# Chapter 19
# Psychrometrics

## 19.1 Characterization of Moist Air

*Psychros* and *metro* are Greek words meaning *cold* and *measure*, respectively, and psychrometrics describes moist air: mixtures of air and water vapor with possibly some liquid water present as well. Psychrometrics is most important for designing proper air conditioning systems for buildings, where the air should be not too dry or moist, to make the environment comfortable; moreover, moisture buildup at (or in!) walls must be prevented.

This chapter describes how to characterize and analyze moist air mixtures, and discusses basic processes for moisturizing and dehumidification in HVAC systems (Heating-Ventilating-Air-Conditioning).

We consider air-vapor mixtures at temperature $T$ and pressure $p = p_\mathrm{a} + p_\mathrm{v}$. Air behaves as an ideal gas, hence the partial pressure of air, $p_\mathrm{a}$, follows the ideal gas law. At the relevant temperatures, the partial pressure of the vapor in moist air, $p_\mathrm{v}$, is so low that the vapor can be described as an ideal gas as well.

The vapor pressure cannot exceed the saturation pressure $p_\mathrm{sat}(T)$. If $p_\mathrm{v} < p_\mathrm{sat}(T)$ there is no liquid water present, but if $p_\mathrm{v} = p_\mathrm{sat}(T)$ some liquid water will be present either in form of droplets (fog), or as an larger amount on the bottom. Figure 19.1 illustrates undersaturated moist air as a mixture of air and vapor, and saturated moist air as a mixture of air, vapor, and liquid water.

Since the saturation pressure $p_\mathrm{sat}(T)$ increases with temperature, warm air can hold more water vapor than cold air. Cooling of moist air can lead to condensation of water, e.g., on cold bottles, or on eyeglasses when one enters the warm and humid air of a house coming in from a cold winter environment.

The humidity ratio $\omega$, also known as specific humidity, is defined as the ratio of vapor and air mass in a sample of moist air of the volume $V$,

$$\omega = \frac{m_\mathrm{v}}{m_\mathrm{a}} = \frac{\frac{p_\mathrm{v}V}{R_\mathrm{v}T}}{\frac{p_\mathrm{a}V}{R_\mathrm{a}T}} = \frac{M_\mathrm{v}}{M_\mathrm{a}}\frac{p_\mathrm{v}}{p_\mathrm{a}} = 0.622\frac{p_\mathrm{v}}{p - p_\mathrm{v}} , \tag{19.1}$$

H. Struchtrup, *Thermodynamics and Energy Conversion*,
DOI: 10.1007/978-3-662-43715-5_19, 

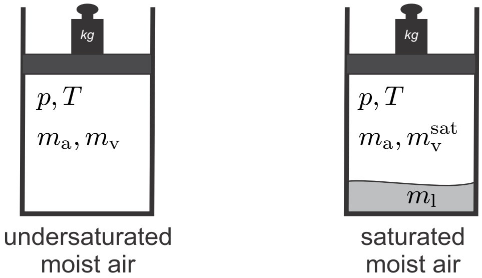


**Fig. 19.1** Undersaturated and saturated moist air

where we have used that $\frac{M_v}{M_a} = 0.622$. The humidity ratio for the saturated state, where $p_v = p_{sat}(T)$, is a function of temperature and pressure

$$\omega_{sat}(T, p) = 0.622 \frac{p_{sat}(T)}{p - p_{sat}(T)} . \tag{19.2}$$

The relative humidity $\phi$ is defined as the ratio between the actual mole fraction of vapor in the sample, and the vapor mole fraction in the saturated state,

$$\phi = \frac{X_v}{X_{sat}(T)} = \frac{p_v}{p_{sat}(T)} , \tag{19.3}$$

where it was used that, for ideal gas mixtures, $X_\alpha = \frac{p_\alpha}{p}$. Whether we perceive moist air as comfortable or not depends on the temperature and the relative humidity. In a dry environment, the human body loses a lot of moisture to evaporation from the skin and in breathing; one must drink a lot to replenish the moisture. In deserts, it helps to cover the body loosely with cloth, to prevent exposure of skin to the dry air, thus limiting evaporation from the skin. In high humidity, the air cannot accept more vapor, and thus sweat does not evaporate which results in difficulty to regulate the body temperature. For buildings, a relative humidity of $\phi \simeq 0.6$ is providing the most pleasant environment.

The enthalpy of a moist air sample is

$$H = H_a + H_v = m_a h_a + m_v h_v , \tag{19.4}$$

where $h_a(T)$ and $h_v = h_g(T)$ are the specific enthalpies of air and water vapor. Since the amount of vapor changes due to evaporation and condensation, it is convenient to base the specific enthalpy of moist air on the dry air mass and we write[1]

[1] The subscript $1 + \omega$ serves to distinguish a specific property per unit mass of dry air, which corresponds to $1 + \omega$ unit masses of moist air. This notation is uncommon in the North-American literature, but it is useful to avoid confusion with proper specific enthalpies.

$$h_{1+\omega}(T,\omega) = \frac{H}{m_a} = h_a(T) + \omega h_v(T) \ . \tag{19.5}$$

Since at these low pressures air and vapor are ideal gases, their enthalpies depend only on temperature $T$, while the enthalpy of moist air, $h_{1+\omega}$, depends also on the humidity ratio $\omega$.

The specific volume of moist air per unit mass of dry air can be computed from the Amagat model as, again with $\frac{M_v}{M_a} = 0.622$,

$$v_{1+\omega} = \frac{V_a + V_v}{m_a} = \frac{\frac{m_a R_a T}{p} + \frac{m_v R_v T}{p}}{m_a} = \left(1 + \frac{\omega}{0.622}\right)\frac{R_a T}{p} \ . \tag{19.6}$$

## 19.2 Dewpoint

In isobaric cooling, see Fig. 19.2, the partial pressures of air and vapor stay constant as long as no water condenses. The dewpoint temperature $T_d$ is defined as the temperature at which vapor starts to condense when moist air is isobarically cooled,[2]

$$p_{sat}(T_d) = p_v \quad \text{or} \quad T_d = T_{sat}(p_v) \ . \tag{19.7}$$

Figure 19.3 illustrates the cooling and condensation process for water in air in a T-s-diagram. Initially, the vapor is at state 1. No water condenses as the vapor is cooled until it reaches the dewpoint (state $d$). In the final state, the air is mixed with saturated vapor (state 2) and saturated liquid (state 3).

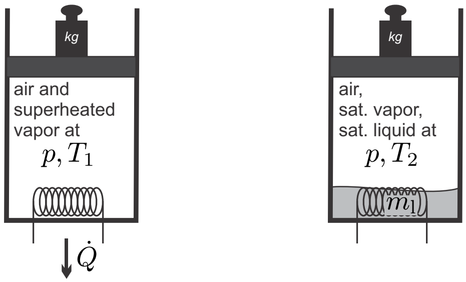


**Fig. 19.2** Isobaric cooling of moist air

[2] Due to the presence of air, the saturation pressure will be slightly different for vapor in air as compared to water alone (see the discussion in later chapters). The difference is small, however, and can be ignored.

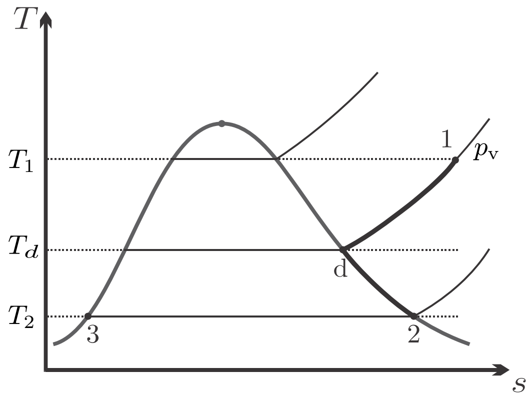


**Fig. 19.3** T-s-diagram of water for the isobaric cooling of moist air

## 19.3 Adiabatic Saturation and Wet-Bulb Temperature

When moist air of temperature $T$ and humidity ratio $\omega < \omega_{\rm sat}$ flows over a surface of water, some water will evaporate and the humidity of the air will increase. Evaporation requires the heat of evaporation $h_{fg}$, which is drawn from the air and the liquid, which therefore cool down as liquid evaporates. If there is sufficient contact between air and water, water will evaporate until the air will finally be saturated.

This effect is an important part of our life: When the body gets hot, humans sweat, the sweat evaporates by drawing the heat of vaporization from the body. The dryer the surrounding air is, the more vapor it can accept, and thus cooling by sweating is more efficient in dry climates. In moist climates, e.g., in tropical rainforests, the air can only accept little or no additional vapor, the sweat cannot evaporate, and no cooling is achieved. In dry climates, patios are cooled by spraying a fine mist of water. The small droplets evaporate immediately in the dry air, and this cools the air. Mothers blow air over their babies' food to cool it, good restaurants serve the meals under covers, so that the food is only in contact with the saturated moist air under the cover, and remains hot.

We study the system depicted in Fig. 19.4. Moist air at $(T, p, \omega)$ flows through a wetted porous material, which provides a large contact surface between air and liquid water. Pressure losses in the flow through the porous material are ignored in the following. The air leaves in saturated state at the so-called wet-bulb temperature $T_{\rm wb}$. We assume a steady state process under adiabatic conditions, that is no heat is added to the air flow from the exterior, and we assume that the make-up water flow from the reservoir is at the wet-bulb temperature $T_{\rm wb}$. The first law for the system then reduces to the equality between incoming and outgoing enthalpy flows, $\sum_{in} \dot{m}_\alpha h_\alpha =$

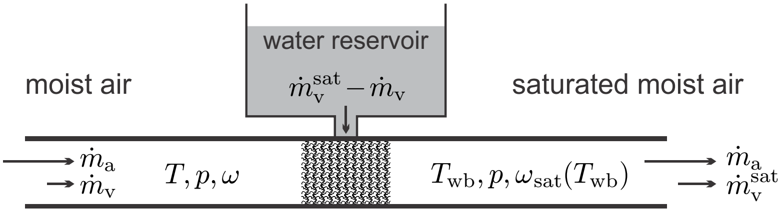


**Fig. 19.4** Saturation of a moist air flow

$\sum_{out} \dot{m}_\alpha h_\alpha$, or, in detail,

$$\dot{m}_a h_a(T) + \dot{m}_v h_v(T) + \left(\dot{m}_v^{sat} - \dot{m}_v\right) h_f(T_{wb}) = \dot{m}_a h_a(T_{wb}) + \dot{m}_v^{sat} h_v(T_{wb}) \ . \tag{19.8}$$

With $\omega = \frac{\dot{m}_v}{\dot{m}_a}$ and $\omega_{sat}(T_{wb}, p) = \frac{\dot{m}_v^{sat}}{\dot{m}_a} = 0.622 \frac{p_{sat}(T_{wb})}{p - p_{sat}(T_{wb})}$ we find an equation for the humidity ratio of the incoming moist air,

$$\omega(T, T_{wb}, p) = \frac{h_a(T_{wb}) - h_a(T) + \omega_{sat}(T_{wb}, p)\left[h_v(T_{wb}) - h_f(T_{wb})\right]}{h_v(T) - h_f(T_{wb})} \ . \tag{19.9}$$

The temperatures of the incoming air, $T$, and of the wet bulb, $T_{wb}$, can be measured easily, and this measurement allows to determine the humidity ratio $\omega$ from the above equation.

Indeed, for the measurement of the wet-bulb temperature it is sufficient to cover a thermometer with a wet cloth, and expose it to air flow. After a while (before the cloth has dried, of course) a steady state is reached, and the thermometer shows the wet-bulb temperature of the air flow. For the measurement in standing air (e.g. in a room), the wet thermometer has to be moved, and one uses a sling psychrometer: two thermometers, one dry, one wet, on a handle are rotated in the air.

## 19.4 Psychrometric Chart

From (19.9) it is evident that determining humidity from the measurement of dry- and wet-bulb temperatures involves some work for finding property data. Instead of working with property tables it is practical to use a psychrometric chart, from which all interesting data for moist air can be extracted. Through the humidity ratio, the equations depend on the total pressure, $p$, and one should take care to use the proper chart. Small daily pressure changes at a location do not affect the results much, but one will have to account for changes of environmental pressure with height. Figure 19.5 shows a chart for $p = 0.8\,\text{bar}$ which would be appropriate for a location at a height of about 2000 m above sea level.

The psychrometric chart has the dry-bulb temperature $T$ on the abscissa and the humidity ratio $\omega$ on the ordinate. The diagram shows lines of constant

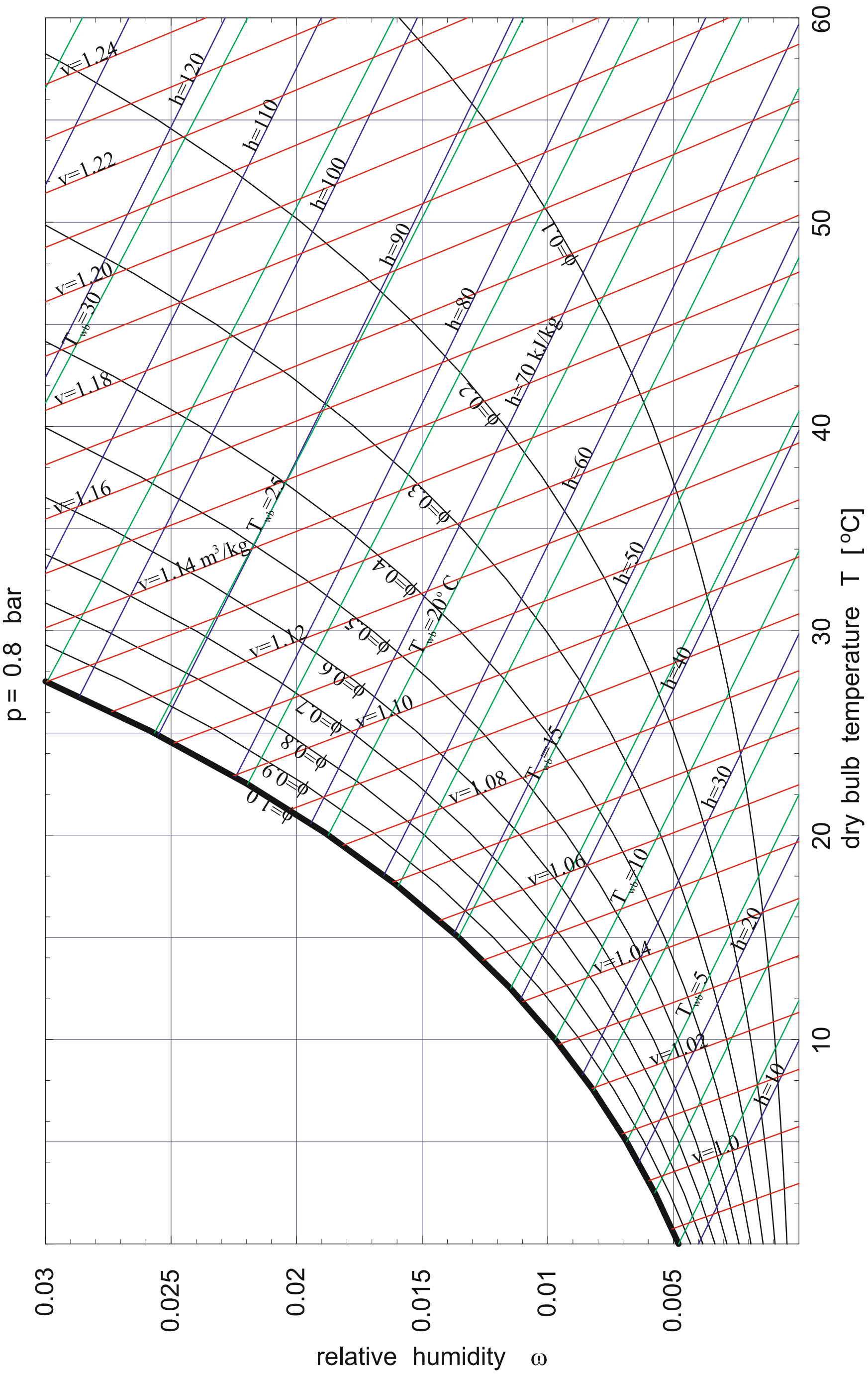


**Fig. 19.5** Psychrometric chart for $p = 0.8$bar

relative humidity $\phi$, constant wet-bulb temperature $T_{\rm wb}$, constant enthalpy $h_{1+\omega}$, and constant specific volume $v_{1+\omega}$. When two of these six quantities are known, all others can be easily read of the diagram. The chart will be used in subsequent sections.

The construction of the psychrometric chart is not difficult. For typical HVAC applications, vapor and dry air can be described as ideal gases with constant specific heats, and liquid water can be described as incompressible liquid with constant specific heat, so that

$$h_{\rm a}(T) = c_p^{\rm a}(T-T_R) \;\;,\;\; h_{\rm v}(T) = c_p^{\rm v}(T-T_R)+h_{fg}\,(T_R) \;\;,\;\; h_f(T) = c_f(T-T_R)\;, \tag{19.10}$$

where $T_R = 273.15\,{\rm K}$, $c_p^{\rm a} = 1.005\frac{\rm kJ}{\rm kg\,K}$, $c_p^{\rm v} = 1.88\frac{\rm kJ}{\rm kg\,K}$, $c_f = 4.18\frac{\rm kJ}{\rm kg\,K}$, and $h_{fg}\,(T_R) = 2500\frac{\rm kJ}{\rm kg}$. This choice of reference values for enthalpies and temperature ($T_R$) ensures that the psychrometric chart is compatible with standard steam tables, for which the triple point enthalpy of the saturated liquid is typically set to zero.

At low pressures, the saturation pressure $p_{\rm sat}(T)$ of water can be described by the Antoine equation

$$p_{\rm sat}(T) = p_{tr}\exp\left[17.0361 - \frac{3974.54}{T/\,^\circ{\rm C} + 233.290}\right]\;, \tag{19.11}$$

where $T$ is the temperature in $^\circ$C, and $p_{tr} = 0.611\,{\rm kPa}$ is the triple point pressure.

To plot the lines of constant $\phi$, $T_{\rm wb}$, $h_{1+\omega}$ and $v_{1+\omega}$, we need, for the given total pressure $p$, the humidity ratio $\omega$ as a function of the dry-bulb temperature $T$, and the quantity in question, i.e., relative humidity, enthalpy, etc. Lines of constant wet-bulb temperature $T_{\rm wb}$ follow immediately by plotting (19.9), which gives $\omega\,(T, T_{\rm wb}, p)$, for fixed values of $T_{\rm wb}$ and $p$. Equations (19.1) and (19.3) can be combined into

$$\omega\,(T,\phi,p) = \frac{0.622}{\frac{1}{\phi}\frac{p}{p_{\rm sat}(T)} - 1} \tag{19.12}$$

which, when plotted for fixed $\phi$ and $p$, gives the lines of constant relative humidity. Solving (19.5) for $\omega$ yields

$$\omega\,(T, h_{1+\omega}) = \frac{h_{1+\omega} - h_{\rm a}\,(T)}{h_{\rm v}\,(T)}\;, \tag{19.13}$$

which gives the lines of constant enthalpy per unit mass of dry air, $h_{1+\omega}$. Finally, lines of constant specific volume per unit mass of dry air follow from (19.6) as

$$\omega\,(T, v_{1+\omega}, p) = \left(\frac{p v_{1+\omega}}{R_{\rm a} T} - 1\right) 0.622\;. \tag{19.14}$$

The psychrometric chart for $p = 0.8\,\mathrm{bar}$ of Fig. 19.5 was obtained from the above equations. The chart exhibits lines of constant enthalpy ($h_{1+\omega} = 10, 20...120\frac{\mathrm{kJ}}{\mathrm{kg}}$), constant relative humidity ($\phi = 0.1, 0.2, ..., 1$), constant wet-bulb temperature ($T_{\mathrm{wb}} = 0, 2.5, 5, 7.5, ...32.5\,°\mathrm{C}$), and of constant specific volume ($v_{1+\omega} = 0.99, 1.00, ..., 1.24$). Lines for enthalpy, volume and wet-bulb temperature have no meaning for $\phi > 1$ and are not drawn. As can be seen from the equations above, the lines of constant wet-bulb temperature, constant enthalpy $h_{1+\omega}$ and constant volume $v_{1+\omega}$ are not straight. Their curvature is so small, however, that in the chart they appear to be straight lines. Also note that enthalpy and wet-bulb temperature lines are not parallel.

Differences in enthalpy values between diagrams obtained from the equations above and those commonly distributed could arise due to different reference states for enthalpies, while enthalpy differences will agree. Figure 19.6 shows a standard ASHRAE[3] chart for $p = 1.01325\,\mathrm{bar}$; the process depicted in the chart will be discussed in the next section.

## 19.5 Dehumidification

When moist air is cooled below its dewpoint, water condenses, and the humidity ratio $\omega$ drops. This process forms the basis for dehumidification systems, in which moist air is cooled below the dewpoint, some water condenses, and then the air is reheated, so that a desired final state is reached, Figure 19.7 shows a schematic for such a process.

We study the process by means of an example, that also shows how to use the psychrometric chart. A mass flow $\dot{m}_{\mathrm{a}} = 10\frac{\mathrm{kg}}{\mathrm{s}}$ of outside air of state 1 ($T_1 = 30\,°\mathrm{C}$, $T^1_{\mathrm{wb}} = 22.5\,°\mathrm{C}$, $p = 1\,\mathrm{atm}$) is to be dehumidified and cooled so that the final state 4 is $T_4 = 20\,°\mathrm{C}$, $\phi_4 = 0.7$. To achieve this state, the flow is first cooled isobarically. As long as the temperature is above the dewpoint, the humidity ratio does not change. At state 2 ($\phi_2 = 1$), water starts to condense. The moist air is cooled further to state 3, while some water condenses. State 3 has the same humidity ratio as the desired final state 4, which is finally obtained by isobaric heating. The process curve is shown in the psychrometric chart, Fig. 19.6.

From the diagram, we read the following data for the process

$$
\begin{array}{llll}
\omega_1 = 0.0145 & , \omega_2 = \omega_1 & , \omega_3 = \omega_4 & , \omega_4 = 0.0105\,, \\
\phi_1 = 0.55 & , \phi_2 = 1 & , \phi_3 = 1 & , \phi_4 = 0.7\,, \\
T_1 = 30\,°\mathrm{C} & , T_2 = 19\,°\mathrm{C} & , T_3 = 14.5\,°\mathrm{C} & , T_4 = 20\,°\mathrm{C}\,, \\
T^1_{\mathrm{wb}} = 22.5\,°\mathrm{C} & , T^2_{\mathrm{wb}} = T_2 & , T^3_{\mathrm{wb}} = T_3 & , T^4_{\mathrm{wb}} = 16.5\,°\mathrm{C}\,, \\
h^1_{1+\omega} = 66\frac{\mathrm{kJ}}{\mathrm{kg}} & , h^2_{1+\omega} = 55\frac{\mathrm{kJ}}{\mathrm{kg}} & , h^3_{1+\omega} = 41\frac{\mathrm{kJ}}{\mathrm{kg}} & , h^4_{1+\omega} = 47\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\
v^1_{1+\omega} = 0.878\frac{\mathrm{m}^3}{\mathrm{kg}} & , v^2_{1+\omega} = 0.847\frac{\mathrm{m}^3}{\mathrm{kg}} & , v^3_{1+\omega} = 0.829\frac{\mathrm{m}^3}{\mathrm{kg}} & , v^4_{1+\omega} = 0.844\frac{\mathrm{m}^3}{\mathrm{kg}}\,.
\end{array}
$$

[3] American Society of Heating, Refrigerating and Air Conditioning Engineers.

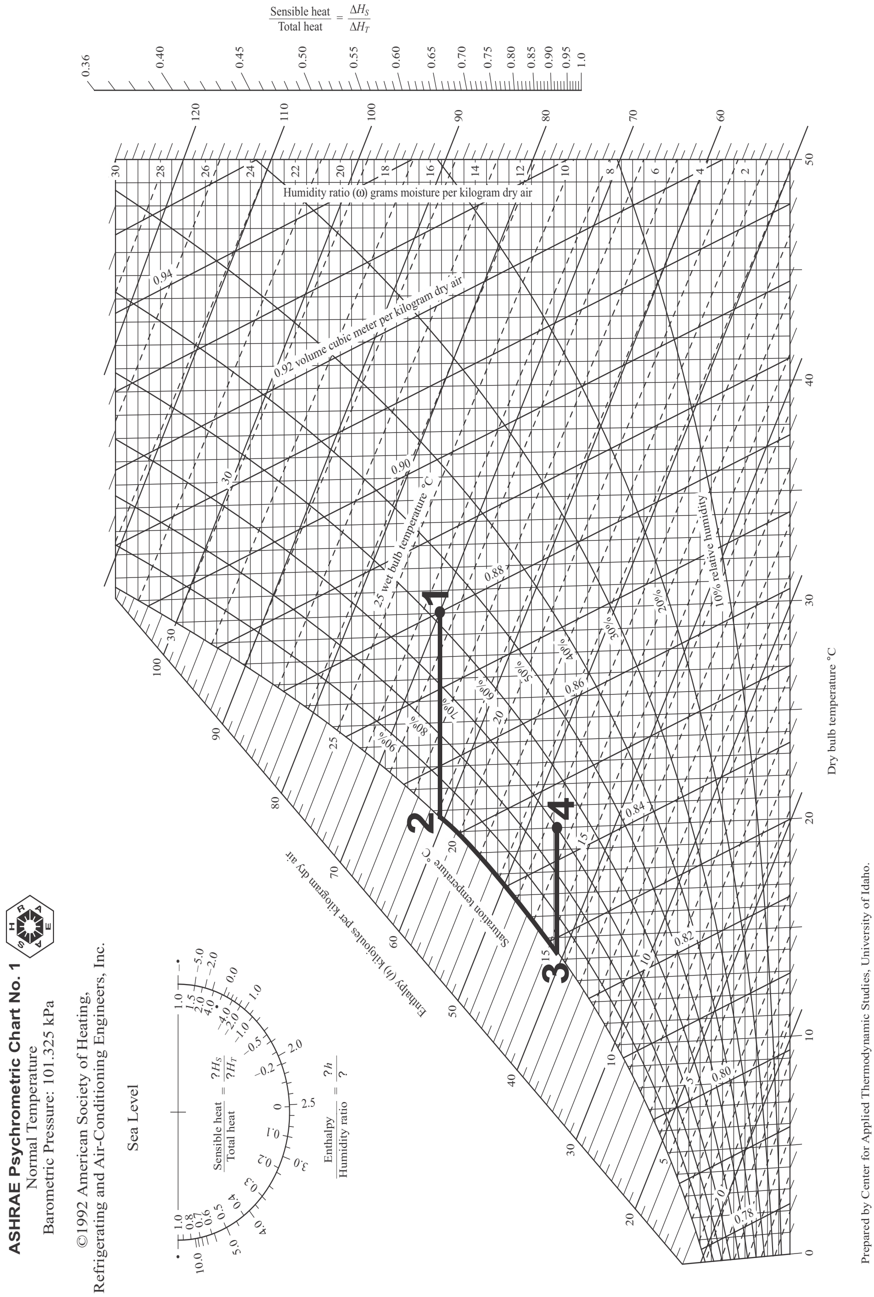


**Fig. 19.6** ASHRAE psychrometric chart for standard atmospheric pressure $p_0 = 1\,\text{atm} = 1.01325\,\text{bar}$. The line 1-2-3-4 depicts the dehumidification process of Example 19.5.

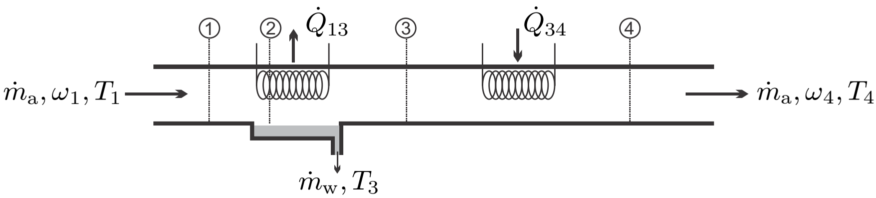


**Fig. 19.7** Dehumidification by cooling (1-2), condensation (2-3), and reheating (3-4)

We assume that the liquid water leaves the system at temperature $T_3$ where it has the enthalpy $h_f(T_3) = c_f(T_3 - T_0) = 60.61\frac{\text{kJ}}{\text{kg}}$, see (19.10). The mass balance for water reads

$$\dot{m}_v^1 = \dot{m}_w + \dot{m}_v^4 \quad \Longrightarrow \quad \omega_1 \dot{m}_a = \dot{m}_w + \omega_4 \dot{m}_a \ ,$$

so that the amount of water removed is,

$$\dot{m}_w = (\omega_1 - \omega_4)\,\dot{m}_a = 0.04\frac{\text{kg}}{\text{s}} \ .$$

The heat exchange rates for cooling and reheating are obtained from the first law, which here reduces to $\dot{Q} = \sum_{out} \dot{m}_\alpha h_\alpha - \sum_{in} \dot{m}_\alpha h_\alpha$ (no work, kinetic and potential energies ignored), and hence

$$\dot{Q}_{13} = \dot{m}_a h_{1+\omega}^3 + \dot{m}_w h_f\,(T_3) - \dot{m}_a h_{1+\omega}^1 = -248\,\text{kW} \ ,$$
$$\dot{Q}_{34} = \dot{m}_a \left(h_{1+\omega}^4 - h_{1+\omega}^3\right) = 60\,\text{kW} \ .$$

The cooling process requires a refrigeration system, some of the heat rejected by the refrigeration system can be used for reheating.

The volume flows entering and leaving the system are

$$\dot{V}_1 = v_{1+\omega}^1 \dot{m}_a = 8.78\frac{\text{m}^3}{\text{s}} \quad , \quad \dot{V}_4 = v_{1+\omega}^4 \dot{m}_a = 8.44\frac{\text{m}^3}{\text{s}} \ .$$

## 19.6 Humidification with Steam

The humidity ratio of dry or moist air can be increased by adding water either as steam or as liquid at pressure $p$. Steam injection increases humidity ratio and temperature, as long as the steam temperature is above the air temperature. Injection of liquid water, e.g., by spraying of fine mist, leads to cooling of the air, due to evaporation. We study both processes by means of examples, beginning with steam.

We study a steam injection process as depicted in Fig. 19.8. A volume flow $\dot{V}_1 = 10\frac{\text{m}^3}{\text{s}}$ of moist air with dry- and wet-bulb temperatures $T_1 = 14\,°\text{C}$,

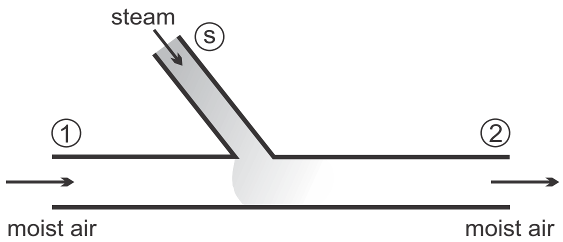


**Fig. 19.8** Humidification of moist air by addition of steam

$T_{\mathrm{wb},1} = 5\,°\mathrm{C}$ flows at a pressure of $p = 0.8\,\mathrm{bar}$. Superheated steam at $T_\mathrm{s} = 211\,°\mathrm{C}$, $p_\mathrm{s} = 0.8\,\mathrm{bar}$ is injected at a rate of $\dot{m}_\mathrm{s} = 306\frac{\mathrm{kg}}{\mathrm{h}}$. We ask for the final state (state 2) of the air.

From the psychrometric chart Fig. 19.5 we find the properties of state 1 as

$$\omega_1 = 0.0032\ ,\ \phi_1 = 0.26\ ,\ h^1_{1+\omega} = 22.5\frac{\mathrm{kJ}}{\mathrm{kg}}\ ,\ v^1_{1+\omega} = 1.035\frac{\mathrm{m}^3}{\mathrm{kg}}\ ,$$

and the enthalpy of the injected steam follows from (19.10) as $h_\mathrm{s} = 2897\frac{\mathrm{kJ}}{\mathrm{kg}}$. The mass flow of dry air is $\dot{m}_\mathrm{a} = \dot{V}_1/v^1_{1+\omega} = 9.66\frac{\mathrm{kg}}{\mathrm{s}}$.

For this continuous flow process, the balances for vapor mass and energy read

$$\omega_1\dot{m}_\mathrm{a} + \dot{m}_\mathrm{s} = \omega_2\dot{m}_\mathrm{a}\ ,$$
$$\dot{m}_\mathrm{a}h^1_{1+\omega} + \dot{m}_\mathrm{s}h_s = \dot{m}_\mathrm{a}h^2_{1+\omega}\ .$$

From these, we find

$$\omega_2 = \omega_1 + \frac{\dot{m}_\mathrm{s}}{\dot{m}_\mathrm{a}} = 0.012\ ,$$
$$h^2_{1+\omega} = h^1_{1+\omega} + (\omega_2 - \omega_1)\,h_s = 48\frac{\mathrm{kJ}}{\mathrm{kg}}\ .$$

With the above values for $\omega_2$ and $h^2_{1+\omega}$ state 2 can be localized in the chart, and we find the following other properties: $\phi_2 = 0.6$, $T_2 = 20\,°\mathrm{C}$, $T^2_\mathrm{wb} = 14.5\,°\mathrm{C}$, $v^2_{1+\omega} = 1.07\frac{\mathrm{m}^3}{\mathrm{kg}}$.

## 19.7 Evaporative Cooling

Next we consider cooling and moisturizing of air by addition of liquid water, again by means of an example, see Fig. 19.9. The initial state is relatively dry air at a pressure of $p = 1\,\mathrm{bar}$, relative humidity $\phi_1 = 0.2$, and temperature $T_1 = 40\,°\mathrm{C}$, so that $\omega_1 = 0.009$. Liquid water at $T_\mathrm{w} = 20\,°\mathrm{C}$ is sprayed into

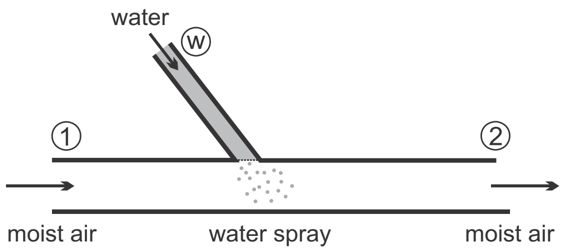


**Fig. 19.9** Evaporative cooling: Water is sprayed into moist air and evaporates

the air, and we ask how much liquid must be added per kg of air in order to lower the temperature to $T_2 = 30\,°\mathrm{C}$. The conservation laws for water mass and energy for this process read

$$\omega_1 \dot{m}_\mathrm{a} + \dot{m}_\mathrm{w} = \omega_2 \dot{m}_\mathrm{a} \; ,$$
$$\dot{m}_\mathrm{a} h_{1+\omega}\left(T_1, \omega_1\right) + \dot{m}_\mathrm{w} h_f\left(T_\mathrm{w}\right) = \dot{m}_\mathrm{a} h_{1+\omega}\left(T_2, \omega_2\right) \; . \qquad (19.15)$$

First, we solve the problem analytically. With the above relations for enthalpies (19.5, 19.10) we find

$$\omega_2 - \omega_1 = \frac{h_\mathrm{a}\left(T_1\right) - h_\mathrm{a}\left(T_2\right) + \omega_1\left[h_\mathrm{v}\left(T_1\right) - h_\mathrm{v}\left(T_2\right)\right]}{h_\mathrm{v}\left(T_2\right) - h_f\left(T_\mathrm{w}\right)} \qquad (19.16)$$
$$= \frac{\left[c_p^\mathrm{a} + \omega_1 c_p^\mathrm{v}\right]\left(T_1 - T_2\right)}{c_p^\mathrm{v}(T_2 - T_R) + h_{fg}\left(T_R\right) - c_f(T_\mathrm{w} - T_R)} = 4.13\frac{\mathrm{g}}{\mathrm{kg}} = 0.00413 \; ,$$

and thus $\omega_2 = 0.0131$.

When one plots this process into the psychrometric chart, one notices that the wet-bulb temperatures of both states are very close. To understand this behavior, we consider two states $\alpha = 1, 2$ with the same wet-bulb temperature $T_\mathrm{wb}$ so that, from (19.9),

$$h_\mathrm{a}\left(T_\mathrm{wb}\right) + \omega_\mathrm{sat}\left(T_\mathrm{wb}\right)\left[h_\mathrm{v}\left(T_\mathrm{wb}\right) - h_f\left(T_\mathrm{wb}\right)\right] = h_\mathrm{a}\left(T_\alpha\right) + \omega_\alpha\left[h_\mathrm{v}\left(T_\alpha\right) - h_f\left(T_\mathrm{wb}\right)\right] \; . \qquad (19.17)$$

By taking the difference of this equation for $\alpha = 1$ and $\alpha = 2$ we find

$$\omega_2 - \omega_1 = \frac{h_\mathrm{a}\left(T_1\right) - h_\mathrm{a}\left(T_2\right) + \omega_1\left[h_\mathrm{v}\left(T_1\right) - h_\mathrm{v}\left(T_2\right)\right]}{h_\mathrm{v}\left(T_2\right) - h_f\left(T_\mathrm{wb}\right)} \; . \qquad (19.18)$$

This almost agrees with the expression (19.16), the only difference is the value of the temperature of the added water in the denominator. If the added liquid water in (19.16) is at the wet-bulb temperature of state 1, then the wet-bulb temperature will stay constant. Since under HVAC conditions the enthalpy of the vapor exceeds the enthalpy of the added liquid by far, the denominators in (19.16) and (19.18) will be very close, and both equations will give almost the same result.

In short, evaporative cooling, that is injection of liquid water into moist air, can be well approximated as a process of constant wet-bulb temperature. The psychrometric chart shows lines of constant $T_{\mathrm{wb}}$ and can be used to evaluate these processes. Moreover, since the lines of constant enthalpy $h_{1+\omega}$ are almost parallel to the lines of constant wet-bulb temperature, some authors suggest to describe evaporative cooling as a constant enthalpy process.

## 19.8 Adiabatic Mixing

We consider the adiabatic and isobaric mixing of two moist air streams of states 1 and 2. Mass and energy balances relate the final state 3 to the incoming streams as

$$\begin{aligned} \dot{m}_{\mathrm{a}}^1 + \dot{m}_{\mathrm{a}}^2 &= \dot{m}_{\mathrm{a}}^3 \,, \\ \omega_1 \dot{m}_{\mathrm{a}}^1 + \omega_2 \dot{m}_{\mathrm{a}}^2 &= \omega_3 \dot{m}_{\mathrm{a}}^3 \,, \\ \dot{m}_{\mathrm{a}}^1 h_{1+\omega}^1 + \dot{m}_{\mathrm{a}}^2 h_{1+\omega}^2 &= \dot{m}_{\mathrm{a}}^3 h_{1+\omega}^3 \,. \end{aligned} \tag{19.19}$$

Elimination of $\dot{m}_{\mathrm{a}}^3$ gives

$$\frac{\dot{m}_{\mathrm{a}}^1}{\dot{m}_{\mathrm{a}}^2} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{h_{1+\omega}^3 - h_{1+\omega}^2}{h_{1+\omega}^1 - h_{1+\omega}^3} \,, \tag{19.20}$$

which implies that in the psychrometric chart the mixed state 3 lies on the line connecting states 1 and 2, see Fig. 19.10 for illustration.

As an example we consider mixing of two streams at $p = 1\,\mathrm{atm}$ and

$$\begin{aligned} T_1 &= 15\,^\circ\mathrm{C} \,, \quad \dot{V}_1 = 30\tfrac{\mathrm{m}^3}{\mathrm{min}} \,, \quad \phi = 1 \,, \\ T_2 &= 30\,^\circ\mathrm{C} \,, \quad \dot{V}_2 = 40\tfrac{\mathrm{m}^3}{\mathrm{min}} \,, \quad \phi = 0.5 \,. \end{aligned}$$

From the psychrometric chart we read

$$\begin{aligned} h_{1+\omega}^1 &= 42\tfrac{\mathrm{kJ}}{\mathrm{kg}} \,, \quad \omega_1 = 0.011 \,, \quad v_{1+\omega}^1 = 0.830\tfrac{\mathrm{m}^3}{\mathrm{kg}} \,, \\ h_{1+\omega}^2 &= 63\tfrac{\mathrm{kJ}}{\mathrm{kg}} \,, \quad \omega_2 = 0.013 \,, \quad v_{1+\omega}^1 = 0.876\tfrac{\mathrm{m}^3}{\mathrm{kg}} \,. \end{aligned}$$

The corresponding mass flows of dry air are

$$\dot{m}_{\mathrm{a}}^1 = \frac{\dot{V}_1}{v_{1+\omega}^1} = 36.15\frac{\mathrm{kg}}{\mathrm{min}} \quad , \quad \dot{m}_{\mathrm{a}}^2 = \frac{\dot{V}_2}{v_{1+\omega}^2} = 45.66\frac{\mathrm{kg}}{\mathrm{min}} \,,$$

and the final state is

$$\omega_3 = \frac{\dot{m}_{\mathrm{a}}^1 \omega_1 + \dot{m}_{\mathrm{a}}^2 \omega_2}{\dot{m}_{\mathrm{a}}^1 + \dot{m}_{\mathrm{a}}^2} = 0.012 \quad , \quad h_{1+\omega}^3 = \frac{\dot{m}_{\mathrm{a}}^1 h_{1+\omega}^1 + \dot{m}_{\mathrm{a}}^2 h_{1+\omega}^2}{\dot{m}_{\mathrm{a}}^1 + \dot{m}_{\mathrm{a}}^2} = 54\frac{\mathrm{kJ}}{\mathrm{kg}} \,.$$

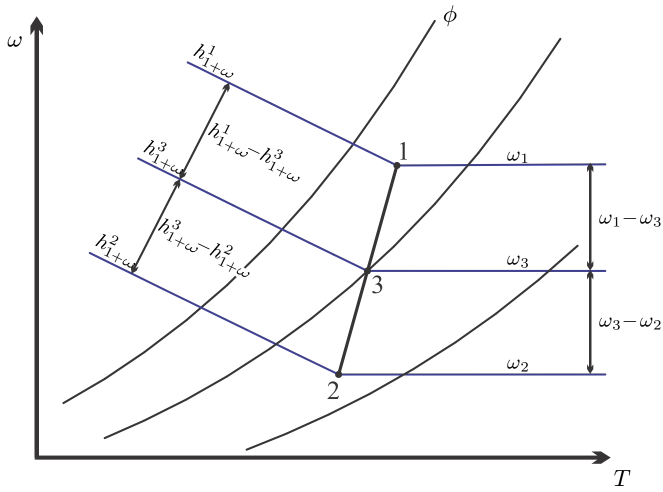


**Fig. 19.10** Adiabatic mixing of moist air streams: In the psychrometric chart, the mixed state 3 is on the line connecting the initial states 1 and 2

A special situation may arise due to the convexity of the saturation line ($\phi = 1$). It can happen that the line connecting the two initial states lies outside the accessible region of the diagram. Figure 19.11 shows this for the special case of mixing of two saturated states. In these cases, some liquid water will fall-out as fog, and the mixture will be in the saturated state. Obviously, formation of fog must be avoided in HVAC applications. The relevant equations are again the conservation laws for air and vapor mass, and for energy, which now read

$$\begin{aligned} \dot{m}_{\mathrm{a}}^1 + \dot{m}_{\mathrm{a}}^2 &= \dot{m}_{\mathrm{a}}^3 \, , \\ \omega_1 \dot{m}_{\mathrm{a}}^1 + \omega_2 \dot{m}_{\mathrm{a}}^2 &= \omega_{\mathrm{sat}} \left( T_3 \right) \dot{m}_{\mathrm{a}}^3 + \dot{m}_{\mathrm{w}} \, , \\ \dot{m}_{\mathrm{a}}^1 h_{1+\omega}^1 + \dot{m}_{\mathrm{a}}^2 h_{1+\omega}^2 &= \dot{m}_{\mathrm{a}}^3 h_{1+\omega}^3 \left( T_3, \omega_{\mathrm{sat}} \right) + \dot{m}_{\mathrm{w}} h_{\mathrm{w}} \left( T_3 \right) \, . \end{aligned}$$

Due to the occurrence of $\omega_{\mathrm{sat}}(T_3)$, these are three non-linear equations for the three unknowns $T_3$, $\dot{m}_{\mathrm{w}}$, $\dot{m}_{\mathrm{a}}^3$, which are best solved numerically.

The air on top of water bodies normally is saturated. When two streams of water at different temperatures meet, fog will occur as a result of the mixing of the two accompanying air flows.

## 19.9 Cooling Towers

Evaporate cooling is used in cooling towers for steam power plants, which require a large amount of heat rejection in the condenser. Figure 19.12 shows

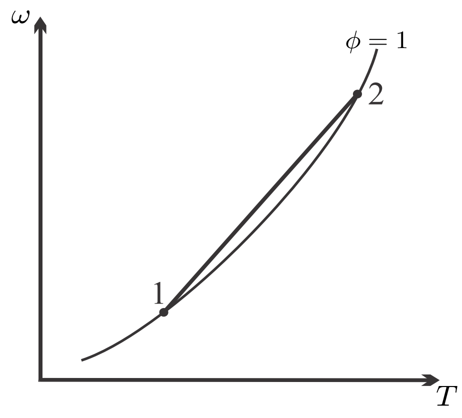


**Fig. 19.11** Adiabatic mixing with fall-out of liquid water

a schematic for a natural draft cooling tower, the hyperbolic shape is chosen for structural strength and low material use.

The cooling water flow $\dot{m}_{\rm cw}$ comes from the condenser of the power plant, where it was heated to $T_1$ while the water circulating in the steam cycle was condensed. The incoming cooling water is sprayed into the cooling tower, where some of it evaporates, which leads to cooling of the liquid. Since moist air is lighter than dry air—the low molar mass of vapor lowers the average molar mass—moist air rises and leaves the tower, while fresh environmental air at $(T_3, \omega_3)$ is drawn in at the bottom. Make-up water at $\dot{m}_{\rm m}, T_5$ is added to compensate the loss of evaporated water and the mass flow $\dot{m}_{\rm cw}$ leaves the cooling tower towards the condenser at $T_2$. Normally, the make-up water is drawn from rivers or lakes, and that is why power plants are build close to these. As the rising moist air equilibrates with the environment, some of the added water might condense, which leads to clouds that normally can be seen above cooling towers.

The balances for air and water mass, and for energy, read

$$\begin{aligned} \dot{m}_{\rm a} &= const.\ , \\ \dot{m}_{\rm cw} + \dot{m}_{\rm m} + \dot{m}_{\rm a}\omega_3 &= \dot{m}_{\rm cw} + \dot{m}_{\rm a}\omega_4\ , \\ \dot{m}_{\rm cw} h_f\left(T_1\right) + \dot{m}_{\rm m} h_f\left(T_5\right) + \dot{m}_{\rm a} h^3_{1+\omega} &= \dot{m}_{\rm cw} h_f\left(T_2\right) + \dot{m}_{\rm a} h^4_{1+\omega}\ . \end{aligned} \tag{19.21}$$

## 19.10 Example: Cooling Tower

As an example we study the cooling tower for a $\dot{W} = 300\,\mathrm{MW}$ power plant with a thermal efficiency $\eta = 0.4$. In the condenser, the cooling water is heated from $T_2 = 30\,°\mathrm{C}$ to $T_1 = 40\,°\mathrm{C}$ (the numbers refer to Fig. 19.12), thus the mass flow of cooling water is

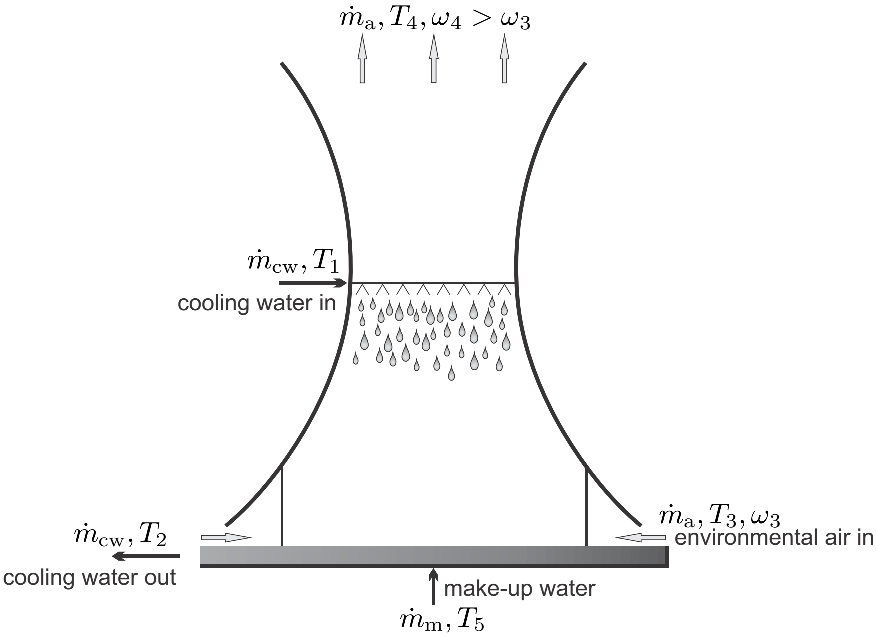


**Fig. 19.12** Air and water flows in a cooling tower

$$\dot{m}_{cw} = \frac{\dot{Q}_{cw}}{h_1 - h_2} = \frac{\dot{Q}_{cw}}{c_f (T_1 - T_2)} = \frac{1-\eta}{\eta} \frac{\dot{W}}{c_f (T_2 - T_1)} = 10.77 \frac{\mathrm{t}}{\mathrm{s}} \; .$$

We ask for the required flows of air and make-up water, which both depend on the state of the incoming and exiting moist air. For further computation we assume that the incoming air is at $T_3 = 25\,^\circ\mathrm{C}$, $\phi_3 = 0.5$, so that $\omega_3 = 0.01$, $h^3_{1+\omega} = 51 \frac{\mathrm{kJ}}{\mathrm{kg}}$, and that the make-up water is at $T_5 = 25\,^\circ\mathrm{C}$, so that, with (19.10), $h_f (T_5) = 104.5 \frac{\mathrm{kJ}}{\mathrm{kg}}$, and $h_f (T_1) = 167.2 \frac{\mathrm{kJ}}{\mathrm{kg}}$, $h_f (T_2) = 125.4 \frac{\mathrm{kJ}}{\mathrm{kg}}$. Furthermore, we assume that the exiting air is saturated, so that $\omega_4 = \omega_{sat} (T_4)$.

The remaining unknowns in this problem are the air mass flow $\dot{m}_a$, the make-up water flow $\dot{m}_m$ and the exit temperature $T_4$. This problem differs from evaporative cooling as discussed above, due to the large amount of warm water sprayed into the air. The heat transfer between air droplets and air could only be described by a detailed heat transfer analysis. To simplify the problem, we assume $T_4 = 30\,^\circ\mathrm{C}$ which implies $h^4_{1+\omega} = 100 \frac{\mathrm{kJ}}{\mathrm{kg}}$ and $\omega_4 = \omega_{sat} (T_4) = 0.0272$. Then we find from the conservation laws

$$\dot{m}_a = \frac{h_f (T_1) - h_f (T_2)}{h^4_{1+\omega} - h^3_{1+\omega} - [\omega_{sat} (T_4) - \omega_3]\, h_f (T_5)} \dot{m}_{cw} = 9.61 \frac{\mathrm{t}}{\mathrm{s}} \; ,$$

$$\dot{m}_m = \dot{m}_a [\omega_{sat} (T_4) - \omega_3] = 0.165 \frac{\mathrm{t}}{\mathrm{s}} \; .$$

The river that provides the make-up water should have a sufficiently large mass flow rate, so that the removal of the make-up water will not disturb the ecological equilibrium of the river. An alternative to cooling towers is the direct use of river or lake water as cooling water. In this case, the heat rejected by the power plant is added to the river or lake. The related increase in water temperature changes the chemical environment, e.g., the amount of oxygen dissolved decreases with increasing temperature (Henry's law, Sec. 22.10), which might disturb the ecological equilibrium more than the removal of some water for use in cooling towers.

## Problems

**19.1. Compression of Moist Air**
Air initially at 1 atm, 25 °C and relative humidity of 60% is compressed isothermally until condensation of water occurs. Determine the pressure at the onset of condensation. Draw the process for the vapor into a T-s-diagram.

**19.2. Compressed Air**
To avoid condensation of water in compressed air lines, it might be necessary to dehumidify the compressed air. To study this, consider a compressor that draws outside air at 93 kPa, 14 °C and relative humidity of 40%, and compresses it to 800 kPa. After compression, the air flows through ducts for distribution, where it is cooled to the workshop temperature of 22 °C. Determine the dewpoint of the compressed air–will there be condensation in the pipes?

**19.3. Air Conditioning**
An air conditioning system provides a volume flow of $3\frac{\mathrm{m}^3}{\mathrm{s}}$ of moist air at 1 atm, 22 °C and 50% relative humidity by conditioning outside air at 34 °C and 50% relative humidity. For this, the outside air is first cooled and dehumidified, and then heated to the final temperature. Assume that the condensate leaves the system at 10 °C and determine the temperature after dehumidification is completed, the amount of heat that must be withdrawn in the cooling process, and the heat added in the heating process per unit mass of dry air.

**19.4. Air Conditioning**
An air conditioning system provides air at 1 atm, 20 °C and 60% relative humidity which is obtained from outside air at 38 °C and 70% relative humidity as follows: The outside air is first cooled and dehumidified, and then heated to the desired final temperature. The pressure stays constant throughout the process. Determine the temperature after dehumidification is completed, and assume that the condensate leaves the system at this temperature. Next, determine the heat that must be withdrawn in the cooling process, and the heat added in the heating process, both per unit mass of dry air. Finally,

determine the mass flow and the volume flow of the air delivered, when the cooling power of the system is 150 kW.

**19.5. Humidification**

An air conditioning system draws $22\frac{m^3}{min}$ outside air at 1 atm, 10 °C and 40% relative humidity. The air is first heated to 22 °C, and then humidified by injection of steam. The air leaves the system at 25 °C and 55% relative humidity. Determine the rate of heat supply in the heating section, the mass flow rate of steam required, and the temperature of the steam.

**19.6. Humidification**

At an elevated location, an air conditioning system draws $20\frac{m^3}{min}$ outside air at 0.8 bar. With a psychrometer it is determined that the dry- and wet-bulb temperatures of the incoming air are 10 °C and 2.5 °C, respectively. To reach the desired state, the air is first heated to a temperature $T_2$, and then humidified by injection of superheated steam at 0.8 bar, 150 °C, where the enthalpy is $2777.84\frac{kJ}{kg}$. The air leaves the system at 22 °C and 55% relative humidity. Determine the mass flow rate of steam required, the rate of heat supply in the heating section, and the temperature of the air before steam is injected.

**19.7. Dehumidification and Mixing**

The outside air of a building is at 26 °C, and 90% relative humidity, the pressure is 1 atm. The air conditioning system of the building is required to provide air at 22 °C and 50% relative humidity. To reach that state, the flow of incoming outside air is split into two streams.

One stream is dehumidified by cooling to 5 °C so that liquid water condenses, and subsequent reheating. The cooling system removes 6 kW from this flow.

Then, the dehumidified stream is mixed with the other stream, so that the desired state is reached.

For the solution use the psychrometric chart.

1. Determine the dry air mass flows of the two streams.
2. Determine the heat required for the reheating of the dehumidified stream.
3. The final air flow should not be faster than $3\frac{m}{s}$, determine the cross section of the duct.

**19.8. Air Conditioning**

An air conditioning system draws a volume flow of $20\frac{m^3}{min}$ of outside air at 30 °C and 90% relative humidity (state 1). The air flow is divided in to two streams, stream A and stream B. Stream A is first dehumidified by cooling to 5 °C (state 2), and then heated to state 3. Stream A and stream B are then mixed adiabatically. The mixture has a dry-bulb temperature of 20 °C at 50% relative humidity (state 4). The pressure is constant at 1 atm throughout the process.

1. Indicate the states 1,2,3,4 in the psychrometric chart.
2. Compute the ratios of dry air mass flows , and the values of the two mass flows.
3. Compute the heat to be removed from (1-2) and added to (2-3) stream A.
4. Is it feasible to do both, heating and cooling with a single refrigeration cycle? Discuss?

**19.9. Evaporative Cooling**

To provide air at a desired state, a volume flow of $10\frac{\mathrm{m}^3}{\mathrm{s}}$ outside air (dry-bulb temperature 15 °C, wet-bulb temperature 10.8 °C) is first heated to 30 °C and then cooled and humidified by spraying of liquid water. The final temperature is 25 °C. Determine the relative humidity at the exit, the mass flow of water added, and the heating rate required. Use the psychrometric chart.

**19.10. Evaporative Cooling**

An air conditioning system draws a volume flow of $50\frac{\mathrm{m}^3}{\mathrm{min}}$ of outside air at 40 °C and 10% relative humidity (state 1). To produce moist air of pleasant conditions, this air is first cooled by evaporative cooling to state 2, and then by heat exchange with a cooling system to the final state, with dry-bulb temperature $T_3 = 20\,°\mathrm{C}$ at $\phi_3 = 60\%$ relative humidity. The pressure is constant at 1 atm throughout the process.

1. Indicate the states 1,2,3 in the attached psychrometric chart.
2. Determine the mass flow of water required for evaporative cooling (1-2).
3. Determine the heat to be removed, $\dot{Q}_{23}$, in kilowatts.
4. A leak occurs in the system, and moist air at state 3 is mixed with outside air. Determine the dry air mass flow of leaked air when the mixture has a dry-bulb temperature of $T_4 = 22\,°\mathrm{C}$.

**19.11. Mixing of Two Moist Air Streams**

Consider the adiabatic mixing of two streams of moist air at $p = 1\,\mathrm{atm}$. Stream 1 is saturated moist air of 20 °C at a volumetric flow rate of $60\frac{\mathrm{m}^3}{\mathrm{min}}$ and stream 2 is moist air of 34 °C, 20% relative humidity. The relative humidity after mixing is 60%. Mark all relevant points on the psychrometric chart.

1. For the incoming flows and for the mixture, determine the values for enthalpy, temperature, relative humidity, humidity ratio, and specific volume.
2. Determine the dewpoint temperature and the wet-bulb temperature of the mixture
3. Compute the volumetric flow of stream 2.

**19.12. Mixing of Air Streams**

Two streams of moist air are mixed adiabatically at 1 atm. One stream has a dry-bulb temperature of 40 °C and a wet-bulb temperature of 32 °C, and the mass flow rate is $8\frac{\mathrm{kg}}{\mathrm{s}}$. The other stream is saturated air at 18 °C with a mass flow rate of $6\frac{\mathrm{kg}}{\mathrm{s}}$. Determine the state of the mixture (temperature, specific humidity, relative humidity, enthalpy, volume flow).

**19.13. Air Conditioning in the Desert**
In the desert: The outside air of a building is at $40^o$C, and 10% relative humidity, the pressure is 1 atm. The air conditioning system of the building is required to provide air at 20 °C and 50% relative humidity. To reach that state, the flow of incoming outside air is split into two streams, A and B:

Stream A is spray-cooled by injection of liquid water to a relative humidity of 100%, the mass flow of water (at 20 °C) injected is $10\frac{\text{kg}}{\text{h}}$.

Stream B is cooled to temperature $T_B$ by a standard refrigeration cycle with COP of 3. Then, the spray-cooled stream A is mixed with stream B, so that the desired end state is reached.

1. Make a sketch of the process, and enter the relevant points in the psychrometric chart.
2. Determine the temperature $T_B$.
3. Determine the dry air mass flows of both streams.
4. Determine the heat removed from stream B, and the power requirement of the refrigerator.

**19.14. Cooling Tower**
In a 500 MW steam power plant, the condenser is cooled by a cooling water flow that enters the condenser at 26 °C, and leaves at 40 °C. The cooling water is cooled back to 26 °C in a natural-draft cooling tower which draws environmental air at 1 atm with dry- and wet-bulb temperatures of 23 °C and 18 °C, respectively, and discharges saturated air at 37 °C. The thermal efficiency of the power plant is 43.5%. Determine mass flow of cooling water, volume flows of air into and out of the cooling tower, and the required mass flow of makeup water.

**19.15. Clouds**
Cumulus clouds are formed when air at the ground is heated, takes up moisture, and then rises due to its buoyancy. While rising, the moist air expands, more or less adiabatically, since the pressure decreases with height. During expansion the temperature of the rising air is decreasing. When the temperature reaches the dew point temperature, water vapor condenses, and a cloud is formed.

In order to compute the height of the clouds, assume that the pressure in the atmosphere is given by the barometric formula 2.26.

Consider a fixed mass of moist air, that occupies a volume $V$, and has enthalpy $H$. Consider the moist air as an ideal gas, so that its enthalpy and volume are given as

$$H = m_\text{a}\left[c_p^\text{a}\left(T_\text{a} - T_R\right) + \omega\left(h_{fg}\left(T_R\right) + c_p^\text{v}\left(T_\text{a} - T_R\right)\right)\right] \,,$$
$$V = m_\text{a}\left(R_\text{a} + \omega R_\text{v}\right)\frac{T_\text{a}}{p} \,.$$

Here, $m_\text{a}$ is the mass of dry air, $\omega$ is the humidity ratio, $c_p^\text{a}$, $c_p^\text{v}$ and $R_\text{a}$, $R_\text{v}$ are the specific heats and gas constants of dry air and vapor, $h_{fg}\left(T_R\right)$ is the heat of evaporation of water at $T_R = 273.15\,\mathrm{K}$, $T_\mathrm{a}$ is the temperature of the rising moist air, and $p$ is the local pressure.

1. Discuss the assumptions behind the equations for enthalpy and volume.
2. Show that the first law for an adiabatic process for the moist air gives $dH = Vdp$.
3. Show that the temperature of the rising moist air is given by
$$T_\mathrm{a}(z) = T_M \left(\frac{p(z)}{p_0}\right)^{\frac{R_\mathrm{a}+\omega R_\mathrm{v}}{c_p^\mathrm{a}+\omega c_p^\mathrm{v}}} ,$$
where $T_M$ is the temperature of the moist air at the ground, just before rising (at $z = 0$).
4. Employ the ideal gas law for the vapor to find its partial pressure in the moist air as
$$p_v(z) = \frac{\omega}{\omega + \frac{R_\mathrm{a}}{R_\mathrm{v}}} p_0 \left[1 - \frac{\alpha}{T_0} z\right]^{\frac{g}{\alpha R_\mathrm{a}}} .$$
5. The saturation pressure for the vapor is given by $p_\mathrm{sat}(T_\mathrm{a}(z))$, and water will condense, when $p_v(z) > p_\mathrm{sat}(T_\mathrm{a}(z))$. Set $T_M = 298\,\mathrm{K}$, $\omega = 0.01$ (or other values), and find the height $z_\mathrm{Cloud}$, where clouds begin to form.