Chapter 4
Introduction to Nuclear Energy

Abstract One may ask the question: Why a book devoted to alternative energy should have three chapters on nuclear energy? After all nuclear power plants and nuclear energy have been the pariahs in every environmentalist's mind for decades. The accidents at the Three-Mile Island, Chernobyl and Fukushima Dai-ichi power plants have contributed to the horrific images of large-scale environmental disasters. The answer to this question is very simple: Global warming, caused by the anthropogenic emission of carbon dioxide is a very serious threat for our planet. Nuclear power plants have the capability to produce a significant part of our electric power at relatively low cost and without any carbon dioxide emissions. A case in point: If in 2008 the United States would have produced 50%, of its electric power from nuclear energy, instead of approximately 25% it actually produced, the country would have exceeded by 150% its quota from the *Kyoto Protocol* without any other changes in the rest of its energy mix. Other OECD countries, such as France and Japan produce more than 70% of their electricity from nuclear power plants. The fundamental concepts of atomic physics with emphasis on the nuclear fission reactions are given in this chapter. At first, the structure of the atom is explained, basic definitions of the atom and the subatomic particles are given succinctly and useful numbers pertaining to the atoms and the nuclear reactions are calculated. Secondly, the nuclear reactions are introduced and the physical principles governing these reactions are explained. Examples of nuclear reactions include radioactive decay and carbon dating. Thirdly, the several ways of interaction of neutrons with nuclei are explained and fission is introduced. The subjects of nuclear fission, chain reactions, nuclear fuels and thermal neutrons are explained in detail. The role of the cross-sections of the naturally occurring nuclear fuels is explained in the fission process as well as in the sustenance of the chain reaction in conventional reactors. The neutron cycle in a nuclear reactor and the striving for the production and conservation of the thermal neutrons are elucidated. Fourthly, the basic concepts of fuel conversion and breeding are given as

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an introduction to breeder reactors. Finally, a few useful numbers are computed on the utilization of natural uranium as a fuel in the conventional nuclear reactors.

4.1 Elements of Atomic and Nuclear Physics

Engineers seeking an insight into the operation of nuclear reactors are interested to know only the results of nuclear reactions and do not need to be concerned with the details of the complex theory of subatomic physics. For this reason a simplified depiction of atoms and nuclei will be given in this chapter, which is sufficient for the understanding of the underlying principles that govern the release of nuclear energy. Central to the theory of energy obtained from nuclear reactions is the famous Einstein equation:[1]

$$E = mc^2, \tag{4.1}$$

where c is the speed of light in vacuum, which is approximately equal to $3*10^8$ m/s.

4.1.1 Atoms and Nuclei: Basic Definitions

Each atom is composed of a *nucleus* and of *electrons*, which are very light particles, have negative charge, $e = -1.602*10^{-19}$ Coulomb (Cb), and revolve around a nucleus in distinct orbits, which are very far from the nucleus. The nucleus is composed of *protons*, heavy particles with positive electric charge and *neutrons*, also heavy particles without any electric charge. Neutrons and protons together are called *nucleons.* The neutrons have almost the same mass as the protons and are electrically neutral. Electrons and protons have equal but opposite charges. The number of electrons and protons is the same in all atoms. Therefore, all atoms are electrically neutral. The number of protons in an atom is usually close to the number of neutrons, but the two numbers are not always equal. Figure 4.1 shows schematically the structure of the atom of carbon-12 (${}_6C^{12}$). The nucleus of this atom is composed of six protons and six neutrons. Also, six electrons revolve around this nucleus at two distinct orbits.

The mass unit by which nuclei and subatomic particles are measured is the a*tomic mass unit, u,* which is equal to $1.6604*10^{-24}$ grams. The masses of the three elementary particles and their electric charges are as follows [1]:

• Mass of an electron:	0.000549 u	charge: $-$ 1.602*10^{-19} Cb
• Mass of a proton:	1.007277 u	charge: + 1.602*10^{-19} Cb
• Mass of a neutron:	1.008665 u	charge: 0

[1] This immensely popularized equation is actually a corollary of Einstein's *Special Theory of Relativity.*

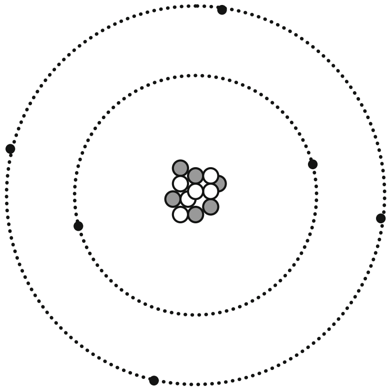


Fig. 4.1 Schematic representation of the atom of carbon-12. Protons (gray) and neutrons (white) are clustered inside the nucleus while the electrons (black) revolve around the nucleus

From Eq. (4.1), it follows that the equivalent energy of 1 u is $1.494*10^{-10}$ Joules (J). In nuclear reactions, the electron-volt (eV) is the commonly used unit of energy. This is equal to the charge of an electron moving through a voltage difference of 1 V, and, it is equal to $1.602*10^{-19}$ J. Therefore, the equivalent energy of 1 u is $9.31*10^{8}$ eV or 931 MeV. This implies that a mass of 1 u will produce 931 MeV if it were entirely converted to energy. For 1 kg, the equivalent energy is $5.62*10^{29}$ MeV or $9*10^{16}$ J, which is a very high amount of energy to be released. The latter is equivalent to the heat obtained from 3,100,000 metric tons of bituminous coal and is sufficient to produce 45,000,000 metric tons of steam.

Protons and neutrons are clustered in the nucleus of an atom. Almost all of the mass of the atom resides in the nucleus. In the case of the carbon-12 atom, which is depicted in Fig. 4.1, only 0.025% of the mass is outside the nucleus and rotates around it with the electrons. Between the nucleus and the electrons there is a great deal of space which is not occupied by any mass particles that is, vacuum. Another characteristic of the atom is that the radii of the orbits of the electrons are very long compared to the radius of the nucleus. If the radius of the nucleus in Fig. 4.1 were of the size of a tennis ball (approximately 7 cm in diameter) then the inner orbit of electrons would have a radius of approximately 600 m and the outer orbit's radius would have been at almost 2,000 m. These dimensions and mass proportions are typical of all atoms and lead us to the conclusion that all matter is concentrated in very small nuclei, with light electrons orbiting the nuclei at extremely long distances. Most of the space we see as a continuum is actually composed of vacuum.

Since electrons are very far from the nucleus, the electrostatic attraction at the outermost orbits is rather weak and an atom may lose one or more electrons, which may attach themselves to another atom or may be shared by several other atoms. In these cases, the atoms are electrically charged and are called *ions*. Ions with positive charges have a deficit of electrons and ions with negative charges have a surplus of electrons.

An atom is characterized by its *atomic number*, Z, which is equal to the number of protons in its nucleus. Oxygen has eight protons and its atomic number is

Z = 8. For carbon, Z = 6, for barium, Z = 56 and for uranium, Z = 92. Since the numbers of protons and electrons in an electrically neutral atom are equal, the atomic number of the atom is also equal to the number of the electrons. Another characteristic number of an atom is the *mass number*, A. The mass number is equal to the sum of the numbers of protons and neutrons in the nucleus. There are atoms that have the same number of protons and, hence, the same number of electrons, but differ in their numbers of neutrons. Such atoms are called *isotopes.* Isotopes have the same atomic number, Z, but different atomic mass, A.

In nuclear physics, an atom is denoted by its chemical symbol (O for oxygen, U for uranium, Pb for lead, etc.) which is accompanied by the atomic number as a left subscript and the mass number as a right superscript. The general notation for an atom, whose chemical symbol is X is: $_ZX^A$. For example, $_{92}U^{235}$ denotes the uranium isotope, which has an atomic number 92 and mass number 235. Since the difference A-Z is equal to the number of neutrons, this isotope has 92 protons and 143 neutrons. Similarly, the symbol $_{92}U^{238}$ denotes the uranium isotope with 146 neutrons and mass number 238, while $_{92}U^{233}$ denotes the uranium isotope with 141 neutrons and mass number 233. Other common isotopes are the following:

For Hydrogen: $_1H^1$ (common hydrogen); $_1H^2$ (deuterium); and $_1H^3$ (tritium), which have 1 proton and 0, 1 and 2 neutrons respectively.
For Oxygen: $_8O^{16}$, $_8O^{17}$ and $_8O^{18}$, all of which are met in the atmosphere.
For Plutonium: $_{94}Pu^{238}$, $_{94}Pu^{239}$, $_{94}Pu^{240}$ and $_{94}Pu^{241}$, all of which have been artificially formed by nuclear reactions.

In the macroscopic world, the mass of the elements is measured in grams (g), kilograms (kg) or gram-atoms. The latter is equal to the atomic mass, A, expressed in grams. Thus, a gram-atom of $_8O^{16}$ is equal to 16 g, while a gram-atom of $_{94}Pu^{239}$ is equal to 239 g. The number of atoms in a gram-atom is a constant for all the elements and is equal to $6.023*10^{23}$ atoms per gram-atom. This very large number is known as the *Avogadro number.*

4.1.2 Atomic Mass, Mass Defect and Binding Energy

The *atomic mass* of an isotope/atom is the mass of this atom, expressed in atomic mass units. Since most of the mass of the atom resides in the nucleus and the masses of protons and neutrons are approximately equal to 1 u, the atomic mass is approximately equal to the mass number, A. Table 4.1 from [1] gives the values of the atomic masses of a few common isotopes.

A glance at Table 4.1 proves that the actual mass of an atom is smaller than the sum of the masses of its constituent protons, neutrons and electrons. Thus, the boron-9 atomic mass is 9.01333 u, while the sum of the masses of its constituent five protons, five electrons and four neutrons is 9.07380 u. There is a small difference between the two numbers, 0.06047 u, which is called the *mass defect* of

Table 4.1 Atomic masses of common isotopes, in u

$_1H^1$	1.007825	$_6C^{12}$	12.00000
$_1H^2$	2.01410	$_6C^{13}$	13.00335
$_1H^3$	3.01605	$_7N^{14}$	14.00307
$_2He^3$	3.01603	$_8O^{16}$	15.99491
$_2He^4$	4.00260	$_8O^{17}$	16.99914
$_2He^5$	5.0123	$_8O^{18}$	17.99916
$_3Li^6$	6.01513	$_{92}U^{234}$	234.0409
$_3Li^7$	7.01601	$_{92}U^{235}$	235.0439
$_5B^9$	9.01333	$_{92}U^{238}$	238.0508

an atom. One way to understand the significance of the mass defect is to relate it to the equivalent energy, which is 56.3 MeV: If the boron atom were to be constructed by its constituents, there would be a mass defect of 0.06047 u. According to the conservation of mass-energy of Eq. (4.1), the mass defect must be released as energy during the formation of the atom. The energy released in such a process is often called the *binding energy.* Since a mass of 1 u is equivalent to 931 MeV, in the case of $_5B^9$, the binding energy is equal to 56.3 MeV, or 6.25 MeV per nucleon of the $_5B^9$ atom. Groups of nucleons that have lower energy are also more stable and do not disintegrate easily. The release of the binding energy holds the protons and neutrons together and makes the nuclei more stable than their atomic particle constituents. Actually, the higher the binding energy per nucleon, the more stable an atom is. For the atoms of $_8O^{16}$ and $_{92}U^{235}$ the binding energy per nucleon is, 7.97 MeV and 7.59 MeV, respectively. This partly explains why the oxygen-16 atom is more stable than the uranium-235 atom.

4.1.3 Nuclear Reactions and Energy Released

There is a plethora of nuclear reactions that take place naturally or artificially. Of relevance to the energy production processes are nuclear reactions that are caused by light subatomic particles, such as neutrons, protons and electrons. An example of an artificial nuclear reaction is the interaction of boron-10 with neutrons, which results in the formation of helium-4 and lithium-7:

$$_5B^{10} + {}_0n^1 \rightarrow {}_3Li^7 + {}_2He^4. \tag{4.2}$$

The nuclei of $_2He^4$ atoms are also called *alpha particles.* Electrons, which are denoted by the symbol $_{-1}e^0$, are called *beta particles* and strong electromagnetic waves are called *gamma particles.* When these three are considered as sources of radiation, one may call them *α-radiation*, *β-radiation* and *γ-radiation*.

A great deal of electromagnetic radiation, or gamma particles, is released in a typical nuclear reaction, because newly formed atoms are at an *excited state,* where the energy level is higher than normal. As these atoms decay to their lower energy level, strong electromagnetic radiation is released, which is rapidly converted to heat in the nuclear reactor.

All nuclear reactions are governed by four fundamental laws of physics:

1. *Nucleon conservation:* The total number of nucleons in the two sides of the reaction is the same. Protons may be converted to neutrons and vise versa by the absorption or emission of an electron. This implies that the sum of the right-hand superscripts in nuclear reactions is the same in both sides. In the example of Eq. (4.2): $10 + 1 = 7 + 4$.
2. *Charge conservation:* The sum of the charges of all nuclear particles in the two sides of the reaction is the same. This implies that the sums of the left-hand subscripts in the two sides of nuclear reactions are equal. In the example of Eq. (4.2): $5 + 0 = 3 + 2$.
3. *Momentum conservation:* The momentum of the particles before and after the reaction is the same because there are no external forces acting on the particles during the reaction.
4. *Mass-energy conservation:* Any mass defect during the reaction is accompanied by a release of energy. The sum of mass plus the mass equivalent of energy, according to Eq. (4.1), applies to the two sides of a nuclear reaction. In nuclear reactions the energy balance (or First Law of Thermodynamics) also includes the energy equivalent of mass.

In the case of the nuclear reaction shown in Eq. (4.2), the masses of the nuclei ${}_5B^{10}$, ${}_3Li^7$ and ${}_2He^4$ are 10.01294 u, 7.01601 u and 4.00260 u respectively, while the mass of the neutron, ${}_0n^1$, is 1.008665 u. Therefore, the reaction results in a mass defect of: $(10.01294 + 1.008665 - 7.01601 - 4.00260) = 0.002995$ u. This is equivalent to an energy release of 2.793 MeV. While this may appear to be an insignificant amount of energy, it must be recalled that it corresponds to only the conversion of a single atom and that the number of atoms corresponding to a macroscopically measured quantity of boron-10 is of the order of the Avogadro number, $6.023*10^{23}$ atoms per gram-atom. Therefore, when a mass of 50 g, or 5 gram-atoms, of boron-10 reacts according to Eq. (4.2) it would release an amount of energy equal to $8.411*10^{24}$ MeV or $1.347*10^{12}$ J. This is equivalent to the amount of energy released by 46.5 metric tons of bituminous coal.

The examples of the conversion of boron-10 and the fact that the conversion of an entire kg of any substance releases energy equivalent to the energy released by the combustion of 3,100,000 metric tons of bituminous coal, give an idea of the tremendous magnitudes of energy that may be released by nuclear reactions. Simply put, nuclear reactions release tremendous amounts of energy that are by far higher than the energy released by other primary energy sources. The reason for these high amounts of energy is the very large number of atoms in a gram-atom, $6.023*10^{23}$. When even small amounts of mass undergo nuclear reactions, the thermal energy released by far exceeds the chemical energy or any other form of energy stored in the mass. Nuclear reactions release very large amounts of energy, and for this reason, if properly and safely harnessed, they can play a very important role in meeting the energy demand of humankind.

4.1.4 Radioactivity

Every chemical element has a number of isotopes. Of these, some are stable and are naturally occurring, while other isotopes are unstable and undergo spontaneous nuclear transformations to become different elements. In general, stable isotopes have approximately the same number of protons and neutrons. When there is a significant imbalance in the numbers of protons and neutrons in the nucleus, the atom is unstable and may undergo a spontaneous transformation to become the nucleus of another element. This spontaneous transformation process is called *radioactivity*. In nuclear terminology, the original nucleus is often referred to as the *parent* nucleus and the product of the transformation as the *daughter* nucleus. The unit of radioactivity is one transformation per second or 1 Becquerel (1 Bq). Oftentimes the unit 1 Curie (1 Ci) is used, with 1 Ci = 3.7×10^{10} Bq. One Ci represents the radioactivity of one gram of radium-226.[2]

The types of transformations that occur with unstable nuclei may be categorized as follows:

1. *Alpha decay*: During this transformation, an alpha particle ($_2He^4$) is emitted by the nucleus. Alpha decay typically occurs with the heavier nuclei that have atomic numbers higher than 82. An example of alpha decay is the conversion of Plutonium-239 to Uranium-235:

$$_{94}Pu^{239} \rightarrow {}_{92}U^{235} + {}_2He^4. \tag{4.3}$$

   It is apparent that, the daughter nucleus, which is created in this transformation, has an atomic mass four units less than the parent nucleus and am atomic number two units less than the parent nucleus. Typically, the daughter nucleus will undergo a type of decay too and this will continue until the last daughter nucleus is a stable isotope. Equation (4.8) at the end of this section shows the long line of transformations, through which uranium-238 decays to the stable lead-206 isotope.
2. *Beta decay*: This is essentially the transformation of a neutron inside a nucleus to form a proton, with an electron being produced simultaneously. This type of radioactive decay occurs in nuclei where the number of neutrons is significantly high in comparison to the number of protons. A very small particle, which is

[2] Radium-226 and the phenomenon of radioactivity were discovered by Marie and Pierre Curie, who were students of Professor Henri Becquerel at the Grande Ecole de la Physique et Chemie Industriele, in Paris. The three shared the Nobel Prize in Physics in 1903.

called a neutrino and denoted by v, accompanies the emission of the electron as in the following example of the conversion of indium-115 to tin-115:

$$_{49}In^{115} \rightarrow {}_{50}Sn^{115} + {}_{-1}e^{0} + v. \tag{4.4}$$

The parent and daughter nuclei of the beta decay have the same mass number. The neutrino is lighter than the electron, interacts weakly with matter and may penetrate very thick layers of materials. Its presence in the nuclear reactions serves to satisfy the momentum conservation and mass-energy conservation principles.

3. *Positron emission decay*: A positron is a light particle of the same mass as the electron, but has positive charge and is denoted by $_{+1}e^{0}$. In general, positron decay occurs when the nucleus has too many protons in comparison to the number of neutrons. Again, the parent and daughter nuclei have the same mass number as in the following example of the conversion of iron-53 to manganese-53.

$$_{26}Fe^{53} \rightarrow {}_{25}Mn^{53} + {}_{+1}e^{0} + v. \tag{4.5}$$

The emitted positron is a short-lived particle among the plethora of electrons that surround the nucleus. Positrons are annihilated by combining with electrons and produce two gamma particles, which are very strong electromagnetic radiation:

$$_{+1}e^{0} + {}_{-1}e^{0} \rightarrow 2\gamma. \tag{4.6}$$

The combined energy of the two gamma particles is equivalent to the combined mass of the annihilated electron and positron. In this case, the equation $E = mc^2$ shows that 1.02 MeV of electromagnetic energy is produced from the reaction of Eq. (4.6).

4. *K-capture*: During this transformation, an electron from the K orbit is captured by the nucleus. The result is the transformation of a proton to a neutron. Simultaneously, a neutrino is produced as well as a significant amount of electromagnetic radiation in the form of gamma particles.

$$_{4}Be^{7} + {}_{-1}e^{0} \rightarrow {}_{3}Li^{7} + v + \gamma. \tag{4.7}$$

Oftentimes the daughter nucleus is unstable and transforms to another daughter nucleus. Thus, a parent nucleus may cause a series of transformations, which is called a *radioactive decay series.* It is typical of the trans-uranium radioactive elements, with $Z > 92$, to decay and produce a variety of daughter elements. The last element in this radioactive decay series is usually lead-206 or another stable isotope. Equation (4.8) shows the radioactive decay series of uranium-238 as a series of alpha and beta decay processes, which involve several isotopes of thorium (Th), protactinium (Pa), radium (Ra), radon (Rn), polonium (Po) and bismuth (Bi).

$$
\begin{aligned}
{}_{92}U^{238} &\rightarrow {}_{90}Th^{234} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{90}Th^{234} &\rightarrow {}_{91}Pa^{234} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{91}Pa^{234} &\rightarrow {}_{92}U^{234} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{92}U^{234} &\rightarrow {}_{90}Th^{230} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{90}Th^{230} &\rightarrow {}_{88}Ra^{226} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{88}Ra^{230} &\rightarrow {}_{86}Rn^{222} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{86}Rn^{222} &\rightarrow {}_{84}Po^{218} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{84}Po^{218} &\rightarrow {}_{82}Pb^{214} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{82}Pb^{214} &\rightarrow {}_{83}Bi^{214} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{83}Bi^{214} &\rightarrow {}_{84}Po^{214} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{84}Po^{214} &\rightarrow {}_{82}Pb^{210} + {}_{2}He^{4} \quad , \quad alpha\,decay \\
{}_{82}Pb^{210} &\rightarrow {}_{83}Bi^{210} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{83}Bi^{210} &\rightarrow {}_{84}Po^{210} + {}_{-1}e^{0} \quad , \quad beta\,decay \\
{}_{84}Po^{210} &\rightarrow {}_{82}Pb^{206} + {}_{2}He^{4} \quad , \quad alpha\,decay.
\end{aligned}
\tag{4.8}
$$

It must be noted that radioactivity is a natural phenomenon and occurs naturally regardless of the man-made nuclear reactors and atomic weapons. There are several radioactive elements in the lithosphere, hydrosphere and atmosphere that contribute to the *natural* or *baseline radioactivity*. Cosmic rays also carry radioactive isotopes and radioactive particles in the earth's atmosphere. Of the naturally occurring radioactive isotopes several are absorbed by humans and animals and become part of their metabolism. For example, potassium-40 exists in the human body and contributes approximately 0.1 μCi of radioactivity or 3,700 transformations per second (3,700 Bq).

4.1.5 Rate of Radioactive Decay: Half Life

The decay of nuclei is a random process. A macroscopic sample of a radioactive isotope consists of a very large number of nuclei and, hence, the decay of the sample may be only described statistically. This implies that one does not know deterministically which atoms will decay within a given time interval or when an individual atom will decay. However, one is certain that, out of a large assembly of atoms, N, a number of them, δN, will decay within the time interval δt. For a large sample of atoms, the rate of decay, λ, is the proportionality constant, which defines the decay process:

$$dN = -\lambda N dt \Rightarrow \lambda = -\frac{1}{N}\frac{dN}{dt}, \tag{4.9}$$

where N is the number of nuclei in the sample, t is the time, and the negative sign denotes that the number of the original nuclei of the isotope decreases. The rate of decay, λ, characterizes an isotope and is a unique parameter for that isotope. Given an initial condition, the last equation may be easily integrated. For example if at $t = 0$ the number of nuclei in the sample is N_0, then at time t the number of remaining nuclei, N, is:

$$N = N_0 e^{-\lambda t}. \tag{4.10}$$

The *half life* of an isotope is also a commonly used parameter to characterize the decay process. The half life, $T_{1/2}$, is defined as the time for the number of nuclei to be reduced to half the original number, or:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}. \tag{4.11}$$

A comparison of Eqs. (4.10) and (4.11) yields the following relationship between the half life and the rate of decay:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}, \tag{4.12}$$

and for the number of the remaining nuclei after time t:

$$N = N_0 \exp\left(-\frac{0.693t}{T_{1/2}}\right). \tag{4.13}$$

Table 4.2 shows the half lives of several common isotopes. It is apparent, that the half lives span several orders of magnitude and range from millions of years to a few seconds. A glance at Eq. (4.13) proves that isotopes with small half lives are depleted rapidly, while isotopes with very long half lives are depleted very slowly. As a consequence, during a finite amount of time, which is much less than the half-life of an isotope, $t \ll T_{1/2}$, the concentration of this isotope may be considered to be constant.

Equations, such as (4.10) and (4.13) may only be interpreted statistically and only when the sample of the material contains a very large number of nuclei. They do not predict which nuclei will disintegrate, but how many nuclei out of a very large sample. These equations have been validated and are considered very accurate in predicting the total number of radioactive nuclei of a given isotope or the total mass of the isotope in a large sample.

It is apparent from Eq. (4.8) that many radioactive processes have several stages and one parent nucleus produces several daughter nuclei. Let us consider such a chain of decay of three radioactive elements: A → B → C with corresponding decay rates λ_A, λ_B *and* λ_C and initial number of nuclei N_{A0}, 0 and 0 respectively. Equation (4.10) yields the amount of isotope A present after a time t is:

Table 4.2 The half life and radioactivity emitted by some common isotopes

Isotope	Half life	Radioactivity
Uranium-235	7.1×10^8 yrs	α, γ
Uranium-238	4.51×10^9 yrs	α, γ
Plutonium-239	2.44×10^4 yrs	α, γ
Thorium-232	1.41×10^{10} yrs	α, γ
Krypron-87	76 min	β
Strontium-90	28.1 yrs	β
Barium-139	82.9 min	β, γ

$$N_A = N_{A0} e^{-\lambda_A t}. \tag{4.14}$$

For isotope B we may stipulate that the rate of change of its nuclei is equal to the rate of the decay of A (which results in the formation of B) minus the decay rate of its own. This yields the following differential equation:

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B = \lambda_A N_{A0} e^{-\lambda_A t} - \lambda_B N_B. \tag{4.15}$$

Given the initial condition $N_{B0} = 0$, the solution of the last equation is:

$$N_B = N_{A0} \frac{\lambda_A}{\lambda_B - \lambda_A} \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right). \tag{4.16}$$

Similarly, the rate of change of the nuclei of isotope C is:

$$\frac{dN_C}{dt} = \lambda_B N_B - \lambda_C N_C = \lambda_B N_{A0} \frac{\lambda_A}{\lambda_B - \lambda_A} \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right) - \lambda_C N_C. \tag{4.17}$$

The solution of this differential equation, subject to the initial condition, $N_{C0} = 0$, would yield the number of nuclei of isotope C as a function of time. If the rate of decay of isotope C is very low in comparison to the rate of decay of B, that is if $\lambda_C << \lambda_B$, an approximate solution to this equation is:

$$\mathrm{N_C} \approx \mathrm{N_{A0}} \left(1 - \frac{\lambda_\mathrm{B}}{\lambda_\mathrm{B} - \lambda_\mathrm{A}} e^{-\lambda_\mathrm{A} \mathrm{t}} + \frac{\lambda_\mathrm{A}}{\lambda_\mathrm{B} - \lambda_\mathrm{A}} e^{-\lambda_\mathrm{B} \mathrm{t}} \right). \tag{4.18}$$

This equation becomes exact if the isotope C is stable.

Figure 4.2 shows the variation of N_A, N_B and N_C when the half lives of the isotopes A, B and C are 2 years, 0.5 years and 100 years respectively. It is apparent from this figure that the fraction of isotope B is significantly lower than the fractions of both A and C because the half life of this isotope is lower than that of the other two isotopes. At the end of the ten year period, isotopes A and B have almost completely decayed and converted to isotope C, which has significantly higher half life.

An interesting application of radioactive decay is *carbon dating,* or the determination of the age of a specimen that was produced by a plant or an animal. The atmospheric nitrogen interacts with cosmic rays and produces the isotope ${}_6C^{14}$,

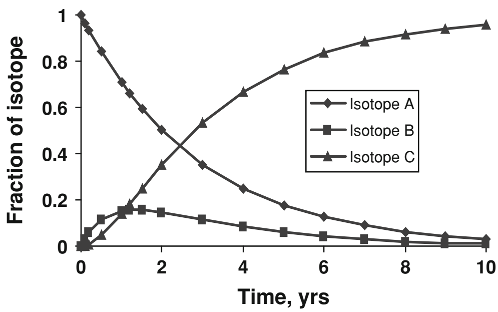


Fig. 4.2 Variation of the mass of isotopes A, B and C vs. time

which is radioactive with a half life of 5,730 yrs. This isotope is chemically converted to carbon dioxide and diffuses in the atmosphere. At any time, 0.1% of the carbon in the atmospheric CO_2 is composed of the isotope ${}_6C^{14}$. CO_2 is typically absorbed by plants and through the food chain it is also absorbed by animals. Therefore, at any time, 0.1% of the carbon atoms in living plants and animals is composed of ${}_6C^{14}$. When the absorption of ${}_6C^{14}$ ceases, due to the harvesting of the plant or the death of the animal, the fraction of this isotope starts decreasing by radioactive decay. The application of Eq. (4.14) yields the time that has elapsed from the harvesting of the plant, which produced the specimen. Following this technique, we may estimate the age of a manuscript written on an old papyrus, in which the current carbon-14 content was determined to be 0.072%. According to Eq. (4.13) the age of the manuscript, t, in years is given by the expression: $0.00072 = 0.001*\exp(-0.693t/5730)$, or $t = 2{,}716$ years.

4.2 Nuclear Fission

The neutron was discovered in 1932 by Chadwick in England. Because it is an electrically neutral particle it is not repelled by the charge of the electrons or the nuclei and, thus, it may penetrate the atoms and interact directly with the nuclei. In 1938 Hahn and Strassmann in Germany observed that barium-139 was produced when uranium-235 interacted with a beam of neutrons. This was the first demonstration of a fission process. Fission occurs when neutrons are captured by heavy nuclei, such as ${}_{92}U^{235}$, ${}_{92}U^{238}$, or ${}_{92}Th^{232}$. The resulting compound nucleus is unstable and soon splits into two large fragments. A few—typically two or three—free neutrons are also released in the process. A typical fission process with uranium 235 as the fuel is shown schematically in Fig. 4.3. The four stages of this process show how the ${}_{92}U^{235}$, produces the unstable nucleus ${}_{92}U^{236}$, which almost immediately splits into ${}_{57}La^{147}$ and ${}_{35}Br^{87}$ plus two free neutrons according to the nuclear reaction:

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow {}_{57}La^{147} + {}_{35}Br^{87} + 2{}_0n^1. \qquad (4.19a)$$

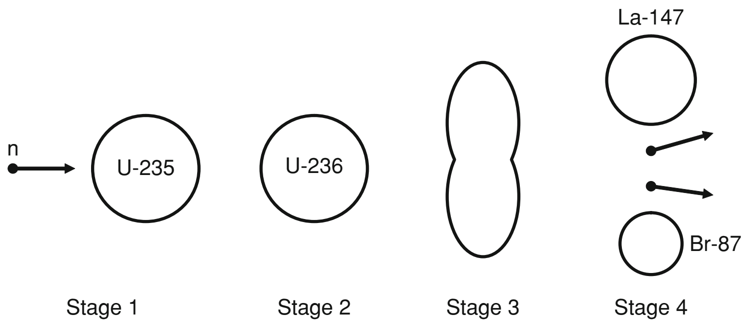


Fig. 4.3 Fission process of ${}_{92}U^{235}$ by neutrons

Another typical reaction of the ${}_{92}\mathrm{U}^{235}$ nucleus produces xenon-140 and strontium-94:

$$ {}_{92}\mathrm{U}^{235} + {}_{0}\mathrm{n}^{1} \rightarrow {}_{92}\mathrm{U}^{236} \rightarrow {}_{54}\mathrm{Xe}^{140} + {}_{38}\mathrm{Sr}^{94} + 2{}_{0}\mathrm{n}^{1}. \tag{4.19b} $$

Plutonium-239, which may be produced by the capture of a neutron by the ${}_{92}\mathrm{U}^{238}$ nucleus, may also undergo fission in several ways. One such fission reaction is:

$$ {}_{94}\mathrm{Pu}^{239} + {}_{0}\mathrm{n}^{1} \rightarrow {}_{56}\mathrm{Ba}^{137} + {}_{38}\mathrm{Sr}^{100} + 3{}_{0}\mathrm{n}^{1}. \tag{4.19c} $$

Conventional nuclear reactors use uranium, which contains both isotopes ${}_{92}\mathrm{U}^{235}$ and ${}_{92}\mathrm{U}^{239}$, as their fuel. Small amounts of plutonium-239 are always formed in conventional reactors and contribute to the thermal energy produced and the neutron flux. The so called *fast breeder reactors* produce larger amounts of ${}_{94}\mathrm{Pu}^{239}$, which later becomes the main fuel of the reactor, as will be explained in more detail in Sect. 4.3.

4.2.1 Interactions of Neutrons with Nuclei

When neutrons penetrate atoms and interact with the nuclei, three types of interactions are possible:

1. *Scattering collisions,* which may be elastic or inelastic. During inelastic scattering collisions, a neutron loses some of its energy to surrounding atoms and slows down. The slowing down of fast neutrons is necessary for the maintenance of the chain reaction in the commonly used reactors. Fast neutrons undergo inelastic collisions and progressively slow down. When the neutrons are in thermal equilibrium with the surrounding nuclei, they are called *thermal neutrons,* and their scattering collisions are considered elastic. It must be noted

that, after a scattering collision, elastic or inelastic, the neutron remains in the reactor and may interact again with nuclei.

2. *Capture.* During capture, a neutron enters and remains in the nucleus. In this case a relatively stable compound nucleus is formed, which has one extra neutron. The compound nucleus may be radioactive and may decay to form another nucleus, as in the following reaction of the production of ${}_{94}\mathrm{Pu}^{239}$ from the neutron capture of ${}_{92}\mathrm{U}^{238}$:

$$\begin{aligned}
&{}_0\mathrm{n}^1 + {}_{92}\mathrm{U}^{238} \rightarrow {}_{92}\mathrm{U}^{239} \\
&{}_{92}\mathrm{U}^{239} \rightarrow {}_{93}\mathrm{Np}^{239} + {}_{-1}\mathrm{e}^0 \\
&{}_{93}\mathrm{Np}^{239} \rightarrow {}_{94}\mathrm{Pu}^{239} + {}_{-1}\mathrm{e}^0.
\end{aligned} \tag{4.20}$$

3. *Fission.* The fission of a nucleus results from the capture of a neutron and the formation of an unstable, bigger nucleus, which immediately splits into two larger fragments and a few neutrons, as shown in Eqs. (4.19a) to (4.19c) and depicted schematically in Fig. 4.3. The produced neutrons may cause further fissions.

The neutron-nuclei interactions of capture and fission are often called *absorption,* because the neutron is initially absorbed by the nucleus. During scattering the neutron does not enter the nucleus and is immediately rejected. After the scattering process the neutron is available to interact with other nuclei and to cause fission.

For the modeling of the interactions of neutrons with bigger nuclei, let us assume that a uniform beam of neutrons is incident upon a slab of a material with area A, where the density of the atoms is N. At a depth z inside the material the flux of neutrons is Φ and at a depth $z + dz$ the flux is $\Phi - d\Phi$, as shown in Fig. 4.4. The amount of nuclei in the volume Adz, with whom the neutrons may interact is $NAdz$. The number of interactions occurring in this volume, dI, must be proportional to the number of nuclei and the intensity of the beam:

$$dI = \sigma \Phi N A dz. \tag{4.21}$$

The constant of proportionality, σ, has the dimensions of area and is called the *microscopic cross-section* of the nucleus, or simply, the *cross section* of the nucleus. This rate of interaction is equal to the difference of the neutron flux from z to $z + dz$, and is equal to the product of $d\Phi$ and the area of the material, A:

$$\sigma \Phi N A dz = -A d\Phi. \tag{4.22}$$

The negative sign in Eq. (4.22) indicates the diminishing neutron flux as it penetrates the material of the slab. The solution of the resulting differential equation, with the boundary condition $\Phi(z = 0) = \Phi_0$, yields the following expression for the strength of the neutron flux at depth z:

$$\Phi = \Phi_0 e^{-\sigma N z}. \tag{4.23}$$

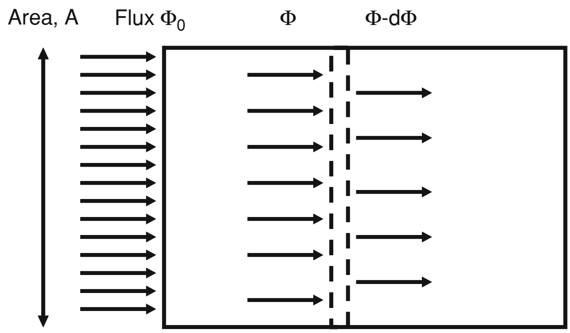


Fig. 4.4 Schematic diagram of the interaction of a flux of neutrons with a material

4.2.2 Cross Sections of Common Nuclei

The cross-section of an isotope is a unique parameter of that isotope. It has the units of area and its numerical value is of the order of magnitude of the actual cross section of a stationary nucleus. The radius of the nuclei is approximately equal to $r_n = 1.3*10^{-13}A^{1/3}$ cm, which implies that the actual cross sectional area of the nucleus of a heavy atom, such as ${}_{92}U^{238}$ or ${}_{92}U^{235}$, is approximately $2*10^{-24}$ cm^2. For this reason cross sections are expressed in units of 10^{-24} cm^2, which are called *barns*. One barn is equal to 10^{-24} cm^2 or 10^{-28} m^2.

The neutron-nucleus interaction is a complex phenomenon, where quantum mechanical interactions play an important role. The cross-section is an average phenomenological parameter that takes into account these interactions and expresses them in terms of parameters that are understood from concepts of classical mechanics. The cross-section is, essentially, the "target area" of the nucleus as seen by the neutron, which is the projectile. Therefore, neutrons have high probability of interacting with nuclei of large cross sections and lower probability to interact with nuclei of smaller cross sections. However, because of the quantum mechanical effects, the area of this "target" depends strongly on the characteristics of the nucleus, the type of interactions that occur, and also on the kinetic energy of the neutrons, which are the "projectiles." There are different cross sections for inelastic scattering, σ_i, capture, σ_c and fission, σ_f. These parameters are functions of the energy of the incident neutrons.

Figures 4.5a and b depict the fission and capture cross sections of ${}_{92}U^{238}$ and ${}_{92}U^{235}$ as functions of the neutron energy. It is observed in the first figure that for the nucleus of ${}_{92}U^{238}$ the fission cross section, σ_f, is approximately 0.6 when the neutron energy is higher than 2 MeV and drops to negligible values at neutron energies less than 1 MeV. The capture cross section of this isotope, σ_c, increases from very low values at 5 MeV, to approximately 1 barn at 1,000 eV. At lower neutron energies, there is a *resonance range* that extends from 1,000 eV to 5 eV, where the capture cross section may reach values as high as 1,000 barn. At electron energies below 2 eV, the capture cross section is approximately constant, with values in the range 10–13 barn. This has significant implications for the operation of a thermal nuclear reactor, where the neutrons must be slowed down to very low energy values (they become thermal neutrons) in order to cause fission. As the neutrons are progressively slowed to lower kinetic energies there is high probability that, in the *resonance range* of 1,000 eV > E > 5 eV, these neutrons will be captured by a $_{92}U^{238}$ nucleus. As a consequence, the captured neutrons will not become available to cause fissions and to produce energy.

Figure 4.5b, which depicts the cross sections for capture and fission of the isotope $_{92}U^{235}$, shows a different dependence of the cross sections on the energies of incident neutrons: At first, the fission cross section of this isotope is significantly higher than the capture cross section in the entire range of neutron kinetic energies. The cross section for fission of this isotope, σ_f, is less than 2 barn in the range of kinetic energy E > 10^5 eV and, thereafter increases at a faster rate to the value 35 barns at E = 80 eV. After a *resonance range* that extends from 80 to 4 eV, and where the resonance cross section values may reach up to 1,000 barn, the fission cross section of $_{92}U^{235}$ continues its monotonic increase to values close to 6,000 barn at 0.001 eV. Secondly, the capture cross section of $_{92}U^{235}$, σ_c, is negligibly small at energies E > 1 eV and increases almost monotonically at lower energies. However, in the whole range of neutron energy, the values of σ_c are at least one order of magnitude lower than σ_f. Therefore, in pure $_{92}U^{235}$, the fission has a significantly higher probability of occurrence than capture. It must be noted that the composition of natural uranium is far from this: natural uranium consists of 99.285% $_{92}U^{238}$ and 0.715% $_{92}U^{235}$. That is, in a given mass of natural uranium there is only 1 nucleus of $_{92}U^{235}$ for every 139 nuclei of $_{92}U^{238}$. Therefore, a neutron in natural uranium is more likely to be captured by a $_{92}U^{238}$ nucleus than to cause the fission of a $_{92}U^{235}$ nucleus. The data for the production of Fig. 4.5a and b were obtained from Bennet [1].

4.2.3 Neutron Energies: Thermal Neutrons

Neutrons are released from nuclear reactions at very high average velocities and, hence very high kinetic energies. For fission-released neutrons typical values of their kinetic energy are close to 2 MeV (two million eV). Because of the complex quantum mechanical interactions between neutrons and nuclei, the kinetic energy of neutrons plays a very important role in determining whether neutrons will cause fissions or what kind of interactions will occur between the neutrons and nuclei. For this reason, neutrons are classified according to their average energies as follows:

1. Fast neutrons with E > 10^5 eV. When neutrons are first released from fission reactions, they are fast neutrons.
2. Intermediate neutrons with 1 eV < E < 10^5 eV.
3. Slow neutrons with E < 1 eV.

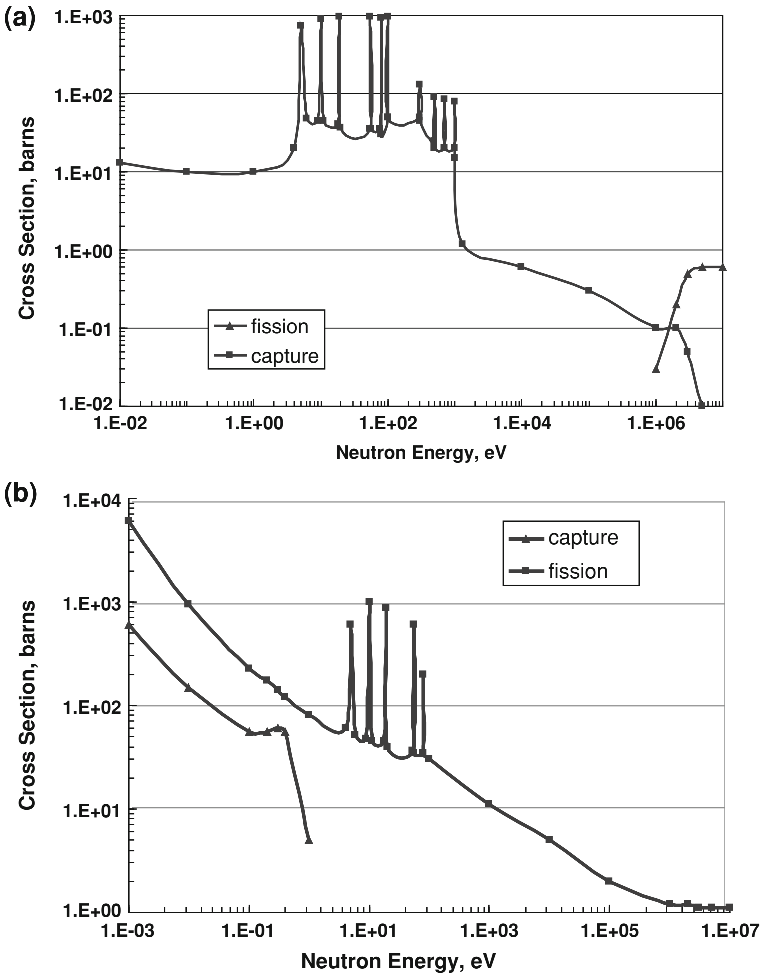


Fig. 4.5 **a** Fission and capture cross-section for uranium-238 **b** Fission and capture cross-section for uranium-235. Data from [1]

The *thermal neutrons* are a category of slow neutrons, which is important in nuclear reactors because they have the right amount of energy to cause a chain reaction of uranium-235 nuclei in nuclear reactors. These neutrons are in thermal equilibrium with the surrounding atoms in a reactor. The state of thermal neutrons is analogous to the state of gas molecules in a container: The molecules of the gas collide with the atoms at the walls of the container and the molecular velocities are significantly high. While individual gas molecules are slowed down or are accelerated by the collisions, there is a dynamic equilibrium of the molecular velocities, which exhibit a distinct distribution, known as the Maxwell-Boltzmann distribution. Similarly, there is a dynamic equilibrium of the velocities, and kinetic

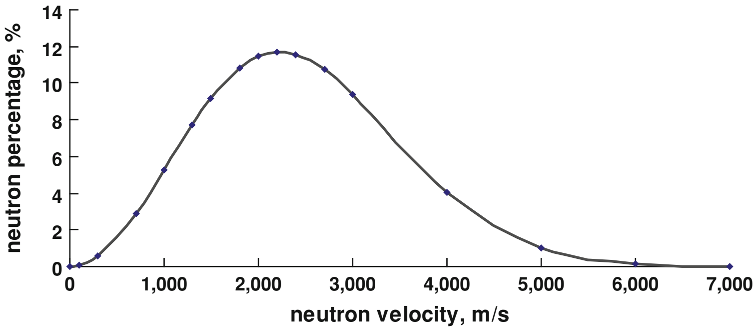


Fig. 4.6 The Maxwell-Boltzmann distribution for thermal neutrons

energies of thermal neutrons enclosed in any material. These neutrons also have a wide range of velocities, which is approximated by the classical Maxwell-Boltzmann distribution:

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(\frac{-mv^2}{2kT}\right)v^2 dv, \tag{4.24}$$

where $n(v)dv$ is the number of neutrons in the range of velocities v and $v + dv$; N is the total number of neutrons; m is the mass of a single neutron, $1.6748 * 10^{-27}$ kg; k is the Boltzmann constant, $1.38 * 10^{-23}$ J/K; and T the absolute temperature, in K. A diagram of the Maxwell-Boltzmann distribution, for neutrons at 300 K is shown in Fig. 4.6. It is observed that the distribution has a maximum, which denotes the most probable velocity of the neutrons. The latter may be calculated by differentiating the distribution function and equating the result to zero. Thus, the most probable velocity and the most probable energy of the thermal neutrons are:

$$v_{mp} = \sqrt{\frac{2kT}{m}}, \quad E_{mp} = kT. \tag{4.25}$$

Similarly, the average velocity of the neutrons, v_{av}, and the average energy, E_{av}, may be calculated from the Maxwell-Boltzmann distribution by integrating the function $vn(v)$ and dividing by the total number of neutrons, N, to yield:

$$v_{av} = \sqrt{\frac{8kT}{\pi m}} = 1.128 v_{mp}, \quad E_{av} = \frac{4kT}{\pi}. \tag{4.26}$$

It must be noted that all general references to neutron velocities refer to the average velocity and all references to neutron energy refer to the kinetic energy that corresponds to the average velocity. The energy of thermal neutrons is 0.025 eV, their average velocity is 2,482 m/s and their most probable velocity is approximately 2,200 m/s.

Table 4.3 shows the values of the most significant cross sections of pure ${}_{92}U^{235}$, pure ${}_{92}U^{238}$, and natural uranium for fast, intermediate and thermal neutrons (data

Table 4.3 Neutron cross sections, in barns, for uranium isotopes

Neutron Energy		$_{92}U^{235}$	$_{92}U^{238}$	Natural U
Fast, 2 MeV	σ_c	0.09	0.14	0.14
	σ_f	1.20	0.018	0.026
	σ_i	2.87	2.3	2.84
Intermediate, 0.3 MeV	σ_c	0.15	0.2	0.2
	σ_f	1.3	0.01	0.02
	σ_i	0.5	0.7	0.7
Thermal, 0.025 eV	σ_c	101	2.75	3.47
	σ_f	577	0	4.16
	σ_i	10	8.3	8.4

from [1]). It is observed in this Table that, since natural uranium is composed mostly of the $_{92}U^{238}$ isotope, its pertinent cross-sections are very close to those of $_{92}U^{238}$.

4.2.4 The Chain Reaction: Probability of Fission

There is no naturally occurring flux of neutrons to cause fissions in a nuclear reactor. The few neutrons that may be produced during the natural radioactive decay of isotopes are not sufficient to maintain the desired reaction rate in a nuclear reactor. The neutron flux, which is crucial for fissions must be artificially created and maintained. Typical nuclear reactors are fueled with nuclear material once every 18 to 24 months and remain hermetically closed. These reactors produce approximately 3,000 MW. Given that a single fission reaction produces approximately 200 MeV of energy, the rate of fission reactions in the nuclear reactors is approximately $9.4*10^{19}$ fissions per second. As a consequence, these nuclear reactions use $9.4*10^{19}$ neutrons per second and these neutrons must be produced continuously inside the reactor.

At the beginning of the refueling process, a neutron source is introduced in the reactor, which produces the first flux of neutrons and acts as the spark for the "ignition" of the fission reactions. A common method for starting the neutron flux is the introduction of a small amount of radium-226, which disintegrates producing alpha particles, and beryllium-9. The alpha particles ($_2He^4$), which are produced during the radioactive decay of radium, are captured by the beryllium to produce cabon-12, a stable nucleus, and a free neutron according to the reaction:

$$_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1. \tag{4.27}$$

A mixture of 1 g of $_{88}Ra^{226}$ and 5 g of $_4Be^9$ constitutes a compact neutron source that provides approximately 10^7 neutrons per second. While this flux may be sufficient for the "ignition" of the reactor, it is simply not sufficient to maintain for long the operation of a typical reactor. For this reason, nuclear reactors are

Table 4.4 Average number of neutrons emitted per fission

Isotope	Neutron energy, eV	n_{av}
$_{92}U^{235}$	0.025	2.44
	1,000,000	2.50
$_{94}Pu^{239}$	0.025	2.87
	1,000,000	3.02
$_{92}U^{233}$	0.025	2.48
	1,000,000	2.55
$_{90}Th^{232}$	1,500,000	2.12
$_{92}U^{238}$	1,100,000	2.46

designed to produce neutrons and maintain a high flux of neutrons by a process called the *chain reaction.*

During the chain reaction the neutron source causes the first fission reactions in the nuclear fuel. As it is apparent from Eqs. (4.19a) typical fission reactions produce a few neutrons (two to three), which are called the first generation neutrons. Some of these neutrons may be used to cause further fissions, which would produce more neutrons, the second generation of neutrons. A fraction of the latter neutrons may cause a third wave of fissions and produce another generation of neutrons and so on. The necessary flux of neutrons is maintained in the nuclear reactor by the neutrons produced during the fissions that took place previously. Figure 4.7 shows schematically the chain reaction. A key parameter of the chain reaction is the *reproduction constant, k*, defined as the number of neutrons in a generation divided by the number of neutrons in the preceding generation. When $k = 1$, the chain reaction is maintained. When $k < 1$, the neutron flux and, hence, the fission reaction rate is decreasing and, if continued, it will lead to the extinction of the fission reactions. When $k > 1$, the neutron flux increases, the fission reaction rate increases and, if sustained, it may lead to an explosive situation. In the last two cases the reactor operators must take action to maintain the controlled chain reaction with $k = 1$. Another important parameter is the average number of neutrons produced by the fission of the isotopes that fuel the reactor. Table 4.4 shows the average number of neutrons emitted per fission reaction, n_{av}, for several isotopes that are commonly used as reactor fuels [2].

While it is rather easy to visualize the chain reaction, its practical realization in fuels other than pure $_{92}U^{235}$ poses several difficulties. The main problem for the establishment of a chain reaction is that when neutrons are produced from a fission reaction they are fast neutrons with energies approximately 2 MeV. A glance at the cross sections of fast neutrons, in Table 4.4, proves that the neutrons produced in each generation are more likely to undergo inelastic scattering or capture than to cause fissions. A concept that is helpful in the understanding of the likelihood of chain reaction realization is the *probability of fission,* which combines the concentration of the various nuclei in the fuel with the cross sections. For a fuel composed of the isotopes $_{92}U^{235}$ and $_{92}U^{238}$, such as natural or enriched uranium, the probability of fission, P_f, is defined by the expression:

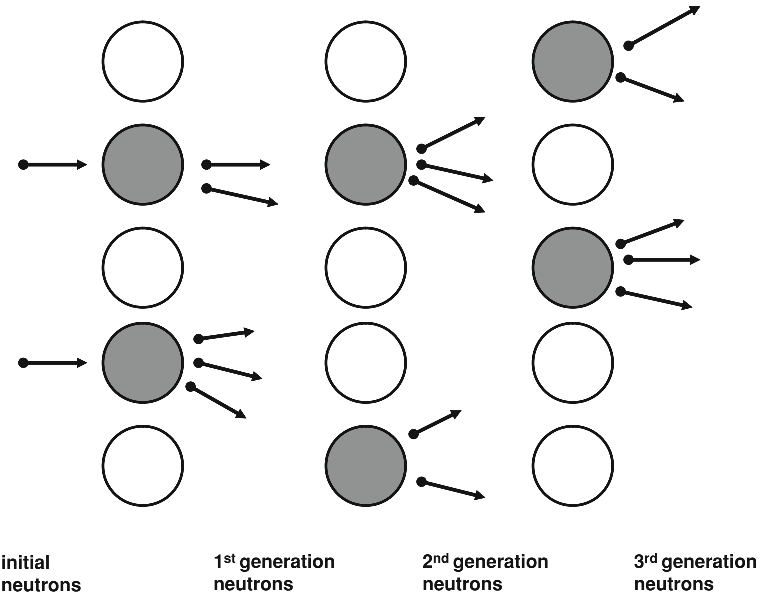


Fig. 4.7 A chain reaction with $k = 1$: Nuclei in grey undergo fission and produce neutrons. Neutrons that do not cause fissions are absorbed by other nuclei or leak outside the reactor

$$P_f = \frac{c_{238}\sigma_f^{238} + c_{235}\sigma_f^{235}}{c_{238}\left(\sigma_f^{238} + \sigma_c^{238} + \sigma_i^{238}\right) + c_{235}\left(\sigma_f^{235} + \sigma_c^{235} + \sigma_i^{235}\right)}, \tag{4.28}$$

where c denotes the concentration of the corresponding nuclei in the bulk of the reactor fuel and the superscripts in the several cross sections denote the pertinent isotopes of uranium. Several values of these cross sections are in Table 4.3 as well as in Figs. 4.5a and b. The probability of fission yields the probability that a given neutron, which is produced in generation j, will cause a fission reaction and, thus, contribute to the production of neutrons in the generation $j + 1$. Since there are n_{av} such neutrons produced by the nuclei in the fuel, and one neutron is needed to cause a fission, a chain reaction will be maintained only if the product $n_{av}{*}\ P_f$ is greater than one. In the opposite case, if $n_{av}{*}\ P_f < 1$, the number of neutrons in the reactor will diminish after each generation and the reactor would cease to operate.

Let us consider the possibility of establishing a chain reaction in a system that is composed entirely of natural uranium, where the concentration of isotope 238 is 0.99285 and the concentration of isotope 235 is 0.00715. When the neutrons are produced by fission reactions, they are fast neutrons with energies close to 2 MeV. For fast neutrons at this energy, and according to the cross section values listed in Table 4.3, the probability of fission by fast neutrons is 0.0076 and the value of the product ($n_{av}{*}\ P_f$) is 0.02, which is significantly lower than 1. This implies that it is not possible to sustain a chain reaction with fast neutrons in natural uranium.

At this energy most neutrons will likely be slowed down by inelastic collisions or will be captured by the nuclei of $_{92}U^{238}$.

The question then arises if it is possible to have a chain reaction with slower neutrons that have undergone a few inelastic collisions. At 0.3 MeV neutron energy the cross section values of Table 4.3 yield: $P_f = 0.021$ and $n_{av}{}^* P_f = 0.052$. Therefore, a chain reaction would not be sustained in natural uranium at this intermediate level of neutron energy. In the case of thermal neutrons, however, the situation becomes different: Because the fission cross-section of the isotope $_{92}U^{235}$ is significantly higher than the other relevant cross sections for thermal neutrons, the probability of fission is calculated to be $P_f = 0.54$ and the product $n_{av}{}^* P_f = 1.32$ is greater than 1. This implies that a sustainable chain reaction with natural uranium is possible with thermal neutrons. The challenge for the scientists and engineers is to build a large-scale system, the thermal nuclear reactor, that would realize and sustain the chain reaction for a long time.

Another way to sustain a chain reaction with faster neutrons is to use *enriched uranium* instead of natural uranium as the reactor fuel. Enriched uranium has a higher concentration of $_{92}U^{235}$ atoms than natural uranium. For example, if the reactor fuel had equal numbers of nuclei of the two uranium isotopes ($c_{238} = c_{235} = 0.5$) and neutrons of 0.3 MeV were used for fission, the probability of fission would have been $P_f = 0.46$ and $n_{av}{}^* P_f = 1.15 > 1$. Hence, a sustained chain reaction would be possible with 50% enriched uranium and neutrons with 0.3 MeV energy.

It must be noted that the process of uranium enrichment requires a significant amount of power and is very expensive: The two isotopes of uranium have the same chemical properties and, thus, they may not be separated by chemical means. Since the isotope $_{92}U^{235}$ is slightly lighter, it diffuses slightly faster through a membrane than $_{92}U^{238}$, if the two isotopes were in gaseous form. For this operation, natural uranium is converted to the gaseous uranium hexafluoride (UF_6). In order to achieve a significant difference in the concentrations many diffusion stages are required and a significant amount of power to pump the hexafluoride through the system of membranes. Alternatively, one may use centrifuging for the separation of the two isotopes: The heavier atoms of $_{92}U^{238}$ in the gaseous hexafluoride will concentrate towards the outer walls of the centrifuge and the lighter $_{92}U^{235}$ atoms concentrate towards the inner part of the centrifuge. Because there is only a very small mass difference between the two isotopes, a large number of centrifuging stages is also required, which consumes large amounts of electric power. Because of this, enriched uranium is very expensive to produce compared to the natural uranium. This is a significant disadvantage for commercial nuclear reactors that require enriched uranium as their fuel.

The condition $n_{av}{}^* P_f \geq 1$ is only a necessary and not a sufficient condition for the establishment of the chain reaction. A reactor contains several components other than the fuel itself (e.g. structural materials, fuel cladding, sensors, safety devices, etc.). The consideration of the fission and capture cross sections of these non fissile materials would lower the probability of fission of the fuel. The significance of these simple calculations is that a sustained chain reaction is possible to be established in natural uranium if the produced neutrons are slowed down and become thermal neutrons. A glance at Fig. 4.5b proves that this is not an easy task, because the neutrons, while slowing down, must pass through the capture resonance range of ${}_{92}U^{238}$. This presents another challenge for the design of the nuclear reactors: *to slow down the neutrons produced from fissions in order to yield as many thermal neutrons as possible, while avoiding the capture of neutrons by* ${}_{92}U^{238}$ *during the deceleration process.*

Contrary to popular thinking, a conventional nuclear reactor that operates with natural or slightly enriched uranium can not explode as an atomic bomb. The reactor simply does not produce the large number of thermal neutrons that are necessary to cause a nuclear explosion. Instead, the design of the reactor must be optimized to "thermalize" neutrons, while avoiding their capture by ${}_{92}U^{238}$ and to ensure that a high percentage of the produced thermal neutrons will cause the fission of the ${}_{92}U^{235}$ nuclei.

4.2.5 The Moderation Process and Common Moderators

The deceleration of fast neutrons in common reactors is accomplished by the *moderator*. Neutrons collide with the atoms of the moderator and impart a fraction of their kinetic energy to the atoms of the moderator. Hence, the deceleration of fast neutrons to thermal neutrons is accomplished by a series of collisions. From classical collision theory, we know that the reduction of the kinetic energy of a small particle is highest when the particle collides with another particle of the same mass. Collisions with much heavier particles are almost like collisions with hard "walls" and the kinetic energy of the small particle remains almost unchanged. Therefore, effective moderators are composed of very light nuclei. Typical moderator materials consist of light nuclei with very low capture cross section and significantly higher inelastic scattering cross section. Interactions of neutrons with the nuclei of these elements would predominantly be inelastic scattering collisions. These would slow down the produced neutrons, while keeping the neutron numbers almost constant because of the very low frequency of capture events. Such elements are hydrogen, both the ${}_{1}H^{1}$ and the ${}_{1}H^{2}$ isotopes, lithium, beryllium and carbon. Boron is not a suitable moderator because its capture cross section is significantly high. Actually, boron is used in the control system of the reactor as an element that captures neutrons.

The scattering and capture cross sections are important parameters of the moderators. Another important parameter that describes the effectiveness of a moderator is the logarithmic energy decrement, ξ, or the average decrease of the logarithm of the neutron kinetic energy per collision. A high value of ξ implies a low number of collisions/interactions needed to convert fast neutrons to thermal neutrons. A low number of collisions in the thermalization process is a desirable design attribute for moderators used in commercial reactors. Table 4.5 (data from

Table 4.5 Properties of common moderators

	H_2O	D_2O[a]	C (graphite)	Beryllium-9
σ_c, barns	0.66	0.001	0.0045	0.0095
σ_i, barns	50	10.6	4.7	6.1
ξ	0.92	0.509	0.158	0.209
Number of collisions to thermalize fission neutrons	20	36	115	87
$\xi\sigma_i$ (moderating power)	46	5.4	0.74	1.27
$\xi\sigma_i/\sigma_a$ (moderating ratio)	1390	5400	165	134

[a] D_2O is the commonly used symbol for heavy water. The molecule of heavy water is composed of one oxygen atom and at least one atom of ${}_1H^2$

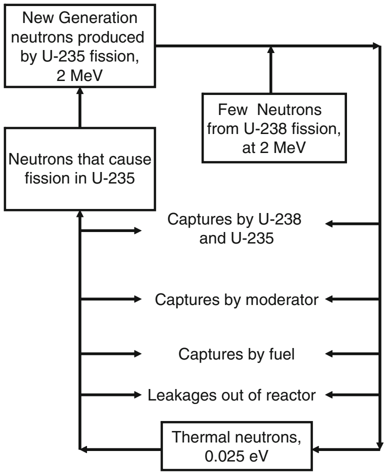
[1]) lists some of the important parameters of several common materials used as moderators.

The diagram of Fig. 4.8 shows the events that may affect the number of one generation of neutrons in a reactor from the neutron production stage to the stage when the new generation of neutrons is produced. This may be called *the neutron cycle* in the reactor. The neutron cycle demonstrates that there are several complex processes that take place in the nuclear reactors, which result in the reduction of the number of neutrons that become available for fission. The choice of the fuel, the materials and the overall design of nuclear reactors strive to achieve the minimization of the number of neutrons that are leaked outside or are absorbed in the reactor.

4.2.6 Fission Products and Energy Released in Chain Reactions

The nuclear reactions in Eqs. (4.19a) and (4.19b) are typical of the fission of ${}_{92}U^{235}$, but they are not unique by any means. The fission products of ${}_{92}U^{235}$ cover a wide spectrum of isotopes with mass numbers from 70 to 165. Figure 4.9 depicts the percentage yield of the fission products, with the exception of neutrons and other very light particles, vs. the mass number of the isotopes produced. It is apparent in this figure that there are two almost symmetric peaks with mass numbers approximately 96 and 135, where the yield is maximum and approximately equal to 6.5%. The yield in this figure may also be interpreted as the probability that an isotope with a certain mass number will be formed from the fission of ${}_{92}U^{235}$. It is also observed that the yield in the middle of the spectrum, where the two nuclei have almost equal mass numbers, 117, is approximately 0.01%. This implies that a nuclear reaction that would produce two nuclei with equal masses has probability of occurrence equal to 0.0001 or 1 in 10,000. Given that approximately 10^{20} fission reactions occur every second in a typical nuclear reactor, Fig. 4.9 shows that there are very large numbers of nuclei of all mass numbers from 70 to 165 that are produced every second. The nuclei, which in their vast majority are radioactive, remain trapped in the reactor until the next refueling

Fig. 4.8 The neutron cycle in a nuclear reactor



process, when the reaction products are removed for reprocessing. Other fissile materials, such as ${}_{94}Pu^{239}$ and ${}_{92}U^{233}$ have similar yield functions with two distinct peaks.

The amount of energy released in every fission reaction of ${}_{92}U^{235}$ depends on the products of the reaction and may be derived from the mass defect of the specific reaction. Given the yield of the product nuclei, as shown in Fig. 4.9, the average energy of the fission of ${}_{92}U^{235}$ may be calculated by averaging over all the probable combinations of the products. Using this method the average energy of the fission of ${}_{92}U^{235}$ is calculated to be approximately 200 MeV, a high percentage of which is manifested as kinetic energy of the fission fragments and is almost immediately dissipated by atomic/molecular collisions into heat. Additional energy is produced from the radioactive decay of the reaction products, also manifested as kinetic energy of the daughter isotopes. Table 4.6 [1] shows the average recoverable energy from the products of the ${}_{92}U^{235}$ fission.

Even though the recoverable energy of the nuclear products is in the form of the kinetic energy of the fragments, this kinetic energy at the molecular level is manifested as thermal energy (heat) at the macroscopic level. The conversion of this energy to electric power is accomplished in a thermodynamic cycle, usually a Rankine or a Brayton cycle and is subjected to the Carnot limitations of thermal energy conversion, which are described in Chap. 3.

4.3 Conversion and Breeding Reactions

The vast majority of nuclear reactors that are currently used are thermal reactors, which primarily use thermal neutrons and the isotope ${}_{92}U^{235}$ as their fuel. This isotope is supplied to the reactors in the form of natural or slightly enriched

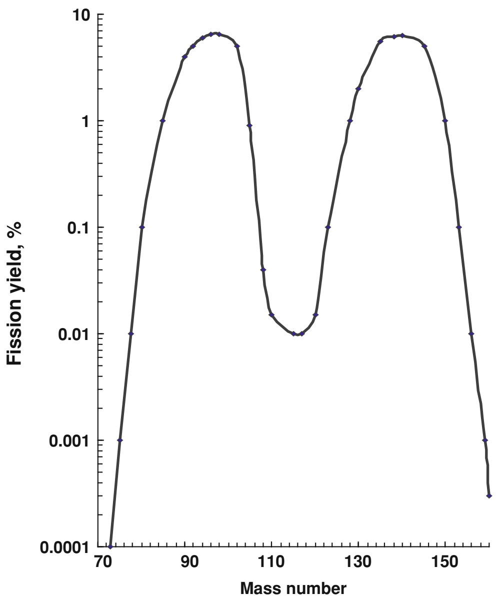


Fig. 4.9 The percentage yield of the fission of uranium-235

Table 4.6 The average recoverable energy of the $_{92}U^{235}$ fission

	Recoverable energy, MeV
Large fission nuclei	168
Fission neutrons	5
Gamma radiation	7
Fission product decay:	
Beta radiation	8
Gamma radiation	7
K-Capture radiation	5
Total energy released	200

uranium. It must be recalled that natural uranium only contains 0.715% of $_{92}U^{235}$. Only 1 out of 140 nuclei in natural uranium ore is a $_{92}U^{235}$ nucleus that may be undergo fission to produce thermal energy in a reactor. The vast majority of the remaining 139 nuclei of the isotope $_{92}U^{238}$ are not used in the fission process. The immutability of these nuclei does not only cause the underutilization of the nuclear energy resources, but also contributes significantly to the piling of the nuclear waste products, which are radioactive and must be stored for thousands of years. With the current mix of nuclear reactors, less than 1% of the nuclear resource is being used. The remaining, which is primarily composed of the isotope $_{92}U^{238}$ is stored as *depleted uranium* and contributes to the accumulation of the nuclear waste. Another naturally occurring isotope $_{90}Th^{232}$ is not utilized at all in thermal reactors for the same reasons. For the better utilization of the natural nuclear resources, scientists and engineers must devise ways to augment the percentage of natural uranium and thorium that are used in the reactors and to design future reactors that may use a larger proportion of the now wasted $_{92}U^{238}$ and $_{90}Th^{232}$ nuclei.

As it was seen in Sect. 4.2.4, the $_{92}U^{238}$ nuclei may undergo fission with fast neutrons. However, the fission cross section of this isotope is not significant enough to sustain the chain reaction in a reactor. A few $_{92}U^{238}$ nuclei actually undergo fission by the fast neutrons produced in the reactor, but inelastic scattering is the dominant process at these neutron energies. A large fraction of the fast neutrons produced would be slowed down before they cause the fission of $_{92}U^{238}$, and therefore, the fission of the $_{92}U^{238}$ nuclei is not a viable option in conventional reactors.

It is apparent from the study of the neutron cycle in a reactor that a significant fraction of the neutrons in a thermal reactor is captured by the nuclei of $_{92}U^{238}$ and form a short-lived isotope, $_{92}U^{239}$, which decays with a half-life of 23 minutes to neptunium-239 ($_{93}Np^{239}$) by electron emission. The latter is also radioactive and decays with a half-life of 55.2 hours to plutonium-239. Therefore, the capture of neutrons by the $_{92}U^{238}$ nuclei results in the production of $_{94}Pu^{239}$, within a relatively short time. Plutonium-239 is a fissile material, similar to $_{92}U^{235}$. The isotope $_{94}Pu^{239}$ has the additional advantage that its fission cross section is significant enough in the range of fast neutrons to allow for the establishment of a chain reaction at the fast neutron range. Similarly, thorium-232, which may be mined in several regions on the planet, when bombarded by neutrons may be finally converted to the fissile isotope $_{92}U^{233}$ according to the following reactions:

$$\begin{aligned} {}_{90}Th^{232} + {}_{0}n^{1} \rightarrow {}_{92}Th^{233} \rightarrow {}_{91}Pa^{233} + {}_{-1}e^{0} \quad (T_{1/2} = 22\ \text{min}) \\ {}_{91}Pa^{233} \rightarrow {}_{92}U^{233} + {}_{-1}e^{0} \quad (T_{1/2} = 27.4\, days). \end{aligned} \tag{4.29}$$

Both isotopes, $_{94}Pu^{239}$ and $_{92}U^{233}$, have significantly long half lives (25,000 years and 160,000 years) to be considered stable isotopes within the timescale of operation of a nuclear reactor. The two naturally occurring isotopes $_{92}U^{238}$ and $_{90}Th^{232}$ are not fissile, but may be converted in suitably designed reactors to produce the fissile isotopes $_{94}Pu^{239}$ and $_{92}U^{233}$, which in turn may be used as nuclear fuel. For this reason, they are called *fertile* isotopes. The process of the production of fissile materials from fertile materials is called *conversion* or *breeding* and the reactors, which enable this process, *breeder reactors.* Despite several technical difficulties with the current generation of the breeder reactors, many scientists have concluded that the wider use of the breeding reactors in the future will enable the utilization of the vast majority of the available nuclear resources for electricity production and will significantly reduce the nuclear waste produced. In addition, since breeder reactors will convert the $_{92}U^{238}$ isotope to the fissile $_{94}Pu^{239}$ nuclei, these breeder reactors will use the depleted and stored uranium and will contribute to the minimization of nuclear waste.

4.4 Useful Calculations and Numbers for Electric Power Generation

At the center of all the calculations on nuclear power is the fundamental equation of mass to energy equivalence $E = mc^2$. Based on this equation it was seen that the conversion of 1 u of mass to energy produces 931 MeV of thermal energy. Also, it was seen that the fission of fissile nuclei, such as those of $_{92}U^{235}$, results in the mass defect of the reaction products, which on average is equivalent to 200 MeV of thermal energy. Based on these numbers, we may calculate several useful parameters for the operation of the nuclear reactors.

At first, a typical large-scale nuclear reactor produces approximately 1,000 MW of electric power. Typical overall efficiencies of the nuclear power plants that utilize such reactors are 32–35%, which implies that the reactors produce approximately 3,000 MW of heat (thermal energy). In order to distinguish the thermal from the electric power output of the reactor the units of these two numbers are often denoted as MW-e and MW-t. Hence the power produced by the power plant would be written as 1,000 MW-e and the heat produced by the reactor as 3,000 MW-t. The latter is referred to as the *rating* of the reactor.

As it was observed in Sect. 4.2.6, at this thermal power the number of reactions per second is:

$3000*10^6/(200 * 1.6*10^{-13}) = 9.375*10^{19} \approx 10^{20}$.

While this appears to be a very large number of reactions, when compared to the Avocadro number ($6.023*10^{23}$ atoms/gram-atom) it represents a rather small number of atoms.

At this rate, the complete consumption of one gram-atom or 235 g of uranium-235, will take $6.023*10^{23}/9.375*10^{19} = 6{,}424$ seconds, or 1.78 hours. During this time the reactor produces 5,340 MWh of thermal energy and 1,780 MWh of electric energy. One single gram of pure $_{92}U^{235}$ produces 22.7 MWh of electric energy or approximately 1 MW for an entire day. This is in contrast to fossil fuel power plants, which require more than 3 metric tons of coal to produce this amount of electric energy.

Accounting for the fact that only 0.715% of natural uranium is the fissile isotope $_{92}U^{235}$, the reactor that produces 3,000 MW-t power would consume 238 grams of natural uranium every 45.9 seconds. During one day the reactor would consume the equivalent of:

$24*60*60*238/45.9 = 447.7$ kg/day of natural uranium.

The yearly consumption of nuclear fuel would be 447.7 * 365 kg or 163.4 metric tons per year. A typical nuclear reactor is fueled (charged) every 18-24 months and operates continuously under normal circumstances. Let us assume that the reactor is initially charged with 1,000 metric tons, or with 52.4 m^3, of natural uranium. At continuous operation, this reactor would consume 16.3% of its fuel in a whole year. The actual amount of mass deficit, Δm, which is converted to energy, is by far smaller than the amount of fuel consumed. During a whole year, the mass deficit for this reactor would be, from $E = mc^2$:

$3000*10^6*365*24*60*60/10^{16} = 9.46$ kg.

This amount of mass deficit is negligible compared to the original weight of 1,000 metric tons the reactor has been charged with. For this reason it is impossible to be measured by weighting the reactor at the beginning and at the end of the year with current measuring techniques.

A quantity of interest with the fuels is the *burnup* of the nuclear fuel, which is defined as the total energy released per metric ton of fuel. Since during one day the reactor under consideration would produce 3,000 * 24 MWh-t of thermal energy and would consume 0.4477 metric tons, the burnup rate of the reactor would be 160,821 MWh per ton of natural uranium. Enriched uranium has a higher percentage of fissile nuclei and its burnup rate is higher.

It must be noted that all the calculations in this section are based on the assumption that only ${}_{92}U^{235}$ undergoes fission in the reactor. In an actual reactor some of the ${}_{92}U^{238}$ undergoes fission by fast neutrons and, also some ${}_{94}Pu^{239}$ is formed from the ${}_{92}U^{238}$ conversion. The produced plutonium is fissile and produces an additional amount of energy. As a consequence, the amounts of the original fuel used during a day or a year would be slightly lower than that of the above computations and will depend on the small amounts of the ${}_{92}U^{238}$ that undergoes fission and the amount of ${}_{94}Pu^{239}$ formed.

Problems

1. Complete the following reactions (identify the elements X,Y,Z and W):

$$
\begin{aligned}
&{}_{92}U^{238} + {}_{0}n^{1} = X +_{-1} e^{0} = Y + 2_{-1}e^{0} \\
&{}_{92}U^{235} + {}_{0}n^{1} =_{38} Sr^{90} + Z + 3_{o}n^{1} \\
&{}_{3}Li^{7} + {}_{0}n^{1} = 2_{2}He^{4} + W
\end{aligned}
$$

2. Use the periodic table of elements to identify the elements with the following atomic numbers: 8, 13, 21, 34, 55 and 89.
3. What is the energy equivalent of 1 lb (0.453 kg) of mass? What is the mass equivalent of the electric energy produced by a 1,000 MW nuclear power plant for an entire year?
4. If four nuclei of ${}_{1}H^{1}$ were to fuse to produces a nucleus of ${}_{2}He^{4}$, what is the mass defect and the corresponding energy released? What is the energy in kJ that would be released from the fusion of 1 kg of hydrogen?
5. What is the mass defect of the nuclei of ${}_{3}Li^{6}$ and ${}_{92}U^{238}$? What is the binding energy per nucleon of these two nuclei and which one of the two isotopes you expect to be more stable?
6. What is the amount of energy released during the reaction:

$$
{}_{3}Li^{7} + {}_{0}n^{1} \rightarrow 2_{2}He^{4} + {}_{-1}e^{0}
$$

If one kmol of lithium reacted (6.023 * 10^{26} atoms), what is the total energy released in kJ? Given that the amount of heat released from the combustion of carbon is $\Delta h = 32{,}770$ kJ/kg, how many metric tons of carbon are needed to provide this energy?

7. It is suggested that nuclear waste, which includes $_{92}U^{238}$, be stored under controlled conditions in mountainous caverns. If one metric ton (1,000 kg) of $_{92}U^{238}$ is stored in the year 2020, how much of the material would remain in the year 3,000 and in the year 5,000? How long would it take for the amount of uranium stored to reach 10% of its initial amount?
8. The Chernobyl accident occurred on April 26, 1986 and all the strontium-90 from the reactor was released to the environment. What is the percentage of this isotope that remained on January 1, 2012 in the environment?
9. The following secondary reactions occur in a nuclear reactor:

$$_{56}Ba^{140} \rightarrow {_{57}La^{140}} + {_{-1}e^{0}}$$

$$_{57}La^{140} \rightarrow {_{58}Ce^{140}} + {_{-1}e^{0}}$$

The half-lives of the three radionuclides Ba, La and Ce are 12.9 days, 1.7 days and 34 days respectively. If at time t = 0, 100 kg of $_{56}Ba^{140}$ are released by accident, determine the approximate amounts of the three elements after 1 day, 5 days and 25 days.

10. Determine the fission cross-sections of uranium-235 at the following neutron energies: 1 MeV, 1 keV, 10 eV, 1 eV, 100 meV, 10 meV and 1 meV.
11. What are the most probable velocity and energy of neutrons at 600 K? At 2,000 K?
12. Is it possible to have a chain reaction with natural uranium and intermediate energy neutrons? Explain your answer thoroughly and state carefully under what conditions it is possible to have a chain reaction with intermediate energy neutrons.
13. What is the probability of fission of 5% enriched uranium (5% $_{92}U^{235}$ and 95% $_{92}U^{238}$) with neutrons at 0.1 eV?
14. Given that at 0.3 MeV the pertinent cross sections for $_{92}U^{235}$ are: $\sigma_f = 1.3$ barn and $\sigma_c = 0.7$ barn, while for for $_{92}U^{238}$ are: $\sigma_f = 0$ barn $\sigma_c = 0.5$ barn, estimate the minimum percentage of $_{92}U^{235}$ in the fuel for which a chain reaction with such neutrons would be possible.
15. Use Fig. 4.9 and the periodic table of elements to identify twelve of the most probable isotopes to be produced in a fission nuclear reactor.
16. A nuclear reactor provides 3,600 MW of thermal power. The reactor operates with 2% enriched uranium. Assuming that only $_{92}U^{235}$ reacts in the reactor, what are the daily consumption of this isotope in the reactor and the burnup rate of the reactor?

References

1. Bennet DJ (1972) The elements of nuclear power. Longmans, London
2. El-Wakil MM (1984) Power plant technology. McGraw Hill, New York