# Chapter 7
# Reversible Processes in Closed Systems

## 7.1 Standard Processes

In Chapter 8 we shall study thermodynamic cycles in closed systems which model thermal engines, including internal combustion engines. The focus will lie on the understanding of the working principles of the cycles, and on the main parameters that determine their efficiency. For this it is customary to base the analysis on reversible processes, which allow a full analysis.

There are a number of processes that are often realized (at least approximately) in thermodynamic systems: processes at constant volume, constant pressure, constant temperature, or adiabatic processes. Typical thermodynamic cycles consist of closed chains of several of these processes. In this chapter we compute work and heat for these standard processes as a reference for the discussion of cycles.

## 7.2 Basic Equations

Figure 7.1 shows, again, a piston-cylinder device as the prototypical closed system. In reversible (quasi-static) processes, the system exchanges energy

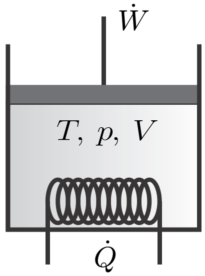


**Fig. 7.1** Closed system with piston work and heat exchange. In this chapter we are interested in reversible processes only, so there is not stirring.

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through heating and piston work only; stirring (propeller work) as an irreversible process is excluded. All movement of the material in the system is so slow that velocity and kinetic energy can be ignored. For a stationary system, potential energy is constant and can be ignored as well. Thus, at all times the system is in homogeneous equilibrium states which are characterized by the temperature $T$, the pressure $p$, and the volume $V$.

We list the relevant equations from previous chapters. Under the above simplifications, the first law for closed systems reduces to

$$\frac{dU}{dt} = \dot{Q} - \dot{W} \ , \tag{7.1}$$

where $\dot{Q}$ is the heat transfer rate, and $\dot{W}$ denotes power. Integration over the duration of the process gives the time-integrated energy balance

$$U_2 - U_1 = Q_{12} - W_{12} \ , \tag{7.2}$$

where

$$Q_{12} = \int_{t_1}^{t_2} \dot{Q} dt \quad \text{and} \quad W_{12} = \int_{t_1}^{t_2} \dot{W} dt \tag{7.3}$$

are the total amounts of heat and work exchanged between the states 1 (at time $t_1$) and 2 (at time $t_2$).

For an infinitesimal step of the process (duration $dt$) we have the differential form of the first law

$$dU = \delta Q - \delta W \ , \tag{7.4}$$

where $\delta Q = \dot{Q} dt$ and $\delta \dot{W} = \dot{W} dt$ are heat and work exchanged during $dt$. The notation implies that work and heat have inexact differentials, since they are process dependent quantities.

For a reversible process in a closed system, the work is just the piston work,

$$\dot{W} = p \frac{dV}{dt} \quad \text{or} \quad W_{12} = \int_{t_1}^{t_2} \dot{W} dt = \int_1^2 \delta W = \int_1^2 p dV \ , \tag{7.5}$$

and the heat can be computed from the second law, which for reversible processes ($\dot{S}_{gen} = 0$) reduces to

$$\dot{Q} = T \frac{dS}{dt} \quad \text{or} \quad Q_{12} = \int_{t_1}^{t_2} \dot{Q} dt = \int_1^2 \delta Q = \int_1^2 T dS \ . \tag{7.6}$$

Thus, for reversible processes, heat and work are the areas below the process curves in the p-V-diagram and the T-S-diagram, respectively, as depicted in Fig. 7.2.

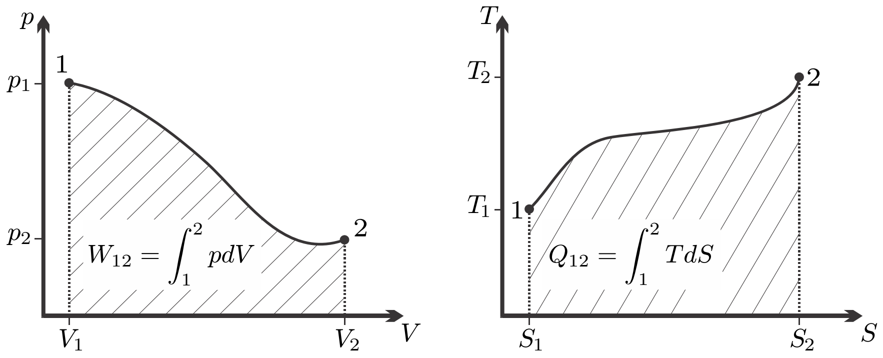


**Fig. 7.2** Heat and work in reversible processes as areas below the process curves in the p-V- and the T-S-diagram

In the following sections we shall compute work and heat per unit mass, which for reversible processes are given by

$$w_{12} = \frac{W_{12}}{m} = u_1 - u_2 + q_{12} = \int_1^2 pdv \ , \tag{7.7}$$

$$q_{12} = \frac{Q_{12}}{m} = u_2 - u_1 + w_{12} = \int_1^2 Tds \ . \tag{7.8}$$

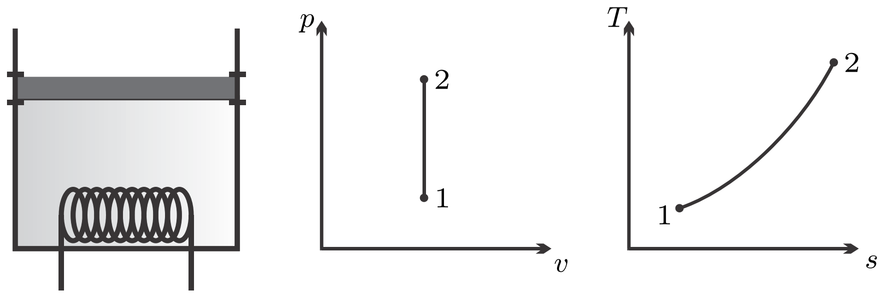


**Fig. 7.3** Isochoric process: Realization, p-v- and T-s-diagrams

## 7.3 Isochoric Process: $v = const.$, $dv = 0$

Isochoric processes (constant volume) can be easily realized by fixing the volume, e.g., by clamping the piston, see Figure 7.3 for process sketch and diagrams.

With $dv = 0$ in the constant volume process, heat and work follow from (7.7, 7.8) as

$$w_{12} = 0 \quad , \quad q_{12} = u_2 - u_1 \; . \tag{7.9}$$

We compute the process curve of an isochoric process in the T-s-diagram for an ideal gas with constant specific heats. From the Gibbs equation and the caloric equation of state we find for the isochoric process

$$Tds = du + pdv = du = c_v dT \; , \tag{7.10}$$

so that upon integration

$$s - s_1 = c_v \ln \frac{T}{T_1} \quad \text{or} \quad T = T_1 e^{\frac{s-s_1}{c_v}} \; . \tag{7.11}$$

Thus, for an ideal gas, the isochoric process in the T-s-diagram follows an exponential, as indicated in the T-s-diagram.

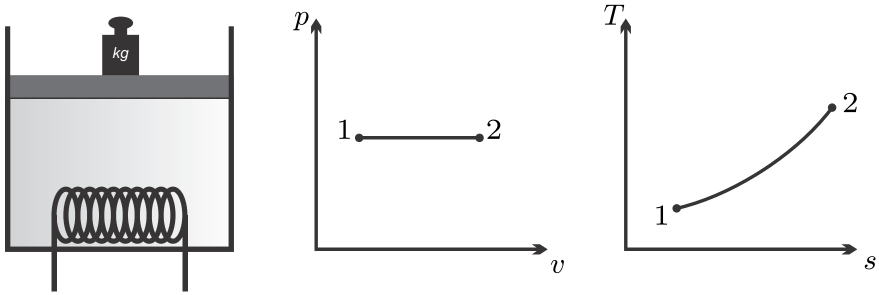


**Fig. 7.4** Isobaric process: Realization, p-v- and T-s-diagrams

## 7.4 Isobaric Process: $p = const.$, $dp = 0$

Isobaric processes (constant pressure) are easily realized by free pistons, where the piston weight controls the pressure; see Fig. 7.4 for process sketch and diagrams.

With $dp = 0$ in the constant pressure process, heat and work follow from (7.7, 7.8) as

$$w_{12} = \int_1^2 pdv = p \int_1^2 dv = p\left(v_2 - v_1\right) \; ,$$
$$q_{12} = \int_1^2 Tds = \int_1^2 (dh - vdp) = \int_1^2 dh = h_2 - h_1 \; . \tag{7.12}$$

Here we have used the Gibbs equation in the form (4.13), $Tds = dh - vdp$.

Again we compute the process curve in the T-s-diagram for an ideal gas with constant specific heats. From the Gibbs equation and the caloric equation of state we find for the isobaric process

$$Tds = dh - vdp = dh = c_p dT \ , \tag{7.13}$$

so that upon integration

$$s - s_1 = c_p \ln \frac{T}{T_1} \quad \text{or} \quad T = T_1 e^{\frac{s-s_1}{c_p}} \ . \tag{7.14}$$

This was used for drawing the curve in the diagram. For an ideal gas, the isobaric process in the T-s-diagram follows an exponential. Since $c_p = c_v + R > c_v$, isobaric lines in the T-s-diagram are not as steep as isochoric lines starting at the same point.

## 7.5 Isentropic Process: $q_{12} = \delta q = ds = 0$

A system that is insulated against heat transfer is adiabatic. However, adiabatic processes are also realized if the process is sufficiently fast, so that there is no time to exchange heat. A pressure disturbance at the boundary, i.e., induced by the moving piston, travels with the speed of sound, and accordingly mechanical equilibrium in the working fluid is assumed rather fast. A temperature disturbance at the boundary, however, diffuses slowly into the working fluid. In other words, pressure equilibration and heat transfer occur on quite distinct time scales. Accordingly, a compression (i.e., pressure increase) or expansion (i.e., pressure decrease) process may be slow enough to allow for pressure equilibration in the system, but at the same time may be so fast that there is no time to exchange heat between the working fluid and the system walls, even if they have different temperatures. Such a process can be modelled to be (approximately) adiabatic.

From the relation between entropy and heat for reversible processes (7.6) follows for an adiabatic process

$$\delta q = 0 = Tds \Longrightarrow ds = 0 \ , \ s = const. \tag{7.15}$$

The reversible adiabatic process is isentropic, see Fig. 7.5 for process sketch and diagrams.

The work is best computed from (7.8) which gives

$$w_{12} = u_1 - u_2 \ . \tag{7.16}$$

We study the isentropic process in the ideal gas in more detail: From (6.21) follows

$$s_2 - s_1 = s^0 \left( T_2 \right) - s^0 \left( T_1 \right) - R \ln \frac{p_2}{p_1} = 0 \tag{7.17}$$

or

$$\frac{p_2}{p_1} = \frac{a \exp \left[ \frac{s^0(T_2)}{R} \right]}{a \exp \left[ \frac{s^0(T_1)}{R} \right]} = \frac{p_r \left( T_2 \right)}{p_r \left( T_1 \right)} \ . \tag{7.18}$$

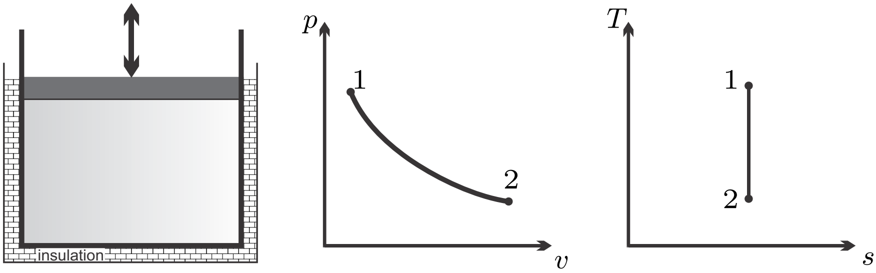


**Fig. 7.5** Isentropic process: Realization, p-v- and T-s-diagrams

$p_r(T) = a \exp\left[\frac{s^0(T)}{R}\right]$ is called the relative pressure and often is tabulated (e.g., for air); $a$ is a constant used for scaling of $p_r(T)$, its value does not affect the relation (7.18). With the ideal gas law $p = \frac{RT}{v}$ we can rewrite (7.18) as

$$\frac{v_2}{v_1} = \frac{b\frac{T_2}{p_r(T_2)}}{b\frac{T_1}{p_r(T_1)}} = \frac{v_r(T_2)}{v_r(T_1)} , \tag{7.19}$$

where $v_r(T) = b\frac{T}{p_r(T)}$ is called the relative volume, and might be tabulated as well; $b$ is another scaling constant. The ideal gas table for air in Fig. 6.17 includes columns for $p_r(T)$ and $v_r(T)$, where $a$ was chosen such that $p_r(298.15\,\mathrm{K}) = 1$, i.e., $a = \exp\left[-\frac{s^0(298.15\,\mathrm{K})}{R}\right]$, and $b = 1$; other tables might use other values of both constants.

In case of constant specific heats, the entropy is given by (6.27), which for an isentropic process gives

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = 0 . \tag{7.20}$$

Solving for the temperature ratio we find, with $R = c_p - c_v$,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} , \tag{7.21}$$

where $k$ denotes the ratio of specific heats $k = \frac{c_p}{c_v}$. By means of the ideal gas law we find the alternative relations

$$\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-k} \quad , \quad \frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{1-k} . \tag{7.22}$$

The above relations can be expressed in compact from as

$$Tp^{\frac{1-k}{k}} = const. \quad , \quad pv^k = const. \quad , \quad Tv^{k-1} = const. \tag{7.23}$$

The value of the ratio of specific heats is $k = 1.667$ for monatomic gases, and, under the cold-gas approximation, $k = 1.4$ for diatomic gases, and $k = 1.333$ for polyatomic gases. Equations (7.23) are the adiabatic relations for ideal gases with constant specific heats.

## 7.6 Isothermal Process: $T = const$, $dT = 0$

Isothermal processes require exchange of heat with a large reservoir at constant temperature. Since heat exchange is slow, isothermal processes must be rather slow and therefore they are not found in the most common thermodynamic cycles. Figure 7.6 shows process sketch and diagrams.

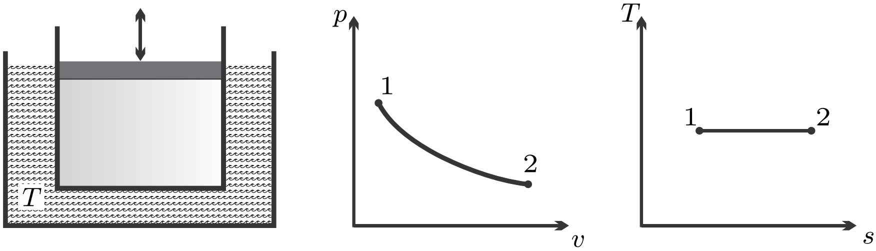


**Fig. 7.6** Isothermal process: Realization, p-v- and T-s-diagrams

Since temperature is constant, heat and work can be computed as

$$\begin{aligned} q_{12} &= \int_1^2 T ds = T \int_1^2 ds = T\left(s_2 - s_1\right) \ , \\ w_{12} &= u_1 - u_2 + q_{12} = u_1 - u_2 + T\left(s_2 - s_1\right) \ . \end{aligned} \tag{7.24}$$

We consider the special case of the ideal gas where $p = \frac{RT}{v}$, which was used to draw the curve in the p-v-diagram. With the thermal equation of state explicitly known, the work can also be determined by integration (recall $T$ is constant!),

$$w_{12} = \int_1^2 p dv = RT \int_1^2 \frac{dv}{v} = RT \ln \frac{v_2}{v_1} = -RT \ln \frac{p_2}{p_1} \ . \tag{7.25}$$

For the ideal gas the internal energy depends only on temperature, that is $du = 0$ when $dT = 0$, and thus the heat exchange is equal to the work,

$$q_{12} = w_{12} = RT \ln \frac{v_2}{v_1} = -RT \ln \frac{p_2}{p_1} \ . \tag{7.26}$$

It is left to the reader to confirm that (7.24) evaluated with the property relations for an ideal gas yields the same result.

## 7.7 Polytropic Process (Ideal Gas): $pv^n = const$

Processes in actual applications might differ from those discussed above. A useful approximate description of a wide variety of processes in ideal gases is offered by the polytropic process, which is a generalization of the adiabatic relations (7.23) to arbitrary exponents $n$,

$$Tp^{\frac{1-n}{n}} = const. \quad , \quad pv^n = const. \quad , \quad Tv^{n-1} = const. \tag{7.27}$$

Special choices for the polytropic exponent $n$ refer to the previously discussed processes as follows

$$\begin{array}{lll} n = 0 & \Rightarrow p = const & \text{isobaric} \\ n = 1 & \Rightarrow pv = RT = const. & \text{isothermal} \\ n = k & \Rightarrow pv^k = const & \text{isentropic (const. } c_p) \\ n = \infty & \Rightarrow v = const. & \text{isochoric} \end{array}$$

Often one uses values of $n$ in the interval $1 \leq n \leq k$ to describe processes that are not fully adiabatic and not fully isothermal, e.g., compression or expansion processes with small heat exchange. Figure 7.7 shows the various processes in the two diagrams.

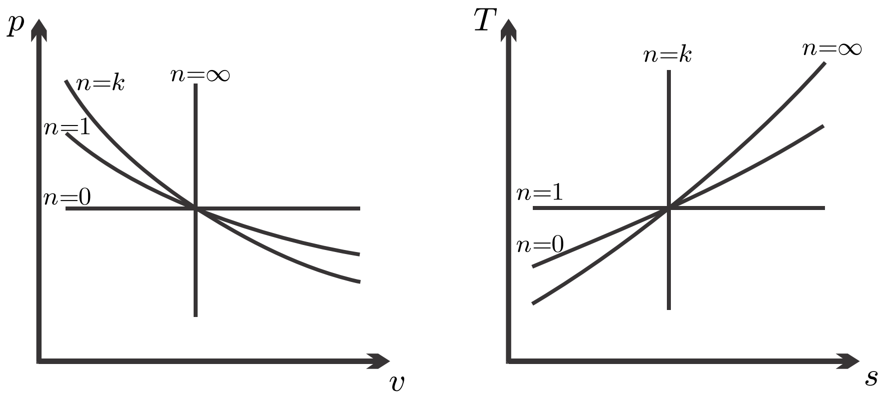


**Fig. 7.7** Polytropic processes with $n = 0, 1, k, \infty$ in p-v- and T-s-diagram

Since $pv^n = p_1 v_1^n$, the work follows by integration as

$$w_{12} = \int_1^2 p dv = p_1 v_1^n \int_1^2 \frac{dv}{v^n} = \frac{p_1 v_1}{1-n} \left[ \left( \frac{v_2}{v_1} \right)^{1-n} - 1 \right] = \frac{R}{1-n} (T_2 - T_1) \ , \tag{7.28}$$

which holds for all $n \neq 1$. The work for the case $n = 1$ (isothermal) can be found from the above by using l'Hôpital's rule:

$$\lim_{n\to 1} w_{12} = \lim_{n\to 1} \frac{RT_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right] = RT_1 \lim_{n\to 1} \frac{\frac{d}{dn}\left(\frac{v_2}{v_1}\right)^{1-n}}{\frac{d}{dn}(1-n)} = RT_1 \ln\frac{v_2}{v_1} \,. \tag{7.29}$$

The heat exchanged follows from the first law, $q_{12} = u_2 - u_1 + w_{12}$.

## 7.8 Summary

For easy reference, we collect the results of this section in a table,

| | | | |
|---|---|---|---|
| isochoric | $dv = 0$ | $w_{12} = 0$ | $q_{12} = u_2 - u_1$ |
| isobaric | $dp = 0$ | $w_{12} = p\,(v_2 - v_1)$ | $q_{12} = h_2 - h_1$ |
| isentropic | $ds = 0$ | $w_{12} = u_1 - u_2$ | $q_{12} = 0$ |
| isothermal | $dT = 0$ | $w_{12} = u_1 - u_2 + q_{12}$ | $q_{12} = T\,(s_2 - s_1)$ |
| isothermal (id. gas) | $dT = 0$ | $w_{12} = RT \ln \frac{v_2}{v_1}$ | $q_{12} = w_{12}$ |
| polytropic (id. gas) | $pv^n = const.$ | $w_{12} = \frac{R}{1-n}\,(T_2 - T_1)$ | $q_{12} = u_2 - u_1 + w_{12}$ |

## 7.9 Examples

### *7.9.1 Isochoric Process for Ideal Gas*

Carbon dioxide is confined in a 10 litre tank at a pressure of $p_1 = 10\,\text{bar}$. In an isochoric heat transfer process the temperature drops from $T_1 = 670\,\text{K}$ to $T_2 = 25\,^\circ\text{C}$. We compute the heat transferred from the system and the entropy change.

The mass of carbon dioxide in the system is ($M_{CO_2} = 44\frac{\text{kg}}{\text{kmol}}$, $R_{CO_2} = 0.189\frac{\text{kJ}}{\text{kg K}}$)

$$m = \frac{pV}{RT} = 79.0\,\text{g} \,.$$

Since volume is constant, the ideal gas law gives the final pressure as

$$p_2 = p_1 \frac{T_2}{T_1} = 4.45\,\text{bar} \,.$$

Specific internal energy and specific entropy in initial and final state can be read from an appropriate table as

$$u_1 = \frac{\bar{u}(T_1)}{M} = 467.9\frac{\text{kJ}}{\text{kg}} \quad , \quad s_1 = \frac{\bar{s}^0(T_1)}{M} - R\ln\frac{p_1}{p_0} = 5.231\frac{\text{kJ}}{\text{kg K}} \,,$$

$$u_2 = \frac{\bar{u}(T_2)}{M} = 156.4\frac{\text{kJ}}{\text{kg}} \quad , \quad s_2 = \frac{\bar{s}^0(T_2)}{M} - R\ln\frac{p_2}{p_0} = 4.579\frac{\text{kJ}}{\text{kg K}} \,.$$

Since the process is isochoric, the work is zero, $w_{12} = 0$. The heat withdrawn is

$$Q_{12} = mq_{12} = m\left(u_2 - u_1\right) = -23.8\,\mathrm{kJ}\,,$$

and the total entropy change is

$$S_2 - S_1 = m\left(s_2 - s_1\right) = -51.48\frac{\mathrm{J}}{\mathrm{kg}}\,.$$

### *7.9.2 Isochoric Heating of Water*

Saturated liquid-vapor mix at 100 °C with quality $x = 0.1$ is isochorically heated until the pressure is 2.5 MPa. We compute the final state, and the heat supplied per unit mass.

From a steam table we find initial volume, specific energy and specific entropy as

$$\begin{aligned} v_1 &= \left[\left(1 - x_1\right) v_f + x_1 v_g\right]_{T=100\,^\circ\mathrm{C}} \\ &= 0.9 \times 0.001044\frac{\mathrm{m}^3}{\mathrm{kg}} + 0.1 \times 1.673\frac{\mathrm{m}^3}{\mathrm{kg}} = 0.168\frac{\mathrm{m}^3}{\mathrm{kg}}\,, \end{aligned}$$

$$\begin{aligned} u_1 &= \left[u_f + x_1 u_{fg}\right]_{T=100\,^\circ\mathrm{C}} = 418.94\frac{\mathrm{kJ}}{\mathrm{kg}} + 0.1\times 2087.5\frac{\mathrm{kJ}}{\mathrm{kg}} = 627.7\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\ s_1 &= \left[s_f + x_1 s_{fg}\right]_{T=100\,^\circ\mathrm{C}} = 1.3069\frac{\mathrm{kJ}}{\mathrm{kg\,K}} + 0.1\times 6.048\frac{\mathrm{kJ}}{\mathrm{kg\,K}} = 1.912\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\,. \end{aligned}$$

The final state is superheated vapor of the same volume at 2.5 MPa. Using steam tables (with interpolation) we find

$$T_2 = T\left(2.5\,\mathrm{MPa}, 0.168\frac{\mathrm{m}^3}{\mathrm{kg}}\right) = 645.8\,\mathrm{K}\,,$$

$$\begin{aligned} u_2 &= u\left(2.5\,\mathrm{MPa}, 0.168\frac{\mathrm{m}^3}{\mathrm{kg}}\right) = 3370.7\frac{\mathrm{kJ}}{\mathrm{kg}}\,, \\ s_2 &= s\left(2.5\,\mathrm{MPa}, 0.168\frac{\mathrm{m}^3}{\mathrm{kg}}\right) = 7.709\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\,. \end{aligned}$$

Since the process is isochoric, the work is zero, $w_{12} = 0$. The heat supplied per unit mass for this evaporation process is

$$q_{12} = u_2 - u_1 = 2743.0\frac{\mathrm{kJ}}{\mathrm{kg}}\,.$$

### 7.9.3 Isobaric Heating of Ideal Gas

200 kg of air are isobarically heated from the initial state $p_1 = 15\,\text{bar}$, $T_1 = 440\,\text{K}$ until the volume has doubled. We compute the final state, the heat supplied, and the work done by the gas.

The initial volume is

$$V_1 = \frac{mRT_1}{p_1} = 16.84\,\text{m}^3\ ;$$

specific energy and enthalpy are (from table)

$$u_1 = u\,(440\,\text{K}) = 316.11\frac{\text{kJ}}{\text{kg}}\ ,\quad h_1 = 442.39\frac{\text{kJ}}{\text{kg}}\ .$$

The initial entropy is

$$s_1 = s^0\,(T_1) - R\ln\frac{p_1}{p_0} = 7.523\frac{\text{kJ}}{\text{kg}\,\text{K}} - 0.287\frac{\text{kJ}}{\text{kg}\,\text{K}}\ln\frac{15}{1.01325} = 6.750\frac{\text{kJ}}{\text{kg}\,\text{K}}\ .$$

With the volume doubled, and the pressure constant, the final temperature is

$$T_2 = \frac{pV_2}{mR} = 2\frac{pV_1}{mR} = 2T_1 = 880\,\text{K}\ .$$

With the temperature known, the other properties are found in the table as

$$u_2 = u\,(T_2) = 658.81\frac{\text{kJ}}{\text{kg}}\ ,\quad h_2 = h\,(T_2) = 911.36\frac{\text{kJ}}{\text{kg}}\ ,$$
$$s_2^0 = s_2^0\,(T_2) = 8.258\frac{\text{kJ}}{\text{kg}\,\text{K}}\ ,\quad s_2 = s_2^0 - R\ln\frac{p_2}{p_0} = 7.485\frac{\text{kJ}}{\text{kg}\,\text{K}}\ .$$

The work done by the system is

$$W_{12} = p\,(V_2 - V_1) = 25.26\,\text{MJ}\ ,$$

while the heat exchanged is

$$Q_{12} = m\,(h_2 - h_1) = 93.79\,\text{MJ}\ .$$

### 7.9.4 Isobaric Cooling of R134a

Superheated cooling fluid R134a at initial state $p_1 = 0.18\,\text{MPa}$ and $T_1 = 40\,^\circ\text{C}$ is isobarically cooled until the temperature is $T_2 = -24\,^\circ\text{C}$. We determine initial and end state properties, the heat transfer per unit mass, and the work per unit mass.

From a vapor table we find the initial data as

$$v_1 = v\,(0.18\,\text{MPa}, 40\,°\text{C}) = 0.1373\frac{\text{m}^3}{\text{kg}}\,,$$

$$u_1 = u\,(0.18\,\text{MPa}, 40\,°\text{C}) = 261.53\frac{\text{kJ}}{\text{kg}}\,,$$

$$h_1 = h\,(0.18\,\text{MPa}, 40\,°\text{C}) = 286.24\frac{\text{kJ}}{\text{kg}}\,,$$

$$s_1 = s\,(0.18\,\text{MPa}, 40\,°\text{C}) = 1.0898\frac{\text{kJ}}{\text{kg}\,\text{K}}\,.$$

The saturation temperature for 0.18 MPa is $T_{\text{sat}} = -12.7\,°\text{C}$ and since the final temperature lies below this value, the final state is compressed liquid. We use the approximations for compressed liquid to determine[1]

$$v_2 \simeq v_f\,(T_2) = 0.00073\frac{\text{m}^3}{\text{kg}}\;,\;\; u_2 \simeq u_f\,(T_2) = 19.21\frac{\text{kJ}}{\text{kg}}\,,$$

$$h_2 \simeq h_f\,(T_2) = 19.29\frac{\text{kJ}}{\text{kg}}\;,\;\; s_2 \simeq s_f\,(T_2) = 0.0798\frac{\text{kJ}}{\text{kg}}\,.$$

Work and heat per unit mass are obtained as

$$w_{12} = p\,(v_2 - v_1) = -24.58\frac{\text{kJ}}{\text{kg}}\,,$$

$$q_{12} = h_2 - h_1 = -266.95\frac{\text{kJ}}{\text{kg}}\,.$$

### *7.9.5 Isentropic Compression of Ideal Gas*

We consider the isentropic (adiabatic reversible) compression of air with a compression ratio $V_1/V_2 = 8$. The initial state of the air is $T_1 = 290\,\text{K}$ and $p_1 = 95\,\text{kPa}$, so that $s_1^0 = s^0\,(T_1) = 7.103\frac{\text{kJ}}{\text{kg}\,\text{K}}$ (from the Table in Fig. 6.17). The final temperature must be obtained from (7.19), which reads

$$8 = \frac{v_1}{v_2} = \frac{v_r\,(T_1)}{v_r\,(T_2)} = \frac{T_1 \exp\left[\frac{s^0(T_2)}{R}\right]}{T_2 \exp\left[\frac{s^0(T_1)}{R}\right]}\,. \tag{7.30}$$

The table includes values for $v_r\,(T)$ which we use now. From the table we find $v_r\,(T_1) = 319.5$, so that $v_r\,(T_2) = v_r\,(T_1)\,/8 = 39.93$. To find $T_2$ from the table, we have to interpolate between 650 K and 660 K, which gives

$$T_2 = 650\,\text{K} + \frac{v_r\,(T_2) - v_r\,(650\,\text{K})}{v_r\,(660\,\text{K}) - v_r\,(650\,\text{K})}10\,\text{K} = 652.4\,\text{K}\,.$$

[1] The saturation pressure at $T_2$ is $p_{\text{sat}}\,(-30°\text{C}) = 84.4\text{kPa}$. The derived approximation for enthalpy adds the term $v_f\,(T_2)\,(p - p_{\text{sat}}\,(T_2)) = 0.069\frac{\text{kJ}}{\text{kg}}$ to enthalpy—here this term contributes little, and can be safely ignored.

To find the proper temperature value when $v_r$ is not provided in a table, one has to use trial and error. A first guess can be obtained from assuming constant specific heats, which yields $\hat{T}_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = 666\,\mathrm{K}$ (with $k = 1.4$). With data from the table we find for $T_2^{(1)} = 650\,\mathrm{K}$ and $T_2^{(2)} = 660\,\mathrm{K}$

$$\frac{T_1 \exp\left[\frac{s^0\left(T_2^{(1)}\right)}{R}\right]}{T_2^{(1)} \exp\left[\frac{s^0(T_1)}{R}\right]} = 7.905 \quad , \quad \frac{T_1 \exp\left[\frac{s^0\left(T_2^{(2)}\right)}{R}\right]}{T_2^{(2)} \exp\left[\frac{s^0(T_1)}{R}\right]} = 8.231 \; .$$

Linear interpolation gives a value below the estimate, $T_2 = 642.9\,\mathrm{K}$.

The internal energies for the two states are read from the table as

$$u_1 = u\left(T_1\right) = 207.74 \frac{\mathrm{kJ}}{\mathrm{kg}} \quad , \quad u_2 = u\left(T_2\right) = 475.97 \frac{\mathrm{kJ}}{\mathrm{kg}} .$$

The work required for compression is

$$w_{12} = u_1 - u_2 = -268.2 \frac{\mathrm{kJ}}{\mathrm{kg}} \; .$$

Since the process is adiabatic, $q_{12} = 0$.

### *7.9.6 Reversible and Irreversible Adiabatic Expansion*

Reversible processes require control over the process at all times. The slow expansion of a gas in a piston-cylinder system is the prototypical example for reversible processes.

To further our understanding, we consider the adiabatic expansion—reversible and irreversible—of air as ideal gas at initial state $p_1 = 10\,\mathrm{bar}$, $T_1 = 500\,\mathrm{K}$ to an end state of half the pressure, so that $p_2 = \frac{1}{2} p_1$.

For the adiabatic reversible case we can refer to the above table which tells us that

$$w_{12} = u\left(T_1\right) - u\left(T_2\right) \; ,$$

where $T_2$ follows from isentropicity of the process,

$$0 = s\left(T_2, p_2\right) - s\left(T_1, p_1\right) = s^0\left(T_2\right) - s^0\left(T_1\right) - R \ln \frac{p_2}{p_1} \; .$$

With the relative pressure $p_r\left(T\right) = a \exp\left[\frac{s^0(T)}{R}\right]$, this relation assumes the form

$$\frac{p_r\left(T_2\right)}{p_r\left(T_1\right)} = \frac{p_2}{p_1} = \frac{1}{2} \; .$$

The table in Fig. 6.17 gives $p_r\,(T_1 = 500\,\mathrm{K}) = 6.202$, hence $p_r\,(T_2) = 3.101$, and interpolation in the table yields $T_2 = 412\,\mathrm{K}$.

The tabulated relative pressure simplifies the determination of the final state. We now show how one has to proceed when $p_r$ is not in the table: The property table gives $s^0\,(T_1 = 500\,\mathrm{K}) = 7.654\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$ and with $R_{air} = 0.287\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$ and $\frac{p_2}{p_1} = \frac{1}{2}$ the above equation gives

$$s^0\,(T_2) = s^0\,(T_1) + R\ln\frac{p_2}{p_1} = 7.455\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\;.$$

Interpolation in the table gives $T_2 = 412\,\mathrm{K}$. The corresponding work per unit mass is

$$w_{12} = u\,(500\,\mathrm{K}) - u\,(412\,\mathrm{K}) = 360.34\frac{\mathrm{kJ}}{\mathrm{kg}} - 295.38\frac{\mathrm{kJ}}{\mathrm{kg}} = 64.96\frac{\mathrm{kJ}}{\mathrm{kg}}\;.$$

We compare the reversible adiabatic process to the fully irreversible process, which was discussed in Secs. 3.13, 4.13. There we saw that the temperature of the ideal gas remained constant. If the initial and final pressures are the same as for the reversible process, we compute the change of entropy as

$$s_2 - s_1 = s\,(T_1, p_2) - s\,(T_1, p_1) = s^0\,(T_1) - s^0\,(T_1) - R\ln\frac{p_2}{p_1} = 0.199\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\;.$$

In the irreversible process no useful work is produced and entropy is generated.

### *7.9.7 Isentropic Expansion of Compressed Water*

Water at $T_1 = 300\,°\mathrm{C}$, $p_1 = 20\,\mathrm{MPa}$ expands in an isentropic (adiabatic and reversible) process to $p_2 = 1\,\mathrm{atm}$. We determine the final temperature, the volume change, and the work.

The saturation temperature for the initial pressure is $T_{\mathrm{sat}}\,(p_1) = 365.8\,°\mathrm{C} > T_1$. Accordingly the initial state is compressed liquid. As will be seen, the process ends in the two phase region. Figure 7.8 shows the p-v- and T-s-diagrams.

From the property table in Fig. 6.16 we find the initial properties

$$v_1 = v\,(20\,\mathrm{MPa}, 300\,°\mathrm{C}) = 0.0013596\frac{\mathrm{m}^3}{\mathrm{kg}}\;,$$

$$u_1 = u\,(20\,\mathrm{MPa}, 300\,°\mathrm{C}) = 1306.1\frac{\mathrm{kJ}}{\mathrm{kg}}\;,$$

$$h_1 = h\,(20\,\mathrm{MPa}, 300\,°\mathrm{C}) = 1333.3\frac{\mathrm{kJ}}{\mathrm{kg}}\;,$$

$$s_1 = s\,(20\,\mathrm{MPa}, 300\,°\mathrm{C}) = 3.2071\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\;,$$

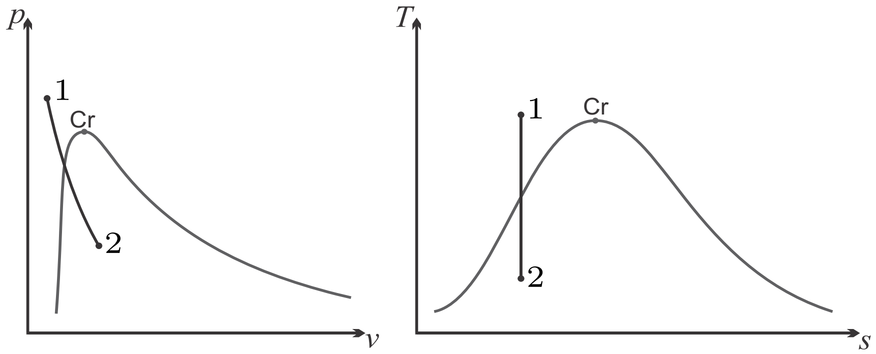


**Fig. 7.8** Isentropic expansion of compressed liquid into the two-phase region in p-v- and T-s-diagram

while the approximations of Sec. 6.9 give

$$v_1 \simeq v_f(T_1) = 0.001404 \frac{\text{m}^3}{\text{kg}} ,$$

$$u_1 \simeq u_f(T_1) = 1332.7 \frac{\text{kJ}}{\text{kg}} ,$$

$$h_1 \simeq h_f(T_1) + v_f(T_1)(p_1 - p_{sat}(T_1)) = 1360.0 \frac{\text{kJ}}{\text{kg}} ,$$

$$s_1 \simeq s_f(T_1) = 3.2534 \frac{\text{kJ}}{\text{kg K}} .$$

Comparison shows that the approximations introduce errors of {3.2%, 2.0%, 2%, 1.4%}. For the computation of the final state, we use the table values.

The final state is given by its entropy, $s_2 = s_1 = 3.2071 \frac{\text{kJ}}{\text{kg K}}$ and its pressure, $p_2 = 1\,\text{atm}$. Since $s_f(p_2) < s_2 < s_g(p_2)$ this state is saturated liquid-vapor mixture with the quality

$$x_2 = \frac{s_2 - s_f(p_2)}{s_{fg}(p_2)} = 0.314 ,$$

and the properties

$$T = 100\,^\circ\text{C} \ , \ v_2 = 0.526 \frac{\text{m}^3}{\text{kg}} \ , \ u_2 = 1074.8 \frac{\text{kJ}}{\text{kg}} ,$$

$$h_2 = 1128.2 \frac{\text{kJ}}{\text{kg}} \ , \ s_2 = 3.2071 \frac{\text{kJ}}{\text{kg K}} .$$

The volume changes quite a bit due to evaporation, $v_2 - v_1 = 0.525 \frac{\text{m}^3}{\text{kg}}$, and the expansion gives the work $w_{12} = u_1 - u_2 = 231.3 \frac{\text{kJ}}{\text{kg}}$.

### 7.9.8 Isothermal Expansion of Steam

Water vapor at $p_1 = 200\,\mathrm{bar}$ and $T = 400\,°\mathrm{C}$ is isothermally expanded to $p_2 = 1\,\mathrm{bar}$. To determine work and heat we require internal energy and entropy at the two states. From a steam table we find

$$u_1 = u\,(20\,\mathrm{MPa}, 400\,°\mathrm{C}) = 2619.3\frac{\mathrm{kJ}}{\mathrm{kg}}\ ,\ s_1 = s\,(20\,\mathrm{MPa}, 400\,°\mathrm{C}) = 5.554\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\ ,$$
$$u_2 = u\,(1\,\mathrm{bar}, 400\,°\mathrm{C}) = 2967.9\frac{\mathrm{kJ}}{\mathrm{kg}}\ ,\ s_2 = s\,(1\,\mathrm{bar}, 400\,°\mathrm{C}) = 8.5435\frac{\mathrm{kJ}}{\mathrm{kg\,K}}\ ,$$

so that heat and work are

$$q_{12} = T\,(s_2 - s_1) = 2012.4\frac{\mathrm{kJ}}{\mathrm{kg}}\ ,$$
$$w_{12} = u_1 - u_2 + q_{12} = 1663.8\frac{\mathrm{kJ}}{\mathrm{kg}}\ ;$$

note that for the computation of heat the thermodynamic temperature of $673.15\,\mathrm{K}$ must be taken.

It is interesting to compare this result with that obtained under the assumption that water vapor can be described as an ideal gas ($R = 0.462\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$), which yields

$$q_{12} = w_{12} = -RT\ln\frac{p_2}{p_1} = 1647.8\frac{\mathrm{kJ}}{\mathrm{kg}}\ .$$

We see clear differences, in particular for the heat $q_{12}$, which are due to real gas effects. Clearly, the assumption of ideal gas behavior of steam at these conditions is not justified.

### 7.9.9 Polytropic Process

A mass of $m = 3\,\mathrm{kg}$ of neon (monatomic ideal gas with $M = 20.18\frac{\mathrm{kg}}{\mathrm{kmol}}$, $R = 0.412\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$, $c_v = \frac{3}{2}R = 0.618\frac{\mathrm{kJ}}{\mathrm{kg\,K}}$) is compressed from $V_1 = 2\,\mathrm{m}^3$, $T_1 = 450\,°\mathrm{C}$ to $V_2 = 0.5\,\mathrm{m}^3$ in a polytropic process with polytropic exponent $n = 1.3$.

The polytropic exponent lies between unity and the ratio of specific heats, $k = 1.67$, and thus the process curve must lie between isothermal and isentropic lines as indicated in Fig. 7.9.

The final temperature follows from the polytropic relation as

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{n-1} = 1095.9\,\mathrm{K}\ .$$

The work can be obtained from (7.28) as

$$W_{12} = \frac{mR}{1-n}\,(T_2 - T_1) = -1536.2\,\mathrm{kJ}\ .$$

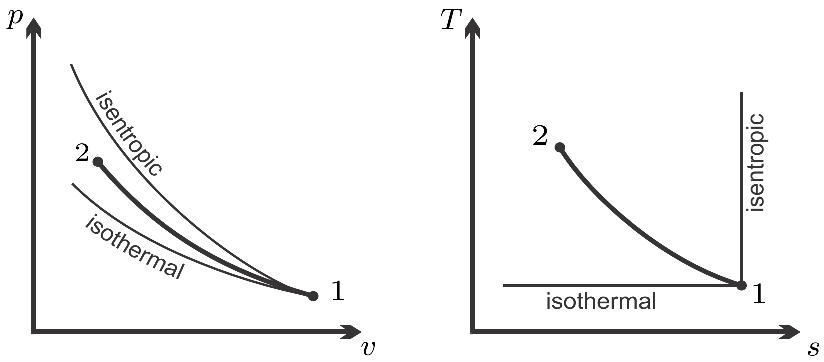


**Fig. 7.9** Polytropic process with $1 < n < k$

The heat removed from the system follows, with $U = mc_v\,(T - T_0)$, as

$$Q_{12} = U_2 - U_1 + W_{12} = m\left[c_v + \frac{R}{1-n}\right](T_2 - T_1) = -845.0\,\text{kJ}\,.$$

To draw the proper curve in the T-s-diagram, it is best to compute the entropy change,

$$\begin{aligned} S_2 - S_1 &= m\left[c_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right] \\ &= mR\left[\frac{1}{k-1} - \frac{1}{n-1}\right]\ln\frac{T_2}{T_1} = -0.942\frac{\text{kJ}}{\text{K}}\,. \end{aligned}$$

Thus, in this compression process, entropy decreases, and temperature grows, as indicated in the diagram.

## Problems

### 7.1. Water in Tank

A closed rigid tank contains 3 kg of saturated water vapor, initially at 140 °C. Heat transfer occurs, and the pressure drops to 200 kPa. Kinetic and potential energy effects are negligible. Determine heat and work exchanged during the process.

### 7.2. Isochoric Heating of Air

2 kg of air are heated in a reversible process at constant volume. The initial temperature and pressure are $T_1 = 20\,°\text{C}$ and $p_1 = 2\,\text{bar}$, and the final temperature is $T_2 = 500\,\text{K}$. Compute heat and work exchanged, and the change in entropy. Draw the process in p-v- and T-s-diagrams.

**7.3. Water in Tank**
A closed rigid tank contains 2 kg of saturated water (liquid and vapor), initially at 0.2 MPa with a quality of 4.65%. How much heat must be added so that the final state is saturated vapor? What is the final temperature, and how much work is required?

**7.4. Heating and Melting of Ice**
2 kg of ice are initially at −20 °C and 1 bar. The ice is isobarically heated, then melted and further heated until a temperature of 20 °C is reached. Determine the heat required for this process, the volume change, and the work. Determine also the heat required to heat the ice to 0 °C and for melting at 0 °C. The heat of melting at 1 bar is $h_{sf} = 333.1 \frac{\mathrm{kJ}}{\mathrm{kg}}$, and the specific heat of ice is $c_{ice} = 2.1 \frac{\mathrm{kJ}}{\mathrm{kg\,K}}$.

**7.5. Freezing of Water**
1.6 kg of liquid water are initially at 15 °C and 1 bar. The water is isobarically cooled, then frozen and further cooled until a temperature of −15 °C is reached. Determine the heat required for this process, the volume change, and the work. The heat of melting at 1 bar is $h_{sf} = 333.1 \frac{\mathrm{kJ}}{\mathrm{kg}}$, and the specific heat of ice is $c_{ice} = 2.1 \frac{\mathrm{kJ}}{\mathrm{kg\,K}}$.

**7.6. Condensation of Steam**
Steam (water vapor) initially at 30 bar, 450 °C is isobarically cooled until the volume is one half of the initial volume.

1. Draw the process in a p-v- and in a T-s-diagram with respect to saturation lines.
2. Determine heat and work for the process when the initial volume was $2\,\mathrm{m}^3$.
3. Now the volume is fixed and heat is supplied. At what temperature is the saturated vapor state reached?

**7.7. Lowering of a Piston**
A freely moving piston with cross section $A = 0.1\,\mathrm{m}^2$ and mass $m = 2\,\mathrm{t}$ closes a cylinder filled with air; the external pressure is 1 atm. The initial state in the cylinder is $V_1 = 0.3\,\mathrm{m}^3$, $T_1 = 500\,\mathrm{K}$. Heat is withdrawn, and the piston moves down as the volume of the gas decreases. The piston movement stops when the volume reaches 2/3 of the original volume, but there is further cooling until the temperature is 270 K. Compute the mass of air in the cylinder, and the total amounts of work and heat exchanged. Draw the process in p-v and T-s-diagrams.

**7.8. Cooling of Air**
10 grams of air at 1400 K, 150 bar are cooled in a closed system. The total heat withdrawn is 7936 J and the final temperature is 600 K. The cooling occurs first at constant pressure (from state 1 to state 2), and then at constant volume (from state 2 to the final state 3). Compute first the temperature at state 2, and then the pressure at state 3. Also determine the work done by the process.

**7.9. Isentropic Compression of Saturated Liquid-Vapor Mixture**
Saturated liquid-vapor mixture of water at 25 °C with a quality of $x = 0.9$ is compressed in an adiabatic reversible process to 175 bar. Determine the temperature of the final state, and work and heat per unit mass.

**7.10. Isentropic Expansion of Air**
Air is isentropically expanded in a closed system from $T_1 = 25\,°\mathrm{C}$ and $p_1 = 1\,\mathrm{MPa}$ to $p_2 = 2.5\,\mathrm{bar}$. Determine heat and work exchanged per unit mass. Draw the process in p-v and T-s-diagrams.

**7.11. Isentropic Expansion**
Neon and air are expanded isentropically from 1000 kPa and 500 °C to 100 kPa in a piston-cylinder device. Which gas has the lower temperature after expansion? Why? Compute the work per unit mass for both.

**7.12. Isentropic Compression**
Which of the two gases—neon or air—has the higher final temperature as it is compressed isentropically from 100 kPa and 450 K to 1000 kPa in a piston-cylinder device? Compute the work per unit mass for both cases.

**7.13. Isentropic Expansion of Superheated R134a Vapor**
Cooling fluid R134a in a closed system is initially at 1.2 MPa, 50 °C. Then the cooling fluid is expanded in an adiabatic reversible process to 0.12 MPa. Determine the temperature of the final state, and work and heat per unit mass.

**7.14. Isentropic Expansion of R134a Vapor**
Cooling fluid R134a in a closed system is initially at 1.6 MPa, 60 °C. Then the cooling fluid is expanded in an adiabatic reversible process to 0.32 MPa. Determine the temperature of the final state, and work and heat per unit mass.

**7.15. Expansion of Air**
Air (ideal gas with variable specific heats) at 1400 K, 50 bar is expanded in a piston-cylinder system until its volume is 12 times the initial volume. Determine work and heat per unit mass (a) when the expansion is isentropic, (b) when the expansion is isothermal.

**7.16. Isothermal Compression of Water Vapor**
In a piston-cylinder system, a mass of 20 kg of water vapor initially at 3 bar, 1200 °C is isothermally compressed to 50 bar.

1. Determine heat and work for this process based on the property tables of water.
2. Assume water vapor at these conditions can be described as an ideal gas and compute work and heat based on this assumption. Compare with the result of the exact calculation and discuss the differences.

**7.17. Evaporation and Expansion**

As part of the processes in a low temperature Carnot engine, R134a undergoes the following process in a piston-cylinder system:

1-2: Isothermal evaporation and heating from saturated liquid state at $T_1 = 60\,°\mathrm{C}$ until the volume is 13 times the initial volume.

2-3: Isentropic expansion to $p_3 = 0.28\,\mathrm{MPa}$.

1. Draw the process in a p-v- and in a T-s-diagram with respect to saturation lines.
2. Determine heat and work for the process when the initial volume was $V_1 = 20$ litres.
3. What would be the thermal efficiency of the corresponding Carnot engine?

**7.18. Polytropic Compression of Oxygen**

Pure oxygen is compressed in a polytropic process with polytropic exponent $n = 1.25$ so that the final volume is half the original volume. The initial temperature is 300 K, the final pressure is 10 bar, and the work done is 40 kJ. Determine the final temperature, the initial pressure, the mass of oxygen, the heat exchanged in the process, and the change in entropy. Draw the process in p-v and T-s-diagrams.

**7.19. Polytropic Compression**

Argon gas, initially at 1 bar, 100 K, undergoes a polytropic process with $n = 1.5$ to a final pressure of 17 bar. Determine the specific work and heat transfer for the process. Argon can be treated as an ideal gas; recall that it is a monatomic gas, so the specific heats are constant.

**7.20. Polytropic Expansion**

Helium gas, initially at 20 bar, 200 K, undergoes a polytropic process with $n = 1.2$ to a final pressure of 2 bar. Determine the specific work and heat transfer for the process. Helium can be treated as an ideal gas, recall that it is a monatomic gas, so the specific heats are constant.

**7.21. Polytropic Compression**

Radon gas (Rn, $M_{\mathrm{Rn}} = 222\frac{\mathrm{g}}{\mathrm{mol}}$) initially at 4 bar, 400 K, is compressed in a piston cylinder system. After compression the measured pressure and temperature are 12 bar and 600 K, respectively. Assume that the process can be described as being polytropic, and determine the polytropic exponent $n$. Then determine the specific work and heat transfer for the process. Radon can be treated as an ideal gas; it is monatomic, hence the specific heats are constant.

**7.22. Compression of Air**

Air at $T_1 = 227\,°\mathrm{C}$, $p_1 = 1\,\mathrm{atm}$ is compressed in a piston-cylinder device to 1/3 of its original volume. Compute the work and the heat transfer per kg of air when the compression process is (a) isothermal, (b) isentropic, (c) isentropic with constant specific heats (cold air approximation), (d) polytropic with $n = 1.4$, (e) polytropic with $n = 1.1$. Draw the process curves in p-v and T-s-diagrams.

**7.23. Irreversible Expansion of Helium**
An adiabatic and rigid container is divided by a membrane so that one third of the container holds 1 kg of helium at 300 K and 100 Pa while the other part is evacuated. The membrane is destroyed, and the gas undergoes a rather fast and irreversible process until it assumes its stable equilibrium state.

1. Compute temperature and pressure in the equilibrium state, and the change of entropy for the process.
2. Design a reversible compression process that will bring the gas back to its original state (i.e. filling 1/3 of the container, 300 K, 100 Pa) and compute the work and heat exchange required.

**7.24. Ice and Saturated Liquid-Vapor Mixture**
An insulated piston–cylinder device initially contains $0.01\,\mathrm{m}^3$ of saturated liquid–vapor mixture with a quality of 0.2 at 120 °C. How much ice at 0 °C must be added isobarically to the cylinder so that after equilibrium is reached the cylinder contains saturated liquid at 120 °C? Hint: The process is isobaric, work is done.