# Chapter 10
# Basic Open System Cycles

## 10.1 Steam Turbine: Rankine Cycle

About 70-75% of the World's electrical energy are produced in steam cycles. An external heat source is used to evaporate pressurized water, and then the high pressure vapor is expanded in steam turbines. The fuel for most steam power plants is coal, followed by nuclear power. Since the heat is supplied externally, many other heat sources can be used, including oil, gas and heat from solar radiation.

The Rankine cycle, which we shall discuss now, has been the basic steam cycle for power generation, it is named after William Rankine (1820-1872). Modern steam power plants use more efficient variations of the Rankine cycle that will be discussed in Sec. 12.2.

Figure 10.1 shows a schematic of the Rankine cycle. Saturated liquid water is pressurized in an adiabatic pump (1-2). In the steam generator, the high pressure water is heated, evaporated and superheated (2-3). The superheated steam is expanded in the steam turbine (3-4), which generates work; part of the turbine work is used to run the pump, the net work is delivered to the generator. The turbine discharges into the condenser (4-1), in which the steam is condensed back to the initial state. Pump and turbine may be irreversible. Figure 10.2 shows the Rankine cycle in the diagrams with respect to saturation lines. Work and heat for the four processes are (see Sec. 9.13)

$$\begin{array}{lll} \text{1-2 adiabatic pump:} & w_{12} = h_1 - h_2 & , \; q_{12} = 0 \, , \\ \text{2-3 isobaric heating:} & w_{23} = 0 & , \; q_{23} = h_3 - h_2 \, , \\ \text{3-4 adiabatic turbine:} & w_{34} = h_3 - h_4 & , \; q_{34} = 0 \, , \\ \text{4-1 isobaric cooling:} & w_{41} = 0 & , \; q_{41} = h_1 - h_4 \, . \end{array} \tag{10.1}$$

The net work of the cycle is

$$w_{\odot} = w_{12} + w_{23} + w_{34} + w_{41} = h_1 - h_2 + h_3 - h_4 \, , \tag{10.2}$$

and the heat supply is

H. Struchtrup, *Thermodynamics and Energy Conversion*,
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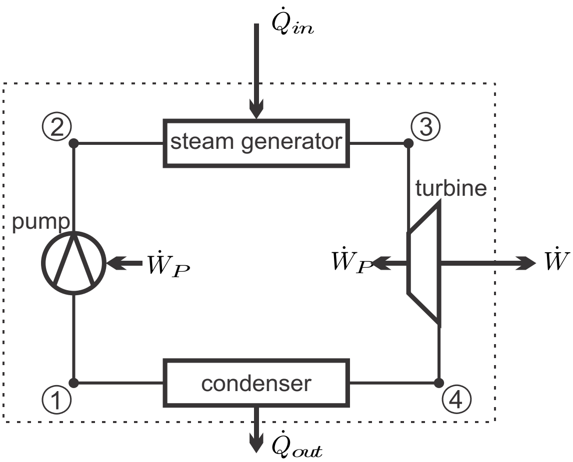


**Fig. 10.1** Schematic of Rankine cycle. The dotted line shows the overall system boundary for the cycle. Part of the turbine work is used to drive the pump.

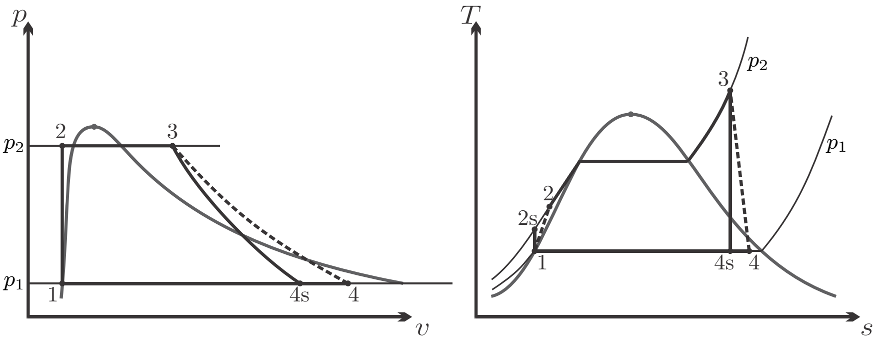


**Fig. 10.2** Rankine cycle: p-v- and T-s-diagrams

$$q_{in} = q_{23} = h_3 - h_2 \; . \tag{10.3}$$

Accordingly, the thermal efficiency of the Rankine cycle is

$$\eta_R = \frac{w_\odot}{q_{in}} = \frac{h_1 - h_2 + h_3 - h_4}{h_3 - h_2} = 1 - \frac{h_4 - h_1}{h_3 - h_2} \; . \tag{10.4}$$

The total power produced, the heat consumed, and the heat rejected by the cycle follow after multiplication with the mass flow $\dot{m}$ as

$$\dot{W} = \dot{m} w_\odot \quad , \quad \dot{Q}_{in} = \dot{m} q_{23} \quad , \quad \dot{Q}_{out} = \dot{m} q_{41} \; . \tag{10.5}$$

In early steam engines the steam was expanded in piston-cylinder devices, not in turbines. Reciprocating piston engines have large load changes, and are more bulky, while turbines are running at constant loads, and can deliver the same amount of power with a significantly smaller footprint.

The condenser is James Watt's (1736-1819) most important contribution—of many—to the improvement of steam engines. The temperature $T_c$ in the condenser is prescribed through heat exchange with the environment, so that $T_c$ is not much above the environmental temperature $T_0$. The condenser pressure is the corresponding saturation pressure $p_{\text{sat}}(T_c)$ which lies substantially below the environmental pressure $p_0$. Since the work delivered by a turbine grows with the pressure ratio,[1] a steam cycle with a condenser will have a significantly larger work output than a cycle that discharges into the environment. The gain of work through the condenser is illustrated in the T-s-diagram of Fig. 10.3.

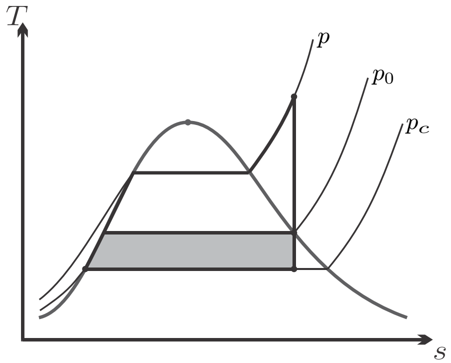


**Fig. 10.3** Rankine cycle with and without condenser. The shaded area is the gain of work through the condenser.

A condenser requires a significant amount of cooling which usually is provided by a cooling water cycle that employs cooling towers. Due to the difficulty of providing sufficient cooling, most steam locomotives do not have condensers, and discharge into the environment. Therefore steam locomotives have low thermal efficiencies, and must be supplied with fresh water frequently.[2] In a steam power plant with condenser the working fluid runs continuously through the system in a closed loop, which allows the use of

[1] This can best be seen for an ideal gas with constant specific heats, for which an isentropic turbine delivers the work $w_T = c_p T_1 \left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right)$.

[2] The water supply for steam locomotives is a driving force in Sergio Leone's wonderful "spaghetti western" *Once Upon a Time in the West* (feat. Henry Fonda, Claudia Cardinale, Jason Robards, Charles Bronson).

purified water to reduce pipe corrosion. Some of the cooling water, however, evaporates in the open cooling towers from which warm moist air rises. As the rising moist air cools down by heat exchange with the surrounding air, some of the water condenses to clouds (Sec. 19.9).

Would the pump be fed with saturated liquid-vapor mix, the sudden collapse of vapor bubbles during the compression process (cavitation) would induce shock waves that lead to material damage and, ultimately, pump failure. Cooling into the compressed liquid region only increases the heat removal, and has no benefit. Thus, the pump should be fed with saturated liquid.

As the steam expands in the turbine, it crosses the saturation line and small liquid droplets form. These droplets hit the fast rotating turbine blades and cause corrosion. On the other hand, a smaller quality $x_4$ reduces the amount of heat rejected in the condenser, and thus improves thermal efficiency. To obtain a good balance between efficiency and prevention of corrosion, one aims at having the quality at the turbine exit at $x_4 = 0.9$ or higher.

The pipes of the steam generator are normally made from standard steel. At the high pressures that occur in the cycle, the temperature for the pipes should not exceed $\sim 560\,^\circ\mathrm{C}$.

Increase of the pressure in the steam generator improves efficiency since the average temperature for heat supply increases. However, when the pressure becomes large, and the turbine inlet temperature is capped at $T_{\max}$, the expansion into the condenser leads to low qualities, and thus damage of the turbine blades due to droplet formation. This is illustrated in Fig. 10.4, where the standard Rankine cycle has the corner points 1-2-3-4'. To shift the point 4' towards values of higher quality requires either a turbine inlet temperature $T_3$ above the maximum temperature $T_{\max}$ for the steam generator, or lower pressure $p_3$. Another alternative, as illustrated in the figure, is to expand the steam in a first turbine to the intermediate pressure $p_5$, reheat back to $T_{\max}$, and then expand in a low pressure turbine to the condenser pressure $p_c$. Net work, heat in and thermal efficiency for the reheat cycle are

$$\begin{aligned} w_{\odot} &= h_1 - h_2 + h_3 - h_4 + h_5 - h_6 \; , \\ q_{in} &= h_3 - h_2 + h_5 - h_4 \; , \\ \eta_{reheat} &= 1 - \frac{h_6 - h_1}{h_3 - h_2 + h_5 - h_4} \; . \end{aligned} \tag{10.6}$$

More complex steam cycles involve multiple turbine stages, and internal heat regeneration to improve efficiency (Sec. 12.2).

## 10.2 Example: Rankine Cycle

As an example we consider a standard Rankine cycle with specifications based on the discussion in the previous section: The condenser temperature is $T_1 = 40\,^\circ\mathrm{C}$, the upper pressure is $p_2 = p_3 = 80\,\mathrm{bar}$, pump and turbine are

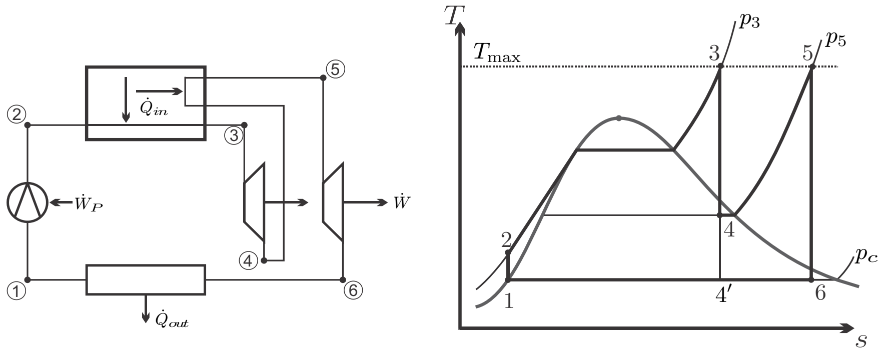


**Fig. 10.4** Schematic and T-s-diagram of Rankine cycle with reheat

irreversible with isentropic efficiencies $\eta_P = 0.85$ and $\eta_T = 0.88$, respectively, and the quality at the turbine exit is $x_4 = 0.9$. Schematic and thermodynamic diagrams for this cycle are as in Figs. 10.1 and 10.2.

We begin with the computation for the pump. State 1 is saturated liquid at $T_1$ with the properties

$$v_1 = v_f\left(T_1\right) = 0.001008 \frac{\mathrm{m}^3}{\mathrm{kg}} ,$$
$$p_1 = p_{\mathrm{sat}}\left(T_1\right) = 7.384\,\mathrm{kPa} ,$$
$$h_1 = h_f\left(T_1\right) = 167.57 \frac{\mathrm{kJ}}{\mathrm{kg}} .$$

A reversible pump between $p_1$ and $p_2$ requires the work

$$w_{p,rev} = h_1 - h_{2s} = -v_1\left(p_2 - p_1\right) = -8.057 \frac{\mathrm{kJ}}{\mathrm{kg}} .$$

The work for the irreversible pump is

$$w_p = h_1 - h_2 = \frac{w_{p,rev}}{\eta_P} = -9.48 \frac{\mathrm{kJ}}{\mathrm{kg}} ,$$

and thus the enthalpy after the pump is

$$h_2 = h_1 - w_P = 177.05 \frac{\mathrm{kJ}}{\mathrm{kg}} .$$

Next we study the turbine. The turbine exit state is

$$T_4 = T_1 = 40\,^{\circ}\mathrm{C} \quad , \quad x_4 = 0.9 ,$$
$$h_4 = \left(h_f + x_4 h_{fg}\right)_{|T_1} = 2333.6 \frac{\mathrm{kJ}}{\mathrm{kg}} .$$

We have to find the corresponding turbine inlet state, for which the pressure $p_3 = 8\,\text{MPa}$ is known, but not the temperature. For the solution we use a trial and error strategy: In the first step, we try $T_3^{(a)} = 550\,°\text{C}$, for which enthalpy and entropy are

$$h_3^{(a)} = h\,(8\,\text{MPa},550\,°\text{C}) = 3521.0\frac{\text{kJ}}{\text{kg}}\,,$$

$$s_3^{(a)} = s\,(8\,\text{MPa},550\,°\text{C}) = 6.8778\frac{\text{kJ}}{\text{kg}\,\text{K}}\,.$$

The isentropic expansion from this point to the condenser pressure yields

$$x_{4s}^{(a)} = \frac{s_3^{(a)} - s_f}{s_{fg}}_{|T_1} = 0.821\,,$$

$$h_{4s}^{(a)} = \left(h_f + x_{4s}^{(1)} h_{fg}\right)_{|T_1} = 2142.3\frac{\text{kJ}}{\text{kg}}\,.$$

From the definition of the turbine efficiency (9.44) we finally find the exit enthalpy for the first guess as

$$h_4^{(a)} = h_3^{(a)} - \eta_T\left(h_3^{(a)} - h_{4s}^{(a)}\right) = 2307.7\frac{\text{kJ}}{\text{kg}}\,.$$

Since this value lies below the target value of $h_4 = 2333.6\frac{\text{kJ}}{\text{kg}}$, for the second try we use a higher temperature, $T_3^{(b)} = 600\,°\text{C}$. Following the same line of arguments we find

$$h_3^{(b)} = h\,(8\,\text{MPa},600\,°\text{C}) = 3642.0\frac{\text{kJ}}{\text{kg}}\,,$$

$$s_3^{(b)} = s\,(8\,\text{MPa},600\,°\text{C}) = 7.0206\frac{\text{kJ}}{\text{kg}\,\text{K}}\,,$$

$$x_{4s}^{(b)} = \frac{s_3^{(b)} - s_f}{s_{fg}}_{|T_1} = 0.839\,,$$

$$h_{4s}^{(b)} = \left(h_f + x_{4s}^{(b)} h_{fg}\right)_{|T_1} = 2187.0\frac{\text{kJ}}{\text{kg}}\,,$$

$$h_4^{(b)} = h_3^{(b)} - \eta_T\left(h_3^{(b)} - h_{4s}^{(b)}\right) = 2361.6\frac{\text{kJ}}{\text{kg}}\,.$$

The last value lies above the target value of $h_4 = 2333.6\frac{\text{kJ}}{\text{kg}}$, and thus the actual temperature $T_3$ lies between the two guesses (550 °C,600 °C). Linear interpolation between $\left(T_3^{(a)}, h_4^{(a)}\right)$, $\left(T_3^{(b)}, h_4^{(b)}\right)$, and $h_4$ gives

$$T_3 = 574\,°\text{C}\,,$$

from which we find

$$
\begin{aligned}
h_3 &= h\,(8\,\mathrm{MPa},574\,^\circ\mathrm{C}) = 3579.1\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
s_3 &= s\,(8\,\mathrm{MPa},574\,^\circ\mathrm{C}) = 6.946\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\,, \\
x_{4s} &= \frac{s_3 - s_f}{s_{fg}}_{|T_1} = 0.829\,, \\
h_{4s} &= \left(h_f + x_{4s} h_{fg}\right)_{|T_1} = 2163.8\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
h_4 &= h_3 - \eta_T\left(h_3 - h_{4s}\right) = 2333.6\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
s_4 &= s_f + x_4 s_{fg} = 7.489\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\,.
\end{aligned}
$$

Although some small inaccuracy can be expected due to interpolation, the enthalpy $h_4$ is at the target value.

With this, all four enthalpy values are determined, and we can compute the net work, the heat in, and the thermal efficiency of the cycle 1-2-3-4 as

$$
\begin{aligned}
w_\odot &= h_1 - h_2 + h_3 - h_4 = 1236.0\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
q_{in} &= h_3 - h_2 = 3402.1\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
\eta &= \frac{w_\odot}{q_{in}} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 36.3\%\,.
\end{aligned}
$$

For a power generation of $\dot{W} = 50\,\mathrm{MW}$, the circulating mass flow is

$$
\dot{m} = \frac{\dot{W}}{w_\odot} = 40.5\frac{\mathrm{kg}}{\mathrm{s}} = 11.2\frac{\mathrm{t}}{\mathrm{h}}\,.
$$

To obtain an idea on the losses to irreversibilities we compute the efficiency for the reversible cycle 1-2s-3-4s,

$$
\eta_{rev} = 1 - \frac{h_{4s} - h_1}{h_3 - h_{2s}} = 41.3\%\,.
$$

Thus, the irreversibilities in pump and turbine reduce the cycle efficiency by 5%.

Another interesting value is the *back-work-ratio*, defined as the portion of the turbine work that is required to drive the pump, for which we find

$$
\mathrm{bwr} = \frac{|w_P|}{w_T} = \frac{h_2 - h_1}{h_3 - h_4} = 0.76\%\,.
$$

Less than one percent of the turbine work is required for the pump. To understand the low back-work-ratio we recall that for a reversible adiabatic process the work for both, pump and turbine, is given by $w = -\int v dp$. The overall pressure difference for both devices are the same, but the volumes for both processes differ considerably: The pump is fed with liquid water, while the turbine is fed with vapor which has a substantially larger volume than the liquid.

## 10.3 Vapor Refrigeration/Heat Pump Cycle

The vapor refrigeration cycle is based on the inversion of the Rankine cycle. However, as we have seen in the last example, the back-work-ratio for the Rankine cycle is very small, that is only a small portion of the turbine work is required for the pump. In the inversion of the cycle the turbine becomes a compressor which consumes power. Due to the small back-work-ratio, only a very small portion of the compressor work could be regained in the inverse pump, and thus one uses a throttling valve instead.

We consider a refrigerator or heat pump, exchanging heat with a cold and a warm external environment at temperatures $T_L$, $T_H$, respectively (recall Sec. 5.4). Heat pump and refrigerator follow the same cycle, but they differ in the external temperatures, and in the temperatures and pressures that occur in the process.

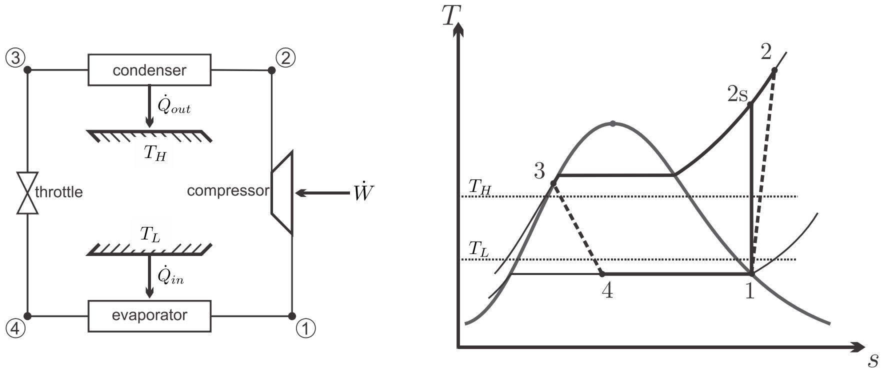


**Fig. 10.5** Schematic and T-s-diagram for a standard vapor refrigeration cycle exchaning heat with environments at $T_L$, $T_H$

The standard vapor compression cycle operates as shown in the schematic and the T-s-diagram of Fig. 10.5: Saturated or superheated vapor at low temperature (state 1) is adiabatically compressed to higher pressure (state 2). The compressed vapor is cooled and condensed (state 3) in the condenser which exchanges heat with the high temperature environment. The liquid at state 3, either saturated or compressed, is then expanded in the throttling device to the lower pressure (state 4). In the throttling process some of the liquid evaporates, and the temperature drops. The low temperature saturated mixture at state 4 receives heat from the low temperature environment and evaporates to the compressor inlet condition (state 1).

Work and heat for the four processes are

$$\begin{array}{lll} \text{1-2 adiabatic compressor:} & w_{12} = h_1 - h_2 < 0 & , \; q_{12} = 0 \, , \\ \text{2-3 isobaric cooling:} & w_{23} = 0 & , \; q_{23} = h_3 - h_2 \, , \\ \text{3-4 adiabatic throttle:} & w_{34} = 0 & , \; q_{34} = 0 \, , \\ \text{4-1 isobaric heating:} & w_{41} = 0 & , \; q_{41} = h_1 - h_4 \, . \end{array} \tag{10.7}$$

The expense for refrigerator and heat pump is the work required to drive the compressor. The gain for the refrigerator is the heat taken in from the cold environment ($q_{in} = q_{41}$), and the gain for the heat pump is the heat rejected into the warm environment ($q_{out} = q_{23}$). Thus, depending on whether we consider a refrigeration device or a heat pump we find the coefficients of performance, as gain/expense,

$$\text{refrigerator:} \qquad \mathrm{COP_R} = \frac{q_{in}}{|w_\odot|} = \frac{h_1 - h_4}{h_2 - h_1} \, , \tag{10.8}$$

$$\text{heat pump:} \qquad \mathrm{COP_{HP}} = \frac{|q_{out}|}{|w_\odot|} = \frac{h_2 - h_3}{h_2 - h_1} \, . \tag{10.9}$$

The total power consumed, the cooling power, and the heating power of the cycle follow after multiplication with the mass flow $\dot{m}$ as

$$\dot{W} = \dot{m} w_\odot \quad , \quad \dot{Q}_{in} = \dot{m} q_{41} \quad , \quad \dot{Q}_{out} = \dot{m} q_{23} \, . \tag{10.10}$$

Working fluids employed for vapor compression cycles must have good temperature-pressure characteristics. For the low temperatures reached, the pressures should be relatively high, and the specific volumes small, so that the pipes and the compressors must not be too voluminous. The critical point should be high, so that the process runs through the 2-phase region. Moreover, the operating temperatures must lie above the triple point, so that solid formation (freezing) will not occur. The heat of evaporation should be large. Finally, at least for household applications, one will prefer a non-toxic and non-flammable working fluid.

Water, obviously, is not suitable, due to extremely low vapor pressures at low temperatures, and the relatively high triple point temperature $T_{tr} = 0.01\,^\circ\mathrm{C}$. Chlorofluorocarbons (e.g., refrigerant R12, $CCl_2F_2$) are now phased out since they destroy the ozone layer, and are presently replaced by fluorocarbons (no chlorine) like R134a ($CF_3CH_2F$). Efficient alternatives are ammonia ($NH_3$) which is poisonous, and propane ($C_3H_8$) and methane ($CH_4$) which are flammable.

## 10.4 Example: Vapor Compression Refrigerator

A refrigerator operating with R134a maintains the cold environment at 0 °C and rejects heat into an environment at 26 °C, the cooling power is 13 kW.

The coefficient of performance for a Carnot refrigerator operating between these temperatures is

$$\mathrm{COP}_{\mathrm{R},C} = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\frac{299}{273} - 1} = 10.5\,,$$

and the Carnot refrigerator would consume a power of $\dot{W}_C = \dot{Q}/\mathrm{COP}_{\mathrm{R},C} = 1.24\,\mathrm{kW}$.

We shall study three increasingly realistic variants of the vapor refrigeration cycle and compare their performance among each other, and with the above COP of the Carnot refrigerator. For all three cycles we assume that the compressor draws saturated vapor, and the throttle draws saturated liquid. The three cycles considered are drawn into the T-s-diagram in Fig. 10.6.

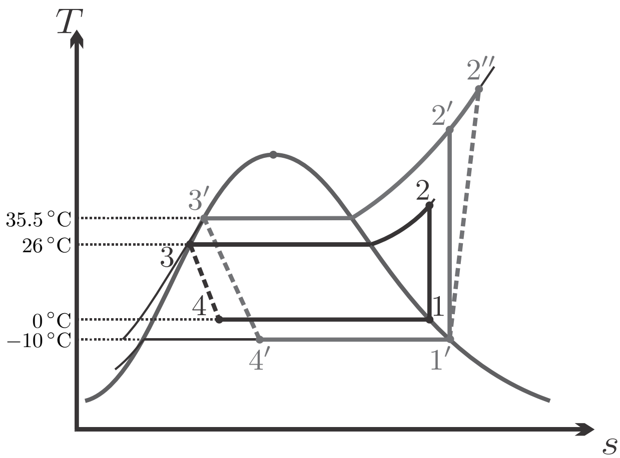


**Fig. 10.6** T-s-diagrams for three refrigeration cycles (1-2-3-4, 1′-2′-3′-4′, 1′-2″-3′-4′)

The cycle 1-2-3-4 operates without temperature difference between the cold environment and the evaporator. For the heat transfer between condenser and warm environment (2-3), there is a finite temperature difference for the cooling before the saturation temperature is reached, but the condensation occurs with infinitesimal temperature difference. Moreover, the compressor is reversible.

The cycle 1′-2′-3′-4′ operates with a temperature difference of about 10 °C to facilitate heat transfer between condenser and evaporator and their respective environments. Moreover, its compressor is reversible, while the cycle 1′-2″-3′-4′ has an irreversible compressor with isentropic efficiency $\eta_C = 0.8$.

We first analyze the cycle 1-2-3-4: From the tables we find the following property data for the corner points:[3]

$$
\begin{aligned}
h_1 &= h_g\left(0\,^\circ\mathrm{C}\right) = 247.23\frac{\mathrm{kJ}}{\mathrm{kg}} \quad , \quad s_1 = s_g\left(0\,^\circ\mathrm{C}\right) = 0.9190\frac{\mathrm{kJ}}{\mathrm{kg\,K}} \;, \\
p_2 &= p_3 = p_{\mathrm{sat}}\left(26\,^\circ\mathrm{C}\right) = 685.3\,\mathrm{kPa} \;, \\
h_2 &= h\left(685.3\,\mathrm{kPa}, 0.9190\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\right) = 263.5\frac{\mathrm{kJ}}{\mathrm{kg}} \;, \\
h_3 &= h_4 = h_f\left(26\,^\circ\mathrm{C}\right) = 85.75\frac{\mathrm{kJ}}{\mathrm{kg}} \;.
\end{aligned}
$$

The COP for this cycle is

$$
\mathrm{COP_R} = \frac{h_1 - h_4}{h_2 - h_1} = 9.93 \;.
$$

This value is below the COP of the Carnot refrigerator since irreversible losses occur in the throttle and in heat transfer over finite temperature difference for the first part (before the 2-phase region is reached) of the cooling process 2-3. With the required cooling power $\dot{Q}_{in} = 13\,\mathrm{kW}$ the mass flow and power consumption are

$$
\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = 0.081\frac{\mathrm{kg}}{\mathrm{s}} \quad , \quad \dot{W} = \frac{\dot{Q}_{in}}{\mathrm{COP_R}} = 1.31\,\mathrm{kW} \;.
$$

Next we consider the cycle 1′-2′-3′-4′ for which we find the data[4]

$$
\begin{aligned}
h_{1'} &= h_g\left(-10\,^\circ\mathrm{C}\right) = 241.3\frac{\mathrm{kJ}}{\mathrm{kg}} \quad , \quad s_{1'} = s_g\left(-10\,^\circ\mathrm{C}\right) = 0.9253\frac{\mathrm{kJ}}{\mathrm{kg\,K}} \;, \\
p_{2'} &= p_{3'} = p_{\mathrm{sat}}\left(35.5\,^\circ\mathrm{C}\right) = 900\,\mathrm{kPa} \;, \\
h_{2'} &= h\left(900\,\mathrm{kPa}, 0.9253\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\right) = 272.4\frac{\mathrm{kJ}}{\mathrm{kg}} \;, \\
h_{3'} &= h_{4'} = h_f\left(900\,\mathrm{kPa}\right) = 99.56\frac{\mathrm{kJ}}{\mathrm{kg}} \;.
\end{aligned}
$$

The COP for this cycle is

$$
\mathrm{COP_{R'}} = \frac{h_{1'} - h_{4'}}{h_{2'} - h_{1'}} = 4.56 \;.
$$

[3] To find $h_2$ one needs to interpolate several times. First find $h\left(600\,\mathrm{kPa}, 0.919\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\right) = 261.97\frac{\mathrm{kJ}}{\mathrm{kg}}$, $h\left(700\,\mathrm{kPa}, 0.919\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\right) = 265.16\frac{\mathrm{kJ}}{\mathrm{kg}}$, then interpolate in pressure.

[4] To reduce the amount of interpolation, we assume a minimum temperature difference of 9.5°C for the condenser.

The finite temperature difference for heat transfer reduces the COP considerably. For this process mass flow and power consumption are

$$\dot{m}' = \frac{\dot{Q}_{in}}{h_{1'} - h_{4'}} = 0.092\frac{\text{kg}}{\text{s}} \quad , \quad \dot{W}' = \frac{\dot{Q}_{in}}{\text{COP}_{\text{R}'}} = 2.85\,\text{kW} \; .$$

For the irreversible compressor 1′-2″ we find the enthalpy at the exit as

$$h_{2''} = h_{1'} + \frac{h_{2'} - h_{1'}}{\eta_C} = 280.2\frac{\text{kJ}}{\text{kg}} \; .$$

The COP for the cycle with irreversible compressor is

$$\text{COP}_{\text{R}''} = \frac{h_{1'} - h_{4'}}{h_{2''} - h_{1'}} = 3.64 \; .$$

The irreversible loss in the compressor reduces the COP further. For this process the mass flow is unchanged, $\dot{m}'' = \dot{m}'$ and the power consumption is

$$\dot{W}'' = \frac{\dot{Q}_{in}}{\text{COP}_{\text{R}''}} = 3.57\,\text{kW} \; .$$

This example shows explicitly how *internal* (throttle, compressor) and *external* (heat transfer over finite temperature differences) irreversibilities lead to a significant reduction of the performance characteristics of a cycle in comparison to the best possible case (here: the Carnot refrigerator).

The realistic cooling cycle 1′-2″-3′-4′ has a COP that is not much more than 1/3 of the Carnot cycle. Advanced cooling cycles use process modifications like cascade refrigeration that increase the COP (Sec. 12.5).

To understand the irreversibilities better, we determine entropy generation and work loss for the four processes in the refrigeration cycle. To compute these, we require the entropies which are

$$\begin{aligned}
s_{1'} &= s_g\left(-10\,^\circ\text{C}\right) = 0.9253\frac{\text{kJ}}{\text{kg}\,\text{K}} \; , \\
s_{2''} &= s\left(900\,\text{kPa}, h = 280.2\frac{\text{kJ}}{\text{kg}}\right) = 0.9499\frac{\text{kJ}}{\text{kg}\,\text{K}} \; , \\
s_{3'} &= s_f\left(900\,\text{kPa}\right) = 0.3656\frac{\text{kJ}}{\text{kg}\,\text{K}} \; , \\
s_{4'} &= s_f\left(-10\,^\circ\text{C}\right) + x_{4'} s_{fg}\left(-10\,^\circ\text{C}\right) = 0.3863\frac{\text{kJ}}{\text{kg}\,\text{K}} \; ,
\end{aligned}$$

where we have used that $x_{4'} = \left[\frac{h_{4'} - h_f}{h_{fg}}\right]_{|T=-10\,^\circ\text{C}} = 0.306$.

The 2nd law gives the entropy generation rates in compressor, condenser, throttle and evaporator as

$$\dot{S}_{gen,comp} = \dot{m}\left(s_{2''} - s_{1'}\right) \ ,$$
$$\dot{S}_{gen,cond} = \dot{m}\left(s_{3'} - s_{2''}\right) - \frac{\dot{Q}_{2''3'}}{T_H} = \dot{m}\left[s_{3'} - s_{2''} - \frac{h_{3'} - h_{2''}}{T_H}\right] \ ,$$
$$\dot{S}_{gen,throt} = \dot{m}\left(s_{4'} - s_{3'}\right) \ ,$$
$$\dot{S}_{gen,evap} = \dot{m}\left(s_{1'} - s_{4'}\right) - \frac{\dot{Q}_{1'4'}}{T_L} = \dot{m}\left[s_{1'} - s_{4'} - \frac{h_{1'} - h_{4'}}{T_L}\right] \ .$$

With $T_L = 273\,\mathrm{K}$, $T_H = 299\,\mathrm{K}$ we find the entropy generation rates

$$\dot{S}_{gen,comp} = 2.263\frac{\mathrm{W}}{\mathrm{K}} \ , \ \dot{S}_{gen,cond} = 1.826\frac{\mathrm{W}}{\mathrm{K}} \ ,$$
$$\dot{S}_{gen,throt} = 1.904\frac{\mathrm{W}}{\mathrm{K}} \ , \ \dot{S}_{gen,evap} = 1.822\frac{\mathrm{W}}{\mathrm{K}} \ .$$

As was shown in Secs. 5.4,5.10, refrigerator work loss is obtained by multiplying entropy generation with the temperature of the environment,[5] $\dot{W}_{\mathrm{loss},R} = T_H \dot{S}_{gen}$, which gives the individual contributions

$$\dot{W}_{\mathrm{loss},comp} = 0.677\,\mathrm{kW} \ , \ \dot{W}_{\mathrm{loss},cond} = 0.546\,\mathrm{kW} \ ,$$
$$\dot{W}_{\mathrm{loss},throt} = 0.569\,\mathrm{kW} \ , \ \dot{W}_{\mathrm{loss},evap} = 0.545\,\mathrm{kW} \ .$$

We see that each of the four sources of irreversibility—two internal, and two external—contributes about one quarter of the total work loss of $\dot{W}_{loss,R} = 2.337\,\mathrm{kW}$. This result clearly shows that for a proper thermodynamic assessment of a system one must consider both, internal *and* external irreversibilities. Note that the work loss is the difference between the actual system work and the work of the Carnot refrigerator (apart from round-off errors).

## 10.5 Gas Turbine: Brayton Cycle

The Brayton cycle (George Brayton, 1830-1892) is an internal combustion cycle based on a gas turbine. Schematic and T-s-diagram are depicted in Fig. 10.7. Environmental air at $p_1 = p_0$, $T_1 = T_0$ enters the compressor in which it is adiabatically compressed to the pressure $p_2$. As the air flows through the combustion chamber, fuel is injected and burnt with some of the air's oxygen which leads to heating of the air. The pressurized hot combustion product expands in the turbine and delivers work; some of the work is required to drive the compressor. The expanded combustion product exhausts to the environment.

[5] Also for a heat pump the work loss is entropy generation times the temperature of the environment, only that the latter is at $T_L$, so that $\dot{W}_{loss,HP} = T_L \dot{S}_{gen}$; re-read Secs. 5.4, 5.10 for details.

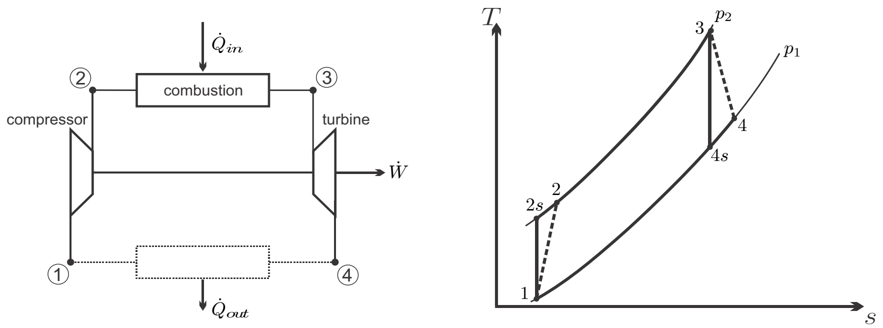


**Fig. 10.7** Schematic and T-s-diagram for standard Brayton cycle. Process 4-1 is the equilibration of the exhaust to envrionmental temperature.

As for the reciprocating internal combustion engines, we shall ignore the addition of fuel, and the change of composition through the chemical reaction, and treat the working fluid as air. Then, the combustion is described as an isobaric heating process. The turbine exhaust is warmer than the environment, and exchanges heat with the environment at $T_0$. The discharge of hot air (state 4) and the intake of cool air (state 1) can be described as an isobaric cooling process at $p_1 = p_4 = p_0$; this process is indicated in the schematic as dotted line.

The Brayton cycle is the equivalent to the Rankine cycle for the ideal gas, and the basic analysis is similar. The main differences are that the Brayton cycle operates without phase change, has no cooler or condenser, and that its back-work-ratio is much larger than that of the Rankine cycle.

Work and heat for the four processes are

$$\begin{array}{lll} \text{1-2 adiabatic compressor:} & w_{12} = h_1 - h_2 \ , & q_{12} = 0 \ , \\ \text{2-3 isobaric heating:} & w_{23} = 0 \ , & q_{23} = h_3 - h_2 \ , \\ \text{3-4 adiabatic turbine:} & w_{34} = h_3 - h_4 \ , & q_{34} = 0 \ , \\ \text{4-1 isobaric cooling:} & w_{41} = 0 \ , & q_{41} = h_1 - h_4 \ . \end{array} \tag{10.11}$$

The net work of the cycle is

$$w_{\odot} = w_{12} + w_{23} + w_{34} + w_{41} = h_1 - h_2 + h_3 - h_4 \ , \tag{10.12}$$

and the heat supply is

$$q_{in} = q_{23} = h_3 - h_2 \ . \tag{10.13}$$

Accordingly, the thermal efficiency of the Brayton cycle is

$$\eta_B = \frac{w_{\odot}}{q_{in}} = \frac{h_1 - h_2 + h_3 - h_4}{h_3 - h_2} = 1 - \frac{h_4 - h_1}{h_3 - h_2} \ . \tag{10.14}$$

The total power produced, the heat consumed, and the heat rejected by the cycle follow after multiplication with the mass flow $\dot{m}$ as

$$\dot{W} = \dot{m} w_{\odot} \quad , \quad \dot{Q}_{in} = \dot{m} q_{23} \quad , \quad \dot{Q}_{out} = \dot{m} q_{41} \,. \tag{10.15}$$

To gain further insight into which parameters are most important, we consider the reversible cycle operating under the cold-air approximation (air as ideal gas with constant specific heats), for which we find the thermal efficiency as

$$\eta_{B,rev} = \frac{h_1 - h_{2s} + h_3 - h_{4s}}{h_3 - h_{2s}} = 1 - \frac{T_{4s} - T_1}{T_3 - T_{2s}} \,. \tag{10.16}$$

The processes 1-2s and 3-4s are isentropic so that

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = P^{\frac{k-1}{k}} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_{4s}} \,. \tag{10.17}$$

This yields

$$\eta_{B,rev} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{P^{\frac{k-1}{k}}} \,. \tag{10.18}$$

The thermal efficiency of the gas turbine increases with the pressure ratio $P = p_2/p_1$.

We proceed by studying operating conditions of gas turbine systems. Obviously, the inlet pressure and temperature, $p_1$ and $T_1$, are set by the environmental conditions, $p_0$ and $T_0$. The highest temperature in the cycle is the turbine inlet temperature $T_3$ which is limited by the materials used for the turbine. For fixed values of $T_1$ and $T_3$, the work per unit mass of air for the gas turbine is

$$w_{\odot} = h_1 - h_{2s} + h_3 - h_{4s} = c_p T_1 \left(1 - P^{\frac{k-1}{k}} + \frac{T_3}{T_1}\left(1 - P^{\frac{1-k}{k}}\right)\right) \,. \tag{10.19}$$

Figure 10.8 shows Brayton cycles in the T-s-diagram. All cycles share the intake pressure $p_1$ and the maximum temperature $T_{\max}$, but their upper pressures $(p_a, p_b, \ldots)$ differ. The specific work is the area enclosed by the cycle in the T-s-diagram, and the figure shows that the cycle with the largest upper pressure $(p_h)$, which has the largest efficiency, has a low work output. Also the cycle with the smallest pressure $(p_a)$, which has the smallest efficiency, has low work output, while a cycle with intermediate pressure $(p_d)$ has the largest work output.

The goal is to produce a certain amount of power $\dot{W} = \dot{m} w_{\odot}$. A gas turbine with small specific work but large efficiency must have a larger mass flow than a turbine with larger specific work. Larger mass flow can only be achieved by building a bigger system, or several smaller systems, which adds capital and maintenance costs. Normally, one will chose to operate a gas turbine close

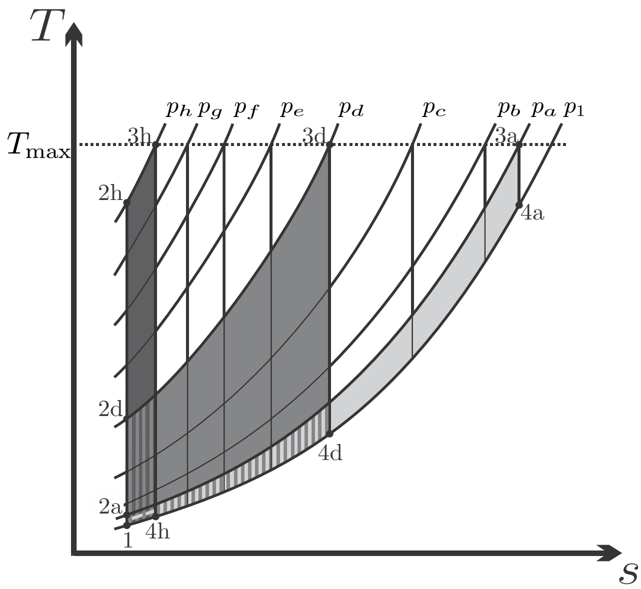


**Fig. 10.8** Brayton cycle between pressure $p_1$ and pressures $p_a \cdots p_h$ with maximum temperature $T_{\max}$. Cycle (1-2d-3d-4d) produces significantly more work than cycles (1-2a-3a-4a) and (1-2h-3h-4h).

to the maximum work output, where the thermal efficiency is smaller, but capital and maintenance costs are smaller as well.

The maximum of the specific work is found, from the condition $\frac{dw_\odot}{dP} = 0$, for the pressure ratio

$$P_{\max} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} \; ; \tag{10.20}$$

the corresponding specific work and thermal efficiency are

$$w_\odot = c_p T_1 \left(\sqrt{\frac{T_3}{T_1}} - 1\right)^2 \quad , \quad \eta_{B,\max} = 1 - \sqrt{\frac{T_1}{T_3}} \; . \tag{10.21}$$

From the above it is clear that increase of the turbine inlet temperature $T_3$ will increase work output *and* thermal efficiency. The use of modern materials and the cooling of turbine blades by pressing cold air through small channels in the blades have led to dramatic increases of turbine inlet temperatures which can be as high as 1700 K for airplanes at take-off.

For simple gas turbine systems, the hot exhaust is just expelled into the environment, where it equilibrates by heat transfer to the environment at $T_0$. This process is associated with external entropy generation, the corresponding work loss is

$$\dot{W}_{\text{loss},41} = T_0 \dot{S}_{gen,41} = T_0 \dot{m} \left( s_1 - s_4 - \frac{h_1 - h_4}{T_0} \right) . \qquad (10.22)$$

Figure 10.8 shows that the turbine exit temperature $T_4$ decreases with increasing pressure ratio. This implies that the external loss becomes smaller with increasing pressure ratio, and hence explains why the cycle efficiency increases.

As can be seen from the T-s-diagrams in Figs. 10.7, 10.8, the exhaust temperature $T_4$ lies above the environmental temperature $T_1$, and may also lie above the pre-combustion temperature $T_2$. In advanced gas turbine cycles, the hot exhaust is used to preheat the compressed gas before combustion (regenerative Brayton cycle, Sec. 13.4), or as heat source for a steam power plant (combined cycle, Sec. 13.6). With this, the hot exhaust is used, and the external loss is reduced.

## 10.6 Example: Brayton Cycle

We assume a gas turbine operating with the compressor inlet temperature $T_1 = 300\,\text{K}$ and the turbine inlet temperature $T_3 = 1400\,\text{K}$. The working medium is air under cold-air approximation, with $R = 0.287 \frac{\text{kJ}}{\text{kg K}}$ and $k = 1.4$.

We first consider a reversible system. From (10.20) we obtain the optimum pressure ratio

$$\frac{p_2}{p_1} = \left( \frac{T_3}{T_1} \right)^{\frac{k}{2(k-1)}} = 14.82$$

and find specific net work and thermal efficiency as

$$w_{\odot} = c_p T_1 \left( \sqrt{\frac{T_3}{T_1}} - 1 \right)^2 = 405.5 \frac{\text{kJ}}{\text{kg}} \quad , \quad \eta_B = 1 - \sqrt{\frac{T_1}{T_3}} = 0.537 .$$

The two remaining temperatures are obtained as

$$T_{2s} = T_{4s} = \sqrt{T_1 T_3} = 648.1\,\text{K} .$$

The back-work ratio for this cycle is

$$\text{bwr} = \frac{|w_C|}{w_T} = \frac{h_{2s} - h_1}{h_3 - h_{4s}} = \frac{T_{2s} - T_1}{T_3 - T_{4s}} = 46.3\% .$$

We compare the above result with that for a gas turbine system with internal irreversibilities operating at the same values for $p_1, p_2, T_1, T_3$ but with isentropic efficiencies for compressor and turbine given as $\eta_C = \eta_T = 0.9$. From (9.43, 9.44) we obtain the temperatures $T_2$ and $T_4$ as

$$T_2 = T_1 - \frac{T_1 - T_{2s}}{\eta_C} = 686.8\,\text{K} \quad , \quad T_4 = T_3 - \eta_T \left( T_3 - T_{4s} \right) = 723.3\,\text{K} .$$

The thermal efficiency of the irreversible gas turbine system is

$$\eta_B = \frac{h_1 - h_2 + h_3 - h_4}{h_3 - h_2} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 40.6\% \, .$$

The internal irreversibilities reduce the thermal efficiency by 25%.

The back-work ratio for the irreversible cycle is

$$\text{bwr} = \frac{|w_C|}{w_T} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{T_2 - T_1}{T_3 - T_4} = 57.2\% \, ;$$

more than 50% of the turbine work is needed to drive the compressor.

## 10.7 Gas Refrigeration System: Inverse Brayton Cycle

The inversion of the Brayton cycle results in a gas cooling system as depicted in Fig. 10.9. Gas is compressed adiabatically (1-2), and then cooled by heat exchange with the warm environment (2-3). The cooled gas is expanded adiabatically in a turbine, and assumes a low temperature (3-4). Finally, the gas is heated by drawing heat from the cold environment (4-1). As always, we consider a cooling system exchanging heat with reservoirs at $T_L$, $T_H$. Then, to have heat transfer in the proper direction, the compressor inlet temperature $T_1$ must not be above $T_L$, and the turbine inlet temperature $T_3$ must not be smaller than $T_H$. These temperature requirements are shown in the T-s-diagram in Fig. 10.9.

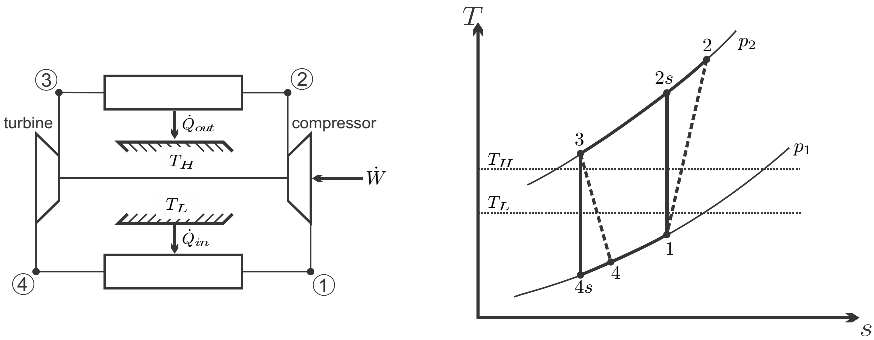


**Fig. 10.9** Inverse Brayton cycle: Schematic and T-s-diagram

The processes in the inverse Brayton cycle are

$$\begin{array}{llll} \text{1-2} & \text{adiabatic compressor:} & w_{12} = h_1 - h_2 \ , & q_{12} = 0 \ , \\ \text{2-3} & \text{isobaric cooling:} & w_{23} = 0 \ , & q_{23} = h_3 - h_2 \ , \\ \text{3-4} & \text{adiabatic turbine:} & w_{34} = h_3 - h_4 \ , & q_{34} = 0 \ , \\ \text{4-1} & \text{isobaric heating:} & w_{41} = 0 \ , & q_{41} = h_1 - h_4 \ . \end{array} \tag{10.23}$$

and the coefficient of performance is

$$\mathrm{COP_R} = \frac{q_{in}}{|w_\odot|} = \frac{h_1 - h_4}{h_2 - h_3 + h_4 - h_1} = \frac{1}{\frac{h_2-h_3}{h_1-h_4} - 1} \ . \tag{10.24}$$

Gas cooling cycles are mainly used at low temperatures, and thus we proceed by assuming constant specific heats for the ideal gas that is used as cooling fluid (cold-gas assumption); then

$$\mathrm{COP_R} = \frac{q_{in}}{|w_\odot|} = \frac{1}{\frac{T_2-T_3}{T_1-T_4} - 1} \ . \tag{10.25}$$

We consider the special case of reversible compressor and turbine, for which we find

$$\mathrm{COP_R} = \frac{1}{\frac{T_{2s}-T_3}{T_1-T_{4s}} - 1} = \frac{1}{\frac{T_{2s}}{T_1} - 1} = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1} \ . \tag{10.26}$$

Here, we used the adiabatic relations for compressor and turbine,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_{4s}} \ . \tag{10.27}$$

Since $T_1 \leq T_L$ and $T_2 > T_H$, the pressure ratio must be sufficiently large,

$$\frac{p_2}{p_1} > \left(\frac{T_H}{T_L}\right)^{\frac{k}{k-1}} \ . \tag{10.28}$$

Larger pressure ratios give smaller COP, but increase the cooling power. In particular for large pressure ratios there are large external irreversibilities associated with the heat transfer over finite temperature differences between the cooling fluid and the two environments (see T-s-diagram in Fig. 10.9). Nevertheless, gas refrigeration cycles offer a relatively simple means to achieve low temperatures ($\sim 130\,\mathrm{K}$). Advanced gas cooling systems use internal heat exchange (regeneration) to increase the COP.

## Problems

### 10.1. Rankine Cycle

A small steam power plant produces 50 MW from a simple Rankine cycle operating between an evaporator pressure of 60 bar and a condenser pressure of 10 kPa. The turbine inlet temperature is 550 °C and the quality at its exit is 0.9. Assume that the adiabatic pump is reversible.

1. Draw a sketch of the cycle, and the corresponding p-v- and T-s-diagrams.
2. Make a list with the values of the relevant enthalpies of the cycle.
3. Determine the isentropic efficiency of the turbine.
4. Determine the mass flow of steam in the cycle and the thermal efficiency of the system.

### 10.2. Steam Cycle

Consider a simple steam power plant that develops a power of 100 MW. The condenser pressure is 10 kPa, the pressure in the steam generator is 8 MPa, and the temperature at the turbine inlet is 500 °C. The isentropic efficiency of the pump is 75%. At the turbine exit the quality is measured as $x = 0.91$.

1. Draw a schematic, a T-s-diagram, and a p-v-diagram (both with respect to saturation lines) of the plant.
2. Determine the enthalpies at the corner points, and the thermal efficiency of the cycle.
3. Determine the isentropic efficiency of the turbine.
4. Determine the mass flow of water through the cycle.
5. In the condenser the heat is transferred to a stream of cooling water which changes its temperature by 10 °C. Compute the mass flow of cooling water (incompressible liquid with specific heat $c_w = 4.2 \frac{\text{kJ}}{\text{kg K}}$).

### 10.3. Geothermal Steam Power Plant

In the Larderello (Italy) steam power plant, steam at 5 bar, 220 °C is produced by geothermal heating. Assume that the processes in the plant follow the basic Rankine cycle with irreversible turbine ($\eta_T = 0.85$) and reversible pump. The condenser temperature is 40 °C.

1. Determine the enthalpies at the corner points.
2. Compute the thermal efficiency, and discuss its value as compared to standard steam power plants, which operate at higher pressures and temperatures.

### 10.4. Reheat Rankine Cycle

A steam power plant with a power output of 80 MW operates on a reheat Rankine cycle:

1-2: adiabatic irreversible pump from condenser pressure $p_1 = 10\,\text{kPa}$ to $p_2 = 10\,\text{MPa}$, state 1 is saturated liquid, isentropic pump efficiency is 95%

2-3: isobaric heating to 500 °C

3-4: adiabatic turbine, expansion to $p_4 = 1\,\mathrm{MPa}$, isentropic turbine efficiency is 80%

4-5: isobaric reheat to 500 °C

5-6: adiabatic turbine, expansion to $p_6 = p_1$, isentropic turbine efficiency is 80%

1. Draw a schematic, and the corresponding p-v- and T-s-diagrams with respect to saturation lines.
2. Determine the thermal efficiency of the system.
3. Determine the mass flow rate.

**10.5. Another Reheat Cycle**
For an ideal (i.e., reversible) Rankine cycle with reheat, the minimum and maximum pressures reached are 10 kPa and 9 MPa, respectively. Moreover, the turbine inlet temperatures of both turbines are 500 °C, the quality at the condenser inlet is 90% and the mass flow is $25\frac{\mathrm{kg}}{\mathrm{s}}$ of steam. Determine:

1. The reheat pressure.
2. The heat input and the power produced.
3. The thermal efficiency.

**10.6. Refrigerators, Heat Pumps**
Draw schematics for vapor refrigeration cycles and heat pumps, as well as the corresponding T-s- and p-v-diagrams (include irreversible processes for compressors). Explain the difference in operating conditions between heat pumps and refrigerators. Also draw T-s-diagrams for Carnot heat pump and refrigerator, and use the diagrams to compute their COP's.

**10.7. Refrigerator**
A vapor-compression refrigeration system uses R134a as working substance. The pressure in the evaporator is 1.4 bar, and the condenser pressure is 7 bar. The temperature at the compressor inlet is −10 °C, and the working fluid leaves the condenser at a temperature of 24 °C. Moreover, the mass flow rate is $0.1\frac{\mathrm{kg}}{\mathrm{s}}$, and the isentropic efficiency of the compressor is 67%.

1. Draw a sketch, a p-v-diagram, a T-s-diagram (with respect to saturation lines).
2. Determine the COP of the system.
3. Compute the refrigeration capacity, and the power consumption.
4. Compute COP and power consumption if the isentropic efficiency of the compressor is 85%.

**10.8. Refrigeration Cycle**
A frozen pizza factory requires a refrigeration rate of 200 kW to maintain the storage facility at −15 °C. Cooling is performed by a standard vapor-compression cycle, using R134a with the following data: condenser pressure:

700 kPa, evaporator temperature: −20 °C, isentropic efficiency of compressor: 75%. The condenser is cooled by liquid water. Use the log p-h diagram for R134a for the solution of this problem.

Determine the mass flow rate of the refrigerant, the power input to the compressor, the COP, and the mass flow rate of the cooling water when its temperature changes by 10 °C.

**10.9. Vapor Refrigeration Cycle**

A refrigerator uses R134a as working fluid which undergoes the following cycle:

1-2: Adiabatic irreversible compression of saturated vapor at $T_1 = -18\,°\mathrm{C}$ to $p_2 = 9\,\mathrm{bar}$; the isentropic efficiency of the compressor is $\eta_C = 0.85$

2-3: Isobaric cooling and condensation to compressed liquid state at $T_3 = 32\,°\mathrm{C}$

3-4: Adiabatic throttling to evaporator pressure $p_4 = p_1$

4-1: Isobaric evaporation to state 1

1. Draw a schematic, and the process in a T-s-diagram, with respect to saturation lines.
2. Make a table with the values of temperature, pressure, and enthalpy at points 1-4.
3. Compute the coefficient of performance (COP).
4. The refrigerator draws a power of 4 kW. Compute the mass flow and the cooling power.

**10.10. Heat Pump**

A heat pump that operates on the ideal vapor-compression cycle with R134a is used to heat water from 15 to 54 °C at a rate of $0.24\frac{\mathrm{kg}}{\mathrm{s}}$. The condenser and evaporator pressures are 1.4 and 0.32 MPa, respectively. Determine the COP and the power input to the heat pump.

**10.11. Heat Pump**

A vapor compression heat pump with R134a as cooling fluid is used to keep a house at 20 °C. The heat pump has a compressor with isentropic efficiency of 85%, and it draws heat from groundwater which has a temperature of 12 °C. The condenser and evaporator pressures are 900 kPa and 320 kPa, respectively, the temperature at the inlet of the throttling valve is 30 °C, and the compressor draws saturated vapor.

Compute the COP, the mass flow, and the power consumption if the heating power is 10 kW. Don't forget to draw schematic and diagrams.

**10.12. Vapor Heat Pump Cycle**

An air conditioning system sucks in a mass flow of $5000\frac{\mathrm{kg}}{\mathrm{h}}$ of outside air at 10 °C, 0.95 bar, and heats it isobarically to 24 °C. The air is heated by means of an standard vapor heat pump cycle (R134a), whose compressor has an isentropic efficiency of 0.85. The condenser pressure is 800 kPa. The evaporator is outside the building, and the minimum temperature difference for heat transfer is 10 °C. Consider the air as ideal gas with constant specific heats ($c_v = 0.717 \frac{\text{kJ}}{\text{kg K}}$, $R = 0.287 \frac{\text{kJ}}{\text{kg K}}$).

1. Make a sketch of the system, and draw the corresponding T-s-diagram.
2. Make a table with the values of pressure, temperature and enthalpy at the corner points.
3. Determine the COP of the cycle.
4. Determine the work required to run the heat pump.

**10.13. Gas Turbine (Ideal Brayton Cycle)**
An ideal Brayton cycle with air as working fluid (variable specific heats) is to be designed such that the minimum and maximum temperatures in the cycle are 300 K and 1500 K, respectively. The pressure ratio is 16.7. Compressor and turbine are both irreversible, with an isentropic efficiency of 0.9 for the turbine and 0.85 for the compressor.

Compute compressor and turbine work per unit mass of air, and the thermal efficiency of the cycle.

**10.14. Brayton Cycle**
This problem compares the calculation with constant and non-constant specific heats, to give an idea of the differences. Air enters the compressor of a simple Brayton cycle gas turbine power plant at 95 kPa and 290 K. The heat transfer rate is $50 \frac{\text{MJ}}{\text{s}}$ and the turbine entry temperature is 1400 K. The pressure ratio is $P = \left(\frac{T_{\max}}{T_{\min}}\right)^{\frac{k}{2(k-1)}}$ for maximum power output. Compressor and turbine are reversible. Compute the power delivered and the thermal efficiency for:

1. Cold air approximation, that is constant specific heats with values at room temperature.
2. Variable specific heats.

Make tables for the values of pressure and temperature at the relevant process points, and draw diagrams and schematic.

**10.15. Gas Turbine (Brayton Cycle)**
A Brayton cycle delivers a power of 150 MW. The working fluid can be considered as air (ideal gas with variable specific heats), and the following data are known: inlet state $p_1 = 0.9$ bar, $T_1 = 280$ K; state after adiabatic compression $p_2 = 17.06$ bar, $T_2 = 690$ K, maximum temperature in the cycle 1600 K, heating rate 354 MW.

Determine the thermal efficiency of the cycle, compressor and turbine work per unit mass of air, mass flow of air, and the isentropic efficiencies of compressor and turbine.

**10.16. Brayton Cooling Cycle**
A small gas-cooling system operates on the inverse Brayton cycle. The cycle uses argon as cooling fluid. The cycle is used for maintaining a small cold space at $-60\,°\mathrm{C}$, and rejects heat into the environment at $25\,°\mathrm{C}$. Both heat exchangers require a temperature difference of at least $5\,°\mathrm{C}$ for operation. The cycle operates between the pressures 1 bar and 4 bar, and isentropic efficiencies of compressor and turbine are 0.75 and 0.85, respectively. Draw schematic and diagrams and determine:

1. The COP.
2. The mass flow required for a cooling power of 0.5 kW, and the required power to run the system.
3. The work losses to irreversibilities in turbine, compressor and both heat exchangers. Discuss the results.

**10.17. Gas Refrigeration Cycle**
A refrigerator for cryogenic applications uses helium as working fluid which undergoes the cycle described below.

1-2: Adiabatic irreversible compression of helium at $T_1 = -75\,°\mathrm{C}$ and $p_1 = 1\,\mathrm{bar}$ to $p_2 = 10\,\mathrm{bar}$, the isentropic efficiency of the compressor is 0.85

2-3: Isobaric cooling until the temperature reaches $T_3 = 30\,°\mathrm{C}$

3-4 : Adiabatic irreversible expansion in turbine to pressure $p_4 = p_1$, the isentropic efficiency of the turbine is 0.8

4-1: Isobaric heating to state 1

Helium is an ideal gas with constant specific heats, with $c_p = 5.196\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$, $R = 2.0785\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$.

1. Draw a schematic, and the process in a T-s-diagram.
2. Make a table with the values of temperature and pressure at points 1-4.
3. Compute the coefficient of performance (COP).
4. The mass flow is $1.5\frac{\mathrm{kg}}{\mathrm{s}}$. Compute the cooling power, and the power needed to run the refrigerator.

**10.18. Gas Cooling System**
A gas refrigeration system operates on the inverse Brayton cycle (1-2: adiabatic compression, 2-3: isobaric heat exchange, 3-4: adiabatic expansion, 4-1: isobaric heat exchange) with air as the working fluid. The compressor pressure ratio is 3. This system is used to maintain a refrigerated space at $-23\,°\mathrm{C}$ and rejects heat to the environment at $27\,°\mathrm{C}$. The isentropic efficiency of the compressor is 0.8, but the turbine exhibits no losses. The temperature difference for heat transfer is $10\,°\mathrm{C}$. Consider air as an ideal gas with variable specific heats.

1. Draw a schematic, and the corresponding T-s-diagram.
2. Make a list with the enthalpies and temperatures at the corner points of the process.
3. Compute the COP.
4. For the computation above, the knowledge of pressure ratio was enough, so nothing was said about the value of $p_1$. Discuss the choice of this pressure (should it be high or low . . .).