Chapter 13
Gas Engines

13.1 Stirling Cycle

13.1.1 The Ideal Stirling Cycle

We have pointed out again and again that effective energy conversion requires the reduction of internal and external irreversibilities as much as possible. An important cause of external irreversibility is heat transfer between the system and its heat sources and sinks, e.g., a stream of combustion gas, or the environment. The related loss can be reduced if heat that is rejected in one process of a cycle can be added elsewhere within the system. This simultaneously reduces the heat rejection and the heat supply from the exterior, thus leading to efficiency improvements. Indeed, *regeneration*, i.e., internal exchange of heat within a system, is the most important tool to reduce external irreversibilities, and increase efficiency. Historically, the first engine which used a regenerator was the Stirling engine.

The idealized Stirling cycle consists of two isothermal and two isochoric processes, taking place at temperatures T_H and T_L, and volumes V_1, V_2, respectively. The working medium is an ideal gas, e.g., air or helium, which is confined permanently in a cylinder. The T-S- and p-V-diagrams are depicted in Fig. 13.1.

We ignore all internal irreversibilities, so that work and heat for the reversible processes within the ideal Stirling cycle are

$$\begin{array}{lll} \text{1-2 isothermal:} & w_{12} = RT_H \ln \frac{V_2}{V_1} > 0 & , q_{12} = RT_H \ln \frac{V_2}{V_1} > 0 \,, \\ \text{2-3 isochoric:} & w_{23} = 0 & , q_{23} = u\left(T_L\right) - u\left(T_H\right) < 0 \,, \\ \text{3-4 isothermal:} & w_{34} = RT_L \ln \frac{V_1}{V_2} < 0 & , q_{34} = RT_L \ln \frac{V_1}{V_2} < 0 \,, \\ \text{4-1 isochoric:} & w_{41} = 0 & , q_{41} = u\left(T_H\right) - u\left(T_L\right) < 0 \,. \end{array} \tag{13.1}$$

H. Struchtrup, *Thermodynamics and Energy Conversion*,
DOI: 10.1007/978-3-662-43715-5_13,

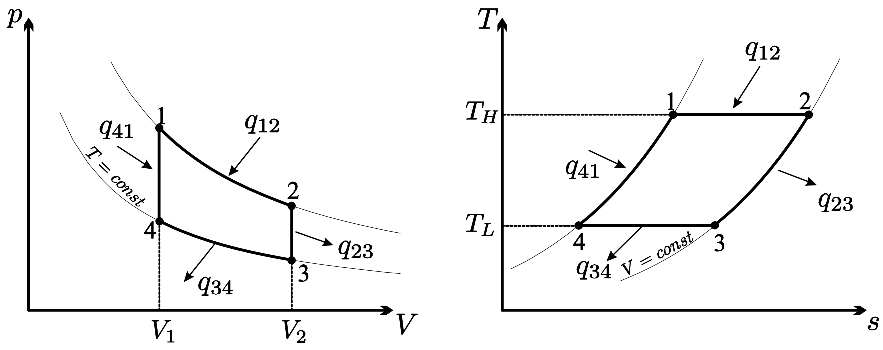

Fig. 13.1 Idealized Stirling power cycle in p-V- and T–s-diagrams

Since internal energy of the ideal gas depends only on temperature, $u = u(T)$, it turns out that the heats exchanged during the isochoric processes are equal, but of opposite sign,[1]

$$q_{23} = u(T_L) - u(T_H) = -q_{41} \ . \tag{13.2}$$

With this, the heat q_{23}, which is rejected during the isochoric cooling process, can be used for the isochoric heating q_{41}. In the Stirling engine, this is done by means of the regenerator, which allows for internal heat exchange. The working principle of the regenerator will be discussed further below.

When a regenerator is employed, the heat q_{41} is exchanged internally, and only the heat q_{12} must be provided from the outside, e.g., by burning a fuel. The thermal efficiency is obtained, with (13.1), as

$$\eta_{\mathrm{St}} = \frac{w_{\odot}}{q_{in}} = \frac{w_{12} + w_{34}}{q_{12}} = 1 - \frac{T_L}{T_H} \ . \tag{13.3}$$

In case that T_L and T_H are the temperatures of reservoirs with which the cycle exchanges heat, the above is just the Carnot efficiency, that is the maximum efficiency for a process operating between reservoirs at temperatures T_H and T_L. It follows that the ideal Stirling process with regenerator is a realization of a Carnot engine, as long as the heat transfer with the reservoirs takes places at infinitesimal temperature difference.

We note that the amounts of heat exchanged during the isochoric processes, q_{23} and q_{41}, can only be equal in size for an ideal gas (with variable or constant specific heats), for which internal energy does not depend on specific volume, $u = u(T)$ and not $u = u(T, v)$. This is different for the Carnot cycle—another realization of a Carnot engine—which has the same efficiency independent of the working medium.

[1] This implies that in the T-s-diagram the areas below the curves 2-3 and 4-1 are equal.

Although the efficiency is independent of the type of ideal gas used, most Stirling engines use helium or hydrogen as working medium. The high heat conductivity of gases with low molecular masses leads to a faster heat exchange and thus allows to operate the engine at a higher frequency.

The power output of the engine depends on the mass m enclosed in the cylinder, and the rotation speed $\dot{n}$ of the engine,

$$\dot{W} = \dot{n} m w_{\odot} \ . \tag{13.4}$$

Fast changes in power demand, as they are necessary for use in cars, can be achieved by changing the amount of working gas in the cylinder, i.e., by pumping additional mass in or out. This, of course, adds to the complexity of the process. It is therefore no surprise that most of today's applications of the Stirling engine consider systems which run at constant load, and generate electricity, in particular with heat supply from solar collectors.

13.1.2 Working Principle of a Stirling Engine

It is difficult, if not impossible, to build an engine that operates on the ideal Stirling cycle. All real Stirling engines approximate the ideal cycle to some extent. There are many different working principles for Stirling engines, and here we discuss the operating principle of a Leybold Stirling engine for use in teaching laboratories, which operates in the same way as the original Stirling engine.[2]

The lab engine, sketched in Fig. 13.2, consists of a glass cylinder in which two pistons—the working piston and the displacement piston—move vertically with a phase shift of 90°. Mounted on top of the cylinder is the heating coil (electrical heating), which maintains the upper part of the engine at high temperature (T_H). The lower part of the cylinder is encased by a second glass cylinder, with cooling water flowing between the two cylinders and through the bottom of the displacement piston to maintain the lower part of the engine at low temperature (T_L).

The displacement piston shifts the gas between the upper high-temperature part of the engine and the lower low-temperature part. The movement of the displacement piston forces the gas through a cylindrical hole in the displacement piston that is filled with copper wool which acts as the regenerator. As the gas passes from the hot part of the engine to the cold part, the gas cools gradually by giving up heat to the copper wool. On the way back the gas takes heat from the regenerator and is thus gradually heated. The working piston seals the gas against the environment and serves to compress or expand it while exchanging work with the environment.

The actual thermodynamic cycle of the Stirling engine differs somewhat from the idealized Stirling cycle, see Fig. 13.3 for a qualitative comparison

[2] This engine is used in the teaching laboratory at the University of Victoria.

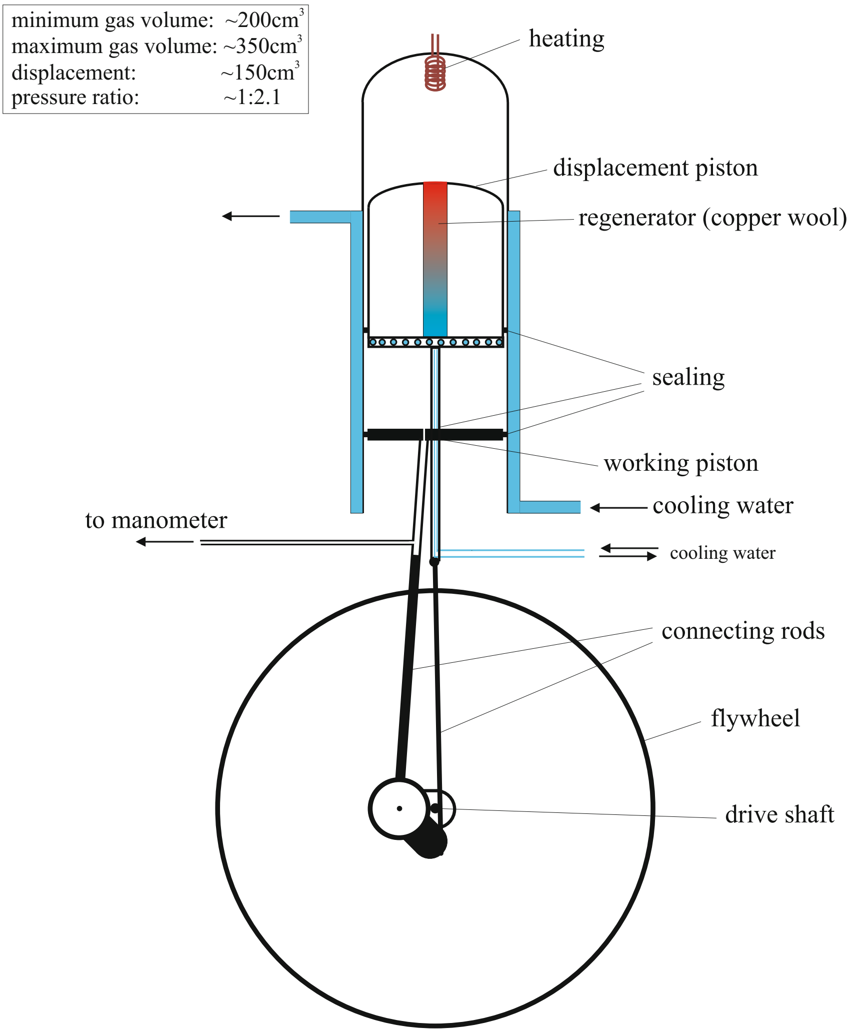

Fig. 13.2 Setup of Leybold-Stirling engine

of the idealized and the real cycle. In order to understand how the Stirling engine approximates the ideal cycle, it is best to study the p-V-diagram in Fig. 13.3, which also shows the displacement of the two pistons as function of the shaft angle.

I. *Isothermal expansion:* The displacement piston is at bottom dead center, almost at rest, so that most of the working gas is in the upper hot zone. The working piston moves downward and the gas expands, the heat supplied is transferred to work.

II. *Isochoric cooling:* The working piston is at bottom dead center, so that the total gas volume is (almost) fixed. The displacement piston is moving upwards and the working gas streams into the lower cold part of the engine. While flowing through the regenerator (copper wool), the gas transfers heat to the regenerator.

III. *Isothermal compression:* The displacement piston is at top dead center, and the working gas is in the lower cool part of the cylinder. The working piston moves upwards compressing the gas. The gas releases heat to the cooling water so that the gas temperature remains nearly constant.

IV. *Isochoric heating:* The working piston is at top dead center, while the displacement piston is moving downwards. The cool gas is streaming upwards through the regenerator, where it receives the energy which was stored in Step II.

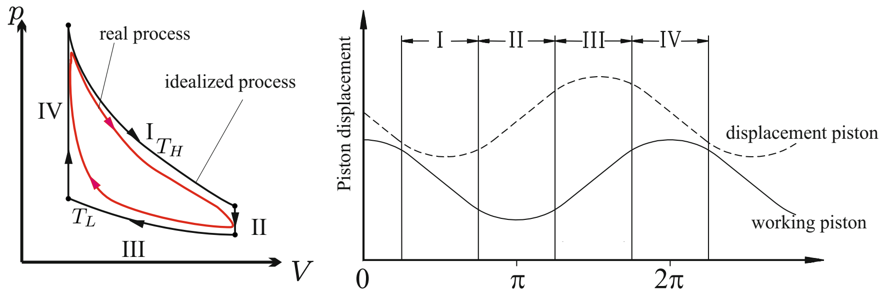

Fig. 13.3 Left: Ideal and real Stirling cycles in p-V-diagram. Right: Piston displacement as function of shaft angle.

The main reasons why the measured p-V-diagram deviates from the ideal one, and assumes the more oval form shown in Fig. 13.3 are:

1. Truly isochoric processes require that the working piston is at rest. However, since it is driven by a crankshaft, the working piston moves sinusoidally.
2. Compression and expansion (I and III) are fast, and do not take place isothermally.
3. The heating coil releases heat into the gas at all times, not only during step I .
4. Part of the working gas remains in the cool part of the engine at all times.
5. The regenerator is not 100% efficient.
6. Heat losses to the environment and friction dissipate energy.
7. Insufficient sealing leads to exchange of gas with the outside.

13.1.3 The Reverse Stirling Cycle

A Stirling engine can also operate as a refrigeration engine or a heat pump. In both cases the engine is driven by a motor, and the process curve in the p-V-diagram runs counter-clockwise, see Fig. 13.4 for the ideal process curve. During the isothermal processes (1-2, 3-4) heat is exchanged with the environment of the engine, while the heat transfer during the isochoric processes (2-3, 4-1) is an internal heat exchange by means of the regenerator.

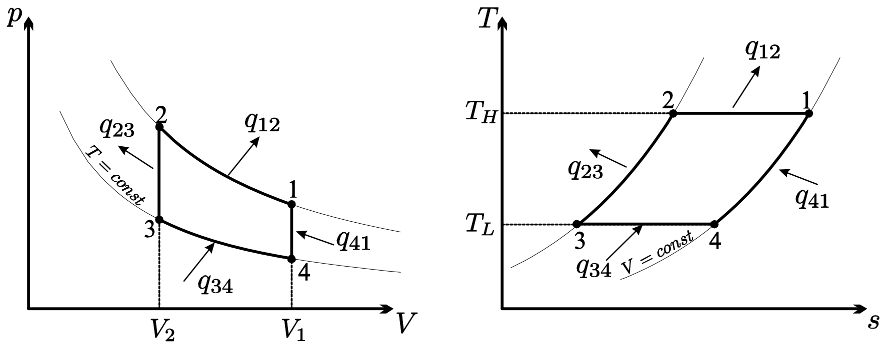

Fig. 13.4 Stirling refrigeration cycle in p-V- and T–s-diagrams

We find for the branches of this cycle the following exchange of heat and work with the environment:

$$\begin{array}{lll} \text{1-2 isothermal: } w_{12} = RT_H \ln \frac{V_2}{V_1} < 0 & , q_{12} = RT_H \ln \frac{V_2}{V_1} < 0\,, \\ \text{2-3 isochoric: } \quad w_{23} = 0 & , q_{23} = u(T_L) - u(T_H) < 0\,, \\ \text{3-4 isothermal: } w_{34} = RT_L \ln \frac{V_1}{V_2} > 0 & , q_{34} = RT_L \ln \frac{V_1}{V_2} > 0\,, \\ \text{4-1 isochoric: } \quad w_{41} = 0 & , q_{41} = u(T_H) - u(T_L) > 0\,. \end{array} \tag{13.5}$$

As for the Stirling heat engine, the heats for the isochoric processes have the same absolute value, $q_{23} = -q_{41}$, and the regenerator ensures internal heat exchange.

The coefficient of performance for the inverse cycle is

$$\mathrm{COP_R} = \frac{q_{in}}{|w_\odot|} = \frac{q_{34}}{|w_{12} + w_{34}|} = \frac{1}{\frac{T_H}{T_L} - 1} \tag{13.6}$$

for a refrigeration engine, and

$$\mathrm{COP_{HP}} = \frac{|q_{out}|}{|w_\odot|} = \frac{|q_{12}|}{|w_{12} + w_{34}|} = \frac{1}{1 - \frac{T_L}{T_H}} \tag{13.7}$$

for a heat pump, respectively. Again, these are the COP's of the respective Carnot engines, i.e., the maximum COP's that can be reached between the temperatures T_H, T_L.

The lab engine can run as both, heat pump and refrigerator. If run as a heat pump, the lower, water cooled, part of the engine is the cold part of the engine (at T_L) and heat is pumped to the upper part of the engine which becomes hot (T_H). When the operating direction of the engine is reversed, the lower, water cooled, part of the engine becomes the hot part of the engine (at T_H) and heat is pumped away from the upper part of the engine which becomes cold (T_L), the engine operates as a refrigerator.

13.1.4 Stirling Engines Then and Now

The Stirling engine was patented in 1816 by Robert Stirling (1790-1878), a minister of the Church of Scotland. At that time, steam engines had a rather low efficiency (2-10%), and were quite unsafe. Boilers exploded often, and the high pressure steam that was released had scalding effects. The Stirling hot-air engine was promising to overcome both problems: The regenerator gave it a good efficiency, and in the unlikely case of a bursting engine, only hot air was released so that the consequences of an accident were far less severe. However, the Stirling engine could never live up to the expectations. While the steam turbine was improved more and more to today's efficiencies of over 45%, and internal combustion engines, i.e., Otto and Diesel engines, prevailed for the use in cars and trucks, Stirling engines almost vanished from the scene.

Unlike an internal combustion engine, a Stirling engine does not exchange the working gas in each cycle, but contains the gas permanently. The heat is supplied outside the engine, so that any heat source is suitable to power a Stirling engine. Thus, a Stirling engine can be driven by carbon fuels (coal, natural gas, gasoline, Diesel oil), hydrogen, solar radiation, nuclear power, waste heat of industrial processes, etc. If a fuel is used, it is burned continuously, with lower emissions than in a reciprocating internal combustion engine.

Stirling refrigeration engines can give very low temperatures, and are widely used for small scale cryogenic cooling. A promising application for Stirling heat engines is the conversion of solar energy into electricity by means of parabolic mirror dishes with Stirling engines in the focus. While these devices can only operate under direct sunlight, they can be far more efficient than photovoltaic cells.

At present it is not possible to build a high efficiency Stirling engine at a competitive price. In order to have a high specific power (kW per litre of engine capacity), the working gas must be highly pressurized (goal: up to 190 bar), causing problems of sealing against the environment and lubrication. It is at the seals where a large portion of mechanical losses occur. The efficiency increases with the temperature difference between the cold and the hot part of the engine. Thus, highly efficient engines require non-standard materials that can operate at temperatures of 750 °C and more.

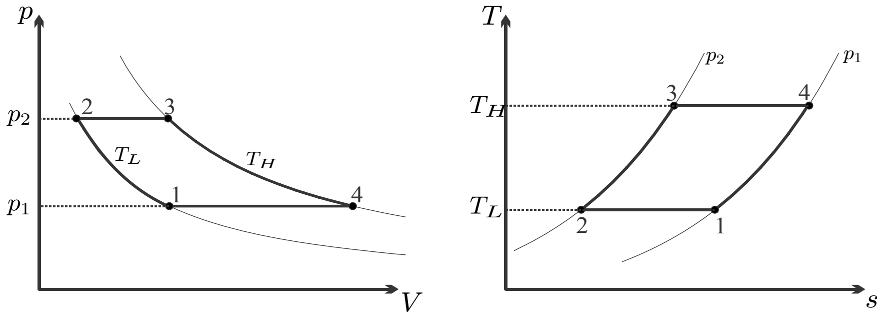

Fig. 13.5 p-V- and T-s-diagrams for the Ericsson cycle

13.2 Ericsson Cycle

The Stirling cycle operates in piston-cylinder assemblies, in closed systems. The Ericsson cycle is very similar, only that it consists of open system devices, namely isothermal turbine and compressor, and isobaric heat exchangers; again, the working fluid is an ideal gas. Figure 13.5 shows the corresponding p-V- and T-s-diagrams. Since we consider open system devices for the realization of the cycle, the corresponding work and heat contributions are

$$\begin{array}{lll}
\text{1-2 compressor at } T_L: & w_{12} = -RT_L \ln \frac{p_2}{p_1} < 0 & , q_{12} = -RT_L \ln \frac{p_2}{p_1} < 0 \,, \\
\text{2-3 heating at } p_2: & w_{23} = 0 & , q_{23} = h(T_H) - h(T_L) > 0 \,, \\
\text{3-4 turbine at } T_H: & w_{34} = -RT_H \ln \frac{p_1}{p_2} > 0 & , q_{34} = -RT_H \ln \frac{p_1}{p_2} > 0 \,, \\
\text{4-1 cooling at } p_2: & w_{41} = 0 & , q_{41} = h(T_L) - h(T_H) < 0 \,.
\end{array}$$

Since the enthalpy of an ideal gas depends only on temperature, but not on pressure, the amounts of heat exchanged on the isobaric legs are equal in magnitude with opposite signs: a regenerator can be used for internal exchange of heat. In fact, the regenerator must be a counter-flow heat exchanger, which leads to the schematic shown in Fig. 13.6.

With the use of the regenerator, only the heat q_{34} must be supplied from the outside, and the thermal efficiency of the ideal cycle becomes

$$\eta_{\text{Er}} = \frac{w_{\odot}}{q_{in}} = \frac{w_{12} + w_{34}}{q_{34}} = 1 - \frac{T_L}{T_H} \,. \tag{13.8}$$

This, again, is the Carnot efficiency, that is the best possible efficiency obtainable from a heat engine operating between reservoirs at temperatures T_H, T_L.

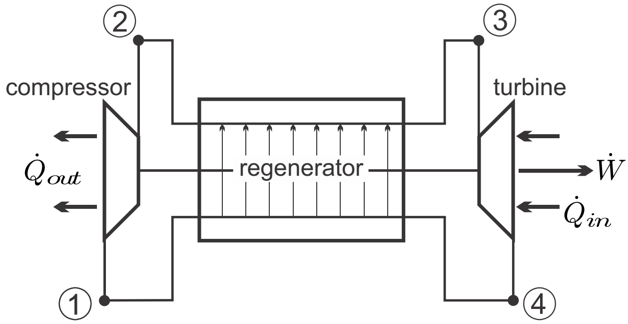

Fig. 13.6 Schematic of Ericsson engine

As the Stirling engine, the Ericsson engine uses external heat supply, which, in principle, allows the use of any heat source.

The Ericsson cycle relies on isothermal turbine and compressor, which require heat exchange during compression and expansion. Since heat transfer is slow, these are difficult to realize: Fast compressors and turbines normally are adiabatic, since the mass flows through too fast so that there is no time for significant heat transfer. Adiabatic compression with intercooling, and adiabatic expansion with reheat, together with regeneration, as discussed in the next sections, are means to bring gas turbine cycles closer to the Ericsson cycle, and thus improve their efficiency.

13.3 Compression with Intercooling

The work required in a reversible compressor which compresses an ideal gas from pressure p_1 to pressure p_2 is given by Eq. (9.25),

$$w_C = -\int_1^2 v dp \,. \tag{13.9}$$

Thus, as discussed earlier, the work is the area to the left of the process curve in the p-v-diagram, see Sec. 9.6; less work is required for smaller specific volume of the compressed substance.

Figure 13.7 shows the curves for an isothermal and an adiabatic compressor in the p-v-diagram, in the reversible case. The adiabatic curve is steeper than the isothermal curve (Sec. 7.7), since in the isothermal process the cooling during compression ensures a smaller specific volume. Therefore the isothermal compressor requires less work than the adiabatic one.

For an isothermal compressor we have $v = RT_1/p$, and the work is $w_C = -RT_1 \ln \frac{p_2}{p_1}$, while for an adiabatic compressor, assuming constant specific heats, we have $v = v_1 (p_1/p)^{1/k}$ and $w_c = c_p T_1 \left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right)$. The difference in work requirement is indicated by the shaded area in the figure.

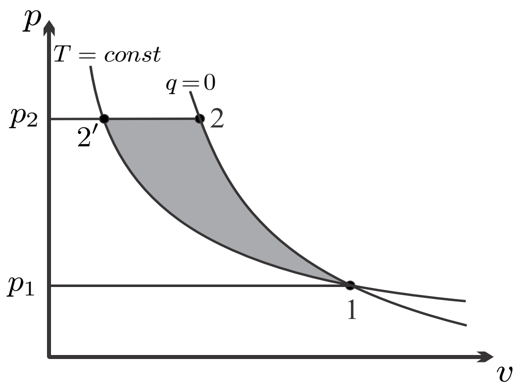

Fig. 13.7 Adiabatic (1-2) and isothermal (1-2') compressor in p-V-diagram. The grey area is the difference in the compressor work.

Obviously, to reduce the work requirement one would aim at isothermal compression. However, isothermal compression requires cooling during compression which is impossible to achieve for the large throughputs required, and cannot be used in practice.

An alternative is offered by multi-stage compressors with intercooling, in which the gas is compressed adiabatically in each stage, and then isobarically cooled to the environmental temperature T_1 before it enters the next stage. Intercooling reduces the gas volume, and thus the work requirement is reduced. As an example, Fig. 13.8 shows a three stage compressor with intercooling. As more stages are used, the process curve approaches the isothermal curve. It should be noted that construction is more costly than for a single stage compressor.

The work savings depend on the pressures chosen for intercooling, which must be optimized. We consider an n-stage compressor which takes in gas at p_1, T_1, and compresses it to the final pressure p_e; between each stage the gas is cooled back to T_1. Stage i compresses from T_1, p_i to p_{i+1}, and requires the work

$$w_{C_i} = \frac{h(T_1) - h(T_{i+1})}{\eta_{C_i}} , \tag{13.10}$$

where η_{C_i} is the isentropic efficiency of stage i. The temperature at the exit of an adiabatic reversible compressor between the same pressures, T_{i+1}, is obtained from the relation $\frac{p_r(T_{i+1})}{p_r(T_1)} = \frac{p_{i+1}}{p_i}$. Accordingly, the reversible work $w_{C,rev}(P_i) = h(T_1) - h(T_{i+1})$ is a function of the pressure ratio $P = \frac{p_{i+1}}{p_i}$ for the stage, and we can write

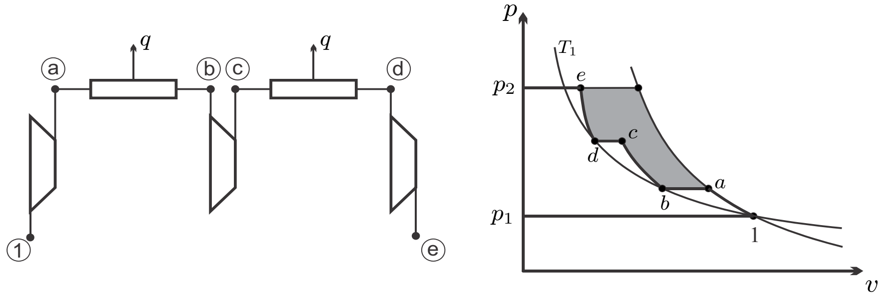

Fig. 13.8 Schematic and p-v-diagram for a compressor with three adiabatic stages and intercooling. The grey area is the amount of work saved in comparison to a single adiabatic compressor.

$$w_{C_i} = \frac{w_{C,rev}\left(\frac{p_{i+1}}{p_i}\right)}{\eta_{C_i}} . \tag{13.11}$$

The total work is just the sum over the individual compressors,

$$w_C = \sum_{i=1}^{n} \frac{w_{C,rev}\left(\frac{p_{i+1}}{p_i}\right)}{\eta_{C_i}} . \tag{13.12}$$

The minimum compression work is obtained from setting $\frac{\partial w_C}{\partial p_j} = 0$ for all intermediate pressures ($j = 1, 2, \ldots, n$). The derivative is evaluated as follows:

$$\begin{aligned}\frac{\partial w_C}{\partial p_j} &= \frac{\partial}{\partial p_j}\left[\frac{w_{C,rev}\left(\frac{p_j}{p_{j-1}}\right)}{\eta_{C_{j-1}}} + \frac{w_{C,rev}\left(\frac{p_{j+1}}{p_j}\right)}{\eta_{C_j}}\right] \\ &= \frac{w'_{C,rev}\left(\frac{p_j}{p_{j-1}}\right)}{\eta_{C_{j-1}}}\frac{1}{p_{j-1}} - \frac{w'_{C,rev}\left(\frac{p_{j+1}}{p_j}\right)}{\eta_{C_j}}\frac{p_{j+1}}{p_j^2} ,\end{aligned} \tag{13.13}$$

where $w'_{C,rev}(P)$ indicates the derivative of $w_{C,rev}(P)$ with respect to the pressure ratio. Setting the above to zero gives the condition for minimum work requirement

$$\frac{p_j}{p_{j-1}}\frac{w'_{C,rev}\left(\frac{p_j}{p_{j-1}}\right)}{\eta_{C_{j-1}}} = \frac{p_{j+1}}{p_j}\frac{w'_{C,rev}\left(\frac{p_{j+1}}{p_j}\right)}{\eta_{C_j}} . \tag{13.14}$$

For the further evaluation we consider only the case where all stages have the same isentropic efficiency, $\eta_{C_j} = \eta_C$, so that

$$\frac{p_j}{p_{j-1}} w'_{C,rev}\left(\frac{p_j}{p_{j-1}}\right) = \frac{p_{j+1}}{p_j} w'_{C,rev}\left(\frac{p_{j+1}}{p_j}\right) . \tag{13.15}$$

The reversible work is a monotonous function of the pressure ratio. It follows that the multi-stage compressor requires minimum work when all stages operate at the same pressure ratio P, i.e.,

$$\frac{p_{i+1}}{p_i} = P \, . \tag{13.16}$$

This implies that all stages consume the same work per unit mass, $w_C(P)$. Multiplication of the pressure ratios of all stages gives, with $p_{n+1} = p_e$,

$$\prod_{i=1}^{n} \frac{p_{i+1}}{p_i} = \frac{p_e}{p_1} = P^n \, , \tag{13.17}$$

so that

$$P = \left(\frac{p_e}{p_1}\right)^{\frac{1}{n}} \quad \text{and} \quad p_{i+1} = p_1 P^i = \sqrt[n]{p_1^{n-i} \, p_e^i} \, . \tag{13.18}$$

Special cases are a two stage compressor, which consumes minimum work for the intermediate pressure $p_m = \sqrt{p_1 p_e}$, and the three stage compressor in the figure, for which the optimum intermediate pressures are obtained as $p_b = \sqrt[3]{p_1^2 p_e}$, $p_d = \sqrt[3]{p_1 p_e^2}$.

In case that the isentropic compressor efficiency depends on the pressure within the compressor, one has to evaluate (13.14). We note that the above argument is valid also for polytropic compressors, as long as the polytropic exponent for all compressors is the same.

13.4 Gas Turbine Cycles with Regeneration and Reheat

13.4.1 Regenerative Brayton Cycle

We return to the discussion of the Brayton cycle, which was introduced in Sec. 10.5. As is evident from the discussion there, in particular from the T-s-diagram, the Brayton cycle expels rather warm exhaust into the environment. Since the exhaust is warmer than the environment, it has a work potential as was shown in Sec. 11.6. If the exhaust is just blown into the environment, this work potential remains unused. The ensuing equilibration between exhaust and environment is an irreversible process—it is an external irreversibility for the gas turbine.

To recover at least a portion of the exhaust work potential, the exhaust can be lead through a regenerator to heat the compressed air before it enters the combustion chamber. With this, less heat must be supplied from the outside, and the efficiency is increased.

Figure 13.9 shows schematic and T-s-diagram for a Brayton gas turbine cycle with regenerator, which is a counter-flow heat exchanger. Since heat

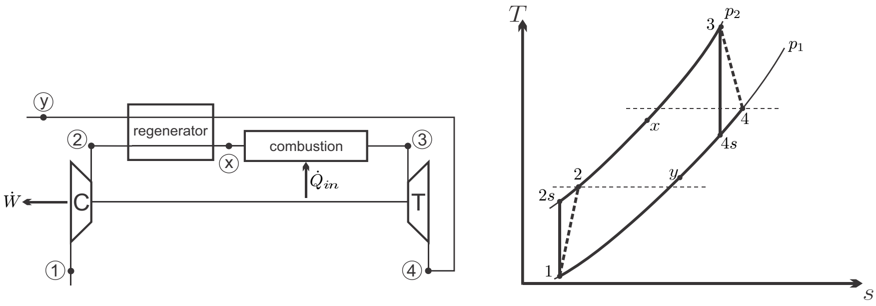

Fig. 13.9 Schematic and T-s-diagram for a Brayton cycle with regeneration

goes from warm to cold, the preheat temperature T_x cannot be larger than the turbine exhaust temperature T_4, while the final exhaust temperature T_y cannot be smaller than T_2. Thus, the use of a regenerator makes only sense when the turbine exhaust temperature T_4 is larger than the temperature after compression, T_2.

A perfect heat exchanger would yield $T_x = T_4$ and this is used to define the regenerator effectiveness as[3]

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \, . \tag{13.19}$$

A 100% effective regenerator would transfer heat at infinitesimal temperature differences, a realistic regenerator operates with finite temperature differences and around 80% effectiveness. The exhaust temperature T_y follows from the energy balance over the regenerator, assuming that no heat is lost to the exterior, as

$$h\left(T_y\right) = h_y = h_4 - h_x + h_2 \, . \tag{13.20}$$

The regenerator reduces the amount of heat that must be supplied from the fuel, which is $q_{x3} = h_3 - h_x$. Accordingly, the thermal efficiency for the depicted cycle $1 - 2 - x - 3 - 4 - y$ is given by

$$\eta_{B,reg} = \frac{h_1 - h_2 + h_3 - h_4}{h_3 - h_x} = 1 - \frac{h_4 - h_1 - \eta_{reg}\left(h_4 - h_2\right)}{h_3 - h_2 - \eta_{reg}\left(h_4 - h_2\right)} \, . \tag{13.21}$$

For $\eta_{reg} = 0$, this reduces to the thermal efficiency of the standard Brayton cycle, $\eta_B = 1 - \frac{h_4 - h_1}{h_3 - h_2}$. For non-zero effectiveness, the efficiency is larger than η_B. This follows from the fact that, because $0 < \eta_B < 1$, $h_4 - h_1 < h_3 - h_2$, which implies that with growing regenerator effectiveness η_{reg} the thermal efficiency $\eta_{B,reg}$ grows as well.

The actual improvement depends on the detailed data of the process.

[3] Note that the working fluid is an ideal gas, where enthalpy is a function of temperature only, $h = h\left(T\right)$.

13.4.2 Example: Brayton Cycle with Regenerator

The impact of the regenerator is best studied by means of examples. We consider a Brayton cycle with compressor inlet temperature $T_1 = 290\,\mathrm{K}$, turbine inlet temperature $T_3 = 1500K$, and pressure ratio $p_2/p_1 = 10$. To simplify the computation, we rely on the cold-air approximation with $k = \frac{c_p}{c_v} = 1.4$, $c_p = \frac{k}{k-1}R$, which gives the temperatures after isentropic compressor and turbine as

$$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 560\,\mathrm{K} \ , \ T_{4s} = T_3 \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = 777\,\mathrm{K} \ .$$

First, we consider the fully reversible cycle, with 100% effective regenerator. For this, according to (13.19, 13.20), the preheat temperature after the regenerator is $T_x = T_{4s}$, and the exhaust temperature is $T_y = T_{2s}$. With the computed temperatures, we find the specific work of the reversible cycle as

$$w_\odot = c_p \left(T_1 - T_{2s} + T_3 - T_{4s}\right) = 454.8\frac{\mathrm{kJ}}{\mathrm{kg}} \ ,$$

and the thermal efficiencies of the cycle without and with regenerator are

$$\eta_B = \frac{T_1 - T_{2s} + T_3 - T_{4s}}{T_3 - T_{2s}} = 48.2\% \ ,$$
$$\eta_{B,reg} = \frac{T_1 - T_{2s} + T_3 - T_{4s}}{T_3 - T_x} = 62.7\% \ .$$

We see that a regenerator can give substantial improvement for cycle efficiency.

With the regenerator, the external loss is reduced, since the exhaust temperature, and thus the external irreversibility, is lowered considerably. Indeed, the exhaust of the cycle without regenerator (temperature $T_x = T_{4s}$) has the work potential

$$w_{ex} = c_p \left(T_{4s} - T_1 - T_1 \ln\frac{T_{4s}}{T_1}\right) = 201.9\frac{\mathrm{kJ}}{\mathrm{kg}} \ ,$$

while exhaust of the cycle with regenerator (temperature $T_y = T_{2s}$) has the work potential

$$w_{ex,reg} = c_p \left(T_y - T_1 - T_1 \ln\frac{T_y}{T_1}\right) = 79.5\frac{\mathrm{kJ}}{\mathrm{kg}} \ .$$

Recall that the work potential of the exhaust is lost, since the exhaust is dumped into the environment. For this example, the regenerator reduces the exhaust loss by about 60%.

All efficiency values in the above example are relatively high, since no internal irreversibilities are accounted for. To study how internal irreversibilities affect the results, we now assume isentropic efficiencies for compressor and turbine of $\eta_T = \eta_C = 0.85$, and a regenerator effectiveness of $\eta_{reg} = 0.8$.

Then, we find the temperatures after compressor and turbine as

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C} = 618\,\mathrm{K} \quad , \quad T_4 = T_3 + \eta_T \left(T_{4s} - T_3\right) = 885\,\mathrm{K} \; ,$$

and the temperatures after heat exchange in the regenerator, from (13.19, 13.20), as

$$T_x = T_2 + \eta_{reg} \left(T_4 - T_2\right) = 832\,\mathrm{K} \quad , \quad T_y = T_4 - T_x + T_2 = 671\,\mathrm{K} \; .$$

The specific work for the cycle is now

$$w_{\odot} = c_p \left(T_1 - T_2 + T_3 - T_4\right) = 288 \frac{\mathrm{kJ}}{\mathrm{kg}} \; ,$$

and the thermal efficiencies for the cycle with and without regenerator are

$$\eta_B = \frac{T_1 - T_2 + T_3 - T_4}{T_3 - T_2} = 32.5\% \; ,$$
$$\eta_{B,reg} = \frac{T_1 - T_2 + T_3 - T_4}{T_3 - T_x} = 43\% \; .$$

The corresponding work potentials of the dumped exhaust are $w_{ex} = 273 \frac{\mathrm{kJ}}{\mathrm{kg}}$ and $w_{ex,reg} = 138 \frac{\mathrm{kJ}}{\mathrm{kg}}$, i.e., the regenerator reduces the exhaust loss by 50%. Nevertheless, the exhaust still has significant work potential, which is about 50% of the work actually delivered by the system. Another heat engine can be used to produce work from the exhaust—see the discussion of the combined cycle further below.

For motivation of the next section we compute the ratio between compressor and turbine work, i.e., the back work ratio, for this cycle as

$$\mathrm{bwr} = \frac{|w_C|}{w_T} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{T_2 - T_1}{T_3 - T_4} = 53.3\% \; .$$

13.5 Brayton Cycle with Intercooling and Reheat

Our discussion of compressors has shown that multi-stage compression with intercooling reduces the work required for compression. Applying this idea in a gas turbine cycle reduces the back work ratio, and also the temperature T_2 behind the compressor. When a regenerator is used, the exhaust temperature T_y is limited by the temperature T_2 after compression, which is lower with

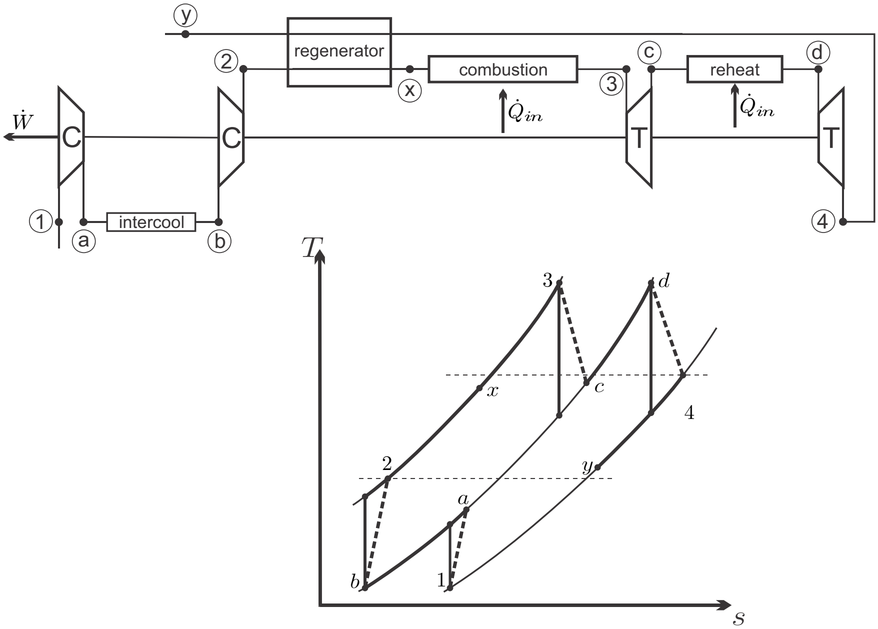

Fig. 13.10 Schematic and T-s-diagram for gasturbine cycle with two-stage compressor with intercooling, two-stage turbine with reheat, and regenerator

multi-stage compression. Lower exhaust temperature T_y lowers the external irreversibility, and thus gives better efficiency.

In short, further efficiency gain can be obtained by using multi-stage compression with intercooling together with regeneration. Figure 13.10 shows schematic and T-s-diagram for a system with two-stage compression and regenerator that also includes two turbines stages with intermediate reheat.

Reheat increases the average temperature for heating, and thus the efficiency. The optimum reheat pressure can be determined by maximizing work, similar to the discussion is Sec. 13.3. If the turbines have the same inlet temperature, and the same isentropic efficiency, the maximum work is obtained when they have the same pressure ratio.

Since reheat increases the turbine exit temperature, reheat will increase the thermal efficiency only if accompanied by regeneration. The thermal efficiency of this cycle is obtained as

$$\eta = \frac{w_{C1} + w_{C2} + w_{T1} + w_{T2}}{q_{comb} + q_{reheat}} = 1 - \frac{h_a - h_1 + h_2 - h_b + h_4 - h_x}{(h_3 - h_x) + (h_d - h_c)} \ . \quad (13.22)$$

With intercooling and reheat, a larger portion of the heat can be exchanged in the regenerator, which reduces the exhaust temperature T_y and the corresponding exhaust loss, thus increasing the thermal efficiency. With more and more intercooling and reheat stages, the process becomes more similar to the Ericsson process.

13.6 Combined Cycle

As we have seen, even in a gas turbine with regeneration there are significant exhaust losses. An alternative to regeneration is using the gas turbine exhaust to provide heat for another heat engine. In the *combined cycle* the gas turbine exhaust is used in a *heat recovery steam generator* (HSRG) to provide heat for a steam power cycle. Figure 13.11 shows schematic and energy flow diagram for the combination of a standard Brayton cycle with a standard Rankine cycle. Real power plants use state of the art regenerative steam cycles (see Section 12.2).[4]

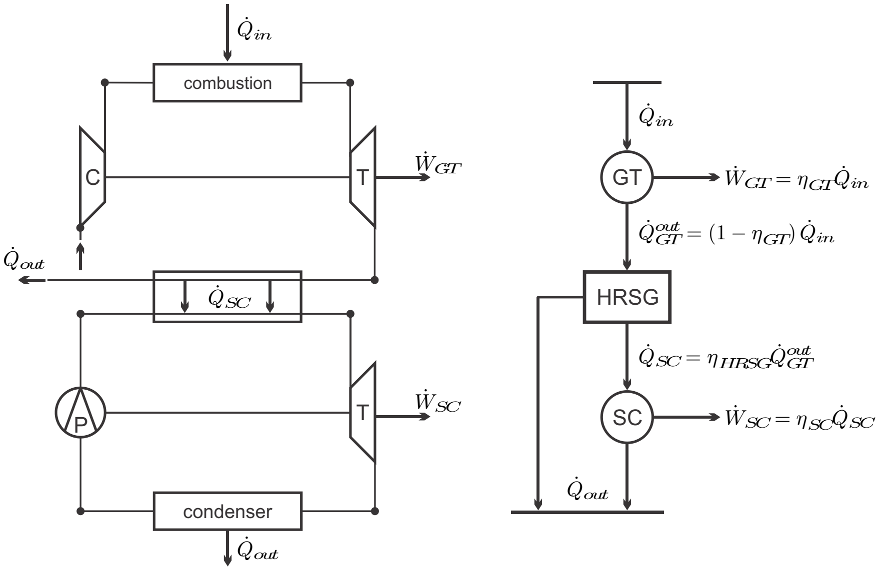

Fig. 13.11 Schematic and energy flows in a combined cycle

In order to evaluate the combined cycle, we consider a gas turbine cycle with thermal efficiency η_{GT} and a steam cycle with thermal efficiency η_{SC}, which are connected by an HSRG with effectiveness η_{HSRG}.

The heat into the system from combustion in the gas turbine is $\dot{Q}_{in}$. The gas turbine cycle produces the work

[4] In case that the gas exhaust from the HRSG still has marked work potential, a regenerator can be added to the gas cycle for preheating of the combustion air (not shown in Figure).

$$\dot{W}_{GT} = \eta_{GT} \dot{Q}_{in} \ ,$$

and rejects the heat

$$\dot{Q}_{GT}^{out} = (1 - \eta_{GT}) \, \dot{Q}_{in}$$

to the HSRG. The latter delivers the heat

$$\dot{Q}_{SC} = \eta_{HSRG} \dot{Q}_{GT}^{out} = \eta_{HSRG} \, (1 - \eta_{GT}) \, \dot{Q}_{in}$$

to the steam cycle, which thus produces the power

$$\dot{W}_{SC} = \eta_{SC} \dot{Q}_{SC} = \eta_{SC} \eta_{HSRG} \, (1 - \eta_{GT}) \, \dot{Q}_{in} \ .$$

The combined power output of both cycles is

$$\dot{W} = \dot{W}_{GT} + \dot{W}_{SC} = (\eta_{GT} + \eta_{SC} \eta_{HSRG} \, (1 - \eta_{GT})) \, \dot{Q}_{in} \ ,$$

which gives the thermal efficiency of the combined cycle as

$$\eta = \frac{\dot{W}}{\dot{Q}_{in}} = \eta_{GT} + \eta_{SC} \eta_{HSRG} \, (1 - \eta_{GT}) \ .$$

The efficiency of the combined cycle is always greater than the efficiency of the gas turbine alone (unless $\eta_{HSRG} = 0$), and is also larger than the efficiency of the steam cycle alone as long as η_{HSRG} is large enough (in particular $\eta > \eta_{SC}$ if $\eta_{HRSG} = 1$).

A gas turbine with $\eta_{GT} = 0.31$ combined with a steam cycle of $\eta_{SC} = 0.45$ by means of a HRSG with $\eta_{HRSG} = 0.9$ has an overall efficiency of 60%. Indeed, combined cycle power plants have the highest available efficiencies among all combustion driven power plants. They allow large upper temperatures in the gas turbines, and reject heat at relatively low temperatures. The only drawback to their use is that they require gaseous or liquid fuels, and cannot be fed directly with coal. Recall that the average efficiency of the World's combustion power plants is about 35% or less. Much better use of fossil fuels could be made by using coal gasification or liquefaction and combined cycle plants.

13.7 The Solar Tower

We mentioned solar power conversion as an application for Stirling engines. Here, we discuss an interesting application for solar power conversion, which relies on the chimney effect, which, in turn, relies on the variation of air pressure with height as expressed in the barometric formula (2.25).

The solar tower, or solar chimney, sketched in Fig. 13.12, works as follows: Solar radiation provides heat $\dot{Q}_{\odot}$ which passes through a glass roof, is absorbed by black mats on the ground, and the warm mats heat up air. The

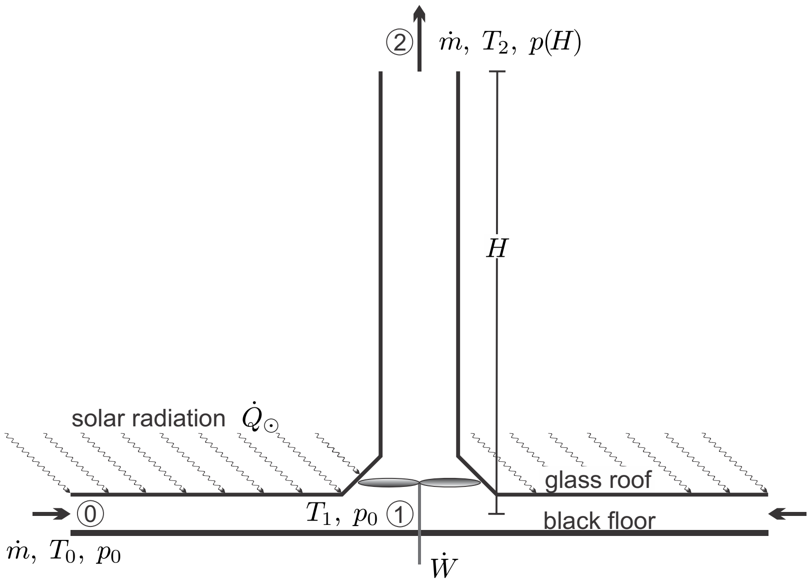

Fig. 13.12 Solar Tower. The sketch shows a cut trough a circular device.

warm air rises through the chimney and drives turbines which are connected to a generator to produce the power $\dot{W}$. When a layer of water is placed below the black mats (e.g., one might use black sacks filled with water), the heat provided from the sun goes partly into air and partly into the water. When the solar heat supply stops after sunset, the warm water heats the air, and the tower still produces electricity until the water has cooled down. An experimental plant with a 180 m tower was build some years ago in Spain, plans to build a tower with a height of 1000 m and a diameter of the glass roof of 6 km in Australia are presently on hold.

We aim at a thermodynamic evaluation of the solar tower, and ask in particular for its thermal efficiency. The temperatures involved are rather low, and thus we can use the cold air approximation, i.e., we assume the specific heats of air to be constants. The exterior air is assumed to have constant temperature T_0, and the pressure depends on height z according to the barometric formula, $p(z) = p_0 \exp\left[-\frac{gz}{RT}\right]$ with p_0 the pressure at the ground.

The incoming air is at $\{T_0, p_0\}$, and as it flows towards the turbines it is heated isobarically until it reaches the temperature T_1 just before the turbines. To compute this temperature, we apply the first law for open systems between the outer rim and the point just before the turbine. Since the radius is large, the flow velocity $\mathcal{V}_0$ at the outer rim can be neglected, and the first law gives

$$\dot{m}\left[h_1 - h_0 + \frac{1}{2}\mathcal{V}_1^2\right] = \dot{Q}_\odot \, .$$

The velocity at turbine inlet is related to mass flow by $\mathcal{V}_1 = \frac{\dot{m}}{\rho_1 A_1}$, where A_1 is the cross section of the chimney at point 1, and the pressure of the flow remains constant, so that $\rho_1 = \rho_0 \frac{T_0}{T_1}$. Thus we find

$$\dot{m}\left[c_p\left(T_1 - T_0\right) + \frac{1}{2}\left(\frac{\dot{m}T_1}{\rho_0 T_0 A_1}\right)^2\right] = \dot{Q}_\odot \,. \tag{13.23}$$

To avoid a detailed discussion of heat transfer mechanisms, we assume the temperature T_1 to be given, so that the above is an equation for the heat supply from the sun, $\dot{Q}_\odot$. Note that due to emission and absorption of radiation the temperature T_1 is limited, as in a greenhouse. A proper radiation heat transfer analysis must be performed to establish the size the glass roof must have, so that the specified temperature T_1 is reached.

Next, we consider the flow between the turbine inlet (Point 1) and the exhaust from the chimney (Point 2). We assume that turbine and chimney are adiabatic, and, for simplicity, that the flow is reversible, so that it is isentropic. The first law gives

$$\dot{m}\left[h_2 - h_1 + \frac{1}{2}\left(\mathcal{V}_2^2 - \mathcal{V}_1^2\right) + gH\right] = -\dot{W} \,.$$

The pressure at H follows from the barometric law, and thus the adiabatic relations give, with $p_1 = p_0$,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \exp\left[-\frac{gH}{RT_0}\frac{k-1}{k}\right] \simeq 1 - \frac{gH}{RT_0}\frac{k-1}{k}\,,$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{k}} = \exp\left[-\frac{gH}{RT_0}\frac{1}{k}\right] \simeq 1 - \frac{gH}{RT_0}\frac{1}{k}\,;$$

the Taylor expansions of the exponentials are well justified for $H = 1000\,\mathrm{m}$, $T_0 = 298\,\mathrm{K}$.

We use all this, and $\mathcal{V}_2 = \frac{\dot{m}}{\rho_2 A_2}$, $c_p = \frac{k}{k-1}R$, to find the power produced as

$$\dot{W} = \dot{m}\left[\left(\frac{T_1}{T_0} - 1\right)gH - \frac{\dot{m}^2}{2}\left(\frac{RT_1}{p_0 A_2}\right)^2\left(1 + \frac{2gH}{RT_0}\frac{1}{k} - \left(\frac{A_2}{A_1}\right)^2\right)\right] \,. \tag{13.24}$$

This equations gives power $\dot{W}$ in dependence of mass flow $\dot{m}$, all other quantities are given by material and construction.

Before we study the full result, we have a look at a further simplification, where the contribution of kinetic energies, that is all terms with factor $\dot{m}^2$, are ignored. In this case (13.24) and (13.23) reduce to

$$\dot{W} = \dot{m}\left(\frac{T_1}{T_0} - 1\right)gH \quad , \quad \dot{Q}_\odot = \dot{m}c_p T_0\left(\frac{T_1}{T_0} - 1\right) ,$$

which yields a thermal efficiency of

$$\eta = \frac{\dot{W}}{\dot{Q}_\odot} = \frac{gH}{c_p T_0} \,.$$

Accordingly, it is beneficial to build the tower as high as possible. For a height of 200 m the efficiency is $\eta = 0.65\%$ and this increases to $\eta = 3.25\%$ when the height is raised to 1000 m. Note that the thermal efficiency is very low nevertheless. Here it must be considered that the main investment is to build the plant, while the energy source—solar radiation—is available for free, as long as no clouds are present, which is, of course, why one would build such a solar tower power plant in a sunny country.

Since the investment costs are high, one will aim to harvest as much power as possible, which will be achieved by optimizing the operating conditions. We return to (13.24) and determine the optimum mass flow to maximize power from the condition $d\dot{W}/d\dot{m} = 0$ as

$$\dot{m}_{\max} = \frac{p_0 A_2}{R T_1} \sqrt{\frac{\frac{2}{3}\left(\frac{T_1}{T_0} - 1\right) gH}{1 + \frac{2gH}{RT_0}\frac{1}{k} - \left(\frac{A_2}{A_1}\right)^2}} \,.$$

The corresponding power output is

$$\dot{W}_{\max} = \dot{m}_{\max} \frac{2}{3}\left(\frac{T_1}{T_0} - 1\right) gH \,.$$

Figure 13.13 shows power output and thermal efficiency as a function of mass flow for the following data: $g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2}$, $H = 1000\,\mathrm{m}$, $T_1 = 345\,\mathrm{K}$, $T_0 = 300\,\mathrm{K}$, $R = 0.287\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$, $c_p = 1.004\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$, $k = 1.4$, $A_1 = 2A_2$, $A_2 = \pi r^2$ with $r = 45\,\mathrm{m}$. These are curves for reversible operation. As for all processes, irreversible processes, mainly in the turbines, will reduce power generation and efficiency. The power curve exhibits the maximum computed above, and the thermal efficiency drops with increasing mass flow. The maximum power output is $\dot{W}_{\max} = 206\,\mathrm{MW}$, where the efficiency is $\eta\left(\dot{m}_{\max}\right) = \frac{2}{3}\frac{gH}{c_p T_0} = 2.2\%$. The corresponding heat intake is $\dot{Q}_\odot\left(\dot{m}_{\max}\right) = 9521\,\mathrm{MW}$. For an average absorbed irradiation of $I = 340\frac{\mathrm{W}}{\mathrm{m}^2}$, the glass roof must have a surface of $A_{roof} = \pi r_{roof}^2 = \frac{\dot{Q}_\odot(\dot{m}_{\max})}{I} = 28\,\mathrm{km}^2$, which corresponds to a radius of 3 km.

13.8 Simple Chimney

In a simple chimney, no work is extracted, $\dot{W} = 0$, the chimney serves to drive an air flow. Equation (13.24), with $\dot{W} = 0$, gives a relation between mass flow and chimney height,

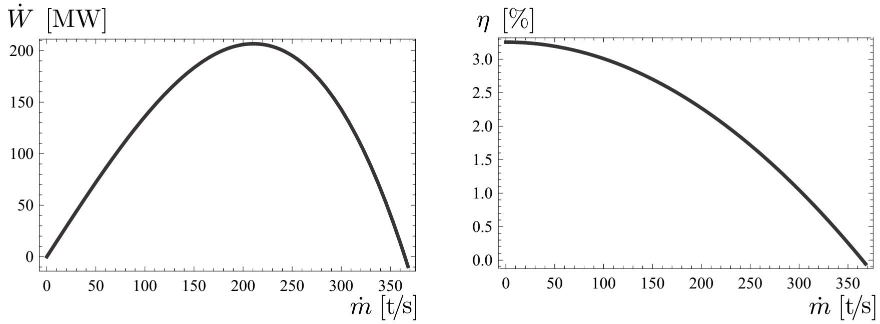

Fig. 13.13 Power and thermal efficiency for a solar tower with height $H = 1000\,\mathrm{m}$ and chimney radius $r = 45\,\mathrm{m}$

$$\dot{m} = \frac{p_0 A_2}{RT_1}\sqrt{\frac{2\left(\frac{T_1}{T_0}-1\right)gH}{1+\frac{2gH}{RT_0}\frac{1}{k}-\left(\frac{A_2}{A_1}\right)^2}} \,.$$

We restrict the attention to cases where $A_2 \ll A_1$ and low enough heights, so that $\frac{2gH}{RT_0}\frac{1}{k} \ll 1$ and

$$\dot{m} = \rho_0 A_2 \sqrt{2gH}\frac{T_0}{T_1}\sqrt{\frac{T_1}{T_0}-1} \,.$$

The draught of the chimney depends on the square root of the height H. The mass flow also has a non-linear dependence on the ratio between the temperatures outside (T_0) and at the foot of the chimney (T_1) with a maximum for $T_1/T_0 = 2$.

In the past, the chimney effect was used for instance to drive the combustion air for a coal power plant. As the above analysis shows, a well working natural draught chimney requires a relatively high temperature ($T_1 = 2T_0$), which implies discharge of large amounts of warm gas, and correspondingly high entropy generation, and work loss. In modern power plants, and other applications, discharge is effected by fans, which allow low exhaust temperatures. High chimneys are still build today, not to increase draught, but to expel the exhaust into higher layers of the atmosphere for better dispersion of the exhaust.

13.9 Aircraft Engines

13.9.1 Thrust and Propulsive Power

While stationary turbines drive generators to produce electrical power, aircraft turbines accelerate the incoming air flow to produce thrust, which is the force F to push the airplane. The engines are powered through the combustion of a fuel.

We consider an airplane moving with the velocity $\mathcal{V}_A$ through environmental air. Due to the motion of the airplane, air is swept into the engine with velocity $\mathcal{V}_A$ and the engine accelerates the flow to the exhaust velocity $\mathcal{V}_E$ measured with respect to the engine. The temperature of the incoming air is T_A and the exhaust is at T_E; inlet and exit pressure are just the environmental pressure, but higher pressures occur inside the engine.

A sketch of the flow through the engine is shown in Fig. 13.14, which also indicates the fuel that is required to drive the engine; the working principle of the engine is discussed later.

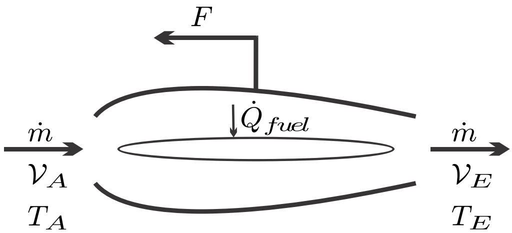

Fig. 13.14 Acceleration of air and thrust in an air engine

According to Newton's second law, the thrust is the rate of change of momentum of the air that passes the engine,

$$F = \dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right) . \tag{13.25}$$

Power is force times velocity, and thus the propulsive power provided by the engine is the product of the force acting on the airplane, i.e., the thrust, and the airplane velocity, that is

$$\dot{W}_P = F\mathcal{V}_A = \dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right)\mathcal{V}_A . \tag{13.26}$$

Before we look inside the engine to discuss its working principles, we take a look at the engine as a whole from different points of view, in order to find criteria for engine performance. Indeed, depending on the point of view of the observer, the engine seems to be performing different tasks:

For an observer resting with the engine, e.g., the pilot or a passenger, the engine consumes fuel, and accelerates and heats air that passes through. This is the frame of reference used in Fig. 13.14. For this observer the first law reads

$$\dot{m}\left[h_E - h_A + \frac{1}{2}\mathcal{V}_E^2 - \frac{1}{2}\mathcal{V}_A^2\right] = \dot{Q}_{fuel} . \tag{13.27}$$

Note, that this observer does not notice thrust and propulsive power.

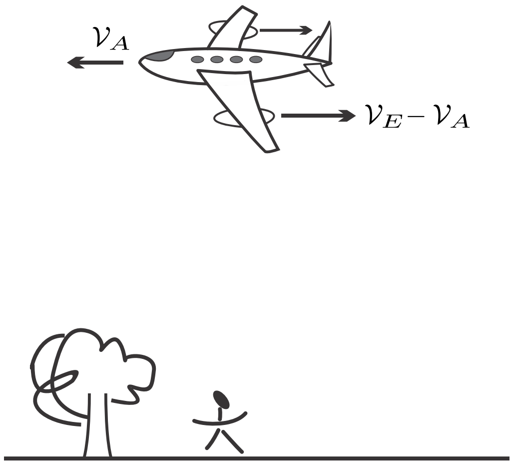

Fig. 13.15 Airplane and engine as observed from the ground

An observer on the ground, as depicted in Fig. 13.15, sees the airplane flying with velocity $\mathcal{V}_A$ through air which is at rest, and observes air expelled from the engine with velocity $(\mathcal{V}_E - \mathcal{V}_A)$ and temperature T_E. To obtain the appropriate form of the first law for this observer we use the identity

$$\frac{1}{2}\left(\mathcal{V}_E^2 - \mathcal{V}_A^2\right) = \frac{1}{2}\left(\mathcal{V}_E - \mathcal{V}_A\right)^2 + \mathcal{V}_A\left(\mathcal{V}_E - \mathcal{V}_A\right) \ , \tag{13.28}$$

which, when inserted into (13.27), gives the energy balance as

$$\dot{m}\left(h_E - h_A\right) + \frac{1}{2}\dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right)^2 + \dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right)\mathcal{V}_A = \dot{Q}_{fuel} \ . \tag{13.29}$$

We introduce the abbreviations

$$\dot{E}_{kin}^{ex} = \frac{1}{2}\dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right)^2 \ ,$$

for the exhaust flow of kinetic energy, and

$$\dot{Q}_H = \dot{m}\left(h_E - h_A\right) \tag{13.30}$$

for the heating rate of the air passing through the engine, that is the amount of heat required to heat the air isobarically from T_A to T_E. With these, and the definition of propulsive power (13.26), the first law (13.29) assumes the compact form

$$\dot{Q}_H + \dot{E}_{kin}^{ex} + \dot{W}_P = \dot{Q}_{fuel} \ . \tag{13.31}$$

This is the first law for the observer on the ground, who understands that the heat $\dot{Q}_{fuel}$ supplied through combustion of the fuel leads to three effects:

(a) Propulsive power $\dot{W}_P$; (b) Acceleration of environmental air, so that the kinetic energy of exhaust is $\dot{E}_{kin}^{ex}$; (c) Heating $\dot{Q}_H$ of the air to temperature T_E.

Of course, both forms (13.27, 13.31) of the first law are equivalent, they just differ by the point of view of the observer. However, the latter form is better suited for evaluation of engine performance.

Subsonic flight and supersonic flight have different aerodynamics, with more propulsive power required in supersonic flows. To optimize speed and efficiency, commercial airliners fly at about 80-90% of the speed of sound. For military applications speed is essential, and many fighter planes fly at supersonic speeds.

13.9.2 Air Engine Efficiency

The engine is build to deliver the propulsive power $\dot{W}_P$. Heating $\dot{Q}_H$ and acceleration $\dot{E}_{kin}^{ex}$ are side effects which must be considered as losses. The exhaust leaving the engine at T_E, $\mathcal{V}_E - \mathcal{V}_A$ has work potential (exergy) against the environment as discussed in Sec. 11.6. Since there is no way to put the exhaust to use after it is expelled from an engine in flight, it just equilibrates with the surrounding air—this is an irreversible loss, i.e., an external irreversibility associated with the process. For efficient use of the fuel one must diminish external losses, that is aim for processes with low exhaust exergy, which means low exhaust temperature T_E and low exhaust velocity $\mathcal{V}_E - \mathcal{V}_A$. We also note that low exit velocities diminish engine noise significantly.

The obvious measure for engine performance is the thermal efficiency for propulsion, defined as

$$\eta_P = \frac{\dot{W}_P}{\dot{Q}_{fuel}} = \frac{\dot{W}_P}{\dot{Q}_H + \dot{E}_{kin}^{ex} + \dot{W}_P} \, , \tag{13.32}$$

where the first law (13.29) was used. For fixed value of $\dot{W}_P$ the thermal efficiency of propulsion grows, when heating $\dot{Q}_H$ and exhaust kinetic energy $\dot{E}_{kin}^{ex}$ become smaller.

A common measure from fluid dynamics for propulsive efficiency is the Froude propulsive efficiency (William Froude, 1810-1879) η_F which asks how much of the gain in kinetic energy produced in the engine, as seen from the observer resting with the engine, is actually converted to propulsive power, that is

$$\eta_F = \frac{\dot{W}_P}{\dot{m}\left(\frac{1}{2}\mathcal{V}_E^2 - \frac{1}{2}\mathcal{V}_A^2\right)} = \frac{\dot{W}_P}{\dot{W}_P + \dot{E}_{kin}^{ex}} = \frac{2\mathcal{V}_A}{\mathcal{V}_A + \mathcal{V}_E} \, . \tag{13.33}$$

The Froude efficiency is a purely mechanical measure, other than the thermal efficiency η_P it does not account for the loss through expulsion of hot exhaust. The Froude efficiency approaches unity when the outflow velocity $\mathcal{V}_E$ approaches the inflow velocity $\mathcal{V}_A$.

For a certain airplane, the required propulsive power $\dot{W}_P$ and flight velocity $\mathcal{V}_A$ are given, while inflow temperature T_A and inflow pressure p_A depend on the local condition of the air the airplane is flying through. Both efficiencies, η_F and η_P, show that an efficient engine for the airplane will have a small increase in velocity $(\mathcal{V}_E - \mathcal{V}_A)$, but a large mass flow $\dot{m}$, so that $\dot{W}_P = \dot{m}\,(\mathcal{V}_E - \mathcal{V}_A)\,\mathcal{V}_A$ has the required value. Moreover, to keep the thermal loss small, the exhaust temperature should be as low as possible.

13.9.3 Turbojet Engine

Standard air turbines operate similar to stationary gas turbines for power generation. While in the latter the turbine serves to drive the compressor and the generator, an air engine has a smaller turbine, which only serves to drive the compressor. After the turbine, the still hot and compressed air is expanded in a nozzle to accelerate the flow to $\mathcal{V}_E$.

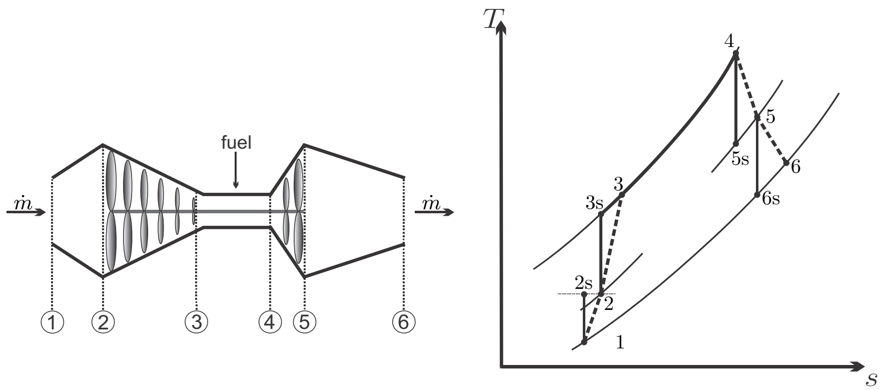

Fig. 13.16 Schematic and T-s-diagram for a standard air engine with diffuser (1-2), compressor (2-3), combustion chamber (3-4), turbine (4-5) and nozzle (5-6)

Figure 13.16 shows a schematic and the T-s-diagram for a simple standard air turbine, i.e., a turbojet engine, consisting of diffuser, compressor, combustion chamber, turbine and nozzle. The diffuser decelerates the inflow to increase the pressure, thus lowering the work required for compression. The turbine is used solely to drive the compressor, the work for both is equal, with different sign. The hot pressurized combustion product is expanded and accelerated in the nozzle. Since the throughput is fast, there is no time to exchange heat, and diffuser, compressor, turbine and nozzle are considered to be adiabatic.

For the discussion of air engines we shall rely, again, on the air standard analysis, that is we ignore any composition changes and treat the working fluid as air. The standard air turbine operates on the following cycle

$$\begin{array}{lll} \text{1-2} & \text{adiabatic diffuser:} & h_2 = h_1 + \frac{1}{2}\mathcal{V}_A^2 \,, \\ \text{2-3} & \text{adiabatic compressor:} & w_C = h_2 - h_3 \,, \\ \text{3-4} & \text{isobaric heating (combustion):} & q_{in} = h_4 - h_3 \,, \\ \text{4-5} & \text{adiabatic turbine:} & w_T = -w_C = h_4 - h_5 \,, \\ \text{5-6} & \text{adiabatic nozzle:} & \mathcal{V}_E = \sqrt{2\,(h_5 - h_6)} \,, \\ \text{6-1} & \text{equilibration with environment.} & \end{array} \tag{13.34}$$

Again, we consider the ideal process under cold-air approximation (constant specific heat) to get insight into the parameters that determine performance. Inlet conditions T_1, p_1, $\mathcal{V}_A$ are given; compressor pressure ratio $P_C = \frac{p_3}{p_2}$ and turbine inlet temperature T_4 are design parameters for the engine. For reversible processes, we find the following relations between the properties at the corner points of the process:

$$\begin{aligned} T_2 &= T_1 + \frac{\mathcal{V}_A^2}{2c_p} \,, & p_2 &= p_1 \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \,, \\ T_3 &= T_2 P_C^{\frac{k-1}{k}} \,, & p_3 &= p_4 = p_2 P_C \,, \\ T_5 &= T_4 + T_2 - T_3 \,, & p_5 &= p_4 \left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} \,, \\ T_6 &= T_5 \left(\frac{p_1}{p_5}\right)^{\frac{k-1}{k}} \,, & p_6 &= p_1 \,. \end{aligned} \tag{13.35}$$

Combining all results yields the exhaust velocity, specific propulsive power, exhaust temperature and heat supply as

$$\begin{aligned} \mathcal{V}_E &= \sqrt{2c_p \left[T_4 \left(1 - P^{\frac{1-k}{k}}\right) + T_1 \left(1 - P^{\frac{k-1}{k}}\right)\right] + \mathcal{V}_A^2} \,, \\ w_P &= \frac{\dot{W}_P}{\dot{m}} = \left(\mathcal{V}_E - \mathcal{V}_A\right) \mathcal{V}_A \,, \\ T_6 &= T_4 P^{\frac{1-k}{k}} \,, \\ q_{in} &= c_p \left[T_4 - T_1 P^{\frac{k-1}{k}}\right] \,; \end{aligned} \tag{13.36}$$

here, $P = \frac{p_2}{p_1} P_C$ is the overall pressure ratio of the engine. For given turbine inlet temperature T_4, the propulsive power has a maximum at pressure ratio $P_{\max} = \left(\frac{T_4}{T_1}\right)^{\frac{k}{2k-2}}$, while propulsive efficiency grows with P.

For a fixed pressure ratio P, exhaust temperature T_6 and heat supply q_{in} grow linearly with the turbine inlet temperature T_4, while exhaust speed $\mathcal{V}_E$ and propulsive power w_P grow slower. Thus, increase of the turbine inlet temperature increases propulsive power, but reduces the propulsive thermal efficiency $\eta_P = w_p/q_{in}$, due to larger external irreversibilities.

From the above follows that engines with high pressure ratio and large turbine inlet temperatures provide large propulsive power. If one is interested mainly in power, and efficiency has lower importance, one will run a engine under these conditions. However, fuel cost is significant, and directly related to efficiency. Most of today's jet engines are high bypass turbofan engines, which are far more efficient, as will be discussed in Sec. 13.9.5.

Standard turbojet engines, or turbofan engines with low bypass ratio, are employed for supersonic propulsion, mostly for military applications. In afterburner engines additional boost is obtained by injecting fuel into the hot nozzle flow, where it burns, and further heats the flow, and thus gives even higher nozzle exit velocities.

13.9.4 Example: Turbojet Engine

An airplane cruises with a velocity $\mathcal{V}_A = 300\,\frac{\mathrm{m}}{\mathrm{s}}$ at about 9000 m altitude where the local pressure and temperature are $p_1 = 32\,\mathrm{kPa}$ and $T_1 = 241\,\mathrm{K}$. The compressor pressure ratio is $P = 12$, and the turbine inlet temperature is 1400 K. We assume isentropic efficiencies of 80% for compressor and turbine, and 95% for diffuser and nozzle, and compute the velocity and temperature of the exhaust gas, propulsive power for a mass flow of $\dot{m} = 50\,\frac{\mathrm{kg}}{\mathrm{s}}$, thermal and Froude efficiencies, and the work potential of the exhaust. The working fluid is air, as ideal gas with variable specific heats. The process is as shown in Fig. 13.16, with irreversible subprocesses. We go through the processes step by step, all numerical values will be entered into a table that is found further below.

Diffuser (1-2): The first law gives $h_2 = h(T_2) = h_1 + \frac{1}{2}\mathcal{V}_A^2$ which determines T_2. The pressure after an isentropic diffuser that reaches T_2 is obtained as $p_{2s} = p_1 \frac{p_r(T_2)}{p_r(T_1)}$; however, this is not the pressure after the irreversible diffuser. The isentropic efficiency of the diffuser $\eta_D = \frac{h_x - h_1}{h_2 - h_1}$ defines the fictitious temperature T_x that would be obtained by isentropic compression to the actual pressure p_2 for the irreversible compressor. We find $h_x = h(T_x) = h_1 + \eta_D(h_2 - h_1)$, and hence T_x. With that, the pressure after the diffuser is $p_2 = p_x = p_1 \frac{p_r(T_x)}{p_r(T_1)}$.

Compressor (2-3): The exit pressure of the compressor is $p_3 = p_2 P$. Moreover, we find T_{3s} from $p_r(T_3) = p_r(T_2) P$, and then h_3 and T_3 follow from the isentropic compressor efficiency as $h_3 = h(T_3) = h_2 + (h_{3s} - h_2)/\eta_C$.

Heating (3-4): As always, the combustion chamber is assumed to be isobaric, $p_4 = p_3$; the temperature T_4 is given.

Turbine (4-5): The turbine is required to drive the compressor, that is $w_T = -w_C$, or $h_4 - h_5 = h_3 - h_2$ from which we find h_5 and then T_5. A reversible turbine expanding to the same pressure p_5 would end with enthalpy $h_{5s} = h_4 + (h_5 - h_4)/\eta_T$ which follows from the definition of the isentropic turbine efficiency η_T. The pressure finally is obtained from the isentropic relation between points 4 and 5s, $p_5 = p_{5s} = p_4 \frac{p_r(T_{5s})}{p_r(T_4)}$.

Nozzle (5-6): The nozzle expands the turbine exhaust to the environmental pressure $p_6 = p_1$. An isentropic nozzle would give the turbine exit temperature T_{6s}, and thus the enthalpy $h_{6s} = h(T_{6s})$, which follow from $p_r(T_{6s}) = p_r(T_5)\frac{p_6}{p_5}$. Exit enthalpy, and temperature, follow from the isentropic nozzle efficiency η_N, one finds $h_6 = h_5 + (h_{6s} - h_5)\eta_N$. The nozzle exit velocity is $\mathcal{V}_6 = \sqrt{2(h_5 - h_6)}$.

The complete data is shown in the following table, which also contains entropy values for inlet and exhaust:

	$p/\,\mathrm{kPa}$	$T/\,\mathrm{K}$	$h/\frac{\mathrm{kJ}}{\mathrm{kg}}$	$p_r(T)$	$\mathcal{V}/\frac{\mathrm{m}}{\mathrm{s}}$	$s^0(T)/\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$
1	32.0	241.0	241.9	0.476	300	6.917
2s	58.2				0	
x	56.5	283.7	284.7	0.841	0	
2	56.5	286.0	286.9	0.865	0	
3s	678	576.6	583.3	10.38	0	
3	678	647.0	657.4		0	
4	678	1400	1516	332.2	0	
5s	175	1005	1052	85.77	0	
5	175	1086	1145	116.8	0	
6s	32	701.1	715.3	21.36	0	
6	32	721.1	736.8		904	8.040

With all data known, we can compute propulsive power and heat supply,

$$\dot{W}_P = \dot{m}(\mathcal{V}_E - \mathcal{V}_A)\mathcal{V}_A = 9.0\,\mathrm{MW}\,,$$
$$\dot{Q}_{in} = \dot{m}(h_4 - h_3) = 42.9\,\mathrm{MW}\,.$$

Thermal propulsive efficiency and Froude efficiency thus are

$$\eta_P = \frac{\dot{W}_P}{\dot{Q}_{in}} = 21.1\% \quad \text{and} \quad \eta_F = \frac{2\mathcal{V}_A}{\mathcal{V}_A + \mathcal{V}_E} = 49.9\%\,.$$

The efficiencies are low, because the exhaust has considerable work potential against the environment, which is not used. Substituting the appropriate values into Eq. 11.18, the work potential of the exhaust, which is just the work lost to external irreversibilities, is[5]

$$\dot{W}_{rev} = \dot{m}\left[h_6 - h_1 - T_1\left[s^0(T_6) - s^0(T_1)\right] + \frac{1}{2}(\mathcal{V}_E - \mathcal{V}_A)^2\right] = 20.3\,\mathrm{MW}\,.$$

[5] Note that inflow (state 1) and outflow (state 6) are at the same pressure; therefore there is no pressure contribution to the entropy difference.

Almost half of the loss (9.12 MW) is due to kinetic energy losses, the remainder is due to thermal losses. Altogether, the external work loss is more than twice the actual propulsive power produced. Both must be reduced to improve engine efficiency.

13.9.5 Bypass Turbofan Engine

The general discussion of air engine efficiency has shown that engines with relatively slow and cold exhaust are more efficient. Since propulsive power is given as $\dot{W}_P = \dot{m}\left(\mathcal{V}_E - \mathcal{V}_A\right)\mathcal{V}_A$, lower exit velocity $\mathcal{V}_E$ must be compensated by larger mass flow, to generate the desired power. Bypass turbofan engines have smaller exit velocities, increased mass flow, and also smaller (average) exit temperature, and thus have small external losses, and high efficiency.

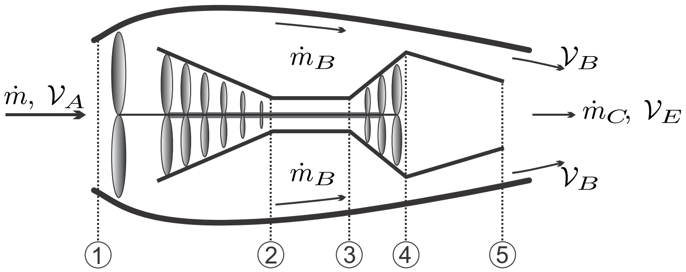

Fig. 13.17 Bypass turbofan engine with fan, compressor, combustion chamber, turbine and nozzle. The fan forces air through the bypass, where it is accelerated

In a turbofan engine, sketched in Fig. 13.17, an additional turbine stage is added to drive the fan. The fan forces air through a duct outside the engine core, called the bypass, where the air is accelerated to velocity $\mathcal{V}_B$. The incoming mass flux $\dot{m}$ is split into two streams, the bypass stream $\dot{m}_B$ and the core stream $\dot{m}_C$. Only the core stream runs through compressor, combustion chamber, turbine and nozzle. The bypass flow is not heated; this lowers the exhaust temperature and improves efficiency.

This arrangement increases the mass flow through the engine and decreases the outflow velocity and temperature, and thus allows to produce propulsive power at higher efficiency. The bypass ratio is defined as the ratio of the inlet cross sections of core and bypass, $M_B = \frac{A_B}{A_C} = \frac{\dot{m}_B}{\dot{m}_C}$, and we expect better efficiency for larger bypass ratio. The fan is particularly important at take-off, where the power demand is high, but the airplane speed $\mathcal{V}_A$ is low.

Again, we study the ideal process under cold-air standard conditions. The power demand for the adiabatic fan follows from the first law. For the ideal process, we have isentropic compression by the fan followed by isentropic expansion in the duct. Inlet and exit pressures agree, as do the respective entropies, and thus the exit temperature equals the inlet temperature. Hence, the fan work is just the difference in kinetic energy for the bypass flow,

$$\dot{W}_F = \dot{m}_B \left[\frac{1}{2}\mathcal{V}_A^2 - \frac{1}{2}\mathcal{V}_B^2 \right] . \tag{13.37}$$

For irreversible fan and duct, the exit temperature would be slightly higher, and enthalpy terms would appear in the first law.

The processes in the core are the same as for the standard turbine, but now we use the numbering as in Fig. 13.17. As long as all processes are ideal, the evaluation is easiest when we begin with the first law balanced over the total core, between states 1 and 5. The turbine work is used to drive the compressor and the fan, but since turbine and compressor are within the control volume, only the fan work appears in the first law, which reads[6]

$$\dot{m}_C \left[h_5 - h_1 + \frac{1}{2}\mathcal{V}_E^2 - \frac{1}{2}\mathcal{V}_A^2 \right] = \dot{Q}_{in} - \left(-\dot{W}_F \right) . \tag{13.38}$$

The heat supply to the compressed air is

$$\dot{Q}_{in} = \dot{m}_C \left(h_3 - h_2 \right) = \dot{m}_C c_p \left[T_3 - T_1 P^{\frac{k-1}{k}} \right] , \tag{13.39}$$

where T_3 is the turbine inlet temperature, and $T_1 P^{\frac{k-1}{k}} = T_2$ is the temperature after the compressor; here, P is the overall pressure ratio of the engine.

The compressed gas at state 3 is expanded reversibly, first in the adiabatic turbine to state 4, and then in the adiabatic nozzle to exhaust state 5, so that the exit temperature is

$$T_5 = T_3 P^{\frac{1-k}{k}} . \tag{13.40}$$

For simplicity, we assume that core and bypass exit velocities are equal, $\mathcal{V}_B = \mathcal{V}_E$. Then, combining the above equations and solving for the exit velocity gives

$$\mathcal{V}_E = \sqrt{\frac{1}{1+M_B} 2c_p \left[T_3 \left(1 - P^{\frac{1-k}{k}} \right) + T_1 \left(1 - P^{\frac{k-1}{k}} \right) \right] + \mathcal{V}_A^2} . \tag{13.41}$$

This result differs from the result for the standard turbine (13.36) only in that the factor $\frac{1}{1+M_B} = \frac{\dot{m}_C}{\dot{m}_C + \dot{m}_B}$ appears in the first term; the previous result is found for $M_B = 0$.

For given turbine inlet temperature T_3, the exit velocity $\mathcal{V}_E$ and the propulsive power $\dot{W}_P = \left(\dot{m}_B + \dot{m}_C \right) \left(\mathcal{V}_E - \mathcal{V}_A \right) \mathcal{V}_A$ both have a maximum for the

[6] $\dot{W}_F < 0$ is defined as the work consumed by the fan, the work delivered from the core to drive the fan is $\left(-\dot{W}_F \right)$.

overall pressure ratio $P_{\max} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2k-2}}$. In modern turbines some of the compressed air is forced through small ducts in the turbine blades for efficient blade cooling. This allows very high turbine inlet temperatures of up to $T_3 = 1700\,\text{K}$, with corresponding pressure ratios of $P_{\max} \simeq 30$ (for $T_1 = 240\,\text{K}$).

At the optimal pressure ratio $P_{\max}$, exit velocity, propulsive power, and heat supply become

$$\mathcal{V}_E^{\max} = \sqrt{\frac{1}{1+M_B} 2c_p \left(\sqrt{T_3} - \sqrt{T_1}\right)^2 + \mathcal{V}_A^2}\,, \tag{13.42}$$

$$\dot{W}_P^{\max} = \dot{m}_C \left(1 + M_B\right) \left(\mathcal{V}_E^{\max} - \mathcal{V}_A\right) \mathcal{V}_A\,, \tag{13.43}$$

$$\dot{Q}_{in}^{\max} = \dot{m}_C c_p \sqrt{T_3} \left(\sqrt{T_3} - \sqrt{T_1}\right)\,, \tag{13.44}$$

so that the propulsive thermal efficiency becomes

$$\eta_{\max} = \frac{\dot{W}_P^{\max}}{\dot{Q}_{in}^{\max}} = \frac{\left[\sqrt{\frac{2c_p}{1+M_B}\left(\sqrt{T_3} - \sqrt{T_1}\right)^2 + \mathcal{V}_A^2} - \mathcal{V}_A\right] \mathcal{V}_A}{\frac{c_p}{1+M_B}\sqrt{T_3}\left[\sqrt{T_3} - \sqrt{T_1}\right]}\,. \tag{13.45}$$

This efficiency depends on turbine inlet temperature T_3 and bypass ratio M_B. Figure 13.18 shows the thermal efficiency $\eta_{\max}$ as function of M_B, T_3. For given turbine inlet temperature T_3, the propulsive efficiency grows with the bypass ratio, while for given bypass ratio there is a optimum for the temperature.

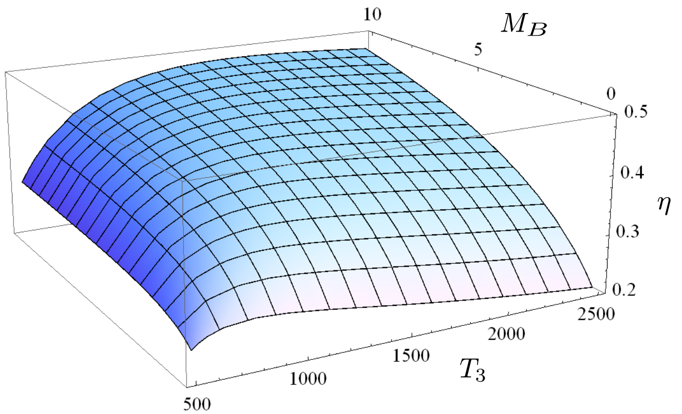

Fig. 13.18 Efficiency $\eta_{\max}$ of bypass turbofan engine over bypass ratio M_B and turbine inlet temperature T_3 (for optimal pressure ratio $P_{\max}$, $\mathcal{V}_A = 280\,\frac{\text{m}}{\text{s}}$)

The bypass ratio is limited, due to size and weight limitations of the engine, and we ask for the optimum turbine inlet temperature T_3 for given bypass ratio M_B, which follows from the condition

$$\left(\frac{\partial \eta_{\max}}{\partial T_3}\right)_{M_B} = 0 \, .$$

The resulting equation between the optimal temperature $T_3^{\rm opt}$ and M_B is best solved for the bypass ratio,

$$M_B = \frac{c_p T_1}{\frac{1}{2}\mathcal{V}_A^2} \frac{\left(\sqrt{\frac{T_3^{\rm opt}}{T_1}} - 1\right)^4}{2\sqrt{\frac{T_3^{\rm opt}}{T_1}} - 1} - 1 \, . \tag{13.46}$$

For an aircraft travelling at speed $\mathcal{V}_A = 280\frac{m}{s}$ through air ($c_p = 1.004\frac{\rm kJ}{\rm kg\,K}$) at $T_1 = 240\,{\rm K}$, the optimal turbine inlet temperature has the values $T_3^{\rm opt} = \{1300\,{\rm K}, 1500\,{\rm K}, 1700\,{\rm K}\}$ for bypass ratios $M_B = \{4.22, 6.78, 9.83\}$, the outflow velocity is $449\frac{\rm m}{\rm s}$.

The optimum efficiency for a bypass turbojet engine with given turbine inlet temperature is obtained from inserting (13.46) into (13.45) as

$$\eta_{\max}^{\rm opt} = \frac{\dot{W}_P^{\max}}{\dot{Q}_{in}^{\max}} = \frac{\left(1 - \sqrt{\frac{T_1}{T_3}}\right)^2}{1 - \frac{1}{2}\sqrt{\frac{T_1}{T_3}}} \, . \tag{13.47}$$

Optimum propulsive efficiency grows with increasing turbine inlet temperature. With the same data as before ($T_1 = 240\,{\rm K}$, $T_3 = 1700\,{\rm K}$) the optimized reversible bypass turbojet engine reaches a thermal propulsive efficiency of 48%.

The optimized values computed above from simplifying assumptions (cold air, air standard, reversible processes, same exit velocity for bypass and core flows) are not too far from those encountered in state-of the art turbofan engines. These engines are quite complex, with 2 or 3 turbine-compressor and turbine-fan pairs running on concentric shafts, and sometimes gears, so that high and low pressure turbines and compressors, and the fan run at optimal rotational speeds.

Noise reduction is an important task in commercial aircraft. Engine noise is high when engine exit flows are supersonic, hence real-life engines must be constructed to have subsonic exit velocities. From the above discussion it is evident that turbofans allow low exit velocities, and hence relatively silent operation.[7]

[7] The above optimization of the bypass engine relied on several simplifying assumptions (cold air standard, all processes reversible) which leads to somewhat inaccurate results. Hence, there is no value in including the requirement for subsonic outflow into the arguments.

In propjet engines, fan and bypass are replaced by a propeller, which provides all thrust, the turbine expands to the environmental pressure and provides the work to run the propeller. Propeller engines are efficient, but for aerodynamic reasons are limited to lower velocities of not more than $600\,\frac{\text{m}}{\text{s}}$, while turbofan engines can operate at subsonic and supersonic flight speeds.

Problems

13.1. Stirling Cycle
A Stirling cycle with 5 g of air as working fluid is heated by solar radiation and rejects heat to the environment. The highest and lowest temperatures reached in the cycle are 1000 K and 300 K, respectively, and the maximum pressure ratio is 10. Draw the process in a T-s-diagram and in a p-v-diagram, then determine the thermal efficiency and the power produced when the engine runs at 400 rpm. Determine the entropy changes, heat and work per unit mass for all four processes.

13.2. Stirling Cycle for Refrigeration
A Stirling engine with helium as working gas is considered for refrigeration purposes. The goal is to withdraw heat at a temperature of $T_L = 150\,\text{K}$ and reject it to the environment at $T_H = 300\,\text{K}$. Helium is a monatomic gas, which is well described as an ideal gas with constant specific heats.

1. Draw T-s-diagram and p-v-diagram for the cycle.
2. For a volume ratio between largest and smallest volume of 3, compute heat and work per unit mass for all four processes, and the coefficient of performance.
3. The computation of specific heat and work is independent of the pressure. Discuss the role of the pressure in the performance of the engine, why is a high pressure desirable?
4. Determine the pressure at all corner points when the highest pressure in the engine is 10 bar.

13.3. Stirling Cycle for Refrigeration
A Stirling engine with argon as working fluid is used for refrigeration purposes. Heat exchange with the cold and warm surroundings takes place at $T_H = 27\,°\text{C}$ and $T_L = -73\,°\text{C}$, respectively. The highest pressure in the cycle is $p_H = 12\,\text{bar}$ and the smallest volume is one third of the largest volume.

1. Plot the Stirling cycle in a T-s-diagram, and in a p-v-diagram, number the corner points.
2. Compute the pressures and specific volumes on all corner points
3. Discuss the regenerator: show that the amounts of heat rejection and heat supply in the two isochoric processes have the same absolute value, but different signs.
4. Compute the coefficient of performance of the cycle.

5. Assume the cylinder of the engine contains an air mass of 40 g, and the engine runs at 300 rpm – what is its refrigeration capacity?

13.4. Compression Modes
Air (ideal gas with variable specific heats) is compressed in a compressor from $p_1 = 1.2\,\text{bar}$, $T_1 = 280\,\text{K}$ to $p_2 = 12\,\text{bar}$. The incoming volume flow is $1\,\frac{\text{m}^3}{\text{s}}$. Determine the power consumption for the following cases:

1. Isothermal reversible compression.
2. Isentropic compression.
3. Polytropic reversible compression with $n = 1.2$.
4. Compression in two isentropic stages with intercooling to T_1 at $p_m = \sqrt{p_1 p_2}$.

Draw a p-v- and a T-s-diagram which shows the four process curves. Hint: For computation of isothermal and polytropic case use that $w_{12} = -\int_1^2 vdp$.

13.5. Two Stage Compressor with Irreversibilities
A two stage compression system with intercooling is used to increase the pressure of an ideal gas. Specifically, the gas enters the system at p_1, T_1, and leaves the first compressor (isentropic efficiency η_{C1}) at pressure p_2. It is then isobarically intercooled to T_1, and compressed to p_4 in the second compressor (isentropic efficiency η_{C2}). Assume constant specific heats, and determine the pressure p_2 that should be chosen to minimize the work requirement of the system.

13.6. Gas Turbine Cycle with Regeneration

1. Draw a schematic for a gas turbine system for electricity generation with irreversible single stage compression, two stages of irreversible expansion with reheat, and a regenerator. Enumerate the relevant corner points of the process.
2. Draw the corresponding T-s-diagram.
3. Express the thermal efficiency in terms of enthalpies.

13.7. Brayton Cycle with Regeneration
A gas turbine running on the Brayton cycle has an efficiency of 35.9%, at pressure ratio 14.7. The turbine inlet temperature is 1288 °C, and the air entering the engine is at 1 bar, 20 °C. The engine produces a net power of 174.9 MW and the mass throughput is $1690\,\frac{\text{t}}{\text{h}}$.

1. Determine the isentropic efficiencies of turbine and compressor.
2. Determine the thermal efficiency for this gas turbine for the case that a regenerator with 80% effectiveness is added to the cycle.

13.8. Optimal Reheat Pressure
Prove the following statement from the text for a reheat turbine with n-stages: If the turbines have the same inlet temperature, and the same isentropic efficiency, the maximum work is obtained when they have the same pressure ratio.

13.9. Brayton Cycle with Intercooling, Reheat and Regeneration
A regenerative gas turbine cycle uses two stage of compression with intercooling, and two stages of expansion with reheating. The pressure ratio for each stage is 3.5, the turbine inlet temperature is 1400 K for both turbines, and between the compressors the air is cooled back to the environmental temperature of 290 K. The isentropic efficiencies of the compressors and turbines are 80% and 85%, respectively, and the regenerator effectiveness is 80%. Determine:

1. The enthalpies at all principal states.
2. The net work and the back work ratio.
3. The thermal efficiency for the system as described, and for the case that no regenerator is present.
4. The work potential of the turbine exhaust, and of the final exhaust.

As usual: draw schematic and diagrams.

13.10. Gas Turbine with Regenerator
A gas turbine with air (non-constant specific heats) as working fluid operates according to the following cycle:

1-2: Adiabatic compression of air at $T_1 = 300\,\text{K}$, $p_1 = 1\,\text{bar}$ to $T_2 = 620\,\text{K}$, $p_2 = 9.74\,\text{bar}$.

2-3: Isobaric heating of the working fluid in the regenerator, the temperature T_3 is 40 K below the temperature of the turbine exhaust, T_5.

3-4: Further isobaric heating in the combustion chamber to $T_4 = 1300\,\text{K}$.

4-5: Adiabatic expansion in turbine to pressure $p_5 = p_1$, with isentropic efficiency of 92%.

5-6: Isobaric cooling in the regenerator.

1. Draw a schematic, and a T-s-diagram.
2. Make a table with pressures, temperatures and enthalpies at the points 1 to 6.
3. Determine the thermal efficiency and the back-work-ratio of the cycle:
   a) when it operates with regenerator
   b) when it operates without regenerator
4. Compute the isentropic efficiency of the compressor.

13.11. Combined Cycle: Gas Turbine and Steam Power Plant
A combined cycle power plant consists of a gas turbine cycle (thermal efficiency 28%), and a steam power plant (thermal efficiency 46%). The exhaust of the gas turbine is used to provide the heat for generating steam in a heat recovery steam generator. Assume that the HRSG has an efficiency of 92%, and compute the overall efficiency of the system.

13.12. Turbojet Engine
A turbojet engine drives an airplane traveling with velocity $290\,\frac{\text{m}}{\text{s}}$ at a height where the pressure is 28 kPa, and the temperature is $-40\,°\text{C}$. The compressor

pressure ratio is 11, and the turbine inlet temperature is 1300 K. The mass flow through the engine is $60 \frac{\text{kg}}{\text{s}}$.

Assume isentropic efficiencies of 82% for compressor and turbine, 95% for the nozzle, and 100% for the diffuser. Determine the velocity of the exhaust gas, the propulsive power, the rate of fuel consumption when the heating value of the fuel is $42000 \frac{\text{kJ}}{\text{kg}}$, the thermal efficiency, and the Froude propulsive efficiency.

13.13. Air Engine

An airplane propelled by a standard turbo-jet engine flies at Mach number $M = 0.9$ in an environment where the pressure is 40 kPa and the temperature is 240 K. The heat added to the air flowing through the engine is $q = 550 \frac{\text{kJ}}{\text{kg}}$ and the hot air leaves the engine at 650 K. The engine inlet has a diameter of 1 m. Assume that the working fluid is air as ideal gas.

Determine outflow velocity, thrust, propulsive power, thermal efficiency, and Froude efficiency of the engine.

13.14. Air Engine

Air at 25 kPa, 225 K enters a turbojet engine in flight at an altitude of 10 000 m, the flight velocity is $290 \frac{\text{m}}{\text{s}}$. The pressure ratio across the compressor is 10. The turbine inlet temperature is 1300 K, and the pressure at the nozzle exit is 25 kPa again. The diffuser and nozzle processes are isentropic, compressor and turbine have isentropic efficiencies of 90% and 95%, respectively, and there is no pressure drop for flow through the combustor.

Consider air as ideal gas with constant specific heats, $R = 0.287 \frac{\text{kJ}}{\text{kg K}}$, $c_p = 1.004 \frac{\text{kJ}}{\text{kg K}}$.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit.

1. Draw a schematic of the engine, and the corresponding T-s-diagram.
2. Make a table with the values of pressure and temperature at each principal state.
3. Compute the velocity at the nozzle exit.
4. Compute the thrust of the engine and the propulsive power for a mass flow rate of $80 \frac{\text{kg}}{\text{s}}$.
5. Determine thermal efficiency and Froude efficiency.

13.15. Bypass Turbofan Engine

A bypass turbo fan engine has a bypass ratio of 5.5 (the mass flow through the bypass is 5.5 times the mass flow through the gas turbine), and propels an aircraft cruising at $250 \frac{\text{m}}{\text{s}}$ in high altitude where the pressure is 30 kPa and the temperature is 230 K. The mass flow through the gas turbine core is $30 \frac{\text{kg}}{\text{s}}$. Assume variable specific heats.

The flow through the bypass consists of isentropic diffuser, fan, nozzle.

The gas turbine process is as follows:

1-2: Compression in isentropic diffuser.

2-3: Isentropic compressor, pressure ratio $p_3/p_2 = 10$.

3-4: Isobaric heating in combustion chamber to 1300 K.
4-5: Turbine TC to drive the compressor.
5-6: Turbine TF to drive the fan.
6-7: Isentropic expansion in nozzle.

1. Make a sketch of the engine, and draw the corresponding T-s-diagram.
2. Determine the power required to drive the fan, when the bypass outflow velocity is $420 \frac{\mathrm{m}}{\mathrm{s}}$.
   Hint: Balance the complete bypass. Pressures at inlet and outlet are equal to environmental pressure. Then, for isentropic operation, the outlet temperature is equal to the inlet temperature (show that!)
3. Determine temperature, pressure, relative pressure, enthalpy, and outflow velocity at all 7 points. Provide a table with the values.
4. Compute the propulsive power of the engine, its thermal efficiency, its Froude efficiency, and the power that could be generated from the exhaust by equilibrating it to the environment.