Chapter 8
Wind Power

Abstract Since the time of the ancient sailboats and windmills, the power of the wind has been harnessed for ship propulsion and the performance of mechanical work. In the modern era, wind has been increasingly used for the production of electric power. In the first decade of the twenty-first century alone, the production of electricity from wind power worldwide has increased by a factor of eight. Similar to solar energy, wind is also a distributed, renewable source of energy. The energy density of the wind is low, but wind is available in all the geographical regions of the world and its geographical distribution is more uniform than that of solar energy. Wind turbines of different sizes and designs are currently used successfully for the production of electric power. The bigger and more efficient types of these engines have blades with lengths between 20 and 50 m, are located at the top of 50–140 m towers and are becoming ubiquitous in the landscape of several OECD countries. Wind is probably the most environmentally benign energy source. In theory, it has the capacity to satisfy the energy needs of entire countries and even that of the whole planet. However, it is also an intermittent source with availability and intensity much less predictable than any other source of alternative energy. This intermittency is a significant drawback and will limit the more widespread use of wind power, unless suitable energy storage systems are developed that would store part of the energy produced during windy periods.

8.1 Wind Patterns

It may be arguably said that wind power is a byproduct of solar energy: The uneven heating of different parts of the globe causes hot air to rise in regions that receive higher amounts of insolation. The rise of hot air creates a small pressure differential, which induces colder air from the surrounding regions to rush in. Thus, horizontal air currents and the wind patterns on the Earth's surface are

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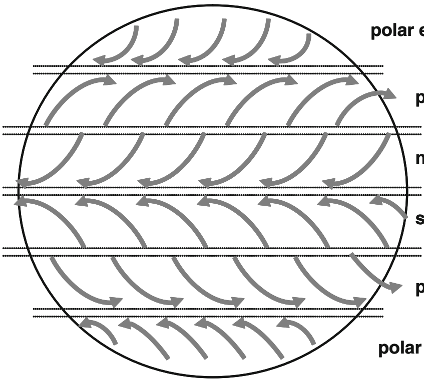


Fig. 8.1 The planetary winds on Earth's surface

created. Depending on the origin and effects of these currents, the currents are classified as *planetary* and *local.*

Planetary winds affect very large regions of the Earth, encompass large masses of air and are primarily caused by the higher amount of solar radiation received by the land masses near the equator. The hotter land masses near the equator cause the air at the tropical regions to rise and move towards the poles. This upward motion of the air masses is affected by the rotational motion of the Earth. The *Coriolis force* with its components on the horizontal plane is developed on the rising masses of air in both the northern and the southern hemisphere. As a result, the air rushing from the temperate zones to fill the relative vacuum at the equatorial zone develops a motion towards the west. This westerly motion causes the north–east *trade winds* in the northern hemisphere and the south–east *trade winds* in the southern hemisphere.[1] Early navigators, such as Columbus and Magellan mastered the effect (but not the causes) of these winds and planned their successful western journeys accordingly. At the same time, the rising warm air moves towards the poles, cools in the upper atmosphere and descends at approximately 30^{o} latitude (in the nautical tradition these are the *horse latitudes*) in both the northern and the southern hemispheres. The downward motion at these latitudes in combination with the Coriolis force develops a general air motion in the temperate zones from the western to the eastern direction, which manifests itself as the *prevailing westerlies,* in both the northern and the southern hemispheres between 30 and 60^{o} latitude. The pattern of the planetary winds is completed with the *polar easterlies*, which blow in both the south and the north Polar Regions. Figure 8.1 is a schematic diagram of the prevailing planetary winds.

Local winds have local effects and are usually caused by the uneven heating of neighboring masses of land or of land and sea. Well known among local winds is

[1] Winds are named from the direction they come from. Thus, a wind blowing from the west to the east is a westerly and a wind blowing from north to south is a northerly wind.

the *sea breeze*, which is caused by the higher temperatures of the land in comparison to a neighboring water surface: as part of the water evaporates, the surface of the water remains cooler than that of the surrounding land. As a result, the air close to the surface of the land rises, a pressure differential—or partial vacuum—is created and cooler air over the water surface rushes to fill this partial vacuum. Breezes are ubiquitous in the coastal areas as well as near large lakes. A lesser known mechanism of air current development is caused by the differential heating on the hills and mountain sides, which causes the air to rise during the day time when solar heating is high (anabatic flow) and to descend during the early evening hours when the mountain or hill sides cool faster than the neighboring areas (katabatic flow). In addition, mountains and hills create their own wind patterns by deflecting masses of air, which is already in motion. Other types of local winds are established according to the type of surface cover, the heating patterns, the climate, and the weather of a region.

Wind patterns are extremely important for the economy of a locality, a region or, a nation, because they are one of the prime determinants of local weather and, especially, of rainfall. Weather and rainfall determine the agricultural production of the area, and by extent, the regional economy. Local and regional wind patterns are intricately entwined into the social and economic fabric of modern societies and nations. Minor perturbations in the prevailing wind patterns may cause significant rainfall disruptions in a region, crop failures and subsequent economic instability. Global warming, which among its other effects is expected to cause significant disruptions in local climatic conditions and wind patterns, is also expected to produce rainfall pattern changes in several regions of the planet. Such changes may cause fresh water scarcity in regions where water is now abundant. This may lead to persistently lower agricultural production and significant economic hardships for the population. For this reason several governments have taken measures to control and limit the emissions of CO_2 and other greenhouse gases.

8.1.1 Early Types of Wind Utilization

Wind power was known and used extensively by ancient civilizations for ship propulsion. Historical records show that the Chinese, Indians, Egyptians and Hittites have been using sails to propel their boats before the second millennium BC. Greek ships sailed to Troy around 1,100 BC in an expedition that was recorded in the epic poems of *Iliad* and *Odyssey*.[2] Actually, propulsion by sail and manpower (rowing) were the only forms of seagoing power until the early nineteenth century and the advent of the steam engine.

[2] Wind energy is often called *Aeolic energy* named after *Aeolos*, the ancient Greek god of the winds.

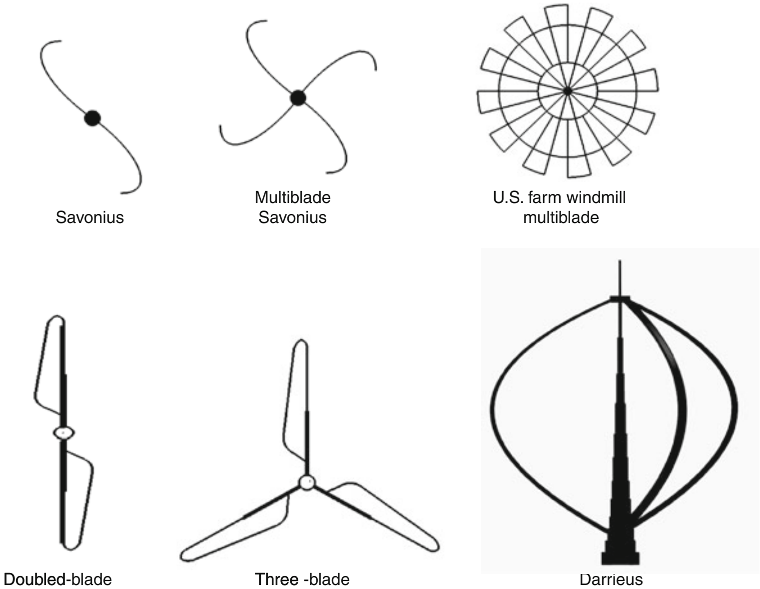


Fig. 8.2 Types of common wind turbines

Land use of wind power for the production of energy and the accomplishment of tasks came a great deal later. The use of windmills was recorded by Arab writers in the Asian regions that are now part of Iran and Afghanistan around 1,000 AD. Windmills were transported to Europe after 1,200 AD and became very common for the grinding of wheat and other food processing tasks. From Europe the use of windmills was transported to America. A type of windmills, the American multi-blade turbine, or *freetail,* was heavily used in the arid southwest part of the United States for pumping water from underground wells. This type of windmill is credited for the population growth and the economic development of the American Southwest.

Wind turbines can be very simple and their early designs demonstrate this simplicity: The *Savonius turbine* is made of an S-shaped metal sheet with a shaft running through its middle. The shaft of this turbine may be either horizontal or vertical but the vertical shaft configuration is the more common. The *Darieus turbine* is mounted on a vertical shaft and has two to four-blades. The blades of this turbine are bent and have an aerodynamics shape. This shape deflects the stream of air and produces a lift force on the blade, which acts in unison with the induced drag and assists in the harnessing of more wind power. A combination of the *Savonius* and the *Darieus* turbines may be constructed on a vertical shaft. Among the horizontal axis turbines are: the four-blade *Dutch windmill,* which was one of the most commonly, used turbines in the past, and the *US farm windmill*, which has as many as twenty-blades. Figure 8.2 depicts the schematic diagrams of these earlier turbine designs as well as a more modern three-blade wind turbine.

The early users of wind power were only concerned that the task the turbine was built would be done and that the turbine would be reliable. Turbine efficiency and optimization of the turbine operation was never a consideration in manufacturing the early engines. For this reason, none of the above devices is designed to convert wind power efficiently to electricity, or for that matter to perform any other task under the maximum possible efficiency. Maximum efficiency is one of the principal considerations of the construction of the engines that are currently built for the conversion of wind power to electricity. Typically, the modern turbines have two or three-blades and their design and operation have been optimized to produce maximum energy from the prevailing local wind conditions.

8.1.2 Wind Power Potential

Slightly more than 2%, of the total solar power received by the Earth, or $3.46*10^{12}$ kW, is converted to wind power. The total solar radiation energy converted into the mechanical energy of the wind is more than $1.1*10^{17}$ MJ per year. This is more than two million times more than the total energy that has been used by the human population in 2010. An immediate conclusion that may be drawn from these numbers is that wind power is an important alternative energy source, which, if properly harnessed, is capable to supply a significant fraction of the energy demand of the Earth's population. Of course, not all the wind power that is generated from the uneven heating of the sun may be harnessed for the production of electricity: Air currents over the sea or at high altitudes are too difficult, if not impossible, to be converted to electricity. However, a significant fraction of wind power is available near the surface of the Earth. This fraction is close to many populated regions and may be harnessed with conventional means. Even if a small fraction of the available power is converted to electricity, it would amount to significant contribution and would alleviate the need to use fossil or nuclear fuels for electric power generation.

The kinetic energy per unit volume of an air stream with velocity V and density ρ is:

$$K.E. = \frac{1}{2}\rho V^2. \tag{8.1}$$

Since the volumetric flow rate through an area A is: $\dot{V} = VA$, and the area swept by the blades of a turbine is a circle of diameter, D, the power of the air stream that is available to a wind turbine is:

$$\dot{W}_{av} = \frac{1}{2}A\rho V^3 = \frac{\pi}{8}D^2\rho V^3. \tag{8.2}$$

Thus, the wind power is proportional to the cubic power of the wind velocity and to the square of the diameter of the wind turbine. This is a significant

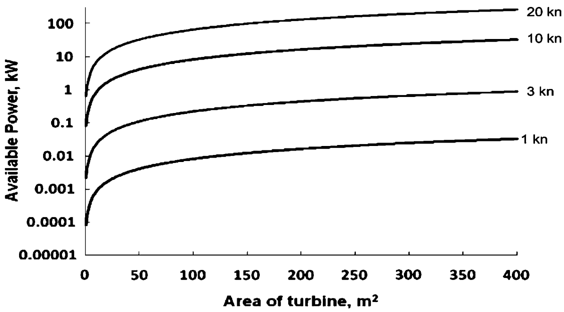


Fig. 8.3 Available power from the wind. 1 kn = 1.15 mph = 1.852 km/h = 0.514 m/s

conclusion for the harnessing of wind power: when the wind velocity is doubled, the power is increased by a factor of eight. Figure 8.3 shows the diagram of the power that is available by the wind as a function of the area swept by the wind turbine and with the wind velocity in knots [3] (kn) as a parameter. It is apparent that the power available from the wind is very high if wind turbines that cover very large areas are constructed and placed in operation at locations with high average wind velocities. A great deal of research and development effort, since the 1970s, has been devoted to the construction of wind turbines with longer blades that cover large areas.

Unlike solar radiation, which is a form of heat, the energy of the wind is kinetic energy, which is a form of mechanical energy. Therefore, the wind turbines, which convert the wind energy to electricity, are direct conversion devices and are not subject to the efficiency limitations (Carnot efficiency) of heat engines. The wind turbines that are currently used for the harnessing of wind power are by far simpler to construct and to operate than any thermal engine. An additional advantage of the wind turbines with respect to thermal engines is that their operation does not require the use of water for cooling purposes.

8.2 Principles of Wind Power

Large or small wind turbines harness the wind power for the production of electricity. The wind turbines are permanently located at a given location and are designed to generate maximum energy annually. Since the wind direction and velocity are subject to rapid and frequent changes, the wind turbines and their associated auxiliary machinery are designed to respond to both. For this reason, wind turbines are usually free to rotate about a vertical axis. The entire wind turbine system is pivoted on bearings and is free to swivel. A separate downwind vane or a simple rudder ensures that the wind turbine system is always

[3] 1 kn = 1.852 km/h. The distance 1.852 km is equal to one minute of arc latitude along any meridian on the surface of the Earth.

perpendicular to the wind. This type of motion is known as *yaw*. The characteristic time of yaw is typically significantly smaller than the characteristic time of wind direction variations. For this reason, in the following sections it will be assumed that the wind turbine is always in a position that the wind is perpendicular to the plane described by the rotation of the blades.

8.2.1 Spatial and Temporal Characteristics of Wind: The Boundary Layer and Exceedance Curves

It is well known from fundamental Fluid Dynamics that when a gas flows over a stationary solid surface, the velocity of the gas close to this surface diminishes and it is actually considered to be equal to zero at the surface. This is the so-called *no-slip condition* on stationary surfaces and is one of the basic principles on which the subjects of Fluid Dynamics and Aerodynamics are based. This no-slip condition applies to environmental flows and to the wind. Thus, the wind velocity approaches asymptotically the value zero on the surface of the Earth and increases with the height, $V(z)$. When a fluid stream flows over a surface its velocity vanishes close to the surface and increases with height to an almost uniform value. This uniform velocity is called the *free-stream velocity,* V_0. All the velocity variation takes place in a region close to the surface, which is called the *boundary layer.* Because the wind blows over large landscapes, the pertinent Reynolds numbers are very large and the flow is invariably turbulent. Hence, a *turbulent boundary layer* is developed over the terrestrial surface when the wind blows.

The boundary layer is the area of the flow where frictional forces and dissipation are of importance. Inertia is the dominant force outside the boundary layer. Two dimensionless parameters characterize the turbulent boundary layer:

a) The friction velocity, v^*, which is given in terms of the wind shear stress at the ground level, τ_w, and the density of air, ρ:

$$v^* = \sqrt{\frac{\tau_w}{\rho}}. \tag{8.3}$$

b) The wall coordinate, z^+, which is given in terms of the friction velocity and the kinematic or the dynamic viscosity of air, v and μ, respectively:

$$z^+ = \frac{zv^*}{v} = \frac{z\sqrt{\tau_w \rho}}{\mu}. \tag{8.4}$$

Very close to the ground and at heights less than 2 m there are two parts of the turbulent boundary layer that are called the *viscous region* and the *buffer layer,* where the viscous effects play an important role. Outside this thin layer, the viscous effects do not affect the velocity profile and the wind velocity may be approximated by the expression:

$$v^{+} = \frac{V}{v^{*}} = \frac{1}{\kappa}\ln(z^{+}) + C. \tag{8.5}$$

where $\kappa = 041$ is the *von Karman constant,* and $C = 5.0$.

The shear stress, τ_w, is usually expressed in terms of the free-stream velocity, V_s, that is the velocity outside the boundary layer and a friction coefficient, f. The latter is an empirical parameter and is obtained from experimental studies:

$$\tau_w = \frac{1}{2} f \rho V_s^2. \tag{8.6}$$

Accordingly, the expressions for the friction velocity and the wall coordinate become:

$$v^{*} = V_s\sqrt{\frac{1}{2}f} \quad and \quad z^{+} = \frac{0.7071\rho z V_s}{\mu} = 0.7071\mathrm{Re}_z. \tag{8.7}$$

It follows that the friction velocity is proportional to the free-stream velocity and that the wall coordinate is effectively proportional to a Reynolds number, which is defined by the height from the surface.

An approximate expression for the ratio of the wind velocities above the ground, at levels z_1 and z_2 is given by the so-called 1/7th power law. This is an approximation to a multitude of experimental data on turbulent boundary layers over solid surfaces. According to this law, the velocity in the boundary layer is proportional to the 1/7th power of the distance from the solid surface, that is $V(z) \approx kz^{1/7}$, where k is a constant. Therefore, the ratio of the velocities at two different heights is given by the simple algebraic expression:

$$\frac{V(z_2)}{V(z_1)} = \left(\frac{z_2}{z_1}\right)^{1/7}. \tag{8.8}$$

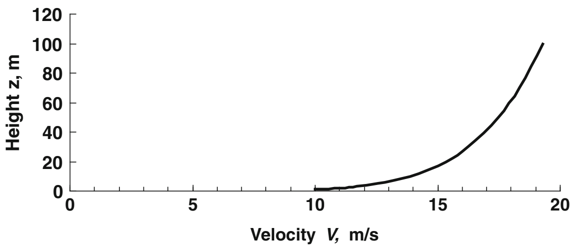
When wind velocities are measured in meteorological stations, by convention they are measured at a height of 30 ft (9.1 m). Hence, $V(z_1 = 9.1\ \mathrm{m})$ is known experimentally, and the last expression may be used to determine any other velocity within the boundary layer. For example, if the measured velocity is 10 kn, the wind velocity at 100 m above the ground is $10*(100/9.1)^{1/7} = 14.1$ kn.

Figure 8.4 shows the power law velocity profile in the first 100 m above the Earth's surface. Two trends are apparent from this figure and from the concept of the boundary layer:

a) Most of the variation in the wind velocity occurs in the very low part of the boundary layer, which is close to the ground; and
b) While the velocity profile flattens above 40 m, there is still wind variability in the upper part of the boundary layer.

For example, the wind velocity increases by 7.6% between the heights 30 and 50 m, where most of the wind turbines are located. Given the cubic dependence of

Fig. 8.4 Velocity distribution in the boundary layer



the power with respect to velocity, this implies that the available wind power increases by 25% between these two heights. The implication of this conclusion is that wind turbines should be placed as high as structural and material considerations allow.

The temporal characteristics of the wind emanate from the fact that the magnitude and direction of wind velocity vary significantly over a period of time. The mean wind velocity, V_m, and the mean power velocity, V_p, are two parameters that apply to the magnitude of the velocity and characterize the velocity variation and the available average power. These two parameters are defined as follows:

$$\mathrm{V_m} = \frac{\int_{t_1}^{t_2} \mathrm{V(t)dt}}{t_2 - t_1} \quad \textit{and} \quad \mathrm{V_p} = \frac{\left[\int_{t_1}^{t_2} [\mathrm{V(t)}]^3 \mathrm{dt}\right]^{1/3}}{t_2 - t_1}. \tag{8.9}$$

The mean power velocity, V_p, yields the average amount of available power for the wind during the time interval $t_1 < t < t_2$ and is derived from Eq. 8.2. Typically, the time interval $[t_1, t_2]$ is taken over one year and the two velocities signify the annually averaged wind velocity and the annually averaged available power.[4] Because, in general, $V(t) > 1$, it follows that $V_p > V_m$. A large number of empirical observations have shown that the two variables are correlated as: $V_p \approx 1.25\ V_m$. Since the available wind power is proportional to the cubic power of V_p, and since $1.25^3 = 1.953 \approx 2$, the expression for the available wind power becomes:

$$\dot{W}_{av} \approx A\rho V_m^3 = \frac{\pi}{4} D^2 \rho V_m^3 \approx \frac{\pi}{8} D^2 \rho V_p^3. \tag{8.10}$$

Wind velocities vary annually from zero, to values significantly higher than the mean wind velocity. The very high velocities however, occur only for short periods of time. Even though the extreme wind velocities may provide significantly higher power temporarily, they have very short duration. An engine designed to operate at the extreme velocities would provide large amounts of power only for a short period annually. When one makes decisions on the

[4] Usually the two integrals are computed from a large number, N, of velocity measurements. Hence, the two integrals are approximated by the sum of the respective integrants and the time interval by the total number of measurements, N.

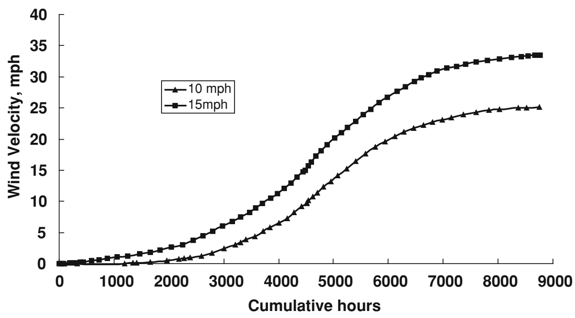


Fig. 8.5 Wind distribution curves for two locations with $V_m = 10$ and 15 mph

construction and operation of a wind turbine the magnitude of the wind velocities as well as the annual hours that certain ranges of velocities occur in the given location become important. For this reason, the *wind distribution curves,* which are also known as *exceedance curves, endurance curves,* and *duration curves,* are used for the construction of wind turbines. These are histograms that show how many hours annually the local wind velocity exceeds a certain value. Wind velocity is typically the abscissa of these diagrams and the hours of the year—0 to 8,760—is the ordinate. The wind distribution curves for two locations with average wind speeds, 10 mph (miles per hour) and 15 mph are shown in Fig. 8.5. These distribution curves provide valuable information on the duration of wind velocities and form the basis for the design and operation of wind turbines, as will be explained in more detail in Sect. 8.3.

8.2.2 *Probability Distributions of Wind Speed and Wind Power*

Rather than rely on the empirical data and the graphical form of the exceedance curves, engineers often develop from the experimental measurements probability distribution functions for the wind speed at a given location. The most suitable and common of these distributions is the *Rayleigh distribution,* which in its general form has the functional relationship:

$$p[x;\sigma^2] = \frac{x}{\sigma^2} exp\left[-\frac{x^2}{2\sigma^2}\right]. \tag{8.11}$$

The mean of the Rayleigh distribution is equal to $\sigma(\pi/2)^{1/2}$. When the latter is substituted for the average velocity, V_m, one obtains the following functional form for the wind velocity distribution:

$$p[V] = \frac{\pi V}{2V_m^2} exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right]. \tag{8.12}$$

This probability distribution function satisfies the two essential conditions of any probability function:

$$\int_0^\infty p[V]dV \equiv 1 \quad and \quad \int_0^\infty Vp[V]dV \equiv V_m. \tag{8.13}$$

From differentiation of Eq. 8.11 it follows that the most probable wind speed at a location is $(2/\pi)V_m = 0.637V_m$.

According to the Rayleigh probability distribution, the average available wind power may be obtained by the following expression:

$$\frac{\dot{W}_m}{\rho A} = \int_0^\infty V^3 p[V]dV = \int_0^\infty \frac{\pi V^4}{2V_m^2} exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right]dV = \frac{6}{\pi}V_m^3. \tag{8.14}$$

Since $6/\pi = 1.910$, the Rayleigh probability distribution implies that the average available power is almost equal to $1.25^3\ {V_m}^3 \rho A$, which is the basis of the approximate Eq. 8.10. It may be also noted that the power probability function has a maximum at $V/V_m = (8/\pi)^{1/2} = 1.60$.

8.2.3 Fundamentals of Wind Power Generation

If one were to convert all the available power of the wind, as expressed by Eq. 8.2, to electricity, the wind would have to leave the turbine with vanishing kinetic energy and, hence, the wind will have to come to a stop downstream of the wind turbine. This would increase the downstream pressure and, naturally, the wind would stop and the turbine would not produce any power. It is apparent from the above that, for the continuous operation of a wind turbine, the air velocity downstream must be finite. Hence, the power extracted by the wind turbine would be less than that given by Eq. 8.2. The simple analysis of the air flow near the wind turbine, which follows, will help derive a realistic value for the maximum power a wind turbine would be expected to produce.

Let us consider an open thermodynamics system, or control volume, which surrounds the wind turbine. The cross-section of the control area at the turbine is equal to the area described by the rotating blades A_b (= $\pi D^2/4$). A schematic diagram of the pressure and velocity distributions within this control volume is given in Fig. 8.6. The inlet and the outlet of the open system are far enough from the turbine so that the influence of the rotating blades on the static pressure is vanishingly small. Hence, the static pressure at both the inlet and the outlet is equal to the atmospheric pressure P_a. The static pressure variations in the entire control volume are small enough and it is assumed that they do not affect significantly the density of the air, which remains almost constant as air flows through the control volume. It is also assumed that the control volume is at a level significantly higher than the ground surface and that the velocity may be considered uniform and

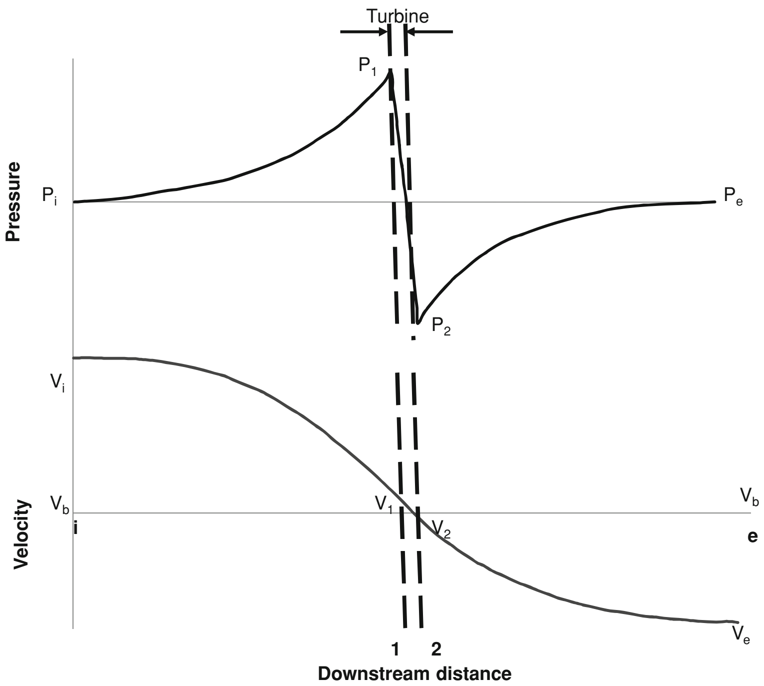


Fig. 8.6 Pressure and wind velocity variations close to the wind turbine

unidirectional. The inlet velocity of the air stream is V_i and the outlet velocity is V_e. It must be noted that the cross-sectional area of the control volume for this problem is not constant. The control volume area varies, in order to accommodate the changes in velocity and to ensure that all streamlines remain within the control volume. Hence, the mass flow rate,

$$\dot{m} = \rho A V. \tag{8.15}$$

is constant in the control volume and the mass continuity equation is satisfied in all cross-sections of the control volume. In the diagram of the Fig. 8.6 the wind turbine occupies the region 1–2, where, because of the extraction of mechanical power, it is expected that the static pressure will drop from P_1 to P_2 and the air stream velocity from V_1 to V_2. The control volume accommodates these changes.

Both the static pressure and the velocity vary continuously in the control volume. The first derivative of the static pressure is not necessarily continuous, because of the extraction of mechanical power at the turbine. The first derivative of the velocity is continuous, as depicted in Fig. 8.6, because the control volume is defined by the streamlines, no mass escapes or is added and the mass conservation equation is satisfied in the direction of the streamlines. Throughout the control volume there is not any source of significant friction and, hence, frictional losses are considered small enough to be negligible. Under these conditions, the momentum conservation equation in the control volume is the same as the Bernoulli equation and may be written as follows for the front section i to 1:

$$\frac{P_i}{\rho} + \frac{1}{2}V_i^2 = \frac{P_1}{\rho} + \frac{1}{2}V_1^2. \tag{8.16}$$

Similarly, Bernoulli's equation is valid at the back of the control volume from 2 to e, where it may be written as follows:

$$\frac{P_2}{\rho} + \frac{1}{2}V_2^2 = \frac{P_e}{\rho} + \frac{1}{2}V_e^2. \tag{8.17}$$

It must be pointed out that Bernoulli's equation is not valid in the region 1–2, because a significant amount of work is extracted by the blades of the turbine. This is a very thin part of the cross-section of the control volume, because the thickness of the blades is very small in comparison to the length of the control volume. One may subtract the last two equations, apply the condition: $P_i = P_e = P_a$ and derive the following expression for the difference of the static pressure, P_1-P_2, immediately before and after the turbine.

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_i^2 - V_1^2 + V_2^2 - V_e^2\right). \tag{8.18}$$

An inspection of the velocity variation in Fig. 8.6 and the fact that both the velocity as well as its first derivative are continuous functions, leads to the conclusion that, for thin turbine blades, points 1 and 2 are very close together and the velocities immediately before and after the wind turbine are approximately equal. If we define the velocity at the center of the wind turbine blade as V_b, we have the condition: $V_b \approx V_2 \approx V_1$. Therefore, the last equation yields the following expression for the static pressure:

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_i^2 - V_e^2\right). \tag{8.19}$$

The axial force that is developed on the blades of the wind turbine is equal to the product of the pressure difference and the area swept by the blades of the turbine, A_b:

$$F_x = A_b(P_1 - P_2) = \frac{1}{2}A_b\rho\left(V_i^2 - V_e^2\right). \tag{8.20}$$

In the absence of any other forces acting on the control volume, the axial force is also the net force developed on the entire control volume. From the momentum conservation principle, the force developed on this volume is equal to the momentum change of the mass flow rate, given by Eq. 8.15. Taking the turbine blade cross-section, where the area is A_b (= $\pi D^2/4$) and the velocity is V_b, as the reference section for the definition of the mass flow rate, the momentum conservation equation yields the following expression for the axial force:

$$F_x = (\dot{m}V)_i - (\dot{m}V)_e = \dot{m}(V_i - V_e) = \rho A_b V_b (V_i - V_e). \tag{8.21}$$

A comparison of the last two equations gives an expression for the unknown velocity at the turbine blade, V_b:

$$V_b = \frac{1}{2}(V_i + V_e). \quad (8.22)$$

Thus, the air velocity at the turbine blades is equal to the arithmetic average of the velocities at the inlet and the outlet of the control volume. Since the power developed by the turbine is equal to the product of the axial force, F_x, and the local velocity, V_b, an expression for the power developed by a turbine with diameter D may be derived as follows:

$$\dot{W} = \frac{\pi D^2}{16}\rho(V_i + V_e)\left(V_i^2 - V_e^2\right). \quad (8.23)$$

The last expression proves that, since V_e must be finite, the actual power obtained from a wind turbine is less than the available wind power, which is given by Eq. 8.2. In a typical wind turbine installation, the inlet velocity V_i is equal to the incoming wind velocity and the latter is determined by nature. The wind turbine and its operation may be designed in a way that the exit velocity, V_e, is optimized to yield maximum power. A simple way to perform this optimization is to differentiate the last expression with respect to the adjustable parameter V_e and ensure that the resulting expression vanishes:

$$\frac{\partial \dot{W}}{\partial V_e} = 0 \Rightarrow 3V_e^2 + 2V_e V_i - V_i^2 = 0 \Rightarrow V_e = \frac{1}{3}V_i. \quad (8.24)$$

Therefore, for the optimum operation of a given turbine, the downstream air velocity should be one-third of the incoming wind velocity. Under this condition the maximum power a wind turbine may produce at continuous operation is as follows:

$$\dot{W}_{max} = \frac{8}{27}\rho A V_i^3 = \frac{2\pi}{27}\rho D^2 V_i^3. \quad (8.25)$$

A figure of merit, or efficiency,[5] for the wind turbines is the ratio of the actual power they produce to the available power in the wind stream:

$$\eta = \frac{\dot{W}}{\dot{W}_{av}}. \quad (8.26)$$

A comparison of Eqs. 8.2 and 8.25 proves that the maximum efficiency a wind turbine may have is $\eta_{max} = 16/27 = 0.593$. Simply put, even an ideal wind turbine

[5] In some publications, the turbine efficiency is called *power coefficient* and is denoted by C_p.

will convert less than 60% of the available power in the wind current to electric power. This is sometimes referred to as *the Betz limit* or *Betz's law*.

8.2.4 Efficiency of Actual Wind Turbines

The steam and gas turbines that are currently used with power plants have isentropic efficiencies close to 80%. These engines are well designed, and also, operate with controlled conditions, such as: almost constant inlet pressure and temperature of the working fluid; several stages; optimum expansion and pressure drop between stages; relatively short blades with low range in the linear blade velocities, ωr, where r is the distance from the blade hub and ω the rotational speed of the turbine; no fluid loses or *spillage effects*; and optimum fluid velocity after each stage. In contrast, wind turbines have: only one stage; are designed to operate with a wide variety of inlet conditions; are open to the atmosphere with the implied fluid losses or *spillage effects*; have very long blades and, therefore, a very wide range of local blade velocities; and must be designed to operate within a wide range of inlet velocities. In addition, Betz's law limits the maximum efficiency of the wind turbines to less than 60%. As a result, wind turbines have significantly lower efficiencies than steam and gas turbines, typically in the range 20–45%.

The efficiency of a wind turbine would depend on several factors such as:

1. The ambient air velocity, which is variable.
2. The rotational design speed of the wind turbine, ω.
3. The type of wind turbine.
4. The size of the wind turbine, or blade diameter D.

A large number of empirical observations with several types of wind turbines have shown that a single parameter, *the tip-speed ratio*, defines very well the efficiency of a wind turbine. The tip-speed ratio (TSR) is the ratio of the velocity at the tip of the blade ($\omega D/2$) to the instantaneous velocity of the wind:

$$TSR = \frac{\omega D}{2V_i} = \frac{\pi n D}{60 V_i}, \tag{8.27}$$

where n is the rotational speed of the engine in revolutions per minute (rpm).

Figure 8.7 depicts the actual efficiencies of several common types of wind turbines as a function of TSR. It is observed that each type of wind turbine has a TSR where optimum operation occurs and that several types of engines, e.g. the Savonius and the American multi-blade types, have a narrow range of operation, outside which the engine does not produce power. The currently used two-blade and three-blade turbines, whose blades are long enough to cover very large areas, have the highest range of efficiencies, close to 40%.

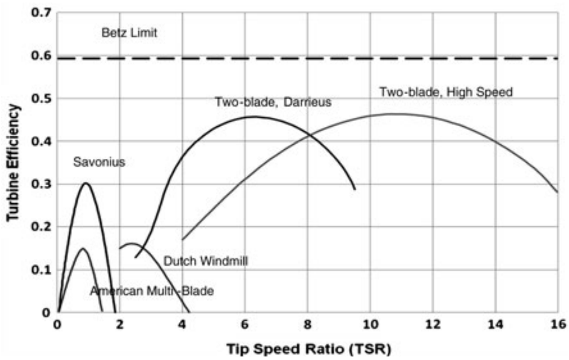


Fig. 8.7 Efficiencies of wind turbines as functions of TSR

8.3 Power Generation Systems: Parts of Common Wind Turbines

As shown in Fig. 8.7, the Savonius, Dutch, and American multi-blade turbines have significantly lower efficiencies than the two-blade engines. In addition, the first engines have high starting torque, which makes them suitable for mechanical work production applications, such as water pumping and grain grinding, but not for power generation. Two or three-blade wind turbines are now used almost exclusively for the production of electric power. The main parts of the systems that comprise these wind turbines are:

1. *The tower*: Since velocities close to the ground are very low and there must be good clearance between the lower part of the blades and the ground, the wind turbines are placed on top of a tower at a significant height above the ground. The height of the tower depends on the diameter of the blade and is of the order of magnitude of the blade diameter, *D*, allowing a clearance of *D*/2, between the ground and the lower part of the blade. Thus, towers are between 30 and 100 m high. The tower is a simple structural element, usually made of reinforced concrete, which is designed to withstand the axial force and resulting moment generated by the wind turbine. It is typically thicker at the lower part and is usually designed as a hollow structure to allow easy access to the top for engine repairs at the turbine hub. Some older (and shorter) towers were designed as trusses made of metal.
2. *The yaw bearings and yaw break*: Because the wind turbine must rotate to face the instantaneous direction of the wind, the entire electricity producing system is pivoted on strong bearings that allow the rotation of the system around a vertical axis. The drag force on a downstream rotating vane or a simple rudder provides the force for this rotation. In order to avoid overshooting in the rotation of the electricity generating system and unnecessary power fluctuations, the yaw break system slows the rotational motion by providing damping.

3. *The rotor blades*: They are the most important part of the generating system, where the wind energy is imparted to the engine. They are very long, typically 30–100 m in diameter. The rotor blades are designed aerodynamically with pitch angles that vary with the distance from the hub and they are made of low-weight and strong materials. Low density woven composites are now typically used for the turbine blades, which are typically hollow. The blades are connected to the hub, which extends to a horizontal metal shaft that becomes the prime mover of the engine. The shaft is supported by a series of bearings. In the more advanced and better optimized engines, a mechanism is put in place that changes the pitch of the blades to produce maximum power at the instantaneous wind velocity. These mechanisms are made of sensors and actuators, which measure the magnitude of the instantaneous wind velocity, adjust the position of the base of the blades inside the hub and, thus change the pitch of the entire blade. The actuator mechanisms are attached to the blades, rotate with them and are supported by their own *pitch-control bearings.*
4. *The gear box:* In order to minimize the centrifugal stresses, the rotational speed of the blades at operating conditions is fairly low, typically of the order of 100 rpm. A gearbox steps up the rotational speed of the prime mover to reach a range 2,000–3,000 rpm and transmits the power to a secondary high rpm shaft, which is connected to the generator. A small fraction of the blade power is dissipated in the gear box by friction. For this reason, larger wind power engines may require a cooling system for their gearbox.
5. *The generator*: Both permanent magnet generators and generators with electromagnets (exciters) are used for the conversion of the mechanical power to electricity. The generators of the more modern and larger engines are rated in MW (typically 1–3 MW) and include power electronics, such as *Variable Speed Constant Frequency* devices (VSCF), which convert the variable frequency of the secondary shaft to a constant frequency. Any power spikes in the system are usually absorbed by the inertia of the rotor.

One of the salient characteristics of wind power systems is that very high power fluctuations occur with relatively low wind velocity changes. For example, an increase of the wind velocity from 8 to 10 m/s (or 25%) would cause a power increase of almost 100%. Frequent power variations of this magnitude are undesirable because they are associated with high stresses on the blades, on the prime mover and gear as well as with strong power fluctuations on the electric grid. These types of problems are minimized by designing the wind turbines to produce almost constant power.

A glance at Figs. 8.5 and 8.7 proves that, if a wind turbine is designed to operate at the maximum range of the prevailing wind speeds, the turbine would produce maximum power, but for a very short time during the year. Therefore, the total energy that would be produced annually would be very small. For this reason, wind turbines are designed to operate and produce their rated power within a range of wind speeds that are significantly lower than the maximum prevailing wind

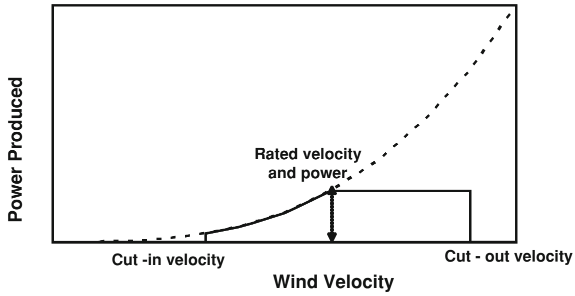


Fig. 8.8 The operation characteristics of a flat-rated wind turbine

velocity. Adjusting the pitch of the blades and the resistance of the prime mover shaft assists in producing constant power in the velocity range between the *rated velocity* and the *cut-out velocity,* which is the upper limit of the velocity range where the engine may operate safely. Such turbines are often called *flat-rated wind turbines.* At the cut-out wind velocity the engine is shut off for safety by *feathering* the blades and removing all the resistance from the prime mover. At this state, the blades may rotate but the engine does not produce any power. On the opposite end, where the wind velocity is very low, the wind turbine is designed to stop producing power at the *cut-in velocity* of the wind. Typically, the cut-in velocity is low and the turbine produces less power than the rated power. There is a range of velocities between the cut-in velocity and the rated velocity, where the power output of the turbine increases according to Eq. 8.2. Figure 8.8 shows the operation of a flat-rated wind turbine. This turbine produces power according to the cubic relationship of Eq. 8.2 between the cut-in and the rated velocities. The power the turbine produces is constant between the rated and the cut-out velocities. And, the turbine does not produce any power when the prevailing wind velocity is higher than the cut-out velocity. It is apparent that, when the wind velocity increases beyond the rated velocity, which is a design parameter, the turbine does not convert all the available power of the wind velocity. Actually, a large fraction of the available power from the wind is not harnessed when the turbine operates at its rated power. This limits significantly the average efficiency of the turbine, when this average is taken over the entire spectrum of wind velocities.

Table 8.1 lists key characteristics and velocity parameters of four types of wind turbines produced by two of the major wind turbine manufacturers. It is observed in this Table that the cut-in, the cut-out and the rated velocities are within narrow ranges in the four types of commercial turbines. The rated velocity of the turbines is in the range 27–30 m/s and the turbines do not produce any power when the wind velocity is more than 60 m/s.

Because of the very high variability of wind velocity, an important parameter in the operation of wind turbines is their capacity factor, CF, defined as the ratio of the average power produced annually to the rated power:

Table 8.1 Key parameters of four commercially available wind turbines (data obtained from www.gewindenergy.com and www.vestas.com)

Rotor diameter, m	104	100	80	70.4
Rated power, kW	3,600	2,500	2,000	1,500
Rated velocity, m/s (mph)	14(30.5)	12.5(27.2)	15(32.7)	13(28.3)
Cut-in velocity, m/s (mph)	3.5(7.6)	3.5(7.6)	4.0(8.7)	4.0(8.7)
Cut-out velocity, m/s (mph)	27(58.8)	25(54.5)	25(54.5)	25(54.5)
Rotor angular speed, rpm	8.5–15.3	N/a	9–19	12–22
TSR	3.3–6.0	N/a	2.5–5.3	3.4–6.2
Efficiency at rated power	0.26	0.27	0.20	0.29

$$CF = \frac{\dot{W}_m}{\dot{W}_r}. \tag{8.28}$$

An analytical expression for the capacity factor in terms of the ratio V_r/V_m, may be obtained using the Rayleigh distribution of Eq. 8.14 and expressing the velocity in terms of the rated velocity V_r, and the temporal pattern of the power produced as depicted in Fig. 8.8. It is noted that an actual flat-rated wind turbine will produce power in the range of wind velocities between the cut-in and cut-out velocities, $[V_{ci}, V_{co}]$. After performing the integration operation, one obtains the following expression for the capacity factor:

$$\begin{aligned} CF = {} & \frac{\pi^2}{16}\left[\exp\left[-\frac{\pi}{4}\left(\frac{V_r}{V_m}\right)^2\right] - \exp\left[-\frac{\pi}{4}\left(\frac{V_{co}}{V_r}\right)^2\right]\right] \\ & + \int_{V_{ci}}^{V_r}\left[\frac{\pi}{2}\left(\frac{V}{V_r}\right)^3\left(\frac{V}{V_m^2}\right)\exp\left(-\frac{\pi}{4}\frac{V^2}{V_m^2}\right)\right]dV \end{aligned}. \tag{8.29}$$

The first two terms of Eq. 8.29 stem from the range of velocities $[V_r, V_{co}]$, where the power produced is constant. Wind turbines are typically designed with rated velocities corresponding to the maximum of the power distribution function. From Eq. 8.14 the latter is: $V/V_m = (8/\pi)^{1/2} = 1.60$. Under this condition, the capacity factor is approximately equal to 0.35. Of this number, the first part of Eq. 8.29 contributes 0.13 and the second part 0.22. This implies that such a wind turbine operates at rated power 13% of the time and at less than rated power 87% of the time. However, because of the significantly lower power produced in the range under the rated power $[V_{ci}, V_r]$ when one looks at the total energy production of the turbine, 37% of the total energy (13/35) is actually produced during the 13% of the time, when the turbine operates at rated power.

8.3.1 Smaller Wind Turbines

While the power systems described in the last section pertain to relatively large power systems (1–3 MW), there are several smaller engines in the market that produce from a few kW to 100 kW. These are classified as small engines and are designed to produce power for local use, not for feeding the electric grid. Small engines are ideally suited for use in remote areas, which are not served by the existing electric grid. Farms are ideal locations for such smaller engines, whose output is used for domestic consumption as well as for pumping and irrigation. In 2010 there were 600,000 small wind turbines installed in the world. At an average power of 1.5 kW, this translates to 900 MW installed electric capacity, or close to the capacity of a large nuclear unit. In rural Texas alone, there were 20 MW of installed small wind turbine capacity. When these systems are also coupled with energy storage systems, they may supply without interruption the energy needs of buildings and small communities in rural areas.

Another type of system that has been marketed more recently is a small wind turbine, which is installed on roofs of houses in urban and suburban areas. Most of these designs have a vertical axis and are based on the Darieus concept. The number of blades on such systems varies from two to eight. These small engines are placed on the rooftops of houses and, depending on their size, may provide 0.5–5 kW of power. This is enough power to satisfy the needs of a household or a small business building. Since vertical axis engines operate independent of the direction of the wind, it is not necessary to pivot these engines for the production of optimum power. Several types of these small engines have aerodynamic blades and use both the drag and the lift components of the wind force for optimum power production. While it is possible to produce a considerable amount of power with the installation of many such small engines in suburban communities, where the average wind velocity at the rooftop level is adequate, the generated noise of the engines has become a significant drawback to their widespread implementation.

8.3.2 Other Wind Power Systems

Another wind engine concept is based on the *Magnus effect* or *lift effect:* When a rotating cylinder is in a crosswind a force is developed on the cylinder, which is perpendicular to the axis of rotation and the direction of the wind. A familiar manifestation of the Magnus effect is the *curveball* in baseball or tennis, where the ball changes direction because of its rotation (spin). Figure 8.9 shows the development and the direction of the *lift* or *Magnus* force on a rotating horizontal cylinder. If the cylinder rotates with angular velocity ω, the radius and length of the cylinder are R and L respectively, the air density is ρ and the wind velocity is V_i, then the Magnus force is given by the expression:

$$\vec{F}_M = 2\pi\rho R^2 L\left(\vec{\omega} \times \vec{V}_i\right). \tag{8.30}$$

where x denotes the vector product. Now, if the rotating cylinder moves in the direction of the Magnus force with a velocity V_c, it produces power, which is given by the scalar product of the force and last velocity vector:

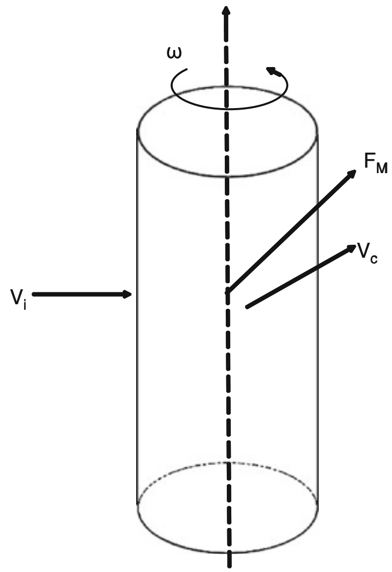


Fig. 8.9 Lift force developed on a rotating cylinder with crosswind

$$\dot{W} = \vec{F}_M \cdot \vec{V}_c = 2\pi\rho R^2 L\left(\vec{\omega} \times \vec{V}_i\right) \cdot \vec{V}_c. \tag{8.31}$$

With the appropriate combination of rotational speed and wind velocity, engines may be developed that would utilize the Magnus effect and produce a significant amount of power. One such engine, the *Madaras engine*, was proposed and developed in the 1930s in New Jersey, U.S.A. It consisted of several vertical cylinders of 30 m height, rotating at 120 rpm (2 rps) and placed on small, flat rail cars that were carried around a circular track. Small electric motors ensured the rotation of the cylinders. Because V_c changes direction as the cars move on their circular track, the direction of the rotation of the cylinders was reversed twice during a full trip around the track. This change in the rotational motion of the cylinders ensured that the developed Magnus force correlated well with the circular motion of the cars to produce power. Electrical generators were driven by the wheels of the cars to produce power, which was to be fed to the grid. The Madaras engine attracted a great deal of attention, but met an inglorious end: while it was in the development stage, it was destroyed by a storm and was not rebuilt.

The main advantage of engines, such as the Madaras engine, is that they produce power regardless of the direction of the wind and they are built on the ground. However, a significant disadvantage of such engines is that the lift force produced is a secondary and much weaker force than the drag force. Thus, the power that may be produced is significantly less than the available power of the wind. This is the main reason why efficient engines based entirely on the Magnus effect have not been built.

8.3.3 The Future of Wind Power

The first decade of the twenty-first century has seen a very significant expansion in harnessing power from the wind for the production of electricity. From approximately 14,500 MW worldwide installed electric capacity in 1999, the capacity in 2009 rose to 115,000 MW, an eightfold increase. Most of the electric capacity has been installed in the European Union. The worldwide installed electric capacity is expected to continue increasing at a high rate in the next decade. This trend is currently fueled by the following factors:

1. Wind power is abundant in several regions of the world and was previously relatively undeveloped.
2. The world population and governments, and especially those in Europe have become more aware of the adverse effects of the fossil fuels.
3. International agreements, such as the Kyoto agreement, mandate higher use from sources such as solar and wind power.
4. Governmental subsidies for clean or "green" power, which make the production of electricity from wind more competitive economically.
5. National or regional regulations and directives that call for a percentage of the electric power to be produced from renewable or "green" sources.

It is expected that, three of the current drawbacks for the utilization of wind power will be overcome, namely:

1. Wind power is diffused and is not harnessed in large power plants, which are the norm in the electricity production industry. A high capacity electric power plant (e.g. 1,000 MW) similar to the large fossil fuel or nuclear power plants, which utilize significant economies of scale, is impossible to construct. The national electric grids and distribution systems are designed for such large power plants and not for the smaller wind power engines. One may place together a large number of smaller wind turbines, 0.5–3 MW each, in *wind farms*, to produce larger amounts of power. In addition, electric grids may be adapted for the lower power inputs.
2. The prime locations for wind power generation are not necessarily in areas where electric transmission lines exist or where the transmission grid has the capacity to carry more power. New or more modern transmission lines may need to be built.
3. The technology of construction and installation of wind turbines is relatively new. This is not a significant disadvantage, because with longer experience, better and more cost-effective systems will be produced in the near future.

The main and most significant drawback, which limits the expansion of wind power, is that it is largely unpredictable and almost intermittent. The discipline of meteorology has advanced sufficiently so that, at present, we are able to predict with relative accuracy the wind speed and available wind power at a given location in the next few hours and, with lesser accuracy, in the next few days. As we move further into the future, e.g. weeks or months, the predictive methods for the strength of wind velocity and power become quantitatively unreliable. For example, science does not have an accurate answer to the question: "What will be the wind speed at the Isle of Jersey one month from today?"

Local governments and companies charged with satisfying the electricity demand of the population in a region or a nation simply cannot rely with any degree of certainty that wind turbines will produce sufficient electric power to satisfy the entire electricity demand at a given day or hour of the year. The available statistical methods are sufficient for reliable correlations to be obtained, but the correlation coefficients are very low for wind power to achieve the level of reliability required for the continuous and uninterrupted supply of the demand for electricity. An electricity generating company may not rely on wind power to supply a large fraction of the demand for electricity for its customers and must always have another option for backup. For this reason, wind power is and will be an alternative option to be harnessed whenever it is available, with the necessary backup of other electric power plants, which may produce electricity continuously during wind-less days or weeks. The absolute necessity of these backup units and the cost associated with them imposes the most significant drawback for the expansion of wind power. The development of new power storage methods and systems will help significantly with this issue. However, energy storage, especially over long time durations, is associated with significant energy losses, as may be seen in Chap. 12. This adds significantly to the cost of wind power, which is more abundant in the spring and fall, while the electric power demand peaks in the summer.

8.4 Environmental Effects

Wind power is one of the most benign and most environmentally friendly forms of electric energy production. It does not involve any chemicals and does not produce any harmful emissions or thermal pollution. The materials that make the towers and the components of the engines are commonly used structural and engineering materials. Hence, the construction and operation of the wind turbines does not impose any environmental threat. There are a few rather minor environmental issues associated with wind power, which are enumerated in the following paragraphs:

1. *Noise pollution*: A rotating engine always produces noise and wind turbines are no exception, especially the parts that operate at higher rpm's. However, large wind turbines are typically located in remote, rural areas with low population density. This mitigates the effect of the noise to human populations, but may have a significant effect on the wildlife in the nearby areas, which migrate because of the noise. Wind farms even if they are located in remote areas

disturb the balance of the local ecosystem. Noise pollution is also the limiting factor for the expansion and more widespread use of small engines in urban or suburban environments.

2. *Bird injuries and mortality*: Flying birds may be caught by the rotating blades and be killed. The overall motion of the blade and the pressure reduction that occurs immediately before the rotating blades, as shown in Fig. 8.6, detract the birds and often kill them. This may have a significant effect on migrating species of birds. Concerns for the bird populations that migrate to the shores of the Gulf of Mexico have put severe restrictions on the wind power development projects near the Gulf, where the sea breeze is always strong and the available wind power averages 500–600 W/m^2.
3. *Aesthetic pollution*: The picturesque landscape of remote, pristine areas is often disturbed by the placement of groups of high towers with wind turbines on top. The aesthetic pollution has raised significant opposition to the development of wind power, especially in areas with tourist interest.
4. *Radio and TV signal interference*: Given that many wind turbines are located near the top or the sides of hills and mountains, their operation interferes with the transmission of television and radio signals, especially with television re-transmission towers. This effect is mitigated by two trends: a) the location of the vast majority of wind turbines is in remote areas, where very few signals need to be transmitted, and b) the recent trend in communications is to transmit signals via fiber optic wires, at least in the most populated areas. This trend will become the norm in the future for all urban areas.

It is apparent that, while not insignificant, the environmental problems associated with wind power are much less harmful to the environment and the ecosystems than the effects of most other renewable energy sources. The further expansion of wind power and the substitution of conventional power sources by wind would be benevolent to the environment.

Problems

1. What is the power per unit area of a 10 knot uniform wind (in W/m^2)?
2. The drag force on a flat surface due to the wind flow is given by the expression:

$$F_D = \frac{1}{2} A\rho C_D V^2, \ with\ C_D = \frac{24}{\mathrm{Re}}\left(1 + 0.32\mathrm{Re}^{0.687}\right)\ and\ \mathrm{Re} = \frac{\rho L V}{\mu}, \quad (8.32)$$

where C_D is the drag coefficient, Re is the Reynolds number and L is a characteristic dimension, which is equal to the square root of the area. The area A is the area that is perpendicular to the wind direction. A sail with 16.5 m^2 area faces the wind perpendicularly. What is the force on the sail when the wind velocity is 6, 10, 25 and 40 knots? What do you observe in these calculations?

3. The friction coefficient on the surface of the sea is f = 0.21. What is the shear stress produced by a wind of 12 knots?
4. A pitot tube at a height of 3 m, measures wind velocity 5.3 m/s. What is the wind velocity at a height of 50 m? If the center of a 45 m diameter wind turbine is placed at 50 m, what is the maximum power this turbine may produce?
5. The wind velocity at a height 3 m is measured to be 2.3 m/s. What is the wind velocity at 20 m? If the air density is 1.21 kg/m^3, what is the mass flow rate of the air that passes through a surface of 10 m width that extends from 2 to 30 m?
6. Starting from first principles, derive analytically *Betz's Law*. Justify every step in your analysis.
7. A 14 knot wind stream comes to a stop at the front of a tall building. What is the pressure increase (stagnation pressure) at the center of the surface of the building?
8. Determine the maximum power a wind turbine may produce at the following wind speeds: 1, 5 and 10 m/s.
9. What is the TSR of a 70 m diameter wind turbine that rotates at 80 rpm when the wind speed is 15 knots?
10. The cut-in velocity of a 30 m diameter flat-rated wind turbine is 1.5 m/s, the rated velocity is 8 m/s and the cut-out velocity 22 m/s. The turbine is placed in a location with the following wind velocity frequency:

$0 < V < 1.5$ m/s 22% of the time.
$1.5 < V < 3$ m/s 12% of the time.
$3 < V < 5$ m/s 12% of the time.
$5 < V < 8$ m/s 28% of the time.
$8 < V < 15$ m/s 12% of the time.
$15 < V < 22$ m/s 10% of the time.
$22 < V$ m/s 4% of the time.

What is the total energy (in kWh) you expect this turbine to produce during a year?

11. Modifications are performed on the turbine of problem 10 and its rated velocity is extended to 15 m/s. What is the annual total energy the turbine will produce?
12. During a 6 h period, the wind velocity varies linearly from 1.2 to 12 m/s. A 40 m diameter flat-rated wind turbine is subjected to this wind. The turbine has cut-in velocity 2 m/s, rated velocity 7 m/s and cut-out velocity 15 m/s. The conversion efficiency of the turbine is 70% of the maximum efficiency. Determine the amount of energy this turbine produces during the 6 h period.
13. Determine the exeedance curve corresponding to a Rayleigh probability distribution with $V_m = 14$ m/s.
14. Determine the most probable velocity and most probable power of a Rayleigh wind velocity distribution with $V_m = 28$ mph.

15. During a particular 24 h period, the wind velocity in Abilene, Texas could have been approximated with a half-sine function, $V = A\sin(\pi t/24)$, where t is measured in hours. The amplitude A was 16.5 m/s during that day. A 38 m diameter wind turbine with cut-in velocity 1.5 m/s, rated velocity 9 m/s and cut-off velocity 21 m/s was placed perpendicular to the wind. If the overall efficiency of the turbine-generator system is constant and equal to 0.32, what is the energy that was produced during that 24 h period?
16. It is apparent that a desirable attribute of wind turbines is to have high rated velocities. Under the conditions described in problem 13, what would have been the total energy produced if the rated velocity for the turbine were 12 m/s? How about if the rated velocity were 17 m/s?
17. A wind tower measures the velocity of wind at a height 30 ft (9.1 m) to be 12 m/s. Calculate the power density of the wind at this height. A large wind turbine with 48 m diameter is placed at this location. The hub of the turbine is 50 m from the ground. Determine the power density of the wind at the hub. Also determine the maximum power that can be delivered by this turbine at this wind velocity.
18. "We can extract so much power from the wind that it would be possible to satisfy the entire electricity demand of humanity using wind power alone." Comment in a 250–300 word essay.