Chapter 15

Transient and Inhomogeneous Processes in Open Systems

15.1 Introduction

So far, we have considered steady state processes in open systems, and time dependent processes in closed systems. In this chapter, to widen the scope a bit, we show some simple applications of space dependent and time dependent open systems.

The full discussion of inhomogeneous processes, steady state or transient, requires the solution of the partial differential equations of hydrodynamics, e.g., the Navier-Stokes equations and Fourier's law of heat conduction. The derivation of the transport equations is the subject of *Non-equilibrium Thermodynamics*, and their solution is a question of mathematics and numerical methods. *Fluid Dynamics*, and *Heat and Mass Transfer* are disciplines which rely heavily on the study of solutions of the appropriate transport equations.

Below, we first discuss one-dimensional co- and counter-flow heat exchangers as a simple application of inhomogeneous systems. Then, as a relatively easy application of open time-dependent systems, we consider filling and discharge processes, e.g., of gas bottles, rooms, or cavities, as long as the content can be assumed to have homogeneous properties.

15.2 Heat Exchangers

15.2.1 Basic Equations

We discuss the principles of simple heat exchange between two flows, which are either running in the same direction (co-flow), or in opposite directions (counter-flow), see Fig. 15.1 for a basic sketch of the set-up. Heat exchange is assumed to be a one-dimensional process, where the temperatures $T_A(x)$ and $T_B(x)$ of the two flows depend only on the space coordinate in flow direction,

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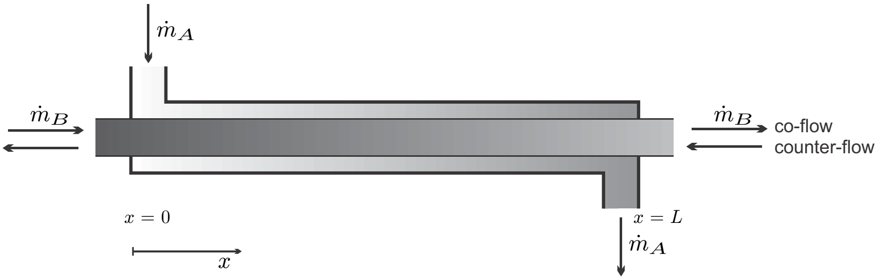

Fig. 15.1 Heat exchange between two flows A and B. Co- and counter flow settings are indicated by the arrows that show the direction of flows.

x. In particular, temperature profiles perpendicular to the flow are ignored, that is the given temperatures are cross-sectional averages.

Heat is transferred between the two flows due to different temperatures $T_A(x)$ and $T_B(x)$. Typically, the inflow temperatures of the flows are given, and one aims at determining the outflow temperatures.

Since the temperatures depend on the location x, we need to balance energy for each location. Figure 15.2 shows small elements of infinitesimal width dx and the corresponding flows and properties for co- and counter-flow settings.

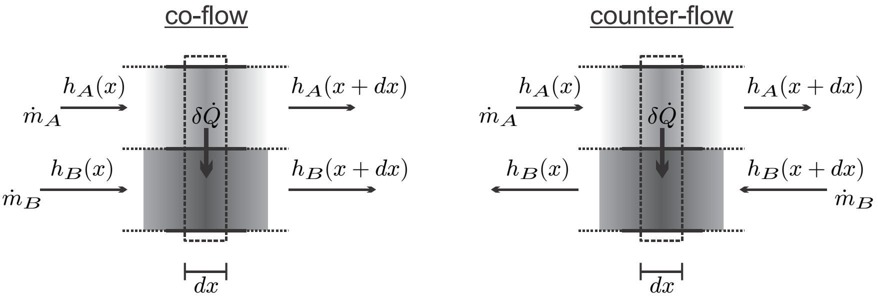

Fig. 15.2 A volume element dx of the heat exchanger, and the corresponding fluxes for co- and counter-flow heat exchangers

The task at hand is to determine the temperature curves in both flows. Applying the first law to each flow in each element gives

$$\begin{aligned} \dot{m}_A \left[h_A(x+dx) - h_A(x)\right] &= -\delta\dot{Q} \ , \\ \pm\dot{m}_B \left[h_B(x+dx) - h_B(x)\right] &= \delta\dot{Q} \ , \end{aligned} \tag{15.1}$$

where in the second equation the upper sign refers to co-flow, and the lower sign refers to counter-flow. The heat exchange between the flows within the element dx is denoted by $\delta\dot{Q}$, and is given by Newton's law of cooling (4.35), which we write here as

$$\delta\dot{Q} = \alpha \left(T_A - T_B\right) dx \ . \tag{15.2}$$

Heat transfer is proportional to the contact area bdx. In the above, the width b is absorbed into the coefficient α and only the length dx of the element is made explicit. The heat transfer coefficient α will be assumed to be constant.

Taylor expansion allows to relate the enthalpy differences to the specific heats and the temperature gradients as

$$\begin{aligned} h_A\left(x+dx\right) - h_A\left(x\right) &= c_p^A \frac{dT_A}{dx} dx \ , \\ h_B\left(x+dx\right) - h_B\left(x\right) &= c_p^B \frac{dT_B}{dx} dx \ \ ; \end{aligned} \tag{15.3}$$

for simplicity we shall assume that the specific heats are independent of temperature.

Combining all of the above yields two coupled differential equations for the temperatures,

$$\frac{dT_A}{dx} = \hat{\alpha}_A \left(T_B - T_A\right) \quad , \quad \frac{dT_B}{dx} = \mp\hat{\alpha}_B \left(T_B - T_A\right) \ , \tag{15.4}$$

with the abbreviations

$$\hat{\alpha}_A = \frac{\alpha}{\dot{m}_A c_p^A} \quad , \quad \hat{\alpha}_B = \frac{\alpha}{\dot{m}_B c_p^B} \ . \tag{15.5}$$

The coupled equations (15.4) can be integrated easily,[1] and the solutions read

$$T_A\left(x\right) = K_2 \exp\left[-\left(\hat{\alpha}_A \pm \hat{\alpha}_B\right) x\right] + \frac{\hat{\alpha}_A}{\hat{\alpha}_A \pm \hat{\alpha}_B} K_1 \ , \tag{15.6a}$$

$$T_B\left(x\right) = \mp\frac{\hat{\alpha}_B}{\hat{\alpha}_A} K_2 \exp\left[-\left(\hat{\alpha}_A \pm \hat{\alpha}_B\right) x\right] + \frac{\hat{\alpha}_A}{\hat{\alpha}_A \pm \hat{\alpha}_B} K_1 \ , \tag{15.6b}$$

where K_1 and K_2 are integrating constants, and, as in all equations in this section, the upper sign is for co-flow, and the lower sign is for counter-flow exchangers.

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[1] Take the difference of both to get an equation for $(T_B - T_A)$ that can be integrated. Then use the result to eliminate T_B in the equation for T_A, and solve for T_A.

15.2.2 Co-flow Heat Exchangers

We consider co-flow heat exchangers (upper sign) first. The known inflow conditions are the temperatures $T_A(0)$ and $T_B(0)$, for which we find from (15.6)

$$T_A(0) = K_2 + \frac{\hat{\alpha}_A}{\hat{\alpha}_A + \hat{\alpha}_B} K_1 \quad , \quad T_B(0) = -\frac{\hat{\alpha}_B}{\hat{\alpha}_A} K_2 + \frac{\hat{\alpha}_A}{\hat{\alpha}_A + \hat{\alpha}_B} K_1 \, . \tag{15.7}$$

Solving this for the constants K_1 and K_2 and inserting these into (15.6) gives the temperature curves for the co-flow heat exchanger as

$$\begin{aligned} T_A(x) &= \frac{T_A(0) + \frac{\hat{\alpha}_A}{\hat{\alpha}_B} T_B(0)}{1 + \frac{\hat{\alpha}_A}{\hat{\alpha}_B}} + \frac{T_A(0) - T_B(0)}{1 + \frac{\hat{\alpha}_B}{\hat{\alpha}_A}} \exp\left[-\left(\hat{\alpha}_A + \hat{\alpha}_B\right) x\right] \, , \\ T_B(x) &= \frac{T_A(0) + \frac{\hat{\alpha}_A}{\hat{\alpha}_B} T_B(0)}{1 + \frac{\hat{\alpha}_A}{\hat{\alpha}_B}} - \frac{T_A(0) - T_B(0)}{1 + \frac{\hat{\alpha}_A}{\hat{\alpha}_B}} \exp\left[-\left(\hat{\alpha}_A + \hat{\alpha}_B\right) x\right] \, . \end{aligned} \tag{15.8}$$

According to these equations, the two temperatures approach a common value exponentially. The common value is obtained in the limit $x \to \infty$ as

$$T_A(\infty) = T_B(\infty) = \frac{T_A(0)}{1 + \frac{\hat{\alpha}_A}{\hat{\alpha}_B}} + \frac{T_B(0)}{1 + \frac{\hat{\alpha}_B}{\hat{\alpha}_A}} \, . \tag{15.9}$$

In a heat exchanger of finite length L, the exit temperatures $T_A(L)$ and $T_B(L)$ differ from this value, see Fig. 15.3.

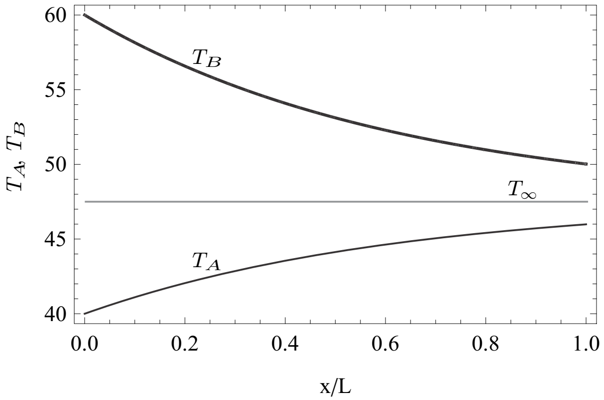

Fig. 15.3 Temperatures T_A and T_B in a co-flow heat exchanger with inflow temperatures $T_A(0) = 40\,^\circ\mathrm{C}$ and $T_B(0) = 60\,^\circ\mathrm{C}$, for $\hat{\alpha}_A/L = 0.7$ and $\hat{\alpha}_B/L = 1$. The asymptotic value $T_A(\infty) = T_B(\infty) = 47.5\,^\circ\mathrm{C}$ is shown as well.

The entropy generation is a useful measure for the work loss associated with the heat exchanger. Since we consider the heat exchanger to be adiabatic to the outside, the entropy generation is just the difference between entropy flowing in and out; for simple incompressible fluids it is computed as

$$\dot{S}_{gen} = \dot{m}_A c_p^A \ln \frac{T_A(L)}{T_A(0)} + \dot{m}_B c_p^B \ln \frac{T_B(L)}{T_B(0)} \ . \tag{15.10}$$

In a co-flow heat exchanger, all heat transfer takes places over finite temperature differences and the entropy generation is always finite. Therefore co-flow heat exchangers always have an associated work loss.

15.2.3 Counter-Flow Heat Exchangers

Now we consider counter-flow heat exchangers (lower sign). The known inflow conditions are the temperatures $T_A(0)$ and $T_B(L)$, for which we find from (15.6)

$$\begin{aligned} T_A(0) &= K_2 + \frac{\hat{\alpha}_A}{\hat{\alpha}_A - \hat{\alpha}_B} K_1 \ , \\ T_B(L) &= \frac{\hat{\alpha}_B}{\hat{\alpha}_A} K_2 \exp\left[-\left(\hat{\alpha}_A - \hat{\alpha}_B\right) L\right] + \frac{\hat{\alpha}_A}{\hat{\alpha}_A - \hat{\alpha}_B} K_1 \ . \end{aligned} \tag{15.11}$$

Solving this for the constants K_1 and K_2 and inserting these into (15.6) gives the temperature curves for the counter-flow heat exchanger as

$$\begin{aligned} T_A(x) &= T_A(0) + \left[T_B(L) - T_A(0)\right] \frac{\exp\left[\left(\hat{\alpha}_B - \hat{\alpha}_A\right) x\right] - 1}{\frac{\hat{\alpha}_B}{\hat{\alpha}_A} \exp\left[\left(\hat{\alpha}_B - \hat{\alpha}_A\right) L\right] - 1} \ , \\ T_B(x) &= T_B(L) + \left[T_B(L) - T_A(0)\right] \frac{\exp\left[\left(\hat{\alpha}_B - \hat{\alpha}_A\right) x\right] - \exp\left[\left(\hat{\alpha}_B - \hat{\alpha}_A\right) L\right]}{\exp\left[\left(\hat{\alpha}_B - \hat{\alpha}_A\right) L\right] - \frac{\hat{\alpha}_A}{\hat{\alpha}_B}} \ . \end{aligned} \tag{15.12}$$

This solution becomes singular for the special case $\hat{\alpha}_A = \hat{\alpha}_B = \hat{\alpha}$. L'Hôpital's rule must be used to find the temperature curves for this case as

$$T_A(x) = T_A(0) + \left[T_B(L) - T_A(0)\right] \frac{\hat{\alpha} x}{1 + \hat{\alpha} L} \ , \tag{15.13}$$

$$T_B(x) = T_B(L) + \left[T_B(L) - T_A(0)\right] \frac{\hat{\alpha}(x - L)}{1 + \hat{\alpha} L} \ . \tag{15.14}$$

Thus, in general, we will observe exponential curves for the temperatures, but straight lines in the case that $\hat{\alpha}_A = \hat{\alpha}_B$. Figure 15.4 shows the temperature curves for three cases with different or equal values of $\hat{\alpha}_A$ and $\hat{\alpha}_B$.

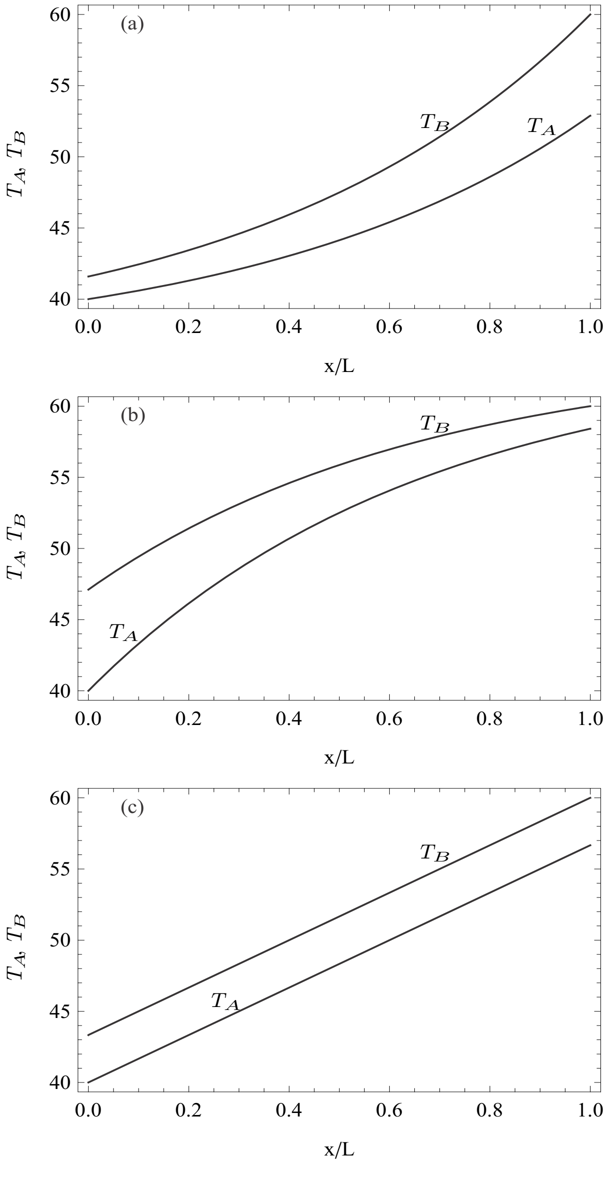

Fig. 15.4 Temperature curves for a counter-flow heat exchanger with $T_A(0) = 40\,^\circ\mathrm{C}$ and $T_B(0) = 60\,^\circ\mathrm{C}$, for the three cases (a) $\hat{\alpha}_A/L = 3.5$ and $\hat{\alpha}_B/L = 5$; (b) $\hat{\alpha}_A/L = 5$ and $\hat{\alpha}_B/L = 3.5$; (c) $\hat{\alpha}_A/L = \hat{\alpha}_B/L = 5$

In particular we note that the exit temperatures are not limited by a common mean value as for co-flow exchange, but can be quite close to the inlet temperature of the other stream. For an exchanger of length L, the exit temperatures of the two streams are

$$T_A(L) = T_A(0) + [T_B(L) - T_A(0)]\,\frac{\exp[(\hat{\alpha}_B - \hat{\alpha}_A)L] - 1}{\frac{\hat{\alpha}_B}{\hat{\alpha}_A}\exp[(\hat{\alpha}_B - \hat{\alpha}_A)L] - 1}\ ,$$

$$T_B(0) = T_B(L) + [T_B(L) - T_A(0)]\,\frac{1 - \exp[(\hat{\alpha}_B - \hat{\alpha}_A)L]}{\exp[(\hat{\alpha}_B - \hat{\alpha}_A)L] - \frac{\hat{\alpha}_A}{\hat{\alpha}_B}}\ . \tag{15.15}$$

We limit our attention to a case where $\hat{\alpha}_B < \hat{\alpha}_A$, and ask for the limiting exit temperatures for infinite length, $L \to \infty$, which are

$$T_A(L_\infty) = T_B(L_\infty)\ , \tag{15.16}$$

$$T_B(0) = T_A(0)\,\frac{\hat{\alpha}_B}{\hat{\alpha}_A} + \left(1 - \frac{\hat{\alpha}_B}{\hat{\alpha}_A}\right) T_B(L_\infty)\ . \tag{15.17}$$

Thus, stream A exits in equilibrium with stream B at $x = L_\infty$, but stream B cannot achieve equilibrium with the incoming stream A at $x = 0$. When $\hat{\alpha}_B > \hat{\alpha}$, the behavior is opposite. The only case where both exiting streams are in equilibrium with the incoming streams, in the case $L \to \infty$, is for $\hat{\alpha}_B = \hat{\alpha}_A = \hat{\alpha}$.

The above discussion already gives indication that a counter-flow heat exchanger works particularly well when $\hat{\alpha}_B = \hat{\alpha}_A = \hat{\alpha}$, which is the case when the mass flows are matched such that $\dot{m}_A c_p^A = \dot{m}_B c_p^B$, see (15.5). The discussion of the entropy generation rate of the heat exchanger sheds more light on this. Again ignoring heat loss to the exterior, the entropy generation is

$$\dot{S}_{gen} = \dot{m}_A c_p^A \ln\frac{T_B(L)}{T_B(0)} + \dot{m}_B c_p^B \ln\frac{T_B(0)}{T_B(L)}\ , \tag{15.18}$$

and Fig. 15.5 shows the reduced entropy generation rate

$$\frac{\dot{S}_{gen}}{\sqrt{\dot{m}_A c_p^A \dot{m}_B c_p^B}} = \ln\left[\left(\frac{T_B(L)}{T_B(0)}\right)^{\sqrt{\frac{\hat{\alpha}_B}{\hat{\alpha}_A}}}\left(\frac{T_B(0)}{T_B(L)}\right)^{\sqrt{\frac{\hat{\alpha}_A}{\hat{\alpha}_B}}}\right]\ , \tag{15.19}$$

as a function of the ratio $\frac{\hat{\alpha}_A}{\hat{\alpha}_B} = \frac{\dot{m}_B c_p^B}{\dot{m}_A c_p^A}$ for various total exchanger lengths L. If the heat exchanger is sufficiently long, the entropy generation develops a minimum for $\frac{\hat{\alpha}_A}{\hat{\alpha}_B} = 1$, which therefore is the optimum condition for running counter-flow heat exchangers.

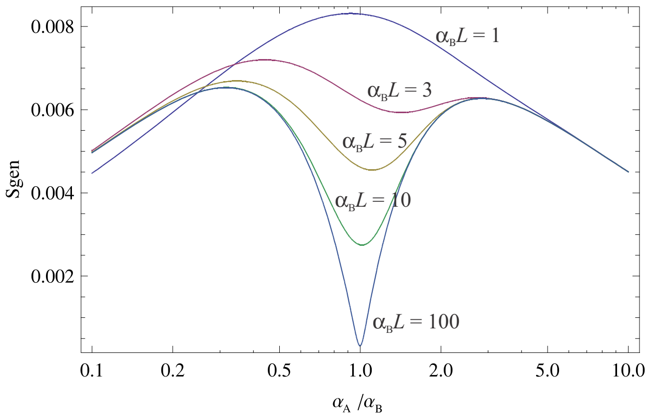

Fig. 15.5 Counter-flow heat exchanger: Reduced entropy generation rate over the ratio $\frac{\hat{\alpha}_A}{\hat{\alpha}_B} = \frac{\dot{m}_B c_p^B}{\dot{m}_A c_p^A}$ for various values of $\hat{\alpha}_B L$; note the logarithmic scale

15.2.4 Summary

In an infinitely long counter-flow heat exchanger running at optimum condition $\frac{\hat{\alpha}_A}{\hat{\alpha}_B} = 1$, both streams have the same temperatures at all locations (with an infinitesimal difference for heat transfer, of course), and no entropy is generated. In co-flow heat exchangers, the entropy generation is always non-zero, since the temperature difference at the common inlet is given by the temperatures of the incoming streams.

Realistic heat exchangers have finite length, and thus some generation of entropy, which is always less in counter-flow heat exchangers as compared to co-flow systems. In the latter, both flows approach an intermediate temperature, while in the counter-flow case, the flows approach an exchange of their temperatures: the exit temperature of one flow is close to the inlet temperature of the other. This certainly is the more desirable outcome.

Entropy generation arguments should be used in the design of heat exchangers, which should be build such that the overall entropy generation in the heat exchanger is as small as possible.

15.3 Heating of a House

As first example for transient processes in open systems, we study the heating of a house in a simplified model. The house of volume V_0 is initially in equilibrium with the outside environment at p_0, T_0. To elevate the temperature in the house to a pleasant value T_H, the house is heated at constant rate $\dot{Q}_{heat}$. Heat transfer through the walls leads to heat loss $\dot{Q}_{loss}$ to the outside. Moreover, the house exchanges air through gaps in doors, windows and walls, and we assume that the pressure stays constant at all times. We ask for the amount of heat that must be supplied to keep the house at the inside temperature T_H, and for the time required to heat the house to that temperature.

The house consists of the mass m_S of structural material (wood, stone, concrete, plaster, ...), which has the average specific heat c_S. To simplify the calculation, we shall assume that the whole structure is at the temperature $T(t)$, that is we ignore any inhomogeneous temperature distribution in the outside walls, which, in reality, would be colder further outside. The mass of air in the house is denoted as $\dot{m}_A(t)$ and is subject to change due to inflow or outflow through gaps.

The mass and energy balances for the house read

$$\frac{dm_A}{dt} = -\dot{m}_A \quad , \quad \frac{dU}{dt} = \dot{Q}_{heat} - \alpha A(T - T_0) - \dot{m}_A h_A \; , \tag{15.20}$$

where α is the heat transfer coefficient for the house, U is the total internal energy (air and structure) and A is the outside surface. Kinetic and potential energies are ignored, and the house and air have the homogeneous temperature T. The last term in the energy balance, $\dot{m}_A h_A$, is the convective outflow of energy. As long as the house is heated, we expect only outflow, due to isobaric expansion of the inside air. In cooling of the house, say at night when the heating is switched off, the air contracts, and outside air (at h_0) flows in.

Before we consider the time dependent process, we have a look at the final steady state, for which $\frac{dm_A}{dt} = \frac{dU}{dt} = 0$ and $T = T_H$, so that the final house temperature is obtained as

$$T_H = T_0 + \frac{\dot{Q}_{heat}}{\alpha A} \; . \tag{15.21}$$

This relation shows that the house temperature is controlled through adjustment of the heating rate $\dot{Q}_{heat}$. Improved insulation reduces the heat transfer coefficient α and thus the heat requirement $\dot{Q}_{heat}$.

With suitable choice of the energy constants, the internal energy of the house and the enthalpy of the air are

$$U = (m_S c_S + m_A c_v)\,T \quad , \quad h = u + RT = c_p T \; , \tag{15.22}$$

where c_v and $c_p = c_v + R$ are the specific heats of air, assumed to be constant. The mass of air in the house follows from the ideal gas law as

$$m_A = \frac{p_0 V_0}{RT} \; . \tag{15.23}$$

Combining the above, including eliminating the air mass by means of (15.23), yields a differential equation for temperature, which after some simplifications reads

$$\left(m_S c_S + c_p \frac{p_0 V_0}{RT}\right) \frac{dT}{dt} = \dot{Q}_{heat} - \alpha A \left(T - T_0\right) \ . \tag{15.24}$$

Separation of variables gives

$$\frac{m_S c_S + c_p \frac{p_0 V_0}{RT}}{\dot{Q}_{heat} - \alpha A \left(T - T_0\right)} dT = dt \ , \tag{15.25}$$

and integration between the initial state (T_0, t_0) and the final state (T, t) gives the solution[2]

$$\frac{p_0 V_0}{R} c_p \frac{\ln\left[\frac{T}{T_0} \frac{\dot{Q}_{heat}}{\dot{Q}_{heat} - \alpha A (T - T_0)}\right]}{\dot{Q}_{heat} + T_0 \alpha A} + \frac{c_S m_S}{\alpha A} \ln\left[\frac{\dot{Q}_{heat}}{\dot{Q}_{heat} - \alpha A \left(T - T_0\right)}\right] = (t - t_0) \ . \tag{15.26}$$

We can use the equilibrium condition (15.21) to write this in a more compact form,

$$m_H c_p \ln \frac{\frac{T_H}{T_0} - 1}{\frac{T_H}{T} - 1} + m_S c_S \ln\left[\frac{T_H - T_0}{T_H - T}\right] = \alpha A \left(t - t_0\right) \ , \tag{15.27}$$

where $m_H = \frac{p_0 V_0}{R T_H}$ is the final mass of air in the house. This equation describes the evolution of the inside temperature T towards T_H over time t, in an implicit way. The first term on the left hand side describes the heating of the air, and the second term describes the heating of the structure.

We consider a small house with a footprint of $10\,\mathrm{m} \times 10\,\mathrm{m}$, a height of $3\,\mathrm{m}$ and a flat roof. The air volume of the house is approximately $300\,\mathrm{m}^3$ which at final conditions (T_H, p_0) corresponds to a mass of $m_H = \frac{p_0 V_0}{R T_H} = 350\,\mathrm{kg}$. Clearly, this mass, and the corresponding heat capacity $m_H c_p$, is much less than mass m_S and heat capacity $m_S c_S$ of the structure.

If we completely ignore the contribution to air heating, we obtain for the temperature an exponential relation,

$$T = T_0 + \left(T_H - T_0\right) \left(1 - \exp\left[-\frac{\alpha A}{c_S m_S} \left(t - t_0\right)\right]\right) \ , \tag{15.28}$$

where we have used (15.21).

In a house, normally first the air is heated, e.g., by radiators or forced air heating, and then heat is transferred from the warm air to the structure. If heat transfer to the structure is slow, the structure heating can be ignored, and the air temperature approaches the final temperature according to

$$T = \frac{T_H}{1 + \left(\frac{T_H}{T_0} - 1\right) \exp\left[-\frac{\alpha A}{c_p m_A} \left(t - t_0\right)\right]} \ , \tag{15.29}$$

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[2] If you cannot do the integral with pencil and paper, you can use an integration table, or a mathematical software package like Mathematica.

where α now is the heat transfer coefficient between inside air (at T) and structure (at T_0).

The approach to the final temperature is determined by the time constants $\frac{\alpha A}{c_S m_S}$ and $\frac{\alpha A}{c_p m_A}$, respectively. Due to the large differences in mass between air and structure, the air heats up much faster than the structure.

15.4 Reversible Filling of an Adiabatic Container

When one inflates a bicycle tire with a hand pump, tire and pump become warm. While some of the heat comes from friction in the pump—no seal is friction free—a large part of the heat is due to the rise of temperature of the air being compressed.

To fix ideas we consider the simplest possible case, namely the filling of an adiabatic container with an ideal gas under the assumption that all processes are fully reversible. The system to be considered consists of the adiabatic container of volume V_0 and an adiabatic reversible compressor which draws outside air at T_0, p_0 and compresses it to the pressure p inside the container, which grows over time. Figure 15.6 gives a sketch of the system considered.

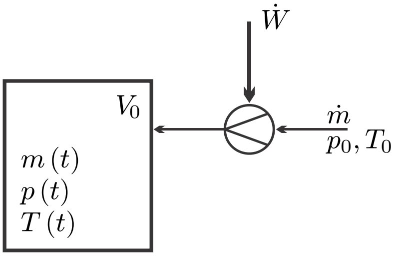

Fig. 15.6 Filling of a container with air

When all processes are reversible, it is convenient to set the system boundary such that the system contains container and compressor. Then, the balances for mass, energy and entropy become

$$\frac{dm}{dt} = \dot{m} \ , \quad \frac{dU}{dt} = -\dot{W} + \dot{m}h_0 \ , \quad \frac{dS}{dt} = \dot{m}s_0 \ , \tag{15.30}$$

where $\dot{W}$ is the power to run the compressor, and $\dot{m}$ is the mass flow pushed into the container. There is no generation of entropy due to the assumption of reversibility. We assume that initially the container is in equilibrium with the environment, so that it contains the mass $m_0 = \frac{p_0 V_0}{R T_0}$, and that the mass flow has the same value at all times, i.e., $\dot{m} = const$. Integration of the mass balance gives the mass in the container at time t as

$$m(t) = \dot{m}t + m_0 \ . \tag{15.31}$$

The entropy balance can be integrated easily, to give

$$S = \dot{m}s_0 t + S_0 \ , \tag{15.32}$$

where $S_0 = m_0 s_0$ is the entropy of the initial filling, and $S = ms$ is the entropy at time t. Combining the last two equations we find

$$S = ms = \dot{m}s_0 t + m_0 s_0 = m s_0 \ . \tag{15.33}$$

Therefore the specific entropy of the gas stays constant,

$$s = \frac{S}{m} = s_0 \ . \tag{15.34}$$

With the entropy of the ideal gas, $s - s_0 = c_p \ln \frac{T}{T_0} - R \ln \frac{p}{p_0}$, and the ideal gas law $p = \frac{mRT}{V_0}$, follows a relation between temperature and mass ($k = c_p/c_v$),

$$T(t) = T_0 \left(\frac{m(t)}{m_0} \right)^{k-1} \ . \tag{15.35}$$

As air mass is added to the container, the pressure rises to

$$p(t) = \frac{m(t) RT(t)}{V_0} = p_0 \left(\frac{m(t)}{m_0} \right)^{k} \ . \tag{15.36}$$

The air inside the container is compressed adiabatically, as is the intake air when it passes through the compressor.

Since the pressure rises, the power to drive the compressor is increasing over time. From the first law, with $U = mu = mc_v T$, $h_0 = c_p T_0$, we find

$$\dot{W} = \dot{m}h_0 - \frac{dU}{dt} = -\dot{m}c_p T_0 \left[\left(\frac{m(t)}{m_0} \right)^{k-1} - 1 \right] \ . \tag{15.37}$$

We ask for the total work required for an n-fold increase of mass, so that $m = nm_0$. The filling time is

$$t_n = \frac{(n-1) m_0}{\dot{m}} \ , \tag{15.38}$$

and the corresponding work is

$$W_n = \int_0^{t_n} \dot{W} dt = -m_0 c_v T_0 \left[n^k - 1 - k(n-1) \right] \ . \tag{15.39}$$

15.5 Reversible Discharge from an Adiabatic Container

In the previous section we filled a container with compressed air. Expansion of the compressed air yields work. Thus, compressing air into a container can be used as a means to store energy. The reversible work potential of the air in the filled container is the closed system exergy of the compressed air (11.31),

$$\Xi = m\left(u - u_E\right) + p_E\left(V - V_E\right) - T_E m\left(s - s_E\right) \ . \tag{15.40}$$

For the evaluation we must consider the appearing quantities with some care: V is the system volume, which for the present notation is the container volume V_0, while $V_E = \frac{mRT_E}{p_E}$ is the volume of the system when expanded to environmental pressure and temperature $p_E = p_0$, $T_E = T_0$. Note that, due to the reversible adiabatic filling process considered, $s = s_0 = s_E$. With this, and the filling state $m = nm_0$, T, p, as given above, the exergy of the compressed air becomes

$$\Xi_n = m_0 c_v T_0 \left[n^k - 1 - k\left(n - 1\right)\right] = -W_n \ . \tag{15.41}$$

So, not surprisingly, the work potential of the compressed air equals the work that is required for its reversible filling (with opposite sign). Thus, if all processes are fully reversible, compressed air storage gives a 100% efficient means of energy storage. All work required to fill the container could be taken out to produce work again, e.g. as electricity.

15.6 Reversible Discharge after Cooling

Real life compressed air storage will have a storage efficiency below 100% due to irreversible losses in compressor and turbine during filling and discharge, and due to energy loss by heat transfer from the hot compressed gas to the environment. To get some insight, we assume that, while filling and discharge happen adiabatically and reversibly, the container loses some heat during storage, so that the temperature drops to T_C. In this case, the state of the air in the container is

$$T = T_C \quad , \quad m_C = nm_0 \quad , \quad p_C = \frac{m_C R T_C}{V_0} \ , \tag{15.42}$$

with the reversible work potential

$$\begin{aligned}
\Xi_C &= mc_v\left(T_C - T_0\right) + p_0\left(V_0 - \frac{mRT_0}{p_0}\right) - T_0 m\left(c_p \ln\frac{T_C}{T_0} - R\ln\frac{p_C}{p_0}\right) \\
&= m_0 c_v T_0\left[\frac{p_C}{p_0} - n\ln\frac{p_C}{p_0} + n\ln n^k - 1 - \left(n - 1\right)k\right] \ .
\end{aligned} \tag{15.43}$$

Since pressure and temperature are lower than directly after filling, the work potential has dropped, $\Xi_C < \Xi_n$.

Realizing the work potential is another question. We study the inversion of the filling process, that is a turbine for the reversible discharge of the air into the environment. Again, we assume an adiabatic process with constant mass flow. During discharge, the state of the air in the container changes due to expansion. For this computation it is best to consider container and turbine separately. With the present state in the container denoted by T, p, u, s and so on, the discharge from the adiabatic container is given by mass and energy balance,

$$\frac{dm}{dt} = -\dot{m} \ , \quad \frac{dU}{dt} = -\dot{m}h \ . \tag{15.44}$$

Integration of the mass balance with the initial mass $m_C = nm_0$ gives the mass at time t as

$$m\left(t\right) = m_C - \dot{m}t \ . \tag{15.45}$$

With $U = mc_vT$ and $h = c_pT$, the first law gives the temperature of the air remaining in the container as

$$\frac{d\ln T}{dt} = -\left(1-k\right)\frac{d\ln m}{dt} \quad \Longrightarrow \quad T = T_C\left(\frac{m}{m_C}\right)^{k-1} \ . \tag{15.46}$$

The temperature drops as mass is discharged, and the same is true for pressure,

$$p = \frac{mRT}{V_0} = \frac{m_C R T_C}{V_0}\left(\frac{m}{m_C}\right)^{k} = p_C\left(\frac{m}{m_C}\right)^{k} \ . \tag{15.47}$$

The discharged gas is then expanded through the turbine from p to p_0. For the reversible adiabatic turbine the entropy is constant, so that the turbine exit temperature is

$$T_e = T\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} = T_C\left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}} \ . \tag{15.48}$$

Interestingly, while the pressure ratio changes throughout the process, the turbine exit temperature remains at the same value.

The power provided by the turbine is

$$\dot{W} = \dot{m}\left(h - h_e\right) = \dot{m}c_p\left(T - T_e\right) = \dot{m}c_pT_C\left[\left(\frac{m}{m_C}\right)^{k-1} - \left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}}\right] \ . \tag{15.49}$$

The discharge will be finished when $p = p_0$, that is

$$\left(\frac{p_0}{p_C}\right)^{\frac{1}{k}} = \frac{m_C - \dot{m}t_{end}}{m_C} \quad \Longrightarrow \quad t_{end} = \frac{m_C}{\dot{m}}\left[1 - \left(\frac{p_0}{p_C}\right)^{\frac{1}{k}}\right] . \tag{15.50}$$

Then, the mass left in the container, and the temperature, are

$$m_{end} = m_C \left(\frac{p_0}{p_C}\right)^{\frac{1}{k}} \ , \ T_{end} = T_C \left(\frac{m_{end}}{m_C}\right)^{k-1} = T_C \left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}} = T_e \ . \tag{15.51}$$

The total work delivered is

$$\begin{aligned} W_C &= \int_0^{t_{end}} \dot{W} dt = \dot{m} c_p T_C \left[\int_0^{t_{end}} \left(\frac{m}{m_C}\right)^{k-1} dt - \left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}} t_{end}\right] \\ &= \cdots = m_0 c_v T_0 \left[\frac{p_C}{p_0} - k\left(\frac{p_C}{p_0}\right)^{\frac{1}{k}} + k - 1\right] . \end{aligned} \tag{15.52}$$

The pressure p_C assumes values between the reversible filling pressure $p_n = p_0 n^k$ and np_0, which is the pressure when the compressed air is thermally equilibrated with the environment at T_0.

When no heat loss occurs ($T_C = T_n$), the turbine exit temperature is just the environmental temperature, $T_e = T_n \left(\frac{p_0}{p_n}\right)^{\frac{k-1}{k}} = T_0$, and it lies below T_0, when the air lost energy to the environment. In the extreme case that $T_C = T_0$, the turbine exit temperature is $T_0 n^{\frac{1-k}{k}} < T_0$. The actual work delivered is below the work potential (15.43), since the air leaving the turbine is colder than the environmental air, and thus there is work potential due to temperature difference between exhaust and environment. When the cold exhaust just mixes with the environmental air, entropy is produced, and this work potential is lost.

The second law efficiency for the discharge process alone is the ratio between the work produced and the initial work potential,

$$\eta_{II} = \frac{W_C}{\Xi_C} = \frac{\frac{p_C}{p_0} - 1 - k\left(\left(\frac{p_C}{p_0}\right)^{\frac{1}{k}} - 1\right)}{\frac{p_C}{p_0} - 1 - k\,(n-1) + n^k \ln\left[n\left(\frac{p_0}{p_C}\right)^{\frac{1}{k}}\right]} . \tag{15.53}$$

The second law efficiency is $\eta_{II} = 1$ for the case without heat loss in storage ($T_C = T_n$ and $T_e = T_0$) and $\eta_{II} = \left[\frac{k}{k-1}\left(n - n^{\frac{1}{k}}\right) - n+\right] / \left[\frac{n^k}{k}\ln n - n + 1\right]$ for complete heat loss ($T_C = T_0$ and $T_e = T_0 n^{\frac{k-1}{k}}$). This efficiency does only account for the work loss in discharge, but not for the loss associated to the heat loss during storage.

The storage efficiency for the complete filling and discharge process is the ratio between the work produced in discharge, and the work required for filling,

$$\eta_{st} = \left|\frac{W_C}{W_n}\right| = \frac{\frac{p_C}{p_0} - 1 - k\left(\left(\frac{p_C}{p_0}\right)^{\frac{1}{k}} - 1\right)}{n^k - 1 - k\left(n-1\right)} \ . \tag{15.54}$$

The storage efficiency is $\eta_{st} = 1$ for the case without heat loss in storage ($T_C = T_n$ and $T_e = T_0$), and $\eta_{st} = \left[n - 1 - k\left(n^{\frac{1}{k}} - 1\right)\right] / \left[n^k - 1 - k\left(n-1\right)\right]$ for complete heat loss ($T_C = T_0$ and $T_e = T_0 n^{\frac{k-1}{k}}$). This efficiency accounts for the work lost in storage and discharge.

The values on both efficiencies depend on the total mass exchange, nm_0, e.g. for $n = 20$, we find for the second law efficiency values in $[0.16, 1]$ and for the storage efficiency values in $[0.22, 1]$.

15.7 Reversible Filling of a Gas Container with Heat Exchange

In the previous sections we assumed that no heat is exchanged during filling and discharge, which would be the case when the processes are rather fast. For a lower filling rate, since real containers are not adiabatic, there will be heat exchange during filling, and here we study a container with heat exchange. We assume that the container is in contact with the environment at T_0, and use Newton's law of cooling. The balances for mass, energy and entropy become

$$\frac{dm}{dt} = \dot{m} \ , \quad \frac{dU}{dt} = \dot{Q} - \dot{W} + \dot{m}h_0 \ , \quad \frac{dS}{dt} - \frac{\dot{Q}}{T} = \dot{m}s_0 \ , \tag{15.55}$$

where $\dot{Q} = \alpha\left(T_0 - T\right)$ is the heat exchange between the compressed gas at T and the environment. As before, we assume that initially the container is in equilibrium with the environment, so that it contains the mass $m_0 = \frac{p_0 V_0}{RT_0}$, and that the mass flow is constant, $\dot{m} = const$. Integration of the mass balance gives the mass in the container at time t as

$$m\left(t\right) = \dot{m}t + m_0 \ .$$

With $S = ms$ and the mass balance, the entropy balance assumes the form

$$m\frac{ds}{dt} = \frac{\alpha\left(T_0 - T\right)}{T} - \dot{m}\left(s - s_0\right) \ . \tag{15.56}$$

The entropy of the ideal gas is

$$s - s_0 = c_v \ln\frac{T}{T_0} + R\ln\frac{V_0/m}{V_0/m_0} = c_v \ln\frac{T}{T_0} - R\ln\frac{m}{m_0} \ . \tag{15.57}$$

After some manipulation, the second law reduces to a differential equation for the gas temperature,

$$\frac{d\ln\frac{T}{T_0}}{dt} = -\frac{\alpha}{mc_v}\left(1-\frac{T_0}{T}\right) - \frac{\dot{m}}{m}\left(\ln\frac{T}{T_0} - (k-1)\left(\ln\frac{m}{m_0}+1\right)\right) \ . \tag{15.58}$$

Solving the first law for power, and use of the appropriate constitutive equation and the differential equation for T results in

$$\dot{W} = \dot{m}c_v T\left[\ln\frac{T}{T_0} - (k-1)\ln\frac{m}{m_0} + k\left(\frac{T_0}{T}-1\right)\right] \ . \tag{15.59}$$

The filling pressure follows from the ideal gas equation, $p = \frac{mRT}{V_0}$. Of course, for the adiabatic case ($\alpha = 0$) the solution of Sec. 15.4 fulfills both equations.

For the non-adiabatic case, the equation for temperature cannot be solved with pencil and paper, but must be solved on a computer. The numerical solution $T(t)$ can then be used to determine the compressor power (15.59) and the total work $W = \int \dot{W} dt$. Figure 15.7 shows, in dimensionless form, mass, temperature, pressure and compressor work as functions of time, comparing the adiabatic case ($\alpha = 0$, dashed), and the case with heat loss to the environment ($\alpha \neq 0$, continuous). Less work is required in the latter case, since, due to the constant heat loss, the temperature, and thus the pressure is lower. For the data given with the figure, the total work requirement for filling (dimensionless) is, $\frac{W}{m_0RT_0} = -34.26$ for the adiabatic case, and $\frac{W}{m_0RT_0} = -22.28$ for the non-adiabatic case.

While non-adiabatic filling requires less work, the work potential of the filling (same mass) is less as well, due to lower pressure. The heat loss to the environment will continue after filling is completed. Accordingly, the filling pressure, and the work potential will drop after filling is completed. We consider the work potential (exergy) in case that the filling temperature has dropped to the environmental temperature T_0. Then the state of the air in the cavern is

$$\frac{m}{m_0} = n \ , \ T = T_0 \ , \ p = \frac{mRT_0}{V_0} \ , \ s = c_p\ln\frac{T_0}{T_0} - R\ln\frac{p}{p_0} + s_0 \tag{15.60}$$

with the work potential (exergy)

$$\Xi_0 = m_0 c_v T_0 (k-1)(n\ln n - n + 1) \ . \tag{15.61}$$

The non-dimensional value corresponding to $n = 11$ (the final state in Fig. 15.7) is $\frac{\Xi_0}{m_0RT_0} = 16.38$. Thus, if the container is filled adiabatically and then cools to environmental temperature, the work potential is only half of the filling work. Of course, irreversibilities during filling will increase the work requirement, and irreversibilities during discharge will reduce the work output below Ξ_0 so that in the end the storage efficiency is relatively low.

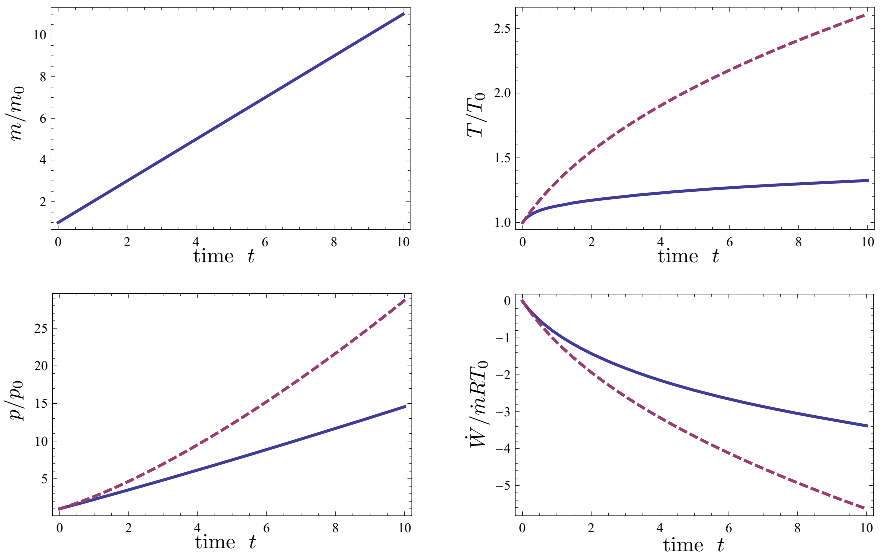

Fig. 15.7 Reversible filling of a container: Dimensionless mass $\frac{m(t)}{m_0}$, temperature $\frac{T(t)}{T_0}$, pressure $\frac{p(t)}{p_0}$, and compressor work $\frac{\dot{W}(t)}{\dot{m}RT_0}$ as functions of dimensionless time $\frac{\dot{m}}{m_0}t$ for: (a) Adiabatic filling ($\alpha = 0$, dashed curves), and (b) Filling with heat loss ($\frac{\alpha}{\dot{m}c_v} = 10$, continous curves)

15.8 CAES: Compressed Air Energy Storage

Renewable energy resources such as wind power or solar energy suffer from the fact that one cannot align the times of energy generation and energy demand. Large scale use of renewable energy therefore requires efficient storage mechanisms for the energy. Compressed air energy storage (CAES) systems are suggested as one means to this end. The idea is to use renewable energy at the time of production to fill a big cavern, e.g. an abandoned salt mine, with compressed air. Later, when energy demand arises, the compressed air in the cavity can be expanded through a turbine to generate electricity.

The filling and discharge problems discussed above can be considered as prototypes of the processes in a compressed air storage system. But, of course, what happens in realistic systems differs from the idealized description considered so far: First of all, the compressor used to fill the cavern, and the turbine used to expand the air, will not be reversible, but both will loose some work to friction. Typically, the temperature of the gas fed into the cavern by the compressor differs from the temperature of the gas in the cavern, entropy is generated as the two mix. As the air in the cavern heats up due to compression there will be heat flow from the hot gas to the colder cavern wall, entropy is generated in heat transfer over finite temperature differences. Moreover, when simply a turbine is used to expand the air leaving the cavern, the exhaust temperature will typically lie below the environmental temperature. More entropy is generated as the cold turbine exhaust mixes with the environmental air. Since entropy generation means work loss, the storage efficiency will always be below 100%.

A simplified CAES system is depicted in Fig. 15.8: An adiabatic irreversible compressor (1-2) pushes air into the cavern, the compressor power is provided by renewable energy, indicated in the figure by a wind turbine. After filling is completed, the air sits a while in the cavern (state C). Cavern temperature $T_C(t)$ and pressure $p_C(t)$ change over time. Since the simple expansion of the air through a turbine leads to cooling and external loss, one typically reheats the air that leaves the cavern. In the system studied here, the reheat occurs by means of a regenerator (C-3) and a combustion chamber (3-4), that is a fuel is used to add heat to the system. The compressed reheated air is then expanded in the turbine (4-5) and the turbine exhaust is run through the regenerator (5-6) before it is discharged into the environment.

With this set-up, a CAES system is not a pure energy storage system but a mix between a storage system and a conventional gas turbine. In a conventional gas turbine the compressor work is provided by the turbine, and thus comes from the fuel. In a CAES system, the compressor work is provided by an external source, either a regenerative source like wind or solar, or surplus electricity from conventional power plants at times of weak demand.

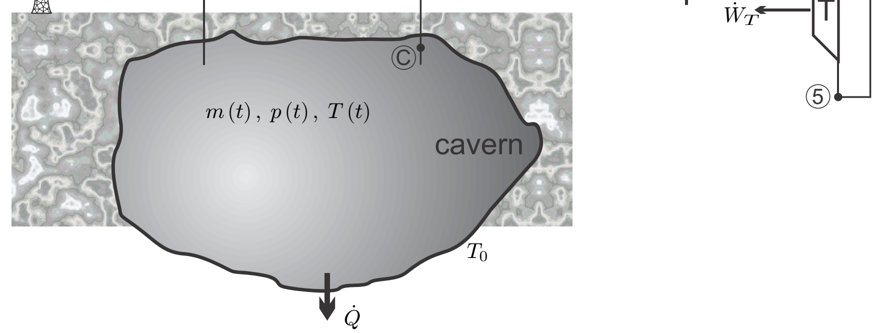

Fig. 15.8 A simple compressed air energy storage system, with reheat and regenerator

In a realistic system, the compressor power depends on the amount of renewable energy available for storage. To simplify the problem we consider constant mass flow $\dot{m}$ and treat air as ideal gas with constant specific heats. Typically, the cavern is not emptied to environmental pressure, but is maintained in a window, $p_{\min} \leq p_C(t) \leq p_{\max}$, to ensure sufficient pressure ratio in expansion and even power generation.

For compressor, regenerator and turbine, we can use the usual expressions for exit properties, work, and heat. The only difference to steady state operation is that due to the changing state in the cavern all temperatures and pressures change over time.

The adiabatic irreversible compressor draws air at environmental state $p_1 = p_0$, $T_1 = T_0$ and compresses it to the cavern pressure $p_C(t)$. The compressor power is given by

$$\dot{W}_C = \dot{m}\frac{1}{\eta_C}c_pT_0\left(1-\left(\frac{p_C}{p_0}\right)^{\frac{k-1}{k}}\right) = \dot{m}c_p\left(T_0 - T_2\right) \ , \tag{15.62}$$

where η_C is the isentropic efficiency. The compressor exit temperature follows from the second equation as

$$T_2 = T_0\left[1-\frac{1}{\eta_C}\left(1-\left(\frac{p_C}{p_0}\right)^{\frac{k-1}{k}}\right)\right] \ . \tag{15.63}$$

The regenerator with effectiveness η_{reg} heats the pressurized air to the temperature

$$T_3 = T_C + \eta_{reg}\left(T_5 - T_C\right) \ , \tag{15.64}$$

where T_5 is the turbine exit temperature. After the regenerator, the air is heated to the turbine inlet temperature T_4 by combustion of fuel. The heat added is

$$\dot{Q} = \dot{m}c_p\left(T_4 - T_3\right) \ .$$

The irreversible adiabatic turbine (isentropic efficiency η_T) expands the heated cavern air to the environmental pressure p_0, and produces the power

$$\dot{W}_T = \dot{m}\eta_T c_p T\left(1-\left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}}\right) = \dot{m}c_p\left(T_4 - T_5\right) \ . \tag{15.65}$$

The turbine exit temperature is

$$T_5 = T_4\left[1-\eta_T\left(1-\left(\frac{p_0}{p_C}\right)^{\frac{k-1}{k}}\right)\right] \ . \tag{15.66}$$

For the description of the cavern state we need the time dependent first law to describe the change. We must distinguish between the periods of filling, resting, and discharge. During filling the mass balance reads

$$\frac{dm}{dt} = \dot{m} \; , \tag{15.67}$$

and the first law reduces to

$$\frac{dU}{dt} = \alpha \left(T_0 - T_C \right) + \dot{m} h_2 \; , \tag{15.68}$$

where we have employed Newton's law of cooling to describe the heat loss to the cavern wall. With the ideal gas law, $T_C = \frac{p_C V_C}{mR}$, the internal energy can be written as

$$U = m c_v T_C = \frac{c_v}{R} m R T_C = \frac{1}{k-1} p_C V_C \; , \tag{15.69}$$

where V_C is the constant cavern volume. With this, the ideal gas law, and the above result for T_2, the first law becomes a differential equation for cavern pressure,

$$\frac{dp}{dt} = (k-1) \frac{\alpha}{V_C} \left(T_0 - \frac{p_C V_C}{mR} \right) + k \frac{\dot{m} R T_0}{V_C} \left[1 - \frac{1}{\eta_C} \left(1 - \left(\frac{p_C}{p_0} \right)^{\frac{k-1}{k}} \right) \right] . \tag{15.70}$$

While the air sits in the closed cavern, it cools down, and the pressure drops. In this case the second term is absent, and pressure changes according to

$$\frac{dp_C}{dt} = (k-1) \frac{\alpha}{V_C} \left(T_0 - \frac{p_C V_C}{mR} \right) . \tag{15.71}$$

During discharge, mass balance and first law become

$$\frac{dm}{dt} = -\dot{m} \; , \quad \frac{dU}{dt} = \alpha \left(T_0 - T_C \right) - \dot{m} h_C \; , \tag{15.72}$$

where h_C is the enthalpy of the air in the cavern. Applying the ideal gas law gives an equation for the change of pressure during discharge,

$$\frac{dp_C}{dt} = \frac{\alpha}{\frac{c_v}{R} V_C} \left(T_0 - \frac{p_C V_C}{mR} \right) - k \frac{\dot{m}}{m} p_C \; . \tag{15.73}$$

The above equations can be easily solved on the computer. To get some idea of the solution behavior, we assume a regular 24 hour cycle of four 6 hour long segments of filling–resting–discharge–resting. The cavern volume is $V_C = 250000\,\mathrm{m}^3$. All other parameters are set somewhat arbitrarily, so that the basic characteristics of the process become evident: The coefficient for Newton's law was chosen as $\alpha = 1000 \frac{\mathrm{kW}}{\mathrm{K}}$, which is high enough to almost equilibrate air and cavern wall over the resting period. The mass in the cavern, and the mass flow $\dot{m}$, are adjusted so that the cavern pressure oscillates between 20 and 50 bar. The turbine inlet temperature, which is controlled by fuel addition, is set to $T_4 = 1000\,\mathrm{K}$. The equation presented above can be solved with an easy numerical stepping system. To have no influence of initial conditions, the equations must be solved over several cycles. Figure 15.8 shows, for one cycle, mass, pressure, temperatures, work and heat for the cavern and the devices.

We point to some distinctive features: During filling, cavern pressure and temperature go up, they reach their maximum when filling stops. Subsequently, both drop, due to isochoric cooling by heat transfer to the cavern walls. In discharge both drop further due to expansion. When discharge is finished, temperature is slightly below the wall temperature, hence it increases during the low pressure rest period. Compressor power and exit temperature increase during the filling period, since the compressor has to work over an increasing pressure ratio. Turbine power decreases during discharge, since the pressure ratio goes down.

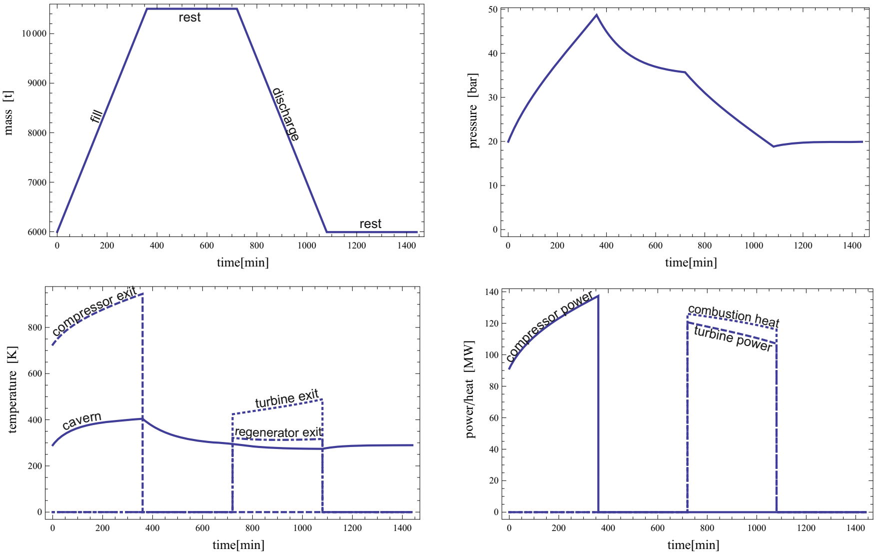

Fig. 15.9 Numerical solution for CAES plant (see text). Upper figures show the mass in the cavern, $m\,(t)$, and the pressure, $p_C\,(t)$. Left lower figure shows exit temperatures of compressor, regenerator and turbine, right lower figure shows compressor and turbine power, and heating rate.

There are several efficiency measures that can be used to evaluate the performance of the system. The simplest is the ratio between overall work produced by the turbine and energy input from renewable energy and fuel,

$$\eta_{overall} = \frac{W_T}{Q + |W_C|} = 0.475 \ . \tag{15.74}$$

Here, W_T, W_C, Q are the time integrals of $\dot{W}_T, \dot{W}_C, \dot{Q}$. This measure assigns the same importance to the renewable energy used to drive the compressor and the heat from the fuel. For the given data, less than half of the energy input is converted into work.

When the interest is mainly about the fuel usage, or the accompanying release of CO_2 into the atmosphere, which both are proportional to the heat required to produce the work output, one will use the thermal efficiency

$$\eta_{thermal} = \frac{W_T}{Q} = 0.940 \ . \tag{15.75}$$

Since the compressor work must not be provided by the turbine, the back work ratio is zero, and the thermal efficiency is about twice that of a conventional Brayton cycle with regenerator. This efficiency assigns no cost to the renewable energy.

The storage efficiency of the system is the percentage of compressor work (input) that is recovered in the expansion (output). The output comes from the fuel and the storage, and it is a question of interpretation how much can be attributed to the fuel. It seems sensible to compare the CAES with a heat engine with a thermal efficiency η_{HE}, so that its net work is $W_{HE} = \eta_{HE} Q$. The difference between the CAES turbine work, W_T and the net work W_{HE} is due to the storage, and therefore we define the storage efficiency as

$$\eta_{storage} = \frac{W_T - \eta_{HE} Q}{W_C} \ . \tag{15.76}$$

Clearly, the efficiency value depends on the type of heat engine one considers for comparison. State of the art combined cycle power plants reach thermal efficiencies of 60%, which gives a rather low storage efficiency $\eta_{storage} = 0.347$. However, the comparison between CAES and combined cycle is somewhat unfair, since both will be used for different purposes. The steam engine in the combined cycle has large start-up and shut-down times, so that the cycle must run at steady states for long periods to provide base power. By construction, the CAES only operates from time to time, and one will run it at peak power demand. Gas turbine systems have short start-up times, and therefore are often used to provide power at times of large demand. Therefore, it is more reasonable to compare CAES to a typical Brayton cycle, which, with regenerator, can have efficiencies of up to $\eta_{HE} = 42\%$. The corresponding storage efficiency is $\eta_{storage} = 0.531$.

Problems

15.1. Heating of a Room

A room of volume $75\,\mathrm{m}^3$ contains air, initially at $T_0 = 273\,\mathrm{K}$. A heater supplies heat at a rate of 2000 W. Since air can leave or enter the room through small gaps in windows and doors, the pressure in the room is equal to the outside pressure of 1 bar at all times. Assume that no air *enters* the room.

Consider air as an ideal gas with constant specific heats, and assume that all properties inside the room are homogeneous.

1. Assuming that there are no heat losses to the environment, compute mass and temperature of the air in the room as a function of time and plot the result.
2. Consider the same problem with heat losses. For this, assume that the heat losses are proportional to the difference between the temperatures inside and outside (assume $T_{outside} = T_0 = 273\,\mathrm{K}$), that is $\dot{Q}_{loss} = \alpha A\,(T_0 - T)$. Here, $\alpha = 90\frac{\mathrm{kJ}}{\mathrm{m}^2\,\mathrm{h}\,\mathrm{K}}$, is the heat transfer coefficient, and $A = 60\,\mathrm{m}^2$ is the wall surface. Plot the resulting curves.

15.2. Heat Transfer Loss in Isobaric Pipe Flow

Consider a mass flow $\dot{m}$ through a pipe of length L at steady state without pressure loss; the fluid enters the pipe at temperature T_1. The heat exchange of the pipe section of length dx with the environment at T_0 can be described by Newton's law as $\delta\dot{Q} = \alpha\,(T_0 - T)\,dx$. For the following, assume that the fluid has constant specific heat c_p and ignore kinetic and potential energies.

1. Compute the temperature of the fluid, $T\,(x)$.
2. Consider a control volume just around the fluid in the pipe, and compute entropy generation and work loss in that volume. Discuss your findings.
3. Consider a control volume around the pipe whose boundaries are at the environmental state and compute entropy generation and work loss. Show that the entropy generation is positive (for any choice of temperatures).

15.3. Friction Loss in Pipe Flow (Incompressible Fluid)

Consider a mass flow $\dot{m}$ through an adiabatic pipe of length L at steady state with pressure loss. The fluid enters the pipe at temperature T_1 and pressure p_1. Due to friction, the pressure drops as $\delta p = -\beta dx$ along the distance dx. Assume that the fluid is incompressible and has constant specific heat c (note: this means $du = cdT$, but not $dh = cdT$!). Ignore potential and kinetic energies.

1. Compute the temperature of the fluid, $T\,(x)$.
2. Consider a control volume just around the fluid in the pipe, and compute entropy generation and work loss in that volume.

15.4. Friction Loss in Pipe Flow (Ideal Gas)

Consider a mass flow $\dot{m}$ through a pipe of length L at steady state with pressure loss; the fluid enters the pipe at temperature T_1 and pressure p_1.

Due to friction, the pressure drops as $\delta p = -\beta dx$ along the distance dx; the pipe is adiabatic. Assume that the fluid is an ideal gas with constant specific heats. Ignore potential and kinetic energies.

1. Compute the temperature of the gas, $T(x)$.
2. Consider a control volume just around the gas in the pipe, and compute entropy generation and work loss in that volume.
3. Compare the results for a gas with those obtained for the incompressible fluid in the previous problem (discuss!).