

# Chapter 11

## Efficiencies and Irreversible Losses

### 11.1 Irreversibility and Work Loss

In the discussion of work losses for closed and open systems we found that irreversibilities reduce the efficiency of energy conversion, see Sec. 5.10. We shall study this now in greater detail. The arguments used in this section are similar to “exergy accounting” (see Sec. 11.8 further below).

We consider a general thermodynamic system as in Fig. 9.1 which is described through the balance laws for mass, energy, and entropy (9.1, 9.9, 9.10). Part of the heat exchange between the system and its surroundings will take place at a temperature  $T_0$ , and we write

$$\dot{Q} = \dot{Q}_0 + \sum_{k=1} \dot{Q}_k \quad , \quad \sum_k \frac{\dot{Q}_k}{T_k} = \frac{\dot{Q}_0}{T_0} + \sum_{k=1} \frac{\dot{Q}_k}{T_k} \quad , \quad (11.1)$$

where  $\dot{Q}_k$  is heat transferred over the system boundary at temperature  $T_k$ ; in particular,  $\dot{Q}_0$  is the heat exchanged at  $T_0$ . With that, the first and second laws of thermodynamics (9.9, 9.10) can be written as

$$\begin{aligned} \frac{dE}{dt} + \sum_{\text{out}} \dot{m}_e \left( h_e + \frac{1}{2} \mathcal{V}_e^2 + gz_e \right) - \sum_{\text{in}} \dot{m}_\alpha \left( h_i + \frac{1}{2} \mathcal{V}_i^2 + gz_i \right) = \\ = \dot{Q}_0 + \sum_{k=1} \dot{Q}_k - \dot{W} \quad , \quad (11.2) \end{aligned}$$

$$\frac{dS}{dt} + \sum_{\text{out}} \dot{m}_e s_e - \sum_{\text{in}} \dot{m}_i s_i = \frac{\dot{Q}_0}{T_0} + \sum_{k=1} \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} \quad . \quad (11.3)$$

The highlighted temperature  $T_0$  can be freely chosen according to the details of the actual system considered. Since most thermodynamic systems interact with the environment, one most often chooses  $T_0$  to be the temperature of the environment, usually  $T_0 = 298 \text{ K}$  for the standard reference

environment. Indeed, the environment acts as an infinite heat reservoir, and no cost is associated with heat drawn from, or dumped into, the environment.

Elimination of the heat exchange with the environment,  $\dot{Q}_0$ , between the first and second laws (11.2, 11.3) yields an equation for power,

$$\begin{aligned} \dot{W} = & -T_0 \dot{S}_{gen} + \sum_{k=1} \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k + \sum_{in} \dot{m}_i \left(h_i - T_0 s_i + \frac{1}{2} \mathcal{V}_i^2 + g z_i\right) \\ & - \sum_{out} \dot{m}_e \left(h_e - T_0 s_e + \frac{1}{2} \mathcal{V}_e^2 + g z_e\right) - \frac{d(E - T_0 S)}{dt}. \end{aligned} \quad (11.4)$$

This equation is the generalization of (5.17) in Sec. 5.10 to include open system boundaries. The equation is valid for any system, open or closed, transient or steady state, that exchanges heat at least at  $T_0$ , and possibly at other temperatures.

Note, that for  $\dot{Q} = \dot{Q}_0 = 0$  there is nothing to be eliminated between the two equations, so that (11.4) is not relevant for fully adiabatic processes.

The factor  $T_0$  relates power loss to entropy generation; the lost power is sometimes denoted as *irreversibility*,

$$\dot{W}_{loss} = T_0 \dot{S}_{gen}. \quad (11.5)$$

Equation (11.4) allows to relate irreversibility as measured by entropy generation  $\dot{S}_{gen}$  to power losses for complete thermodynamic systems, such as power and refrigeration cycles, in contact with the environment. Since  $\dot{S}_{gen} \geq 0$  and<sup>1</sup>  $T_0 > 0$ , this equation shows that irreversibilities reduce work output for a power producing system (where  $\dot{W} > 0$ , e.g., a power plant) or increase work demand for a power consuming system (where  $\dot{W} < 0$ , e.g., a heat pump or a refrigerator).

The task of a thermal engineer can be described as to improve efficiency of a thermal system as much as possible. This requires to identify and reduce—as much as possible—causes of losses, i.e., irreversibilities. This will lead to a redesign of the system, which in turn leads to a change of the inflow, outflow and boundary conditions of the system.

A proper understanding of the losses associated with a system requires that *all* sources for irreversibility are considered. For a proper accounting of losses, the system boundary should be wide enough to include all causes for loss, so that internal and external irreversibilities are accounted for.

The redesign should start with removing the main causes for losses. If we indicate the different causes for entropy generation by Greek indices, we can write

$$\dot{W}_{loss} = \sum_{\alpha} T_0 \dot{S}_{gen}^{\alpha}. \quad (11.6)$$

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<sup>1</sup>  $T$  is the thermodynamic temperature, of course!

The importance of the various entropy generating processes can be evaluated by a relative measure, e.g., the ratio between lost power and the total power exchange for the process

$$\chi_\alpha = \frac{T_0 \dot{S}_{gen}^\alpha}{|\dot{W}|}. \quad (11.7)$$

One will not be able to avoid losses altogether, and thus will have to accept values of a few percent (or more) for  $\chi_\alpha$ . Nevertheless, *any* reduction in irreversibilities that can be attained at reasonable cost for redesign and construction will lead to increased power production or decreased power consumption, and thus offer substantial savings in the long run of operation.

## 11.2 Reversible Work and Second Law Efficiency

A sometimes useful measure<sup>2</sup> for the performance of a system is the second law efficiency  $\eta_{II}$ , which compares the actual performance to the reversible work, i.e., the best case scenario for the same process parameters.

The *reversible work*  $\dot{W}_{rev}$  is the power that would be obtained from a reversible process operating under the same boundary conditions as the system considered, and exchanging heat with the environment at  $T_0$ . Its value follows from (11.4) simply by setting the irreversibility to zero,  $T_0 \dot{S}_{gen} = 0$ ,

$$\begin{aligned} \dot{W}_{rev} = & \sum_{k=1} \left( 1 - \frac{T_0}{T_k} \right) \dot{Q}_k + \sum_{\alpha, in} \dot{m}_\alpha \left( h_\alpha - T_0 s_\alpha + \frac{1}{2} v_\alpha^2 + g z_\alpha \right) \\ & - \sum_{\alpha, out} \dot{m}_\alpha \left( h_\alpha - T_0 s_\alpha + \frac{1}{2} v_\alpha^2 + g z_\alpha \right) - \frac{dE - T_0 S}{dt}. \end{aligned} \quad (11.8)$$

Since all irreversibilities reduce the process performance, for a work generating process the reversible work is the maximum work that could be obtained, and for a work consuming process it is the minimum work required. The actual work can be expressed as the difference between reversible work and work loss,  $\dot{W} = \dot{W}_{rev} - \dot{W}_{loss}$ .

The second law efficiency is defined as the ratio between the actual work of the process and the reversible work. For a power producing system one defines

$$\eta_{II} = \frac{\dot{W}}{\dot{W}_{rev}} = \frac{\dot{W}_{rev} - \dot{W}_{loss}}{\dot{W}_{rev}} = 1 - \frac{T_0 \dot{S}_{gen}}{\dot{W}_{rev}} < 1 \quad (\dot{W} > 0), \quad (11.9)$$

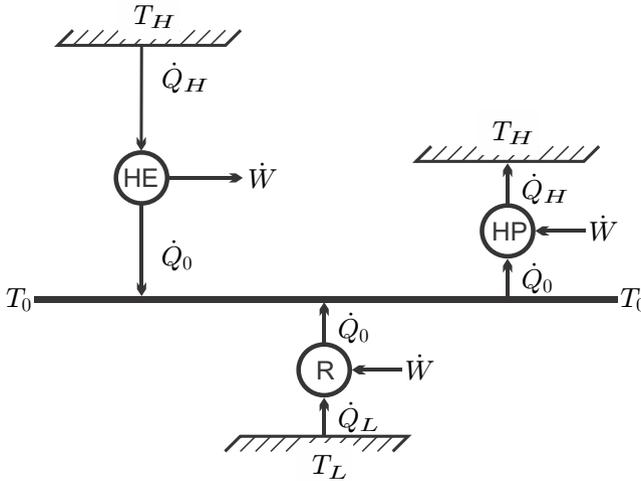
and for a power consuming system, one defines

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<sup>2</sup> All efficiencies are useful only when they are defined and applied in a meaningful way!

$$\eta_{II} = \frac{|\dot{W}_{\text{rev}}|}{|\dot{W}|} = \frac{|\dot{W}_{\text{rev}}|}{|\dot{W}_{\text{rev}}| + \dot{W}_{\text{loss}}} = \frac{1}{1 + \frac{T_0 \dot{S}_{\text{gen}}}{|\dot{W}_{\text{rev}}|}} < 1 \quad (\dot{W} < 0) . \quad (11.10)$$

The simplest examples are given by steady state engines operating between two reservoirs at constant temperatures, and we consider heat engine, refrigerator, and heat pump, as depicted in Fig. 11.1. Note that all engines exchange heat with the environment at  $T_0$ , while the temperatures of the other reservoirs are different. The fully reversible engines operating between two reservoirs are Carnot engines.



**Fig. 11.1** Heat engine (HE), refrigerator (R), and heat pump (HP) in contact with the environment at  $T_0$

For a heat engine that receives the heat  $\dot{Q}_H$  from a hot reservoir at  $T_H > T_0$ , and rejects heat into the environment at  $T_0$ , actual and reversible work are  $\dot{W} = \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_H - T_0 \dot{S}_{\text{gen}} > 0$  and  $\dot{W}_{\text{rev}} = \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_H > 0$ . For the second law efficiency we obtain

$$\eta_{II} = \frac{\dot{W}}{\dot{W}_{\text{rev}}} = \frac{\dot{W}/\dot{Q}_H}{\dot{W}_{\text{rev}}/\dot{Q}_H} = \frac{\dot{W}/\dot{Q}_H}{1 - \frac{T_0}{T_H}} = \frac{\eta}{\eta_C} , \quad (11.11)$$

where  $\eta = \dot{W}/\dot{Q}_H$  is the thermal efficiency of the actual engine, and  $\eta_C = \dot{W}_{\text{rev}}/\dot{Q}_H = 1 - \frac{T_0}{T_H}$  is the thermal efficiency of a fictional Carnot heat engine operating between the same temperatures. It is straightforward to conclude that for a more complex power producing system  $\eta_{II} = \eta/\eta_{\text{rev}}$  where  $\eta_{\text{rev}}$  is the thermal efficiency associated with the reversible work.

For a refrigerator that removes the heat  $\dot{Q}_L$  from a cold space at  $T_L < T_0$  and rejects heat into the environment at  $T_0$ , actual and reversible work are  $\dot{W} = \left(1 - \frac{T_0}{T_L}\right) \dot{Q}_L - T_0 \dot{S}_{gen} < 0$  and  $\dot{W}_{rev} = \left(1 - \frac{T_0}{T_L}\right) \dot{Q}_L < 0$ . For the second law efficiency we obtain

$$\eta_{II} = \frac{|\dot{W}_{rev}|}{|\dot{W}|} = \frac{\dot{Q}_L / |\dot{W}|}{\dot{Q}_L / |\dot{W}_{rev}|} = \left(\frac{T_0}{T_L} - 1\right) \text{COP}_R = \frac{\text{COP}_R}{\text{COP}_{R,C}}, \quad (11.12)$$

where  $\text{COP}_R = \dot{Q}_L / |\dot{W}|$  is the actual coefficient of performance, and  $\text{COP}_{R,C} = \dot{Q}_L / |\dot{W}_{rev}| = 1 / \left(\frac{T_0}{T_L} - 1\right)$  is the COP of the Carnot refrigerator.

For a heat pump that supplies the heat  $\dot{Q}_H$  to a warm space at  $T_H > T_0$  and draws heat from the environment at  $T_0$ , actual and reversible work are  $\dot{W} = \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_H - T_0 \dot{S}_{gen} < 0$  and  $\dot{W}_{rev} = \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_H < 0$ . Also in this case the reversible heat pump is a Carnot engine. For the second law efficiency we obtain

$$\eta_{II} = \frac{|\dot{W}_{rev}|}{|\dot{W}|} = \frac{|\dot{Q}_H| / |\dot{W}|}{|\dot{Q}_H| / |\dot{W}_{rev}|} = \left(1 - \frac{T_0}{T_H}\right) \text{COP}_{HP} = \frac{\text{COP}_{HP}}{\text{COP}_{HP,C}},$$

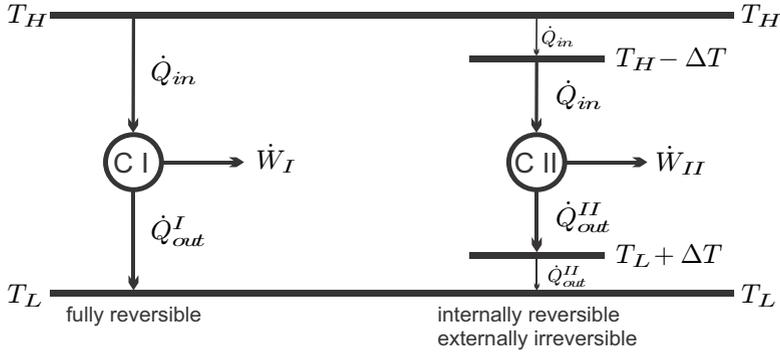
where  $\text{COP}_{HP} = |\dot{Q}_H| / |\dot{W}|$  is the coefficient of performance, and  $\text{COP}_{HP,C} = |\dot{Q}_H| / |\dot{W}_{rev}| = 1 / \left(1 - \frac{T_0}{T_H}\right)$  is the COP of the Carnot heat pump.

To summarize, we state that the second law efficiency gives a qualitative measure for the overall quality of a process by comparing actual performance to that of a fictional reversible system with the same parameters..

### 11.3 Example: Carnot Engine with External Irreversibility

Fully reversible (e.g., Carnot) engines cannot be build since any real process is associated with some irreversibilities. These can be reduced, but not avoided. For instance, heat transfer requires finite temperature differences, which are accompanied by entropy generation, i.e., work loss. As an example we study a reversible Carnot heat engine with external irreversibilities that occur in transferring heat in and out of the engine, see Fig. 11.2.

We compare a fully reversible engine (I), which does not require temperature differences for heat transfer, and an engine (II) that requires finite temperature differences. When both engines take in the same amount of heat, their power outputs are



**Fig. 11.2** Heat engine between two reservoirs: Fully reversible cycle (I), and internally reversible cycle with external irreversibilities (II)

$$\dot{W}_I = \left(1 - \frac{T_L}{T_H}\right) \dot{Q}_{in} \quad , \quad \dot{W}_{II} = \left(1 - \frac{T_L + \Delta T}{T_H - \Delta T}\right) \dot{Q}_{in} ; \quad (11.13)$$

obviously  $\dot{W}_I > \dot{W}_{II}$ . The heat rejected to the environment by the two engines is

$$\left| \dot{Q}_{out}^I \right| = \dot{Q}_{in} - \dot{W}_I = \frac{T_L}{T_H} \dot{Q}_{in} \quad , \quad \left| \dot{Q}_{out}^{II} \right| = \dot{Q}_{in} - \dot{W}_{II} = \frac{T_L + \Delta T}{T_H - \Delta T} \dot{Q}_{in} . \quad (11.14)$$

For engine II, the entropy generation due to irreversible heat transfer at the higher and lower temperatures are

$$\dot{S}_{gen}^H = \left( \frac{1}{T_H - \Delta T} - \frac{1}{T_H} \right) \dot{Q}_{in} = \frac{\dot{Q}_{in}}{T_H} \frac{\Delta T}{T_H - \Delta T} > 0 ,$$

$$\dot{S}_{gen}^L = \left( \frac{1}{T_L} - \frac{1}{T_L + \Delta T} \right) \left| \dot{Q}_{out}^{II} \right| = \frac{\left| \dot{Q}_{out}^{II} \right|}{T_L} \frac{\Delta T}{T_L + \Delta T} = \frac{\dot{Q}_{in}}{T_L} \frac{\Delta T}{T_H - \Delta T} > 0 ;$$

the corresponding work loss to heat transfer is

$$\dot{W}_{loss} = \dot{W}_I - \dot{W}_{II} = T_L \frac{\dot{Q}_{in} \Delta T}{T_H - \Delta T} \left( \frac{1}{T_L} + \frac{1}{T_H} \right) = T_L \left( \dot{S}_{gen}^L + \dot{S}_{gen}^H \right) . \quad (11.15)$$

While the internally reversible engine II has the optimum efficiency with respect to its boundary temperatures  $T_L + \Delta T$  and  $T_H - \Delta T$ , the engine does not have optimum efficiency with respect to the available boundary temperatures  $T_L$  and  $T_H$ , due to external irreversibilities in heat transfer.

This simple example shows once more the importance of considering external losses. In order to obtain a comprehensive picture of thermodynamic performance, one cannot restrict the attention to the performance of an

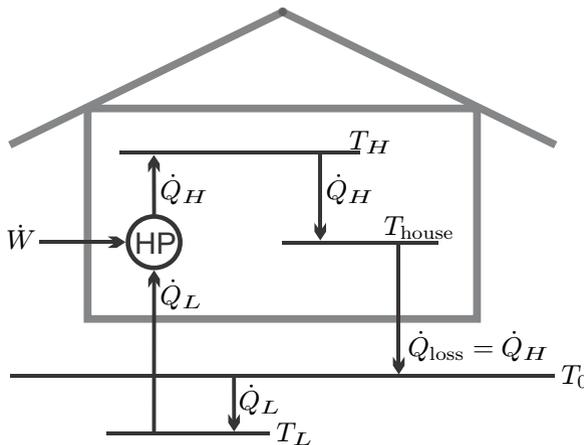
isolated system, say a heat engine, but one has to account for the interaction of the system with its surroundings as well.

A perfect heat exchanger, which operates at infinitesimal temperature difference, and hence does not generate entropy, requires an infinite exchange surface, and thus can only be thought of, but not be build. Thus, in any existing heat exchanger the temperature difference is finite, and entropy is generated. The art of building heat exchangers, or choosing heat exchangers for a particular system, is to make the temperature difference, and thus the work losses, as small as technology and purchase or construction costs allow.

### 11.4 Example: Space Heating

Another instructive example is given by the heating of a house with a device powered by electricity, e.g., a resistance heater or a heat pump. The house sits in an external environment at temperature  $T_0$ , and its temperature is maintained at  $T_{\text{house}} > T_0$ . Due to the temperature difference between the warm house and the cooler environment there is a heat loss  $\dot{Q}_{\text{loss}}$  to the environment. The heating system must provide the same amount of heat to compensate for the loss, so that the inside temperature remains at  $T_{\text{house}}$ .

We first examine the case where the heat is provided by a heat pump which draws the power  $\dot{W}$  from the electrical grid, draws heat  $\dot{Q}_L$  from the environment, and provides the heat  $\dot{Q}_H = \dot{Q}_{\text{loss}}$  at a temperature  $T_H > T_{\text{house}}$ ; the temperature difference ensures heat transfer from the heat pump to the house. Figure 11.3 shows the energy flows and the temperature levels for this process.



**Fig. 11.3** Energy flows for the heating of a house by means of a heat pump

There are two sources of entropy generation, that is, ultimately, work loss: the heat pump itself, and the heat transfer from the house to the environment. The latter is given, from (11.4), as

$$\dot{S}_{\text{loss}} = \left( \frac{1}{T_0} - \frac{1}{T_{\text{house}}} \right) |\dot{Q}_{\text{loss}}| .$$

This loss is independent of the method used for heating, and can be reduced by reducing the heat loss  $\dot{Q}_{\text{loss}}$ , or the inside temperature  $T_{\text{house}}$ . By Newton's law of cooling the heat loss is given by

$$\dot{Q}_{\text{loss}} = \alpha A (T_{\text{house}} - T_0) ,$$

where  $A$  is the outer surface of the house, and  $\alpha$  is an overall heat transfer coefficient. The heat loss can be reduced by better insulation of the exterior walls and the roof, which gives smaller value for  $\alpha$ , but also by reduction of the inside temperature  $T_{\text{house}}$ . For a house exposed to an environment at  $0^\circ\text{C}$ , the heating cost can be reduced by 10% when the setting of the thermostat is reduced from  $22^\circ\text{C}$  to  $20^\circ\text{C}$ .

Next we consider losses associated with the heat pump. To ensure heat transfer from the environment and into the house, the heat pump operates between temperatures  $T_L < T_0$  and  $T_H > T_{\text{house}}$ . The first law relates power and heat transfer rates as

$$|\dot{W}_{\text{HP}}| + |\dot{Q}_L| = |\dot{Q}_H| = |\dot{Q}_{\text{loss}}| .$$

The heat supplied to the house,  $|\dot{Q}_H|$ , is the sum of the power to run the heat pump  $|\dot{W}_{\text{HP}}|$ , which must be paid for, and the heat intake from the environment  $|\dot{Q}_L|$ , which is freely available.

The combined first and second law applied to the heat pump gives the required work as (with  $\dot{W}_{\text{HP}} = -|\dot{W}_{\text{HP}}|$ )

$$|\dot{W}_{\text{HP}}| = \left( 1 - \frac{T_0}{T_{\text{house}}} \right) |\dot{Q}_{\text{loss}}| + T_0 \dot{S}_{\text{gen}} ,$$

where  $\dot{S}_{\text{gen}} = \dot{S}_{\text{gen}}^{\text{int}} + \dot{S}_{\text{gen}}^{\text{ext}}$  denotes the associated internal and external irreversibilities. The external irreversibilities due to heat transfer from the environment to the cold side of the heat pump, and heat transfer from the hot side of the heat pump to the house, are

$$\dot{S}_{\text{gen}}^{\text{ext}} = \left( \frac{1}{T_L} - \frac{1}{T_0} \right) |\dot{Q}_L| + \left( \frac{1}{T_{\text{house}}} - \frac{1}{T_H} \right) |\dot{Q}_H| .$$

If these are inserted explicitly into the above equation for power, it assumes the form

$$|\dot{W}_{HP}| = \left(1 - \frac{T_L}{T_H}\right) |\dot{Q}_{loss}| + T_L \dot{S}_{gen}^{int} ;$$

here  $T_L \dot{S}_{gen}^{int}$  is the work loss to internal irreversibilities which are unavoidable in a real-life heat pump.

The most efficient heating method is given by a fully reversible heat pump which would require the work  $|\dot{W}_{rev}| = \left(1 - \frac{T_0}{T_{house}}\right) |\dot{Q}_{loss}|$ . Real heat pumps have higher power consumption, and require finite temperature differences for heat exchange, so that irreversible processes are present.

The least efficient heating method are resistance heaters, for which  $\dot{Q}_L = 0$ , and the heating rate is equal to the power,  $|\dot{W}_{RH}| = |\dot{Q}_H|$ .

The coefficients of performance (COP) for heat pump and resistance heater are

$$COP_{HP} = \frac{|\dot{Q}_{house}|}{|\dot{W}_{HP}|} = \frac{1}{\left(1 - \frac{T_0}{T_{house}}\right) + \frac{T_0 \dot{S}_{gen}}{|\dot{Q}_{loss}|}} > 1, \quad COP_{RH} = \frac{|\dot{Q}_{house}|}{|\dot{W}_{RH}|} = 1 .$$

Thus, vendors for resistance heaters are right when they claim that their product has an “efficiency” of 100%, but they conceal that a heat pump can have a much higher COP.

A Carnot heat pump operating without temperature differences in heat transfer has the maximum coefficient of performance,  $COP_{HP,C} = \frac{1}{1 - \frac{T_0}{T_{house}}}$ . When the outside temperature is 0°C and the house is kept at 20°C, the maximum COP is 14.65.

If the heat pump requires temperature differences of 5°C for heat transfer, but is internally reversible, its coefficient of performance is  $COP_{HP} = \frac{1 - \frac{T_L}{T_H}}{1 - \frac{T_0}{T_{house} + \Delta T}} = \frac{1}{1 - \frac{T_0 - \Delta T}{T_{house} + \Delta T}} = 9.93$ . In this case, the second law efficiency is

$$\eta_R^{II} = \frac{COP_{HP}}{COP_{HP,C}} = 0.68 .$$

A resistance heater has a  $COP_{RH} = 1$  independent of the temperatures, and, for the same temperatures, the second law efficiency

$$\eta_R^{II} = \frac{COP_{RH}}{COP_{HP,C}} = 1 - \frac{T_0}{T_{house}} = 0.068 .$$

Obviously, heat pumps have a considerably smaller power demand than resistance heaters, and thus are a better choice for space heating than resistance heaters. Indeed, resistance heaters are common only where electricity is cheap, e.g., in Norway and British Columbia where hydropower is the main source for generation.

## 11.5 Example: Entropy Generation in Heat Transfer

Heat exchange over finite temperatures is irreversible, the associated entropy generation is related to a work loss. The reason for the work loss is that any temperature difference could be used to drive a heat engine. In heat exchange, there is no engine, hence the loss. As we have seen, energy at higher temperature is more valuable, since more work can be extracted (larger Carnot efficiency). Heat transfer over finite temperature difference conserves the amount of energy transferred (heat), but after transfer the energy is at lower temperature, where the energy is less valuable, since less work can be extracted. We now discuss why the lost work for a system in contact with the environment is  $T_0 \dot{S}_{gen}$  (11.5).

We consider heat transfer  $\dot{Q}$  between reservoirs at  $T_H$  and  $T_L$ . The associated entropy generation is

$$\dot{S}_{gen} = \dot{Q} \left( \frac{1}{T_L} - \frac{1}{T_H} \right) .$$

A Carnot heat engine operating between the two reservoirs and receiving the heat  $\dot{Q}$  from the hot reservoir could produce the power

$$\dot{W}_C = \left( 1 - \frac{T_L}{T_H} \right) \dot{Q} = T_L \dot{S}_{gen} .$$

Note that the Carnot engine involves heat exchange, which must be done by perfect heat exchangers operating at infinitesimal temperature difference, which is impossible in practice.

We recall that the computation of work loss and reversible work relies on the assumption that all boundary parameters remain unchanged. The hypothetical Carnot engine consumes the heat  $\dot{Q}_H = \dot{Q}$ , but rejects the heat

$$\left| \dot{Q}'_L \right| = \dot{Q} - \dot{W}_C = \frac{T_L}{T_H} \dot{Q}$$

into the cold reservoir, which therefore receives less heat than in the case of pure heat conduction—the difference is just the portion of heat that is converted to power. In order to compensate for this, there must be a second reversible engine employed which rejects the heat

$$\Delta \dot{Q} = \dot{Q} - \left| \dot{Q}'_L \right| = \left( 1 - \frac{T_L}{T_H} \right) \dot{Q} = \dot{W}_C$$

at the temperature  $T_L$ .

If  $T_0 > T_L$ , a reversible heat engine operating between  $T_0$  and  $T_L$  is employed. An engine that delivers the work  $\dot{W}' = \left( \frac{T_0}{T_L} - 1 \right) \Delta \dot{Q}$  rejects the required heat  $\Delta \dot{Q}$ , and the reversible work is

$$\dot{W}_{rev} = \dot{W}_C + \dot{W}' = \frac{T_0}{T_L} \left( 1 - \frac{T_L}{T_H} \right) \dot{Q} = T_0 \dot{S}_{gen} .$$

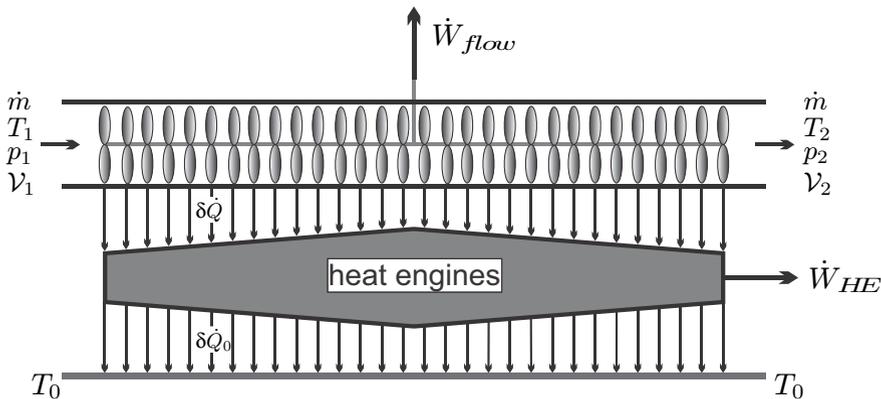
If  $T_0 < T_L$ , a portion of the work  $\dot{W}_C$  must be used to drive a Carnot heat pump operating between the environment at  $T_0$  and  $T_L$ . To deliver the heat  $\Delta\dot{Q}$ , the heat pump requires the power  $\dot{W}'' = \left( 1 - \frac{T_0}{T_L} \right) \Delta\dot{Q}$ . Accordingly, the reversible work is, again,

$$\dot{W}_{rev} = \dot{W}_C - \dot{W}'' = T_0 \dot{S}_{gen} .$$

In both cases, the second engine is exchanging heat with the environment at  $T_0$ . It should be noted that fully reversible engines are not available, so that the discussed systems are only of theoretical interest. Nevertheless, wherever heat transfer over finite temperature differences occurs, there is the potential to do work. Whether it is feasible to do this depends on the individual circumstances of the heat transfer process, in particular on the temperature difference. The larger the temperature difference, the larger is the entropy generation, and the bigger is the work potential.

### 11.6 Work Potential of a Flow (Exhaust Losses)

Many engines, in particular combustion engines, discard warm or hot exhaust. We ask how much work could be extracted by bringing the exhaust into equilibrium with the environment by reversible processes. For this we consider Fig. 11.4, which shows a system to extract work from an available mass flow  $\dot{m}$  at  $T_1, p_1, \mathcal{V}_1$ . The figure indicates that we can obtain work from propellers inside the flow and by heat transfer through reversible engines which discard



**Fig. 11.4** A system to extract work out of a flow by equilibrating it with the environment

heat into the environment at  $T_0$ . We consider steady state operation in a one-inlet-one-exit system for which the combined law (11.4) reduces to

$$\dot{W} = -T_0 \dot{S}_{gen} - \dot{m} \int_1^2 \left( dh - T_0 ds + d \left( \frac{1}{2} \mathcal{V}^2 \right) + g dz \right). \quad (11.16)$$

With the Gibbs equation  $T ds = dh - v dp$  we can write instead

$$\dot{W} = -T_0 \dot{S}_{gen} + \dot{m} \int_2^1 \left( v dp + d \left( \frac{1}{2} \mathcal{V}^2 \right) + g dz \right) + \dot{m} \int_2^1 \left( 1 - \frac{T_0}{T} \right) T ds. \quad (11.17)$$

The first term on the right,  $-T_0 \dot{S}_{gen}$ , describes irreversible losses anywhere in the system, the second term is the reversible flow work extracted from the propellers inside the flow, and the third term is the reversible work available from the external heat engines. Indeed, with  $\dot{m} T ds = \delta \dot{Q}$  the last term can be written as  $-\int_1^2 \left( 1 - \frac{T_0}{T} \right) \delta \dot{Q}$  where  $-\delta \dot{Q}$  is the heat supplied to the heat engines on an infinitesimal step of the flow. Thus, if the entropy generation vanishes, the heat engines are a series of infinitesimal Carnot engines.

The maximum work is extracted when all processes are reversible, so that  $\dot{S}_{gen} = 0$ , and when no external irreversibilities occur, which requires that the exhaust is in equilibrium with the environment, i.e.,<sup>3</sup>  $T_2 = T_0$ ,  $p_2 = p_0$ ,  $\mathcal{V}_2 = \mathcal{V}_0 = 0$ . Then,

$$\dot{W}_{rev} = \dot{m} \psi_1 = \dot{m} \left[ h_1 - h_0 - T_0 (s_1 - s_0) + \frac{1}{2} \mathcal{V}_1^2 + g (z_1 - z_0) \right]. \quad (11.18)$$

Here we have defined the flow exergy (or availability)  $\psi$  as the maximum work per unit mass that can be extracted from a flow by equilibrating it with the environment.

The exhaust of a turbine or nozzle has work potential as measured by its exergy. If this work potential is not used, exergy is destroyed and entropy produced. For the case of a single outflow into the environment, the corresponding entropy generation is

$$\dot{S}_{gen} = \frac{\dot{m} \psi}{T_0}. \quad (11.19)$$

## 11.7 Heat Engine Driven by Hot Combustion Gas

As a relevant application we consider the maximum amount of work that can be obtained from combustion of a fuel in a fully reversible process. We will

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<sup>3</sup> The environment is at rest and work could be extracted from any flow faster than the environment. Thus, for no external irreversibilities to occur, one must set  $\mathcal{V}_2 = 0$ . However, then one could not remove the exhaust—for real applications one will assume  $\mathcal{V}_2 \ll \mathcal{V}_1$ .

compare the result to that of a case where a single reversible Carnot engine is employed to convert combustion heat into power.

Fuel must be mixed with air and, after its work is done, the combustion product must be removed. Thus, for combustion processes one does not have a single hot reservoir, but the heat is taken from a stream that is gradually cooled as heat is withdrawn.

To simplify the discussion, we ignore the amount of fuel mass added, and treat the combustion product as air; we shall also simplify for constant specific heats to obtain explicit formulae for work and efficiencies. Pressure losses are ignored, and the combustion is assumed to take place at environmental pressure  $p_0$ .

First we consider the isobaric combustor: fuel and air enter the combustor at environmental temperature  $T_0$ , and the hot combustion gas leaves at the flame temperature  $T_F$ . Since we ignore the mass flow of fuel, the heat added to the air is

$$\dot{Q}_{\text{fuel}} = \dot{m} [h(T_F) - h(T_0)] = \dot{m} c_p (T_F - T_0) , \quad (11.20)$$

where the flame temperature  $T_F$  depends on the amount of fuel added. The amount of heat supplied by combustion of the fuel is the product of the mass flow of fuel and the fuel's heating value  $q_{\text{HV}}$  (measured in kJ/kg fuel),  $\dot{Q}_{\text{fuel}} = \dot{m}_{\text{fuel}} q_{\text{HV}}$ . Thus, the cost associated with the process is proportional to  $\dot{Q}_{\text{fuel}}$ .

$\dot{Q}_{\text{fuel}}$  is also the maximum heat available to convert into power in a heat engine. If  $\dot{Q}_{\text{fuel}}$  is completely consumed by the heat engine, the exhaust of the power plant is at  $T_0$ . The maximum amount of work that can be obtained from the hot combustion gas<sup>4</sup> follows from (11.18), after ignoring kinetic and potential energies, as

$$\begin{aligned} \dot{W}_{\text{rev}} &= \dot{m} [h(T_F) - h(T_0) - T_0 (s(T_F, p_0) - s(T_0, p_0))] \\ &= \dot{m} c_p \left[ T_F - T_0 - T_0 \ln \frac{T_F}{T_0} \right] , \end{aligned} \quad (11.21)$$

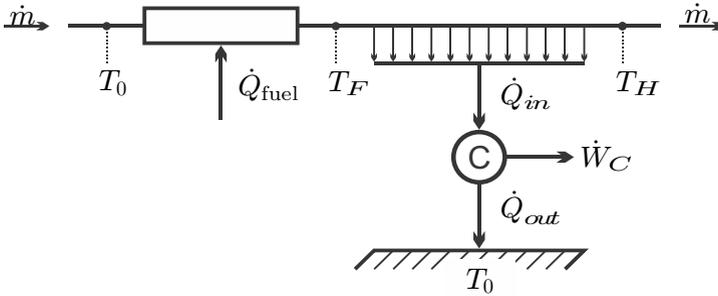
with the corresponding thermal efficiency

$$\eta_{\text{max}} = \frac{\dot{W}_{\text{rev}}}{\dot{Q}_{\text{fuel}}} = 1 - \frac{\ln \frac{T_F}{T_0}}{\frac{T_F}{T_0} - 1} . \quad (11.22)$$

This thermal efficiency for power extraction from a hot gas flow, valid only for constant specific heat, is the equivalent to the Carnot efficiency, which describes processes between reservoirs at constant temperatures.

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<sup>4</sup> This is also the maximum amount of work that can be obtained from the fuel in a combustion process, but, since combustion is irreversible, it is *not* the maximum amount of work available from the fuel—this will be discussed later, when reacting mixtures are discussed.



**Fig. 11.5** Carnot engine heated by continuous stream of combustion gas

Now we compare this hypothetical best case to the case where the heat extracted from the flow is transferred into a single Carnot engine operating between the temperatures  $T_0$  and  $T_H$ . The engine is heated by the hot combustion gas that enters its heat exchanger at flame temperature  $T_F$  and leaves at  $T_H$ . For now we assume that the exhaust at  $T_H$  is not further utilized, but dumped into the environment. The Carnot engine is internally reversible, but obviously there are external irreversibilities associated with the heat transfer into the engine, and with heat transfer between the exhaust gas and the environment. Figure 11.5 shows the corresponding system, including the air temperatures before and after combustion, and before and after heat exchange with the Carnot engine. The heat withdrawn from the combustion gas and added to the Carnot engine is

$$\dot{Q}_{in} = \dot{m} [h(T_F) - h(T_H)] . \quad (11.23)$$

Accordingly, the work produced by the—reversible—Carnot engine is

$$\begin{aligned} \dot{W}_C &= \left(1 - \frac{T_0}{T_H}\right) \dot{Q}_{in} = \left(1 - \frac{T_0}{T_H}\right) \dot{m} [h(T_F) - h(T_H)] \\ &= \dot{m} c_p \left(1 - \frac{T_0}{T_H}\right) (T_F - T_H) . \end{aligned} \quad (11.24)$$

The temperature  $T_H$  at the hot side of the engine is a variable of the process. A larger value of  $T_H$  increases the thermal efficiency of the Carnot engine, but also leads to a larger exergy of the exiting flow, and thus to a larger external loss. Small values of  $T_H$  lead to small thermal efficiency, and to large temperature difference between engine and hot gas flow, which implies large entropy generation and work loss in heat transfer. Closer examination shows that for a given flame temperature  $T_F$  and the combustion flow  $\dot{m}$ , the power produced by the Carnot engine,  $\dot{W}_C$ , has a maximum at  $T_{H,\max} = \sqrt{T_0 T_F}$ , where the power produced is

$$\dot{W}_{C,\max} = \dot{m}c_p \left( \sqrt{T_F} - \sqrt{T_0} \right)^2 . \tag{11.25}$$

Next we consider the efficiency of this conversion process. There are several efficiency measures that can be defined. The thermal efficiency of the Carnot engine alone is based on the heat intake of the engine,

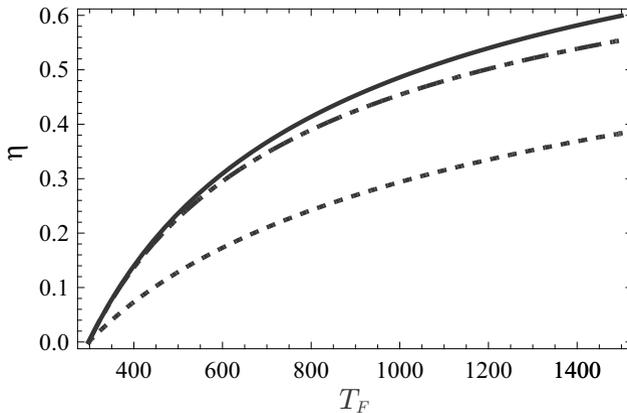
$$\eta_{C,\max} = \frac{\dot{W}_{C,\max}}{\dot{Q}_{\text{in}}} = 1 - \frac{T_0}{T_{H,\max}} = 1 - \sqrt{\frac{T_0}{T_F}} . \tag{11.26}$$

Interestingly, this is just the thermal efficiency (10.21) of the ideal Brayton cycle at maximum work. However, this efficiency is not a good measure for the system performance, since it ignores that more heat is available from the exhaust gas at  $T_H$ , which is not further used, and, according to the assumptions made, dumped into the environment.

Since the total heat available from cooling the hot flow to environmental temperature is<sup>5</sup>  $\dot{Q}_{\text{fuel}} = \dot{m}c_p (T_F - T_0)$ , the proper efficiency measure for the conversion of combustion heat into power in this set-up is

$$\eta_{\text{comb}} = \frac{\dot{W}_{C,\max}}{\dot{Q}_{\text{fuel}}} = \frac{(\sqrt{T_F} - \sqrt{T_0})^2}{T_F - T_0} = 1 - \frac{2\sqrt{T_0}}{\sqrt{T_F} + \sqrt{T_0}} . \tag{11.27}$$

Figure 11.6 shows the efficiencies  $\eta_{\max}$ ,  $\eta_{C,\max}$  and  $\eta_{\text{comb}}$  for temperatures  $T_F$  between  $T_0 = 298$  K and 1500 K. We compute the efficiencies for  $T_F = 1500$  K as  $\eta_{\max} = 0.599$ ,  $\eta_{C,\max} = 0.554$ , and  $\eta_{\text{comb}} = 0.383$  ( $T_H = 668$  K for the Carnot engine).



**Fig. 11.6** Efficiencies  $\eta_{\max}$  (continuous),  $\eta_{C,\max}$  (dash-dotted), and  $\eta_{\text{comb}}$  (dashed) as defined and discussed in the text

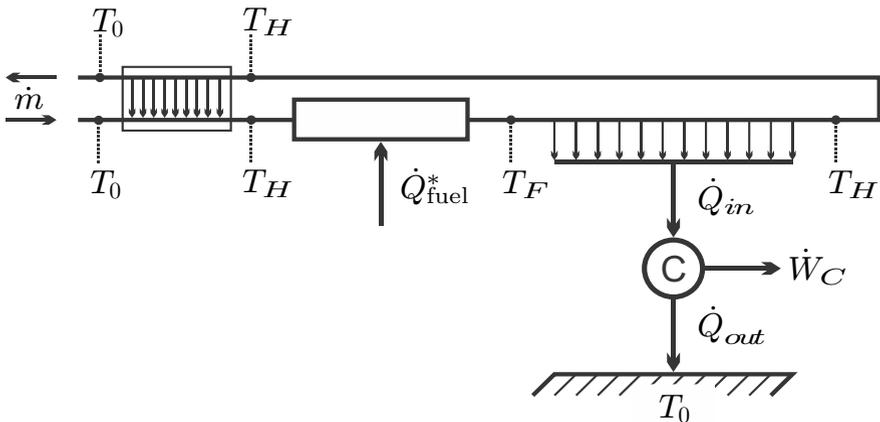
<sup>5</sup> Recall that the fuel cost is proportional to  $\dot{Q}_{\text{fuel}}$ .

The efficiency  $\eta_{\max}$  describes the best possible, i.e., fully reversible, system to produce work from the combustion gas. This efficiency can be used to define the second law efficiency for other processes as  $\eta^{\text{II}} = \frac{\eta}{\eta_{\max}}$ . For the data given above, we find the second law efficiencies  $\eta_{C,\max}^{\text{II}} = 0.925$ , and  $\eta_{\text{comb}}^{\text{II}} = 0.639$ .

The fully reversible system has the highest efficiency,  $\eta_{\max}$ . The single Carnot engine loses work to irreversibilities in heat transfer between hot gas and engine, and in the dumping of hot exhaust into the environment; accordingly it has a significantly lower efficiency  $\eta_{\text{comb}}$ . The efficiency  $\eta_{C,\max}$  considers only the actual heat entering the engine,  $\dot{Q}_{in}$ , not the total heat produced in combustion,  $\dot{Q}_F$ . This efficiency is higher than  $\eta_{\text{comb}}$  since it ignores the energy lost with the hot exhaust at  $T_H$ .

It is particularly important to note that the efficiency  $\eta_{C,\max}$ , which ignores the exhaust losses, leads to a far better impression of the quality of the process than the use of the efficiency  $\eta_{\text{comb}}$  which includes the exhaust losses. The important lesson to be learned here, is that one has to be rather careful about efficiency values presented anywhere, since one can always find an efficiency measure that lets a process appear to be better than it actually is, simply by excluding some, or all, external irreversibilities associated with the process.

The discussed process invites the question what one could do to utilize the exhaust at  $T_H$ . For an answer, we recognize that the fuel is needed to heat the air before it exchanges heat with the heat engine. Fuel consumption can be reduced by using the exhaust air (at  $T_H$ ) to preheat the incoming air before combustion. The heat exchanger used for this is called a regenerator, and Fig. 11.7 shows the system with the regenerator added. When we assume perfect heat exchange between the incoming and the exiting air streams (both



**Fig. 11.7** Carnot engine heated by continuous stream of combustion gas with regenerator added

with mass flow  $\dot{m}$ ), the incoming air can be heated to  $T_H$  while the exiting air cools down to  $T_0$ . In this case, only the heat

$$\dot{Q}_{\text{fuel}}^* = \dot{m} [h(T_F) - h(T_H)] = \dot{m} c_p (T_F - T_H) = \dot{Q}_{\text{in}} \quad (11.28)$$

must be provided from the combustion of fuel. As indicated, this is just the heat going into the heat engine,  $\dot{Q}_{\text{in}}$ , so that now all heat from the fuel is used in the engine. The exhaust is in equilibrium with the environment, and hence there is no external entropy generation—and no external loss. The only irreversibility in the considered process is the heat transfer between the hot gas and the engine. The efficiency for the engine with regenerator, and optimal choice for  $T_H$ , is

$$\eta_{C,reg} = \frac{\dot{W}_{C,max}}{\dot{Q}_{\text{fuel}}^*} = \frac{\dot{W}_{C,max}}{\dot{Q}_{\text{in}}} = \eta_{C,max} . \quad (11.29)$$

Regeneration will be discussed in more detail in subsequent chapters, including the consideration of imperfect heat exchange.

## 11.8 Exergy

Exergy, also known as “availability”, is defined as the maximum amount of work that can be extracted from a flow or an amount of substance by only exchanging heat with the environment until equilibrium with the environment is reached.

Thus, exergy describes work potential, and can be a useful concept to answer questions like whether it is worthwhile to harvest energy from a system. As an example one might think of the exhaust of fuel fired power plants as discussed in Section 11.6.

Exergy analysis is now an accepted method within the field of thermodynamics. We shall introduce the concept in this section, but since we prefer to focus on entropy generation and lost work arguments, we shall use exergy only occasionally.

To compute exergy, the combined first and second law (11.4) is simplified for the case where heat is exchanged only with the environment at  $T_0$  and all processes are reversible, that is by setting  $\sum_{k \neq 0} \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - T_0 \dot{S}_{gen} = 0$ .

We shall distinguish between flow exergy, and closed system exergy.

In a closed system, where all mass flows vanish, Eq. (11.4) reduces further to

$$-\frac{d(E - T_0 S)}{dt} = \dot{W} . \quad (11.30)$$

Integrating between the actual state  $\{E_a, S_a\}$  and the final equilibrium state  $\{E_0, S_0\}$ , and subtracting the work done to the environment, which has constant pressure  $p_0$ , yields the closed system exergy as

$$\begin{aligned}\Xi_a &= \int \dot{W} dt - \int_a^0 p_0 dV = - \int_a^0 (dE - p_0 dV - T_0 dS) \\ \Xi_a &= E_a - E_0 + p_0 (V_a - V_0) - T_0 (S_a - S_0) .\end{aligned}\quad (11.31)$$

The specific closed system exergy is

$$\xi = \frac{\Xi}{m} = e - e_0 + p_0 (v - v_0) - T_0 (s - s_0) . \quad (11.32)$$

Flow exergy  $\psi$  is defined as the maximum work per unit mass that can be extracted from a single flow in a steady state process as it is equilibrated with the environment, that is heat is exchanged only with the environment and the outflow is in equilibrium with the environment. Thus, from (11.8),

$$\psi = \frac{\dot{W}}{\dot{m}} = h - h_0 - T_0 (s - s_0) + \frac{1}{2} (\mathcal{V}^2 - \mathcal{V}_0^2) + g (z - z_0) = \xi + (p - p_0) v . \quad (11.33)$$

With these definitions, the combined first and second law (11.4) can be written in form of an exergy balance. Use of the mass balance (9.1) to eliminate some terms with the constant factor  $(e_0 + p_0 v_0 - T_0 s_0)$ , yields

$$\frac{d\Xi}{dt} + \sum_{out} \dot{m}_e \psi_e - \sum_{in} \dot{m}_i \psi_i = \sum_{k \neq 0} \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left(\dot{W} - p_0 \frac{dV}{dt}\right) - T_0 \dot{S}_{gen} . \quad (11.34)$$

This equation describes the change of exergy of a system due to convective transport ( $\dot{m}\psi$ ), heat transfer ( $\dot{Q}_k$ ) at temperatures  $T_k \neq T_0$ , work ( $\dot{W} - p_0 \frac{dV}{dt}$ ), and exergy destruction due to irreversibilities ( $-T_0 \dot{S}_{gen}$ ).

The combination  $\left(\dot{W} - p_0 \frac{dV}{dt}\right)$  is called the useful work. Note that for typical open systems, e.g., turbines, the volume  $V$  stays constant, so that  $p_0 \frac{dV}{dt}$  is zero. Moreover, most relevant closed system engines are reciprocating, so that for one cycle  $\oint p_0 \frac{dV}{dt} dt = p_0 \oint dV = 0$ . This is reflected in our discussion of the Otto and Diesel cycles, where we did not consider the work done on the environment.

## Problems

### 11.1. A Heat Engine

A heat engine that operates between two reservoirs at  $T_H = 500^\circ\text{C}$  and  $T_L = 25^\circ\text{C}$  produces 1.25 MW of power from a heat intake of 2.5 MW.

Compute the heat rejected, the thermal efficiency, the entropy generation rate, the work loss to irreversibilities, and the 2nd law efficiency of the engine.

### 11.2. A Heat Pump

An off-the-shelf heat pump system has a COP of 3.4 for operation between  $25^\circ\text{C}$  and  $-5^\circ\text{C}$ . Determine the entropy generation per kW of heating, the

percentage of power consumed required to overcome irreversibilities, and the 2nd law efficiency of the system.

### 11.3. A Refrigerator

A refrigeration system has a COP of 2 for operation between  $20^\circ\text{C}$  and  $-10^\circ\text{C}$  and consumes 1.5 kW of electrical power. Determine the second law efficiency of the system, the entropy generation rate, and the amount of power required to overcome irreversibilities.

### 11.4. Heating of House

A small house is exposed to an environment of  $T_0 = -5^\circ\text{C}$ , the temperature inside is to be maintained at  $T_h = 22^\circ\text{C}$ . The heat loss is given by Newton laws of cooling as  $\dot{Q}_{loss} = \alpha A(T_h - T_0)$ , where  $A = 260\text{ m}^2$  is the outside surface of the house and  $\alpha = 1 \frac{\text{W}}{\text{m}^2\text{K}}$  is an overall heat transfer coefficient. Determine the amount of electrical work for heating the house for the following cases:

1. With a resistance heater.
2. With an internally and externally reversible Carnot heat pump.
3. With an externally irreversible Carnot heat pump with 15 K temperature difference for heat transfer.

### 11.5. Space Heating

A friend who is going to build a house asks you for advice on heating systems. His contractor has offered the following choices: (a) baseboard resistance heaters, (b) heat pump with hot water radiators (circulating water heated to  $60^\circ\text{C}$ ), (c) heat pump with warm water floor heating (circulating water heated to  $35^\circ\text{C}$ ).

Based on your knowledge of thermodynamics, which option should your friend chose? Present your arguments. Assume outside temperature  $-10^\circ\text{C}$  and inside temperature  $20^\circ\text{C}$ .

### 11.6. A Cycle

A closed piston-cylinder engine with helium as working medium operates on the following reversible cycle

- 1-2: Isentropic compression from  $p_1 = 10\text{ bar}$ ,  $T_1 = 300\text{ K}$
- 2-3: Isobaric heat addition until  $T_3 = 1200\text{ K}$
- 3-1: Isochoric cooling to the initial state

1. Draw p-V-diagram and T-s-diagram for the cycle.
2. Determine the thermal efficiency of the cycle and the net work output per unit mass.
3. How much work per unit mass could be obtained from the heat rejected into the environment in the best case? Assume the environment is at  $T_0 = 300\text{ K}$ ,  $p_0 = 1\text{ bar}$ .

### 11.7. Exhaust of a Car Engine

The engine of a car delivers a net work of  $848.4 \frac{\text{kJ}}{\text{kg}}$  from an heat intake of  $1520.4 \frac{\text{kJ}}{\text{kg}}$  (reversible operation). The state at the end of the expansion stroke of the engine is 1146.6 K and 3.822 bar. In the engine, air in this state is exhausted into the environment which is at 300 K, 1 bar.

The exhaust process is modelled as isochoric cooling to the environment. Use the combined first and second law to compute the amount of work that could be obtained from the exhaust in a fully reversible process and compare to the work delivered by the engine.

What would be the thermal efficiencies for the engine alone, and for a system that also provides the work obtainable from the exhaust?

### 11.8. Exhaust of a Car Engine (Continuous)

The state at the end of the expansion stroke of a car engine is 1140 K and 3.8 bar. In the actual engine, air in this state is exhausted into the environment which is at 300 K, 1 bar.

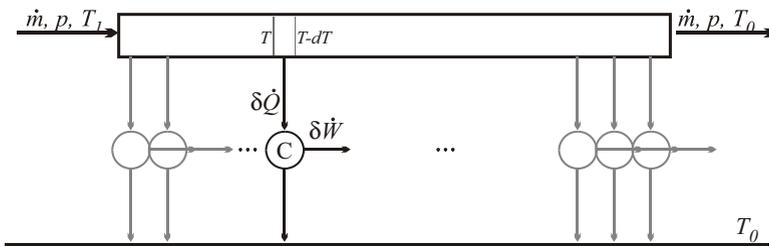
Assume that the exhaust is leaving the engine as a continuous steady flow, and determine the work that could be obtained from the exhaust in a fully reversible process. Compare to the work delivered by the engine.

### 11.9. Use of Waste Heat

A chemical plant rejects 2 MW of waste heat at 350 °C and 4 MW at 200 °C. Moreover the plant consumes 5 MW of electrical energy. In the past, the waste heat was just dumped into the environment (20 °C), but now there is a plan to use it for power production to reduce the electricity bill. Estimate what percentage of the electric power consumed in the plant could be produced by a suitable system, when 20% of the maximum possible is lost to irreversibilities.

### 11.10. Reversible Heat Transfer

Consider a steady state isobaric flow (pressure  $p$ , mass flow  $\dot{m}$ ) of a fluid that enters the system at a temperature  $T_1$  and leaves at the environmental temperature  $T_0$ . The heat withdrawn is used to drive a series of infinitely many infinitesimal Carnot engines. All processes are fully reversible. Compute the total power output of the system.



**11.11. Entropy Generation in Mixing**

In an adiabatic mixing chamber, a mass flow of  $200 \frac{\text{kg}}{\text{s}}$  compressed liquid water at 10 bar,  $40^\circ\text{C}$  is mixed isobarically with saturated steam so that the exiting flow is in the saturated liquid state. Determine the mass flow of saturated steam that must be added, the entropy generation rate, and the work loss for the process (with respect to standard environment).

**11.12. Entropy Generation in Steam Generator**

The temperature in the boiler of a big steam power plant is constant at  $700^\circ\text{C}$ . The pipes of the steam generator run through the boiler, with the inlet state being compressed liquid at 200 bar,  $50^\circ\text{C}$ , and the exit state being at 200 bar,  $550^\circ\text{C}$ . For a mass flow of  $1150 \frac{\text{t}}{\text{h}}$ , determine the heating rate, the entropy generation rate and the associated work loss (with respect to standard environment).

**11.13. Entropy Generation in Steam Generator**

Consider a 250 MW nuclear power plant with thermal efficiency of 0.32. The steam generator is kept at a pressure of 17.5 MPa. The incoming feedwater is in the compressed liquid state at  $40^\circ\text{C}$ , and the exiting steam is superheated vapor at  $400^\circ\text{C}$ . The heat is provided by a counter-flow of molten sodium (ideal incompressible liquid, mass density  $0.927 \frac{\text{g}}{\text{cm}^3}$ , specific heat  $1.26 \frac{\text{kJ}}{\text{kg K}}$ ) which enters at  $500^\circ\text{C}$  and leaves at  $350^\circ\text{C}$ .

1. Determine the mass flows of steam and sodium.
2. Determine the total entropy generation rate in steam generator, and the corresponding work loss.

**11.14. Entropy Generation in Condenser**

The condenser of a small steam power plant is kept at a temperature of  $45^\circ\text{C}$ . The inlet state is at a quality of 90%, and the exit state is saturated liquid. The condenser rejects heat to the environment which is at  $5^\circ\text{C}$ . For a mass flow of  $75 \frac{\text{t}}{\text{h}}$ , determine the cooling rate, the entropy generation rate and the associated work loss.

**11.15. Entropy Generation in Throttling**

Cooling fluid R134a in compressed liquid state at 1 MPa,  $26^\circ\text{C}$  is throttled adiabatically to a pressure of 0.14 MPa. For a mass flow of  $1 \frac{\text{kg}}{\text{s}}$ , determine the entropy generation rate, and the associated work loss.

**11.16. Work Potential of a Hot Rock**

A 2t block of granite (specific heat  $c = 0.79 \frac{\text{kJ}}{\text{kg K}}$ ) is initially at a temperature of  $500^\circ\text{C}$ . How much work could be obtained from equilibrating the rock with the environment at  $15^\circ\text{C}$ ?