 - 6 - 0 - 0 = 9$.

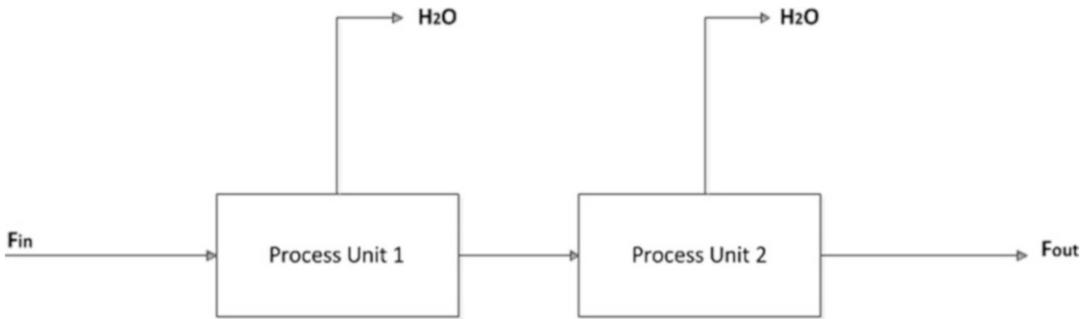


Fig. 7.40 Concentration process with two units in series

(b) Determine the degrees of freedom of the process.

$$DF = 9.$$

13. **Are there enough data?** [8]. In the following concentration process, units 1 and 2 remove H_2O . At the feed the mass flow rate of the wet material is 100 kg/h with a solid concentration of 10 % w/w (the remainder is H_2O). At the output of unit 2 the solid concentration is 80 % (w/w) (Fig. 7.40). (a) How much H_2O was removed in the whole process? (b) What is the mass flow rate of the end product (kg/h)?

Step I

Reading and understanding

A simple observation suggests that there are not enough data to answer the questions. An interesting reflection here is that to completely solve a problem, you need $DF = 0$, but occasionally, although $DF > 0$, you can still answer the questions of the problem. If $DF > 0$, then you cannot calculate the value of all variables, but you might be able to calculate some of them, and if they are the ones requested in the problem statement, then you can solve the problem.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

Our first step will be to complete the flow diagram provided with the problem statement.

F_1 : Mass flow rate of wet material (kg/h) (first unit)

x_{S1} : Mass fraction of solids on wet material

F_2 : Mass flow rate of semidried material (kg/h) (first unit)

x_{S2} : Mass fraction of solids on semidried material

F_3 : Mass flow rate of H_2O (kg/h) (first unit)

F_4 : Mass flow rate of H_2O (kg/h) (second unit)

F_5 : Mass flow rate of product (kg/h) (second unit)

x_{S5} : Mass fraction of solids on end product.

Thus, $NV = 8$, $NSV = 3$, and $NR = 0$.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.41, the process consists of two process units, and each stream has two components. Thus,

$$NMB = 4.$$

and so $DF = 8 - 4 - 3 - 0 = 1$.

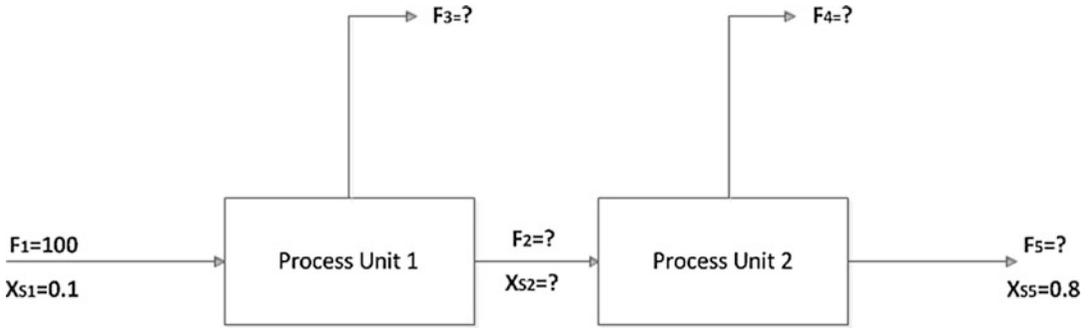


Fig. 7.41 Concentration process with two units in series including all variables and available data

As expressed in step I, we cannot calculate all the unknowns (five in total). We can formulate four independent equations, and we have five unknowns. But as stated, we might be able to calculate some of them.

Step IV

Mathematical formulation including all available data

As a first attempt, we will explore an analysis of system S_T .

System S_T

Total mass balance:

$$100 - F_3 - F_4 - F_5 = 0. \quad (7.68)$$

Solids mass balance:

$$0.1 \times 100 - 0.8 \times F_5 = 0. \quad (7.69)$$

Step V

Solution, results, analysis, and discussion

From (7.69) we get $F_5 = 12.5$ kg/h, and replacing F_5 in (7.68) we get $F_3 + F_4 = 87.5$ kg/h, where $F_3 + F_4$ represents all the H_2O removed in the process.

(a) How much H_2O was removed in the whole process?

$$F_3 + F_4 = 87.5 \text{ kg/h.}$$

(b) What is the mass flow rate of the end product in kilograms per hour?

$$F_5 = 12.5 \text{ kg/h.}$$

As expressed and discussed in step I, we cannot calculate all the unknowns but we might be able to answer the specific questions. For example, we cannot calculate F_3 and F_4 individually, but the question was related to the total amount of H_2O removed ($F_3 + F_4$), and, as shown, it was possible to calculate that.

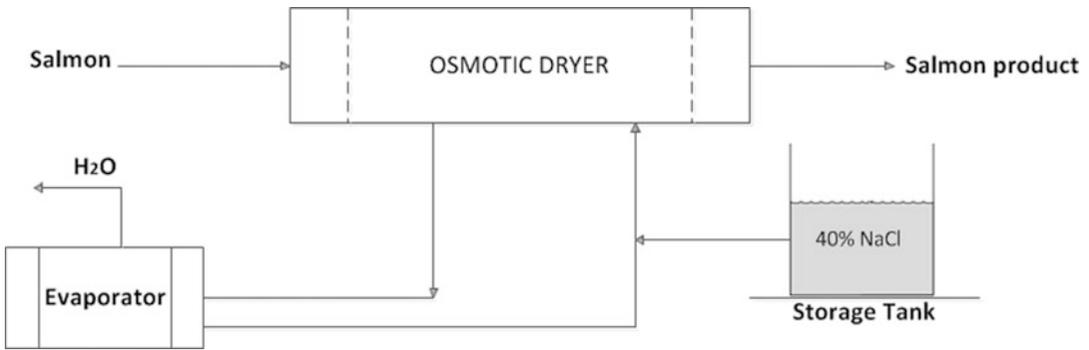


Fig. 7.42 Osmotic dehydration of salmon

14. Osmotic dehydration of salmon [10]. The salting of salmon through a osmotic-dehydration process has been experimentally tested at the pilot plant scale. In the osmotic-dehydration unit, salmon exchange water and receive salt from the solution. The experimental process is depicted in Fig. 7.42. The object of the process is to obtain a salt (NaCl) concentration of 3 % w/w on the salmon product. 1,000 kg/h of salmon are fed to the osmotic-dehydration unit and the salmon composition is 67 % H₂O, 12 % fat, 19 % proteins, 2 % inert solids (all w/w), and no salt (0 %). Preliminary results show that H₂O in salmon product (after processing) is 65 % w/w. As shown in Fig. 7.42, to maintain NaCl concentration at 30 %, in the osmotic-dehydration unit, part of the brine is concentrated in a single-effect evaporator and then mixed with a brine solution with 40 % NaCl that comes from a storage tank. (a) What is the mass flow rate that comes from the storage tank? (b) How much H₂O is evaporated in the single-effect evaporator?

Step I

Reading and understanding

First, what is an osmotic dehydration process? In osmotic dehydration the product (in this case salmon) is in direct contact with a low-water-activity solution (e.g., concentrated salt) in which a two-way mass transfer is established: (a) water is transferred from the product to the solution and (b) in the opposite direction, solute (in this case salt) is transferred from the solution to the salmon tissue. Now it is clear why it is necessary to inject salt into the system (storage tank) because the salmon product carries 3 % salt at the end of the process.

In terms of minimizing variables, it would be advisable to put together as one variable fat, proteins, and inert material.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1 : Mass flow rate of feed stream (kg/h)

x_{S1} : Solids mass fraction at feed stream (fat, proteins, inert, but excluding salt) (w/w)

x_{H_2O1} : H₂O mass fraction at feed stream (w/w)

x_{NaCl1} : NaCl mass fraction at feed stream (w/w)

F_2 : Mass flow rate salmon product (kg/h)

x_{S2} : Solids mass fraction at outlet stream (fat, proteins, inert, but excluding salt) (w/w)

x_{H_2O2} : H₂O mass fraction at outlet stream (w/w)

x_{NaCl2} : NaCl mass fraction at outlet stream (w/w)

F_3 : Mass flow rate of brine solution (out of osmotic unit) (kg/h)

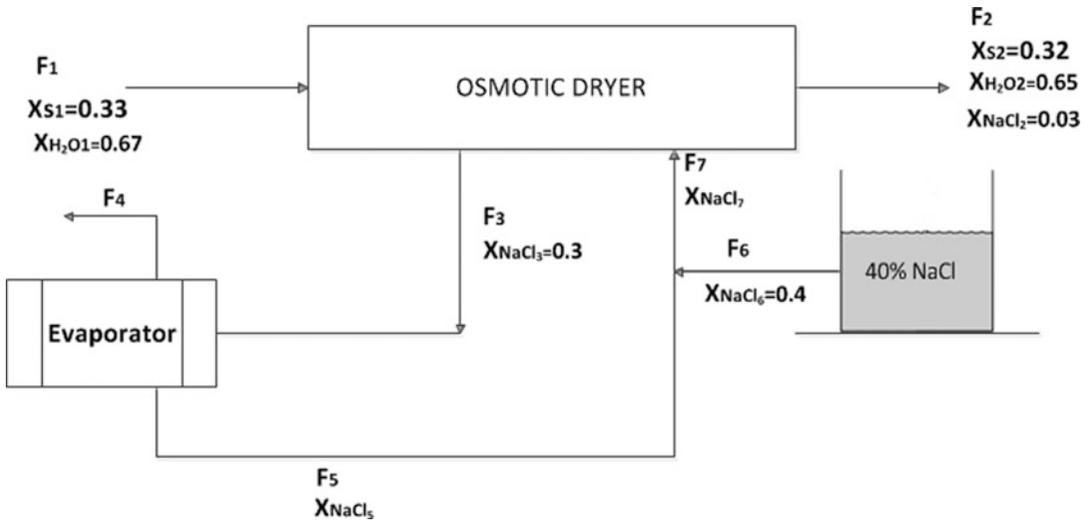


Fig. 7.43 Osmotic dehydration of salmon including all variables and available data

$x_{\text{NaCl}3}$: NaCl mass fraction of brine solution (out of osmotic unit) (w/w)

F_4 : Mass flow rate of H₂O from single effect evaporator (kg/h)

F_5 : Mass flow rate of concentrated brine from evaporator (kg/h)

$x_{\text{NaCl}5}$: NaCl mass fraction of concentrated brine from evaporator (w/w)

F_6 : Mass flow rate of brine solution from storage tank (kg/h)

$x_{\text{NaCl}6}$: NaCl mass fraction of brine solution from evaporator (w/w)

F_7 : Mass flow rate of mixed brine solution (kg/h)

$x_{\text{NaCl}7}$: NaCl mass fraction of mixed brine solution (w/w)

To minimize the number of variables, we have defined solids as fat, proteins, and inert materials (all together).

Thus, $NV = 17$, $NSV = 9$ (F_1 , x_{S1} , x_{H2O1} , x_{NaCl1} , x_{H2O2} , x_{S2} , x_{NaCl2} , x_{NaCl3} , and x_{NaCl6}), and $NR = 0$. We have included x_{S2} as a known variable at the output stream because the mass fraction of H₂O is 0.65 (65 %) and the mass fraction of NaCl is 0.03 (3 %). Thus, $x_{S2} = 1 - 0.65 - 0.03 = 0.32$.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.43, the process consists of two process units (osmotic dehydrator and an evaporator) plus an “added” mixer. In the osmotic-dehydration unit we have three components, and so we can formulate three independent material balance equations. In the evaporator and mixer we have two components (H₂O and NaCl), so we can formulate two equations for each equipment item. Therefore, $NMB = 7$.

$$DF = NV - NMB - NSV - NR = 17 - 7 - 9 - 0 = 1.$$

Thus, we cannot calculate all the unknowns (of which there are eight).

Step IV**Mathematical formulation including all available data**

As expressed in step III, we can formulate seven independent equations ($NMB = 7$) and the process has eight unknowns. We cannot calculate all unknowns but, as shown in the previous problem (problem 13), we might be able to calculate the unknowns that are related to questions (a) and (b) i.e., F_4 and F_6 , respectively.

First, we will write three mass balances for the system S_T because in these material balances, both F_4 and F_6 variables are involved.

System S_T

Total mass balance:

$$1,000 + F_6 - F_2 - F_4 = 0. \quad (7.70)$$

Solid mass balance:

$$0.33 \times 1,000 - 0.32 \times F_2 = 0. \quad (7.71)$$

NaCl mass balance:

$$0.4 \times F_6 - 0.03 \times F_2 = 0. \quad (7.72)$$

Step V**Solution, results, analysis, and discussion**

From step IV we have three equations (7.70)–(7.72) and three unknowns (F_2 , F_4 , and F_6). Within these three unknowns are the required calculations of F_4 and F_6 .

From (7.71) and (7.72) we can directly obtain F_2 and F_6 , respectively. Therefore, $F_2 = 1.03 \times 10^3$ kg/h and $F_6 = 77.34$ kg/h. Replacing F_2 and F_6 in (7.70) we get $F_4 = 46.09$ kg/h.

This is an intricate problem. In our experience, students “suffer” a lot with this problem, but if you follow the proposed procedure, not only will it seem very simple, but indeed it is simple.

15. Glutamic acid purification [10⁺]. Glutamic acid ($C_5H_9NO_4$) is a nonessential amino acid (the human body is capable of producing it), but it is extensively used as a food additive due to its taste-enhancing properties and because it gives food an umami, or savory, taste. Umami is a term introduced by Kikunake Ikeda in 1908 and since 1985 has been recognized as the fifth basic taste. A processing plant for the purification of glutamic acid starts with a flow rate of a liquor that contains 1,000 lb/h of water, 5.0 lb/h of impurities, and an unknown amount of glutamic acid. A schematic representation of the process is given in Fig. 7.44.

The liquor is concentrated in the evaporator where 950 lb/h H_2O are removed. Then to favor crystallization a diluted solution of 85.14 lb/h containing 10/11 parts H_2O and 1/11 parts of the substance w/w (considered as impurity) is added to the precipitator.

In the centrifuge, crystals are separated from the residual liquor. Crystals are wet with a layer of liquor, where the liquor layer represents 11.09 % w/w of the clean crystals. Then the residual liquor is recycled and mixed with the liquor fed to the process, and part of the residual liquor purged to avoid the accumulation of impurities in the process. Finally, crystals are fed to a dryer where all the remaining water is removed. The dryer is fed with 490 lb/h dry air with a humidity of 0.001 lb H_2O /lb dry air and leaving the dryer with a humidity of 0.011 lb H_2O /lb dry air. The final product contains 1 % impurities. (a) What is the efficiency of the process, defined as follows: (kilogram per hour of glutamic acid in final product/kilogram per hour of glutamic acid fed) $\times 100$. (b) What is the flow rate and composition of purge.

the mass flow rate of component i [water (H_2O), glutamic acid (GA), and impurities (I)] in stream j (1, 2, 3...10, R , P), where R stands for recycle and P for purge.

Then we can determine that $NV = 31$, $NSV = 4$, and $NR = 10$. It is clear from Fig. 7.45 that we have in total 31 variables and 4 of them are specified, but why $NR = 10$?

From the problem statement we know that the diluted solution added to the crystallizer contains 10/11 parts water and 1/11 parts substance w/w (two relationships). Also, H_2O , impurities, and glutamic acid are in the same proportion in all streams with the mother liquor (recycle, purge stream 7, and mother liquor in stream 8. From that we get six independent relationships as follows:

$$F_{IP}/F_{H2OP} = F_{IR}/F_{H2OR}, F_{IP}/F_{H2OP} = F_{I7}/F_{H2O7}, F_{IP}/F_{H2OP} = F_{I8}/F_{H2O8}, \\ F_{GAP}/F_{H2OP} = F_{GAR}/F_{H2OR}, F_{GAP}/F_{H2OP} = F_{GA7}/F_{H2O7}, F_{GAP}/F_{H2OP} = F_{GA8}/F_{H2O8}.$$

In addition, the liquor layer represents 11.09 % w/w of the clean crystals (one relationship), and the final product contains 1 % impurities (one relationship). Therefore, in total we have ten relationships.

Step III

Analysis of degrees of freedom in process

Observing the flow diagram we can formulate three independent material balances in each piece of equipment (mixer, evaporator, crystallizer, and centrifuge), for a total of 12 equations. The dryer can be separated into two systems, and so we can formulate three equations for the mother liquor and crystals and one for the air. Finally, at the purge there is a division, and so we can formulate one more independent equation.

Therefore, $NMB = 17$, and DF will be

$$DF = 31 - 17 - 4 - 10 = 0.$$

Step IV and V

Mathematical formulation including all available data, solution, results, analysis, and discussion

As was mentioned earlier, we need to focus, first, on solving the problem; it is not necessary to answer all unknowns (27 in total). Then, we will first calculate the unknowns related to the questions. What are they?

(a) What is the efficiency of the process, defined as follows: (kg/h glutamic acid in final product/kg/h glutamic acid fed) $\times 100$. Thus, efficiency = $100 \times (F_C + F_{GA10})/F_{GA1}$.

(b) What is the flow rate and composition of the purge, thus the mass flow is ($F_{H2OP} + F_{IP} + F_{GAP}$) and the compositions are $x_{H2OP} = F_{H2OP}/(F_{H2OP} + F_{IP} + F_{GAP})$; $x_{IP} = F_{IP}/(F_{H2OP} + F_{IP} + F_{GAP})$; $x_{GAP} = F_{GAP}/(F_{H2OP} + F_{IP} + F_{GAP})$.

This indicates that our strategy should be focused on calculating F_{GA1} , F_C , F_{GA10} , F_{H2OP} , F_{IP} , F_{GAP} .

First, looking at system S_T (global) for H_2O , we have three unknowns, F_{H2O5} , F_{H2OP} , and F_{H2O9} , where we can formulate the following equations.

Global mass balance for H_2O

$$1,000 - 950 + F_{H2O5} - F_{H2O9} - F_{H2OP} = 0. \quad (7.73)$$

Relationship for $F_{\text{H}_2\text{O}_5}$:

$$F_{\text{H}_2\text{O}_5} = (10/11) \times 85.14; \text{ therefore, } F_{\text{H}_2\text{O}_5} = 77.40 \text{ lb/h.}$$

Mass balance for H_2O in dryer

H_2O entering with air + H_2O from mother liquor – H_2O leaving with air = 0, and given that the mass flow rate of dry air does not change,

$$490 \times 0.001 + F_{\text{H}_2\text{O}_9} - 490 \times 0.011 = 0; \text{ thus, } F_{\text{H}_2\text{O}_9} = 4.90 \text{ lb/h.}$$

Replacing $F_{\text{H}_2\text{O}_5}$ and $F_{\text{H}_2\text{O}_9}$ in (7.73) we get $F_{\text{H}_2\text{O}_P} = 1.23 \times 10^2 \text{ lb/h.}$

Now our focus should be to obtain F_{GA_1} , F_{C} , $F_{\text{GA}_{10}}$, F_{IP} , F_{GAP}

Global mass balance for impurities:

$5 + F_{\text{I}_5} - F_{\text{I}_{10}} - F_{\text{IP}} = 0$, where $F_{\text{I}_5} = (1/11) \times 85.14 = 7.740 \text{ lb/h}$. Therefore, $5.0 + 7.740 - F_{\text{I}_{10}} - F_{\text{IP}} = 0$; then

$$12.7 - F_{\text{I}_{10}} - F_{\text{IP}} = 0. \quad (7.74)$$

The final product contains 1 % impurities. Thus,

$F_{\text{I}_{10}}/(F_{\text{C}} + F_{\text{GA}_{10}}) = 0.01$. Note that $F_{\text{GA}_{10}} = F_{\text{GA}_8}$. Therefore, we can write

$$F_{\text{I}_{10}}/(F_{\text{C}} + F_{\text{GA}_8}) = 0.01. \quad (7.75)$$

Crystals are wet with a layer of liquor, where the liquor layer represents 11.09 % w/w of the clean crystals. Thus,

$(F_{\text{GA}_8} + F_{\text{I}_8} + F_{\text{H}_2\text{O}_8})/F_{\text{C}} = 0.1109$, where $F_{\text{I}_8} = F_{\text{I}_{10}}$ and $F_{\text{H}_2\text{O}_8} = F_{\text{H}_2\text{O}_9} = 4.9 \text{ lb/h}$. Then

$$(F_{\text{GA}_8} + F_{\text{I}_{10}} + 4.9)/F_{\text{C}} = 0.1109. \quad (7.76)$$

Finally, from $F_{\text{IP}}/F_{\text{H}_2\text{O}_P} = F_{\text{I}_8}/F_{\text{H}_2\text{O}_8}$ and $F_{\text{GAP}}/F_{\text{H}_2\text{O}_P} = F_{\text{GA}_8}/F_{\text{H}_2\text{O}_8}$, and recalling that $F_{\text{H}_2\text{O}_8} = F_{\text{H}_2\text{O}_9} = 4.9 \text{ lb/h}$, $F_{\text{GA}_{10}} = F_{\text{GA}_8}$ and $F_{\text{H}_2\text{O}_P} = 1.23 \times 10^2 \text{ lb/h}$, we can write

$F_{\text{IP}}/122.5 = F_{\text{I}_8}/4.9$, but $F_{\text{I}_8} = F_{\text{I}_{10}}$. Thus,

$$F_{\text{IP}}/122.5 = F_{\text{I}_{10}}/4.9, \quad (7.77)$$

$$F_{\text{GAP}}/122.5 = F_{\text{GA}_8}/4.9. \quad (7.78)$$

We have five equations (7.74)–(7.78) and five unknowns ($F_{\text{I}_{10}}$, F_{IP} , F_{C} , F_{GA_8} , and F_{GAP}). Therefore,

$$F_{\text{I}_{10}} = F_{\text{I}_8} = 0.47, \quad F_{\text{IP}} = 12.25, \quad F_{\text{C}} = 48.96, \quad F_{\text{GA}_8} = F_{\text{GA}_{10}} = 0.0397, \quad \text{and } F_{\text{GAP}} = 0.9924.$$

Now doing a global mass balance for glutamic acid we obtain

$$F_{\text{GA}_1} - F_{\text{C}} - F_{\text{GA}_{10}} - F_{\text{GAP}} = 0. \quad (7.79)$$

Replacing F_{C} , $F_{\text{GA}_{10}}$ and F_{GAP} we get $F_{\text{GA}_1} - 4.9 \times 10 - 0.040 - 0.99 = 0$, then $F_{\text{GA}_1} = 50$.

- (a) Efficiency = $100 \times (F_C + F_{GA10})/F_{GA1} = 100 \times (4.9 \times 10 + 0.040)/50 = 98 \%$.
 (b) Flow rate of purge = $(F_{H2OP} + F_{IP} + F_{GAP}) = 122.5 + 1.2 \times 10 + 0.99 = 1.36 \times 10^2$ lb/h.
 (c) $X_{H2OP} = F_{H2OP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 122.5/1.36 \times 10^2 = 0.902$ (90.2 %),

$$X_{IP} = F_{IP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 1.2 \times 10/1.36 \times 10^2 = 0.0902$$
 (9.02 %),

$$X_{GAP} = F_{GAP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 0.99/1.36 \times 10^2 = 0.0073$$
 (0.73 %).

One lesson from this example is that we need to be flexible in the way we define variables. In this case, the input data were given in a particular way and then we defined the variables accordingly. In addition, in this example and then in real-world problems, it is critical to have a good flow diagram and read and follow the problem statement together with the diagram to get a feel for the problem.

7.10 Proposed Problems

Before taking on the exercises, it is advisable that the student become familiar with the function and purpose of the equipment in each process stage related to the problem. Then, after the problem has been critically analyzed and solved, it is recommended that you search for information (e.g., on the Web) and briefly describe the equipment used in each problem and then add an example of its industrial utilization.

- Salt solution [3].** A processing plant requires a stream with a flow rate of 100 kg/h with a salt concentration of 1 % (w/w). For this purpose, the plant has unlimited quantities of water and a stream with 3 % salt (w/w). In what proportion should the water and stream be mixed with 3 % salt to obtain a desired stream of 100 kg/h with 1 % salt?
A: 2:1 (kg/h water stream /kg/h stream with 3 % salt)
- Apple juice [3⁺].** As a general rule, fresh juices have better color and flavor than processed juices. An undisputed advantage of processed juice is its extended shelf life and convenience. An important aspect to consider when processing fruit juice is trying to retain its natural properties, such as color, flavor, and aromas. 100 kg/h of an apple juice with moisture of 92 % (w/w) will be mixed with 200 kg/h of an apple juice with 94 % moisture (w/w). After being well mixed, they are passed through an evaporator to obtain a concentrated product with 50 % solids (w/w). How much water should be removed in the evaporator?
A: 260 kg/h
- Mixing separation process [5].** Two streams with three components, A, B, and C, are fed to a process unit. At the outlet of the equipment there is one stream containing components A, B, and C. In a similar way as was analyzed in problem 1 (Sect. 7.9) we can determine that this system has six degrees of freedom. (a) Can we arbitrarily fix the composition of the three streams (say, x_{A1} , x_{B1} , x_{A2} , x_{B2} , x_{A3} , and x_{B3})? (b) If your answer to (a) is no, explain.
A: (a) No. (b) For example, if you fix x_{A1} and x_{A2} , then you can fix x_{A3} , but you are limited to a value between x_{A1} and x_{A2}
- Vacuum dryer [3].** Usually, a vacuum dryer is used for materials (e.g., foods) that could get damaged if exposed to high temperatures. Furthermore, the vacuum prevents oxidation and hazardous conditions in certain materials that may be explosive in the presence of oxygen. It is also used when one must achieve very low humidity levels. In addition, dehydration time is normally lower when compared with atmospheric dryers. To further increase the solid

concentration of the previous product (use the data from exercise 2), it will be further processed in a vacuum dryer to obtain a final product with 66.67 % solids (w/w). How much water should be removed in the vacuum dryer?

A: 10 kg/h

5. **Single-effect evaporator [5].** A stream of water and component A have a flow rate of 10 L/h, where the density of A is 1.2 kg/L and its concentration is 0.20 kg A/L_{solution}. The stream is passed through a single-effect evaporator where 20 % by weight of the input flow is removed (water). (a) What is the mass flow rate at the entry of the evaporator? (b) What is the density of the solution at the entry of the evaporator? (c) How many kilograms of water are evaporated per hour? (d) What is the density of the solution at the outlet of the evaporator? (e) What is the concentration of component A at the outlet w/v?

A: (a) ~10.34 kg/h, (b) ~1.034 kg/L, (c) ~2.067 kg/h, (d) ~1.042 kg/L, (e) 0.252 kg A/L_{solution}

6. **Humid air [2].** Humid air contains 0.025 lb water vapor per pound of dry air. How many pounds of water vapor does humid air contain per pound of humid air?

A: ~0.024 pounds of water vapor per pound of humid air

7. **Continuous process [5].** A continuous process is fed with 10 lb/min of humid air containing 0.02 pounds of water vapor per pound of dry air. At the outlet, the air contains 0.03 g of water vapor per gram of humid air. How much water was removed or added to the air in 1 h of operation?

A: 6.43 lb water were added to the air in 1 h.

8. **Continuous process [6].** Humidification is an operation directed at increasing the amount of vapor present in a gas stream. The vapor may be increased by passing the gas through a liquid that evaporates into the gas. The transfer process through the gas stream takes place by diffusion at the interface. A continuous process is fed with A lb/min of humid air containing a pounds of water vapor per pound of dry air. At the outlet, the air contains b grams of water vapor per gram of humid air, where b > a and both $\lll 1$. (a) Is the equipment a dryer or a humidifier? (b) How much water was removed or added in 1 h of operation?

A: (a) Humidifier. (b) $60A(b(1 + a) - a) / ((1 + a)(1 - b))$ pounds of water were added in 1 h. You can check this formula using the data from exercise 7.

9. **Air mixture [4].** 10 lb/min of humid air containing 0.02 pounds of water vapor per pound of dry air are mixed with 10 lb/min of humid air that contains 0.03 grams of water vapor per gram of humid air. What is the humidity of the air mixture at the outlet on a dry basis?

A: ~0.0255 pounds of water vapor per pound of dry air

10. **Air mixture [5].** A lb/min of humid air containing a pounds of water vapor per pound of dry air are mixed with B lb/min of an air that contains b grams of water vapor per gram of humid air. What is the humidity of the air mixture at the outlet on a dry basis?

A: $(Aa + Bb(1 + a)) / (A + B(1 + a - b - ab))$ pounds of water vapor per pound of dry air. You can check this formula using the data from exercise 9.

11. **Humidification [4].** Humidification is an operation directed at increasing the amount of vapor present in a gas stream. The vapor may be increased by passing the gas through a liquid that evaporates into the gas. The transfer process through the gas stream takes place by diffusion at the interface. In a certain process, it is necessary to increase the humidity of a gas stream. (a) How much water is required to increase the humidity of 100 kg of a gas having an original humidity of 1 % (w/w, wet basis) until it reaches a humidity of 2 % (w/w, wet basis)? (b) What is the mass of the final gas stream with 2 % humidity after the process?

A: (a) 1.20 kg; (b) 1.01×10^2 kg

12. **Humidification [6].** In a certain process, it is necessary to increase the humidity of a gas stream. (a) How much water is required to increase the humidity of A kg of a gas stream having an

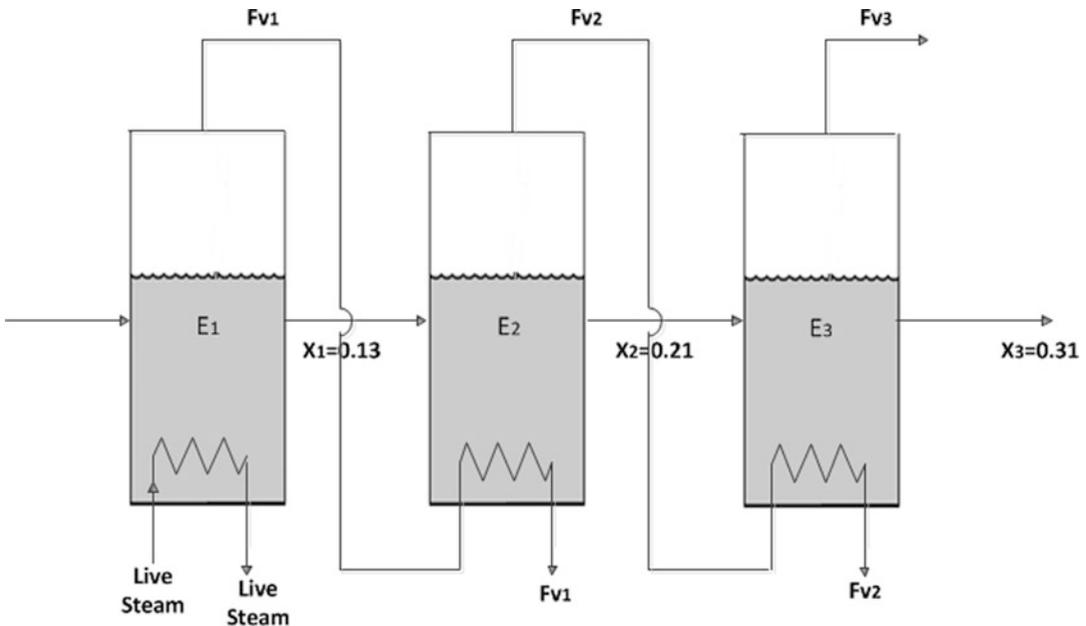


Fig. 7.46 Multi-effect evaporator

original humidity of a % (w/w, wet basis) until it reaches a humidity of b % (w/w, wet basis) where $b > a$? (b) What is the mass of the final gas stream with b % humidity after the process?
A: (a) $A(b - a)/(100 - b)$; (b) $(A(100 - a))/(100 - b)$. You can check this formula using the data from exercise 11.

13. **Multieffect evaporator [5].** In an evaporation process, the fluid is heated to its saturation temperature, and then additional energy is applied to start the liquid evaporation. Evaporation occurs at constant temperature and requires a large amount of energy so that the molecules in the liquid state pass to the vapor state. Unlike dryers, what is achieved in the evaporator is concentration, where normally a diluted "juice" with 5–10 % solids is concentrated to 30–50 % solids. Indeed, evaporation is a concentration process and not a dehydration process. Tomato juice will be concentrated in a three-stage multieffect evaporator (Fig. 7.46). 1,000 kg/h of tomato juice are fed to the system with an initial concentration of 6 % solids (w/w). The object of the operation is to obtain a commercial concentrate of 31 % solids (w/w). The outlet solid concentration of stages 1–3 are 13, 21, and 31 % (w/w), respectively. (a) How much water was removed in each stage of the system? (b) What is the flow rate of the concentrate at the outlet in kg/h?

Note: It is reasonable to assume that tomato juice is composed of two phases, solids (mainly soluble carbohydrates) and water.

A: (a) ~ 538.5 , ~ 175.8 , and ~ 92.17 kg/h, respectively; (b) ~ 193.6 kg/h

14. **Tomato concentrates [6].** The agro industry is a very important industrial sector worldwide, especially for countries like New Zealand and Chile. In addition, process optimization has always been a noble objective of engineers entrusted with the responsibility of developing and improving processes throughout the food industry. In trying to optimize the quality of tomato concentrate [31 % solids (w/w)], some modifications have been proposed to the operation of a three-effect evaporator (problem 13). A good and creative friend of yours is proposing to include a side stream so not all the juice passes through the three stages of the multieffect evaporator. He is

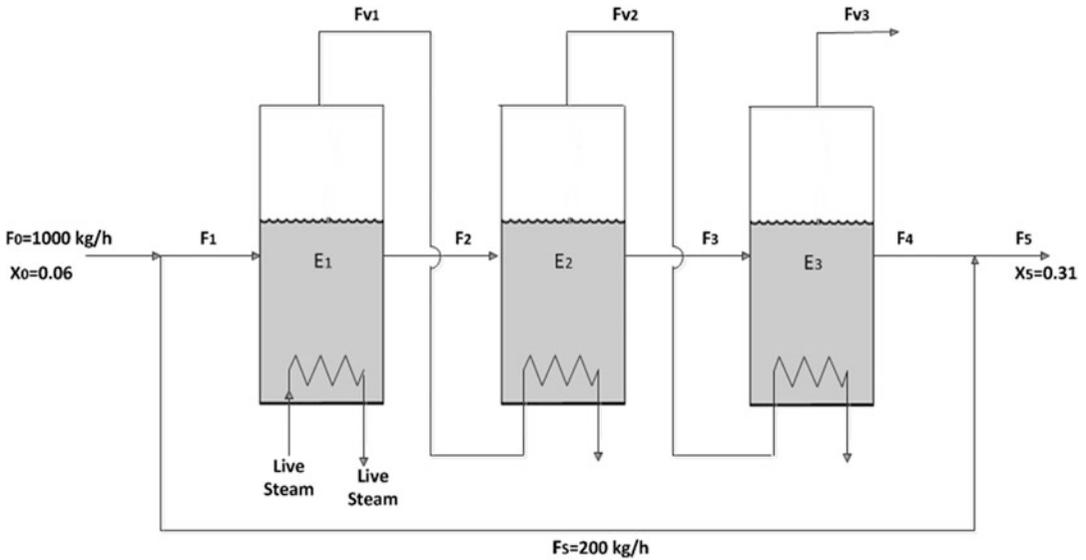


Fig. 7.47 Multi-effect evaporator for tomato concentration

proposing to have a side stream of 200 kg/h, as shown in Fig. 7.47, and is assuring you a very high quality of the tomato concentrate in terms of color and consistency. He argues that the process, including the side stream, has 15 variables (NV), 9 independent material balances (NMB), and 6 degrees of freedom. He points out that so far, he has used just three of them ($X_0 = 0.06$, $F_0 = 1,000$ kg/h, and $X_5 = 0.31$), so he still has three degrees of freedom. Where will he decide to have one of the degrees of freedom to be the side stream of 200 kg/h? What do you think?

A: Your friend is right in his analysis of the degrees of freedom, but, as was stated in Sect. 7.8.2.1, there are limitations on the value that can be assigned, in this case to the side stream. Yes, your friend can fix the side stream, but with limitations. As calculated in problem 13, the output flow [31 % solids (w/w)] is ~193.6 kg/h. Then your friend's proposition is impossible to implement because the side stream should be less than 193.6 kg/h. What is the maximum theoretical value of the mass flow rate of the side stream? ~142 kg/h

15. **New idea [5].** After listening to your powerful arguments (problem 14), your creative friend acknowledges that you are right, but he comes back with a new idea. He still wants to have a side stream of 200 kg/h, but he is now aware of your technical and correct analysis. Now, for the same inputs and outputs (feed stream = 1,000 kg/h, and input and output solids of 6 and 31 %), he has designed a five-effect evaporator with a side stream of 200 kg/h, but this time the side stream will be mixed with the output of stage 3 (Fig. 7.48). In addition, the amount of removed vapor will be the same in each effect. (a) What do you think? (b) If you are now convinced of your friend's proposition, then what are the values of the solid concentration in each stage?

A: (a) We do not know whether or the new idea is a good one, but at least it is feasible. (b) 7.5 %, 10.03 %, 15.15 %, 11.62 % (after mixing with side stream), and 16.9 %

16. **Strawberry jam with pectin [5].** Pectin is a fiber that is normally found in acid fruits. One of their many uses is as a gelling agent, and it is used for the manufacture of marmalades. It is sold as a powder (white). You have been asked to perform the calculations needed to prepare 100 kg of strawberry jam (65 % (w/w) solids). Since this is the first time you will prepare a jam, you will be using a recipe you inherited from your grandmother. According to the recipe, the main ingredients

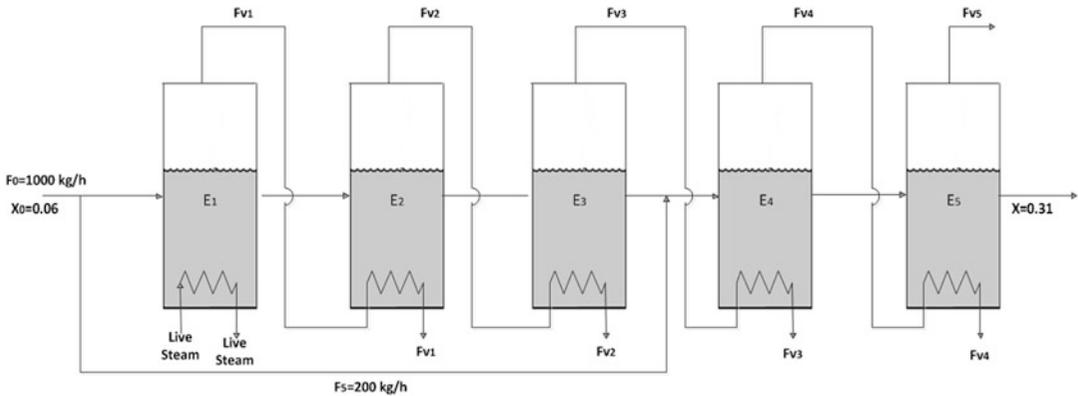


Fig. 7.48 Multi-effect evaporator for tomato concentration with a side-stream

are crushed fruit, sugar, and a gelling agent, as mentioned, usually pectin. The recipe indicates the addition of 120 g of pectin per 100 kg of jam and 55 parts sugar per 45 parts crushed fruit. The commercial sugar has 5 % w/w moisture, and the strawberry has 12 % w/w solids. If it can be assumed that the amount of water in the pectin is negligible, then: (a) How many kilograms of strawberries are needed to prepare 100 kg of jam? (b) How many kilograms of water are evaporated in the "cooking" process?

A: (a) ~50.64 kg. (b) ~12.66 kg

17. **Quince jam [5].** Now you are asked to prepare some quince jam. Since experimentation is the mother of science, again you use the notes of your beloved grandmother. In the detailed recipe preparation steps, first you need to wash the quinces, peel and cut them into chunks, removing the heart, put them in a pot with a little water, and cook for 30 min. Then grind it all up and add the same amount of quince pulp and sugar. Finally mix well and cook for about 10 min. Unfortunately, there is no indication about the composition of the quince pulp and how much pectin is needed. But you have been learning to be practical and not complicated, so you use the same amount of pectin that was recommended in the recipe for strawberry jam –120 g of pectin per 100 kg of jam. After some research you find out that the composition of quince pulp is 92 % w/w H₂O and the rest is solids. Assume that the sugar is 5 % w/w of H₂O and that the amount of H₂O in the pectin is negligible. If the jam is 62 % w/w solids, then how many kilograms of sugar are needed to prepare 100 kg of quince jam?

A: ~60 kg

18. **Generic formula [6⁺].** Given that most jams follow a similar recipe, you want to develop a generic formula to avoid doing these calculations each time. If the fruit pulp has **a** % w/w H₂O, then the sugar has **b** % w/w H₂O, the pectin has no H₂O, and **c** g of pectin are added per **g** kg of jam; finally, **e** parts pulp are used for **f** parts sugar. Thus: (a) How many kilograms of sugar are needed to prepare **g** kg of jam with **h** % w/w solids? (b) How many kilograms of fruit are needed to prepare **g** kg of jam with **h** % w/w solids?

A: (a) $\frac{g \left[1 - \frac{100-h}{100} \right] - \frac{c}{1000}}{\frac{e}{f} + 1 - \frac{ae}{100f} - \frac{b}{100}}$ (kg) You can check this formula using the data from exercise 17.

(b) $\frac{e}{f} \times \frac{g \left[1 - \frac{100-h}{100} \right] - \frac{c}{1000}}{\frac{e}{f} + 1 - \frac{ae}{100f} - \frac{b}{100}}$ (kg)

19. **Side stream [5].** Tomato juice will be concentrated in a continuous one-stage evaporator. The process includes a side stream to avoid excessive browning in the final product

Fig. 7.49 Single-effect evaporator with a side-stream

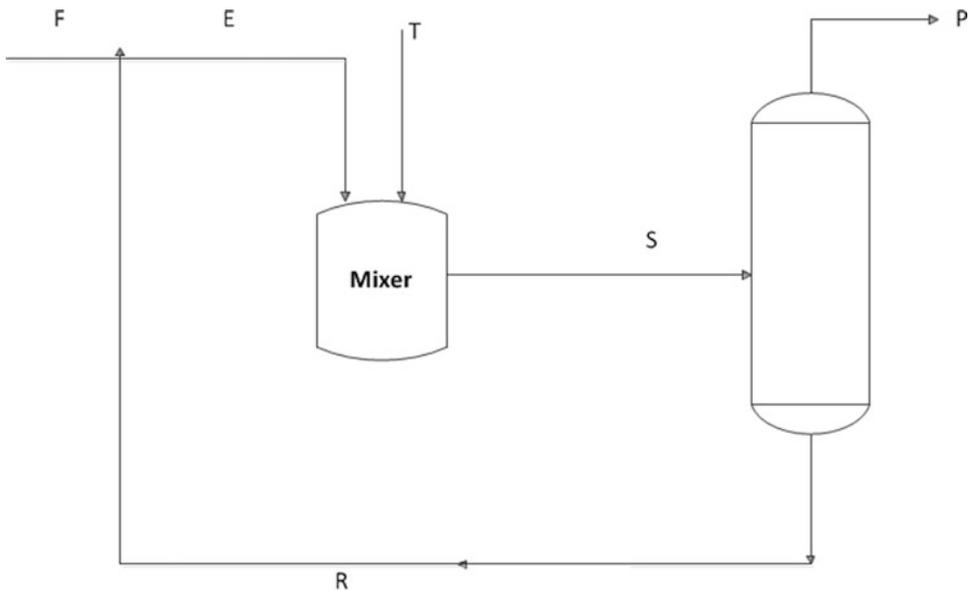
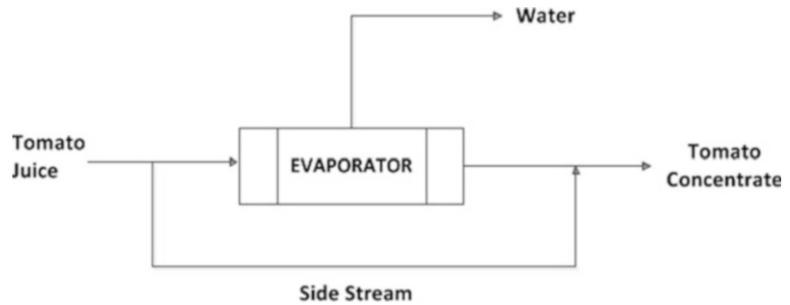


Fig. 7.50 Schematic diagram for a new process

(Fig. 7.49). The process is fed with 1,000 kg/h tomato juice (6 % w/w solids) and the side stream flow rate corresponds to 12 % of the feed flow. If you have to obtain a concentrate with 30 % w/w of solids, then: (a) How many kilograms per hour of concentrate are obtained? (b) What is the percentage w/w of solids at the evaporator outlet?

A: (a) 200 kg/h; (b) 66 % w/w

20. **Separation [4].** A stream of 100 kg/h with 40 % w/w A, 30 % w/w B, and 30 % w/w water should be separated into two flows, one containing A and possibly water (but not B) and a second flow containing B and possibly water (but not A). In addition, the ratio of the stream containing A to the stream containing B is 2:3. (a) What is the flow rate of each stream? (b) What is the composition of each flow?

A: (a) 60 kg/h (the one containing B) and 40 kg/h (the one containing A). (b) 50 % w/w B and 50 % water; 100 % A

21. **New process [5].** The process shown in Fig. 7.50 is the base of the production in MBT Enterprises. The manager gives you the following information to carry out a complete analysis of the material balance of the whole process. The feed flow of the process is $F = 15 \text{ m}^3/\text{h}$ ($\rho = 1 \text{ g/mL}$). This flow is composed of water and solids, both with a density of 1 g/mL. At the

outlet of the mixer, the flow rate is 20,000 kg/h and the added water (T) is 2,000 kg/h. The manager believes that this is enough information for you to carry out the material balance analysis. You carefully analyze the given information and then tell the manager that in fact you don't have enough information and argue that the process has more degrees of freedom than the manager thinks. Specifically, you ask for two more data points. Without objection, the manager does two additional measurements and then tells you the following information: the solid concentration on flow E is 20 % w/w and the solids concentration in flow F is 18 % w/w.

(a) How many degrees of freedom did the process have before and after obtaining the additional data? (b) Was it reasonable to ask for more information? (c) Complete all the information for each flow (including composition).

A: (a) First, two degrees of freedom, so 0. (b) Yes, you received three pieces of data (flows F, S, and T) but five were needed, (c)

Stream	Flow rate (kg/h)	% w/w of solids
F	15,000	18
E	18,000	20
T	2,000	–
S	20,000	18
P	17,000	15.9
R	3,000	30

22. **Fish meal [4].** Ten tons of fish meal are processed in a dryer where the moisture (w/w) is decreased from 15 to 8 % (w/w). In the drying process, the hot air is at 350 °C and has a humidity of 0.012 g H₂O/g dry air. At the outlet, the air is at 80 °C with a humidity of 0.075 g H₂O/g humid air. Determine the volume of air fed.

A: $\sim 2.0 \times 10^7$ L

23. **Evaporation-crystallization [5].** Crystallization is the process of forming crystals from a solution. Strictly speaking, it is one of the techniques used to separate a "solid" from a solution. To perform crystallization, usually the temperature is lowered, thus decreasing the solubility of the solid in the solution. An alternative process is to remove water by evaporation. A solution contains 20.0 % w/w Na₂SO₄, 15.0 % w/w Na₂CO₃, with the rest being water. This solution is passed through an evaporator that removes 10.0 % of the original mass and then crystallizes 20.0 % Na₂SO₄ and 30.0 % Na₂CO₃. The crystals formed are Na₂CO₃ per ten molecules of H₂O and Na₂SO₄ per ten molecules of H₂O. Calculate the composition of the residual solution.

A:

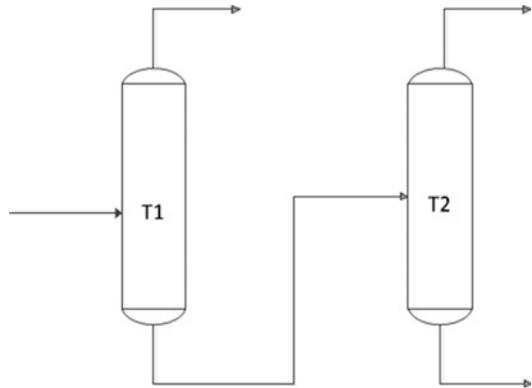
Component	Composition % w/w
Na ₂ SO ₄	23.3
Na ₂ CO ₃	15.3
H ₂ O	61.5

24. **Stream mixing [8].** Two streams of unknown flow rate are mixed to form one stream of a kg/h. If a soluble salt is added to the first stream, it reaches a salt concentration of 4.76 % w/w, and the salt content in the outlet mixed stream is 0.62 % w/w. What is the ratio of these two streams?

A: 7

25. **Distillation [4].** Distillation is a technique used for separation of components in a single liquid stream. This separation is based on the different boiling points of the components. At the industrial level, one of the best known processes is the production of gasoline. Different types of distillation include, for example, simple distillation and fractional distillation. A mixture of

Fig. 7.51 Distillation process with two columns in series



benzene, toluene, and xylene is separated in a fractional distillation process into two columns. The first column delivers benzene as a product at the top and toluene and xylene as product at the bottom. Then the bottom product is fed to a second column. The second column delivers toluene at the top and xylene as the bottom product. A flow diagram of the process is shown in Fig. 7.51. If the feed of the first column is 25,780 lb-mol/day of a mixture whose molar composition is 37.54 % benzene, 22.34 % toluene, and 40.12 % of xylene, then what is the composition w/w at the bottom of the first column?

A: 66.13 % xylene and 33.88 % w/w toluene

26. **Distillation [5].** A process of two distillation columns arranged in series is designed to separate a mixture of three components—benzene, toluene, and xylene. The product will be three streams, each one rich in one of these chemical species. 1,275 kmol/h of a mixture of 30 mol% benzene (B), 25 % toluene (T), and the remaining xylene (X) are fed to the process. In the first column, at the bottom, the product contains 99 % xylene and no benzene. This stream recovers 98 % of the total xylene that was fed. The product at the top of the first column is fed to the second column. In the second column, the top product contains 99 % benzene and no xylene. The benzene recovered in this stream represents 96 % of the benzene fed to the column. What is the molar composition in each output stream?

A:

Column	Top % molar	Bottom % molar
1	54.1	–
	44.28	1
	1.62	X: 99
2	B: 99	4.56
	1	T: 92.03
	–	3.41

Benzene, Toluene and Xylene

27. **Distillation [4].** A media culture that for practical purposes can be considered as a mixture of ethanol (C_2H_5OH) and water will be distilled. The process basically consists of two columns connected in series (Fig. 7.52). The alcohol distillate (end product) obtained through the top of the second column must contain 94 % w/w ethanol. The feed flow of the first column is 100 kg/h, and the flow rate at the top is 20 kg/h. The alcohol concentration at the bottom of the first column is 2 % w/w. The flow at the bottom of the second column is 5 kg/h with 6 % w/w alcohol. (a) What is the composition of the feed flow? (b) What is the composition of the top flow of column 1? (c) How many kilograms per hour of end product are obtained?

Fig. 7.52 Distillation process with two columns in series

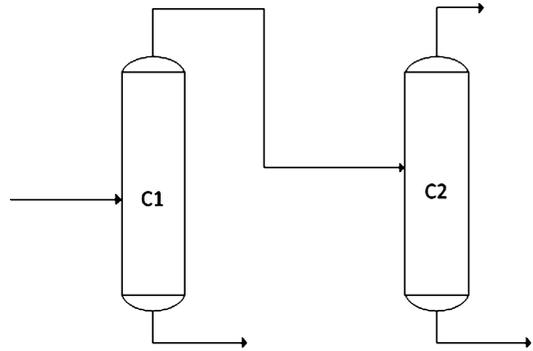
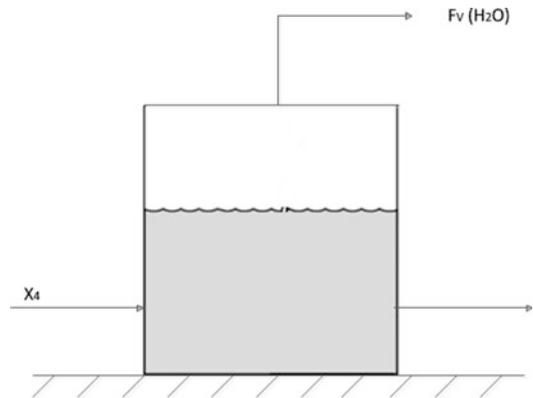


Fig. 7.53 Single-effect evaporator at laboratory scale



A: (a) 16 % w/w alcohol and 84 % w/w H₂O. (b) 72 % w/w alcohol and 28 % w/w H₂O. (c) 15 kg/h

28. **Multieffect evaporator [5].** In order to design an industrial process to evaporate much of the water present in the product X_4 , an experiment was conducted in a laboratory-scale evaporator (simple) as shown in Fig. 7.53:

It was experimentally shown that this evaporator is capable of evaporating 80 % of the water initially contained in the product. The X_4 product contains 80 % w/w water, and it is expected that the final product will not be more than 10 % w/w water and no less than 6 % w/w. As the water content of the product obtained in the laboratory equipment is greater than 10 %, in practice, it will be necessary to build an evaporator as depicted in Fig. 7.54:

From experience, we know that each evaporator added to the process is less efficient than the preceding one. The percentage of water that is removed at each stage is determined by the formula $X = 90 - 10i$, where X is the percentage of H₂O removed in stage i and i is the stage (1,2,3,..., N). (a) How many evaporators (stages) do there have to be so that the product meets the requirements established for the X_4 end product? (b) What is the final concentration of H₂O in the end product?

A: (a) 3 (b) 8.76 % w/w. If you add a fourth evaporator, the product X_4 will not meet the requirements. Verify this!

29. **Dryer [9].** 10,000 lb/h of a solid material having a moisture content of 15 % w/w is dried to reach a moisture content of 2 % w/w (Fig. 7.55). The fresh air (Z) contains 0.012 lb of H₂O/lb dry air at a temperature of 70 °F. The air leaving the dryer is 100 °F with a humidity of 3 % w/w. Part of this

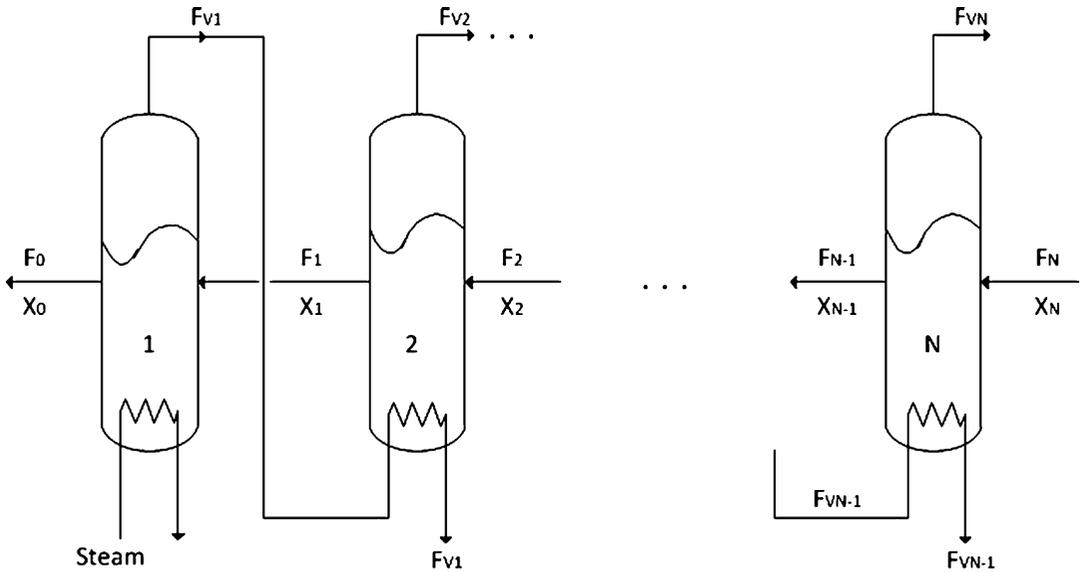
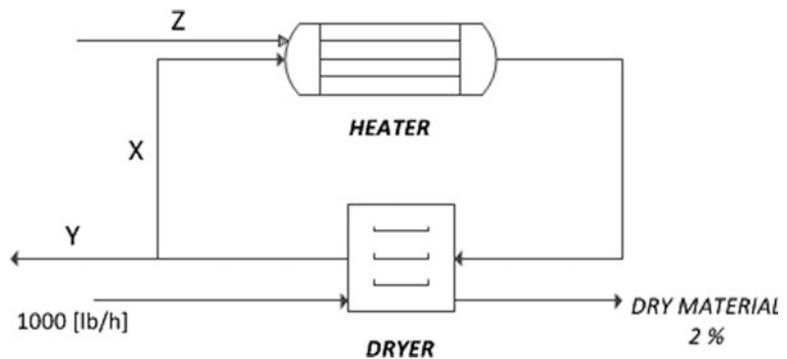


Fig. 7.54 Multi-effect evaporator

Fig. 7.55 Dryer including recycle



air is recycled (X) and mixed with fresh air (Z), then the mixture passes through a heater and leaves it at 200°F . Then the mixed stream leaving the heater is fed to the dryer. The ratio of the recycle stream (X) to the purged stream (Y) is 2:3. The process operates at 1 atm. (a) Calculate the mass flow rate of fresh air (Z). (b) Calculate the air humidity at the dryer inlet. (c) Calculate the volume flow rate of fresh air (Z) in ft^3/h .

A: (a) $\sim 70,925$ lb/h. (b) 1.92 %. (c) $967,514.1$ ft^3/h (2.74×10^7 L/h)

30. **Purification [4].** We have designed a process for drying and purifying a particular commodity. The raw material contains A, B, and water. The final product must contain at least 98 % w/w of component A. The process consists of two dryers in series and at the last stage is a separator. The industrial process requires a production of 1,000 kg/h of component A. If the raw material contains 50 % w/w of component A and each dryer removes 70 % of the water entering the dryer, and at the last stage the separator removes all of B, then: (a) What is the composition w/w of the raw material? (b) How many kilograms of water are removed in each dryer?

A: (a)

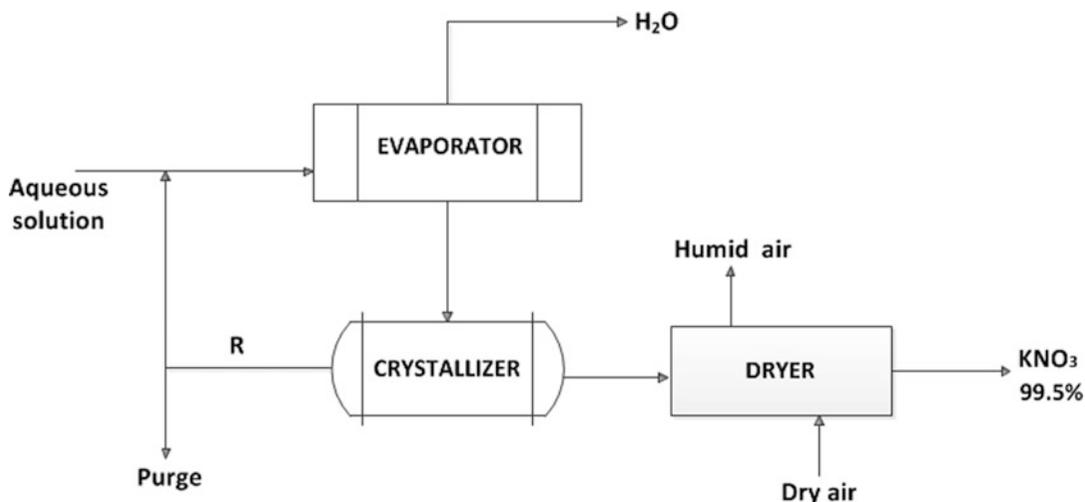


Fig. 7.56 KNO_3 purification in a system including evaporation, crystallization and dehydration

	Composition % w/w
A	50.00
B	38.66
H₂O	11.34

(b) First dryer: 158.7 kg/h, second dryer: 47.62 kg/h

31. **KNO_3 purification [9].** KNO_3 is an odorless, white, and hygroscopic crystalline powder. It can be obtained from nature or from the neutralization of K_2CO_3 (or KOH) with aqueous HNO_3 . Its most common uses are in fireworks, explosives, glass, fertilizers, food preservation, toothpaste, gunpowder, and as a diuretic. In a similar process to the glutamic acid purification (problem 15, Sect. 7.9), it is desirable to obtain KNO_3 with 99.5 % purity from 1,000 kg/h of an aqueous solution containing 15 % KNO_3 and 1.5 % insoluble impurities. The proposed process is depicted in Fig. 7.56. The fresh feed and the recycle are mixed and sent to an evaporator in which KNO_3 is concentrated to 50 % ($\sim 150^\circ\text{C}$). This solution is cooled to 38°C in a crystallizer yielding two separate streams: one consisting of pure crystals of KNO_3 and carrying a certain amount of liquor, the other composed of only liquor. This latter stream is in part recycled and the rest is purged to prevent the accumulation of impurities. This stream contains 0.6 kg $\text{KNO}_3/\text{kg H}_2\text{O}$ and 12 % impurities. The wet crystals are completely dehydrated in a dryer. (a) How many degrees of freedom does this process have? (b) What is the product mass flow rate (kg/h)? (c) What is the mass flow rate of the purge? (d) What is the efficiency of the process (kg KNO_3 in the product/kg KNO_3 fed)?

A: (a) $\text{DF} = 0$. (b) 110.8 kg/h. (c) 120.4 kg/h. (d) 0.735 (73.5 %)

32. **Crystallization and interpolation [9].** According to the dictionary, to interpolate is to introduce (something additional or extraneous) between other things or parts; interject; interpose; intercalate. In mathematics, of course, there are several ways or methods to interpolate. The simplest one, and in certain specific cases a good method, is linear interpolation, where you trace a straight line between two data points to estimate an unknown datum (see example below).

1,000 lb/h of a saturated NaHCO_3 solution at 50°C are fed to a crystallizer. The target of the operation is to crystallize 50 lb/h of NaHCO_3 . Determine the temperature at which the

Table 7.1 Solubility of NaHCO₃ in water as a function of temperature

Temperature (°C)	Solubility (g NaHCO ₃ /100 g H ₂ O)
60.0	16.40
50.0	14.45
40.0	12.70
30.0	11.10
20.0	9.60
10.0	8.15

crystallizer should be operated. From a manual of properties, we have the following data for the solubility of NaHCO₃.

A: 14 °C (obtained through a linear interpolation). (Note *linear interpolation* refers to the estimation of a value between two discrete data points. For example, if you want to estimate the solubility of NaHCO₃ at 14 °C, then using linear interpolation you should proceed as follows using the data reported in Table 7.1:

Temperature (°C)	Solubility (g NaHCO ₃ /100 g H ₂ O)
20	9.6
14	S
10	8.15

Graphically:

$$\frac{9.6 - S}{20 - 14} = \frac{9.6 - 8.15}{20 - 10},$$

$S = 8.73$ [g NaHCO₃/100 g of H₂O].

33. **Waste treatment** [7]. A waste conditioning plant for post-coprocessing features three stages: two streams are fed to the process. The first stream (F_1) contains 80 % sawdust, 5 % heavy metals, and 15 % H₂O. The second stream (F_2) contains 72 % plastic, 8 % heavy metals, and 20 % water. Stream F_2 is three times larger than stream F_1 . These two streams (which correspond to the feed) are joined by a third stream (a recirculation stream), and then the output stream is the input of a crusher. The output stream from the crusher is fed to an electromagnetic filter, which removes part of the heavy metals. The stream that leaves the electromagnetic filter is composed of 21 % water, and thus is the input of a trommel that has three output streams. The first output stream is a fine solid substitution fuel (FSSF), which is 40 (ton/net shift), takes 66.04 % of the total sawdust from the feed stream and 7 % of the total plastic of the feed stream and has a heavy metal content of 0.05 %. The second output stream is a thick solid substitution fuel (TSSF) of 140 (ton/net shift), and its content of heavy metals is 0.07 %. The third is the recycle stream, 7 (ton/h), with 0.8 % sawdust and 0.09 % heavy metals. Of the 8 h that correspond to a working day, the plant works just 5 h (the net shift is therefore 5 h) and there are three shifts per day. Answer the following questions: (a) What is the mass flow rate (ton/h) of the feed streams? (b) What is the composition of the FSSF and TSSF streams? (c) What is the amount of effective working hours per day? (d) What is the mass flow rate ton/year of the heavy metal recovery? (e) What is the percentage of heavy metals removed at the electromagnet filter? (f) If the calorific power of the TSSF is 4,100 kcal/kg, determine how much energy in kilojoules per month can be obtained. (g) If the calorific power of the FSSF is 3,800 kcal/kg, express this stream in kilowatts (kW).

$$F_{6,\text{FSSF}} = 8 \text{ ton/h} \quad F_{7,\text{TSSF}} = 28 \text{ ton/h}$$

$$x_{6,\text{a}} = 64.04 \% \quad x_{7,\text{a}} = 9.41 \%$$

$$\text{A: (a) } F_1 = 9.7 \text{ ton/h; } F_2 = 29.09 \text{ ton/h} \quad (\text{b) } x_{6,\text{HM}} = 0.05 \% \quad x_{7,\text{HM}} = 0.07 \%$$

$$x_{6,\text{p}} = 18.33 \% \quad x_{7,\text{p}} = 69.57 \%$$

$$x_{6,\text{w}} = 17.59 \% \quad x_{7,\text{w}} = 20.95 \%$$

$$(\text{c}) 5 \left[\frac{\text{h}}{\text{shift}} \right] 3 \left[\frac{\text{shift}}{\text{day}} \right] = 15 \left[\frac{\text{h}}{\text{day}} \right]$$

$$(\text{d}) F_4 = 2.79 \left[\frac{\text{ton}}{\text{h}} \right] \left(\frac{15}{1} \left[\frac{\text{h}}{\text{day}} \right] \right) \left(\frac{365}{1} \left[\frac{\text{day}}{\text{year}} \right] \right) = 1.53 \times 10^4 \left[\frac{\text{ton}}{\text{year}} \right]$$

$$(\text{e}) \eta = \frac{F_4}{F_3} \times 100 = \frac{2.79 \text{ [ton/h]}}{2.82 \text{ [ton/h]}} \times 100 = 98.9 \%$$

$$(\text{f}) 28 \left[\frac{\text{ton}}{\text{h}} \right] \left(\frac{1,000}{1} \left[\frac{\text{kg}}{\text{ton}} \right] \right) \frac{4,100}{1} \left[\frac{\text{kcal}}{\text{kg}} \right] \left(\frac{4,184}{1} \left[\frac{\text{kJ}}{\text{kcal}} \right] \right) \left(\frac{15}{1} \left[\frac{\text{h}}{\text{day}} \right] \right) \left(\frac{30}{1} \left[\frac{\text{day}}{\text{month}} \right] \right) = 2.2 \times 10^{11} \left[\frac{\text{kJ}}{\text{month}} \right]$$

$$(\text{g}) 8 \left[\frac{\text{ton}}{\text{h}} \right] \left(\frac{1,000}{1} \left[\frac{\text{kg}}{\text{ton}} \right] \right) \frac{3,800}{1} \left[\frac{\text{kcal}}{\text{kg}} \right] \left(\frac{4,184}{1} \left[\frac{\text{kJ}}{\text{kcal}} \right] \right) \left(\frac{1}{3,600} \left[\frac{\text{h}}{\text{s}} \right] \right) = 3.5 \times 10^4 \text{ [kW]}$$

34. **Banana milk** []. To prepare a delicious banana milk on an industrial scale, there is an experimental process that consists of five unit operations. A stream of banana (whose components are fruit, peel, and moisture) is fed to a peeler, where 99.9 % of the peel is removed. This stream should become pulp, so it is sent to an extruder press that mills the banana. Meanwhile, a stream of the pure additive R is added, which serves to avoid pulp oxidation. The resulting flow of 30 kg/h has a composition of 2 % w/w of additive R and is then fed to an agitated mixer. To the same mixer is fed a dairy stream of 200 kg/h that contains 70 % milk (the rest is water) and a sweet stream that contains 30 % w/w sugar (the rest is water). For each 10 kg/h of dairy flow, 1 kg/h of the sweet flow is fed. The stream that leaves the agitated mixer has 6 % fruit and 0.005 % peel and enters a thermal process in which 5 % of the water and 3 % of the milk evaporate. The resulting flow of the heat treatment (hot banana milk) finally joins the mixer, where the flavoring S is added. The final banana milk contains 30 % water. Calculate: (a) the composition and flow of the initial stream of bananas, (b) the annual flow of peels that are separated in the process (ton/year), (c) the mass lost during the thermal process (kg/h), (d) the composition of the final product stream and its flow (kg/h), and, (e) in the case where the density of the final product is 1.3 (g/mL), calculate how many 0.2 L packages of banana milk are produced per month.

$$F_2 = 41.886 \text{ kg/h}$$

$$(\text{a}) \quad x_{2,\text{f}} = 35.81 \%$$

$$x_{2,\text{c}} = 29.84 \%$$

$$x_{2,\text{w}} = 34.35 \%$$

$$(\text{b}) F_3 = 12.4875 \left[\frac{\text{kg}}{\text{h}} \right] \left(\frac{24}{1} \left[\frac{\text{h}}{\text{day}} \right] \right) \left(\frac{365}{1} \left[\frac{\text{day}}{\text{year}} \right] \right) \left(\frac{1}{1,000} \left[\frac{\text{ton}}{\text{kg}} \right] \right) = 109.39 \left[\frac{\text{ton}}{\text{year}} \right]$$

$$F_{12} = 279.9 \text{ kg/h}$$

$$x_{12,\text{f}} = 5.36 \%$$

$$x_{12,\text{c}} = 0.006 \%$$

$$x_{12,\text{w}} = 30 \%$$

$$(\text{c}) F_9 = 8.61 \text{ kg/h} \quad (\text{d}) \quad x_{12,\text{R}} = 0.214 \%$$

$$x_{12,\text{a}} = 2.14 \%$$

$$x_{12,\text{l}} = 48.52 \%$$

$$x_{12,\text{S}} = 13.76 \%$$

$$(e) 279.9 \left[\frac{\text{kg}}{\text{h}} \right] \frac{1}{1.3} \left[\frac{\text{L}}{\text{kg}} \right] \left(\frac{24}{1} \left[\frac{\text{h}}{\text{day}} \right] \right) \left(\frac{30}{1} \left[\frac{\text{day}}{\text{month}} \right] \right) \frac{1}{200} \left[\frac{\text{box}}{\text{mL}} \right] \left(\frac{1,000}{1} \left[\frac{\text{mL}}{\text{L}} \right] \right) = 775,091 \left[\frac{\text{box}}{\text{month}} \right]$$

35. **Liquid industrial waste [10+].** One of the three streams that enter a mixer has a composition of 5 % w/w solids, 10 % w/w bacteria, and 7 % w/w heavy metals. The resulting stream from the mixer has a mass flow rate of 1,000 kg/h and a composition of 9.6 % w/w bacteria. Then this stream is fed to a settler, whose dense stream is 20 % of the input flow. The clarified stream is fed to a sorter, which generates a recycle of 10 %, which is the second stream that enters the mixer. The flow of the clarified stream that leaves the plant (leaves the sorter) has a composition of 1 % w/w solids. The dense flow has a composition of 20 % w/w solids and is then fed to a centrifuge that removes 30 % w/w moisture at a mass flow rate of 50 kg/h, which is the third stream that enters the mixer. The output stream of the centrifuge has 7 % of bacteria and is fed to an evaporator, which eliminates 60 % w/w humidity, generating a final stream with 40 % w/w solids. Calculate: (a) the composition and mass flow rate of the clarified stream that leaves the plant (leaves the sorter), (b) the composition and mass flow rate of the stream that leaves the evaporator, (c) the composition and mass flow rate of the input and output streams of the settler, and (d) the Composition of the third stream that enters the mixer.

$$F_6 = 720 \frac{\text{kg}}{\text{h}} \quad F_9 = 90.75 \frac{\text{kg}}{\text{h}} \quad F_2 = 1,000 \frac{\text{kg}}{\text{h}} \quad F_3 = 800 \frac{\text{kg}}{\text{h}} \quad F_4 = 200 \frac{\text{kg}}{\text{h}}$$

$$(a) \quad x_{6,s} = 1 \% \quad x_{6,b} = 10.63 \% \quad x_{6,HM} = 7.8 \% \quad x_{6,w} = 80.57 \%$$

$$(b) \quad x_{9,s} = 40 \% \quad x_{9,b} = 11.57 \% \quad x_{9,HM} = 4.96 \% \quad x_{9,w} = 43.47 \%$$

$$(c) \quad x_{2,s} = 4.8 \% \quad x_{2,b} = 9.6 \% \quad x_{2,HM} = 7.04 \% \quad x_{2,w} = 78.56 \%$$

$$x_{3,s} = 1 \% \quad x_{3,b} = 10.63 \% \quad x_{3,HM} = 7.8 \% \quad x_{3,w} = 80.57 \%$$

$$x_{4,s} = 20 \% \quad x_{4,b} = 5.48 \% \quad x_{4,HM} = 4.02 \% \quad x_{4,w} = 70.5 \%$$

$$F_5 = 50 \frac{\text{kg}}{\text{h}}$$

$$(d) \quad x_{5,s} = 7.4 \% \quad x_{5,b} = 1.0 \% \quad x_{5,HM} = 7.0 \% \quad x_{5,w} = 84.6 \%$$

36. **Chemical process [].** You are a process engineer at a factory that produces chlorine soda. On the process line, a mixer is fed with 450 kg/h of a stream that contains a solution of 5 % w/w NaOH and 10 % w/w NaCl, and another stream of 550 kg/h that contains a solution of 7 % w/w NaOH and 8 % w/w NaCl. The resulting stream is fed to an evaporator in which the output stream is 20 % w/w NaOH. This stream is fed to a crystallizer/separator that produces two streams to eliminate the excess of NaCl solution. The scrap stream removes 70 % of the NaCl and contains no NaOH. The stream of product leaving the crystallizer/separator is composed of 30 % NaOH. Calculate: (a) What is the performance of the process, expressed in kilograms of solution of product/kilograms of solution fed]? (b) What is the composition and mass flow rate of the product stream? (c) How much water evaporated in the process? (d) What is the waste stream composition?

$$F_7 = 203.3 \frac{\text{kg}}{\text{h}}$$

$$(a) \quad \eta = \frac{F_7}{F_3} = \frac{203.3 \frac{\text{kg}_{\text{PRODUCT}}}{\text{h}}}{1,000 \frac{\text{kg}_{\text{FEED}}}{\text{h}}} = 0.2033 \rightarrow 20.33 \% \quad (b) \quad x_{7,\text{NaOH}} = 30 \% \quad (c) \quad F_5 = 695 \frac{\text{kg}}{\text{h}}$$

$$x_{7,\text{NaCl}} = 13 \%$$

$$x_{7,w} = 57 \%$$

$$(d) \quad F_6 = 101.7 \frac{\text{kg}}{\text{h}}$$

$$x_{7,\text{NaCl}} = 61.3 \%$$

$$x_{7,w} = 38.7 \%$$

37. **Drying apples [4].** A dryer is fed with 45 kg/h of apples with a moisture content of 85 % w/w; at the output the moisture content of the apples is 17.65 % w/w on a dry basis. The fresh air at the input has a moisture content of 0.03 kg H₂O/kg dry air, and at the outlet it has moisture of 5.21 % w/w on a wet basis. Determine the input mass flow of air (wet) to be used for this operation.
A: 1,529.1 kg humid air/h (1,484.56 kg dry air/h)
38. **Inoculum [5].** A batch fermentation uses as inoculum a portion of the previous batch, equivalent to one-tenth the total volume. The process proceeds as follows: once a batch is done, a valve at the bottom of the fermenter is opened until the inoculum needed for the next batch is left. At that time, the valve is closed, and then the other valve is opened to feed fresh substrate at a concentration of 40 g/L to complete the volume of the batch. Fermentation stops when reaching a biomass and substrate concentration of 10 g/L. (a) Calculate how many grams of substrate are consumed per gram of biomass generated. (b) How many grams of substrate are consumed per gram of biomass generated if at the end of the fermentation the substrate concentration is 0?
A: (a) 3 g substrate/g biomass. (b) 4 g substrate/g biomass
39. **New juice [6].** A small jam factory is developing a new product, a fruit juice. The product will be prepared with a mixture of fruit pulp to make the juice. Pulp of pineapple and strawberry will be mixed, five parts of strawberry for three parts of pineapple. To the mixture will be added sugar that make up 10 % of the total weight of the pulp (without sugar). For the opening of a restaurant, the jam factory will sell 150 jars of juice (1 L each). To produce 1 L of juice, 200 g pulp mix (with sugar) are needed. (a) How many kilograms of raw materials should the factory buy? (b) If the factory wants to prepare a second batch and the stock of raw materials is 32 kg strawberry pulp, 26 kg pineapple pulp, and 9 kg sugar, what is the maximum amount (liters) of juice the factory can produce?
A: (a) Sugar 2.72 kg, strawberry 17.05 kg, and pineapple 10.23 kg. (b) 281.6 L
40. **Filter aid [9].** To separate cells from a culture, a stream containing a concentration of 25 g cells/L is passed through a filter. To facilitate the operation, a filter aid is used. The filter aid is fed in a proportion of 0.08 kg filter aid/kg cell. It is known that the solid cake has 40 % moisture and the entire filter aid remains in the solid cake. Furthermore, the filter aid corresponds to a 15th part by weight of the solid cake. Determine the concentration of cells in the permeate stream. Assume that the density of the culture, solid cake, and permeate is equal to that of water.
A: 0.0092 g/L
41. **Dairy company [8].** A dairy company receives 85,000 L of milk per day, which is divided into three streams to produce whole, low-fat, and skim milk. The low-fat milk and skim milk are obtained by removing part of the fat. The removed fat will be used in another process. The fat stream is equivalent to one-tenth the weight of the milk was used to produce the skim milk. It is further known that the ratio of fed milk to total milk obtained is 1.02 w/w. (a) How many kilograms of milk goes to each line? (b) How much fat is available per day for further processing? Assume that the milk fed to the process has a density of 1.034 g/cm³ and that its fat content is 4 % w/w. In addition, the low-fat milk has a 2 % w/w fat and skim milk contains no fat.
A: (a) 20,680 kg/day (22.7 %) of whole milk, 48,253 kg/day (57.6 %) low fat milk, and 17,233 kg/day (19.7 %) skim milk. (b) 1,724 kg/day
42. **Bioethanol production [9].** In the first phase of a production process of bioethanol (second generation) (Wiche, 2010) 31,800 kg/h of wood (50 % moisture) are fed to a mill. In the mill, wood chips are obtained that are then fed to a tank to be impregnated with sulfuric acid. In this tank, chips are mixed with sulfuric acid solution (0.25 % w/w) at a ratio of one part chips per two parts acid solution. It can be assumed that during the process, the moisture of the chips does not change. Then the mixture undergoes a separation process in which two streams are

obtained, one of impregnated wood 0.87 H₂SO₄ mg/g of moisture wood and the other a solution of H₂SO₄. Seventy-five percent of the acid solution is recycled and the rest is purged. The recycle stream is mixed with H₂SO₄ at 95 % w/w and fresh water. This latter stream is fed to a pond, where it is mixed with the chips for impregnation. Moreover, the impregnated stream from the separator passes through a steam explosion process, which is fed in a ratio of 0.7 kg steam/kg dry wood. Then, in a cyclone, the exploded wood is separated from all the steam that was fed. The steam carries 2.7 % of the dry mass of wood that was fed to the system. Determine (a) the flow of acid and fresh water required per hour, (b) the flow of steam for the explosion process, and (c) the flow of wood and its moisture after the explosion process.

A: (a) 52.7 kg H₂SO₄/h and 15,857.6 kg H₂O/h. (b) 11,130 kg steam/h. (c) 31,370.7 kg wood/h, 50.68 % moisture

43. **Biogas [5].** To use a biogas stream (68 % mol methane and 32 % mol carbon dioxide), the stream is passed through a separation system. First, the biogas begins to accumulate in a gas cylinder (pressure of 1.02 atm) until it reaches a height of 5 m. Then a valve is opened, allowing the passage of biogas into a system where it is separated into two streams, one rich in methane (96 % molar) and stored in a rigid tank 8 m³ at 20 °C, and another that contains only carbon dioxide and is fed to a rigid tank 240 m³ at 30 °C, where it is mixed with air to a pressure of 1.5 atm. This mixture of air and carbon dioxide is then used to feed a greenhouse. Determine the molar flow and composition of each stream. Assume that the gas cylinder has a radius of 4.5 m and the gas temperature is 20 °C.

A: From the gas cylinder: 13,504 mol/batch (68 % CH₄ and 32 % CO₂); methane-rich stream: 9,565.35 mol/batch (96 % CH₄ and 4 % CO₂); from separator to mixer with air: 3,938.69 mol/bath (100 % CO₂); stream to greenhouse: 14,489.25 mol/batch (27 % CO₂, 57 % N₂, and 16 % O₂).

44. **Lignin recovery [8].** To take advantage of some of the components obtained in the production of bioethanol, Wiche and collaborators proposed the recovery of lignin, its subsequent use as fuel, and furthermore allowing the other stream to be fermented. For this, two washing steps, in series, are carried out with alkali. In each washing process, two streams are obtained, one rich in lignin and another that is continuous in the main processing. The amount of NaOH fed for the first wash is equal to 25 % of the weight of lignin fed to the process. In addition, water is added until the concentration of solids is 5 % w/v. The amount of NaOH fed in the second wash is equivalent to 42 % by weight of the lignin remaining in the process stream. In both washing steps, the amount of lignin removed is the same, in the first washing the amount of lignin is 6.4 % of the weight of the stream, and in the second washing the amount of lignin is 1.2 % of the weight of the stream. Also, lignin liquor contains 95 % of NaOH fed at each washing step, and other components in each stream are distributed as fractions of the streams. All the cellulose remains in the main processing line. Both washing steps remove a total of 90 % of the lignin fed to the process.

ADDITIONAL DATA

Fed stream	
Flow rate	33,300 kg/h
Density	996.4 kg/m ³
Moisture	66.6 %
Lignin	10.2 %
Cellulose	20.0 %
Other components	3.2 %

- (a) Determine the amount of NaOH and water to be fed for the first and second washings. (b) Characterize (the flow and composition of) both (together) lignin liquor streams. (c) If the second washing is also carried out with 5 % solids w/v, will it be necessary to add or remove water? If so, how much?

A: (a) 849.15 kg/h for the first and 784.6 kg/h for the second. (b) 151,254.8 kg/h (2.02 % lignin, 1.05 % NaOH, 0.44 % other components, and 96.48 % H₂O). (c) 1,200 kg/h of H₂O should be retired before the second washing.

45. **Double-drum dryer and spray dryer [5].** A factory that produces dried tomatoes receives fresh tomatoes as raw material. Fresh tomatoes have about 1 % proteins and 6 % carbohydrates (the rest is H₂O). 3,000 kg/h of tomatoes are fed into a press, generating two streams, one of pulp and the other of juice. The juice stream is 40 % in weight of feed stream. The juice has 0.3 % proteins and 0.7 % carbohydrates. Then the juice is processed in a spray dryer that removes 99 % of the water, transforming it into powder. Furthermore, the pulp obtained from the press is fed to a double-drum dryer, which delivers an output stream with 7 % moisture. Then the dried product from both dryers is fed to a mixer. Determine (a) the process yield, expressed as kilograms of dry product/kilogram of tomatoes fed; (b) the composition and flow rate of the output current; and (c) the amount of water that evaporated during the process in kilograms per hour.

A: (a) 7.9 %. (b) 76.02 % carbohydrates, 12.67 % proteins, and 11.31 % H₂O, flow rate = 236.78 kg/h. (c) 2,763.22 kg/h.

References

Wiche, P., 2010. Análisis de Ciclo de Vida de la Producción de Bioetanol de Segunda Generación. Memoria de Título para el grado de Ingeniero Civil Bioquímico. Pontificia Universidad Católica de Valparaíso, Chile.

Additional Web References

Mass Balance Tutorial <https://www.youtube.com/watch?v=HOvOdAVIjW0>

Open Systems - Mass and Energy Balance - Steady Nozzle <https://www.youtube.com/watch?v=t6FFAC4DVA4>

Energy Balance Examples for Open Systems http://www.et.byu.edu/~rowley/ChEn273/Topics/Energy_Balances/Energy_Balance_Open_Systems/Open_Sys_Examples.htm

Material Balance Problem Approach <https://www.youtube.com/watch?v=BJ4Tzhi48h0>

Introduction to Degrees of Freedom <https://www.youtube.com/watch?v=W8HyPgmUF0>

Degree of Freedom Analysis on a Single Unit <https://www.youtube.com/watch?v=6Rx2ry1P6ME>

Multiple Unit System: Degree of Freedom Analysis <https://www.youtube.com/watch?v=VqvjssZku5Y>

Elementary Mass Balances in Chemical Engineering <https://www.youtube.com/watch?v=Wpj0XJzqPcQ>