Chapter 42
Bearing Capacity

In this chapter the case of a strip footing on cohesive material, considered in Chaps. 39 and 40, is extended to a general type of shallow foundation, on a soil characterized by its cohesion c, friction angle ϕ and volumetric weight γ. The soil is assumed to be completely homogeneous. Although the formulas were originally intended to be applied to foundation strips of buildings, at a shallow depth below the soil surface, they are also applied to large caisson foundations used in offshore engineering for the foundation of huge oil production platforms. Although many scientists have contributed to the analysis, the usual reference is to the Danish geotechnical engineer J. Brinch Hansen (1970).

42.1 Bearing Capacity of Strip Foundation

An important problem of foundation engineering is the computation of the maximum load (the *bearing capacity*) of a strip foundation, i.e. a very long foundation, of constant width, at a certain depth below the soil surface. The influence of the depth of the foundation is accounted for by considering a surcharge at the foundation level, to the left and the right of the applied load. For the simplest case, of a strip of infinite length, on weightless soil, the first computations were made by Prandtl (1920), see Fig. 42.1, on the basis of the assumption that in a certain region at the soil surface the stresses satisfy the equilibrium conditions and the Mohr–Coulomb failure criterion. In this entire region the soil then is on the verge of yielding. This analysis is a direct generalization of the problem considered in Chap. 40 of a strip load on the surface of a cohesive material. The foundation pressure is denoted by p. The surcharge q, next to the foundation, is supposed to be given. It can be used to represent the effect of the depth of the foundation (d) below the soil surface. In that case $q = \gamma d$, where γ is the unit weight of the soil.

Prandtl's solution, which will not be derived in detail here, again uses a subdivision of the soil into three zones, see Fig. 42.1. In zone I the horizontal stress is supposed to be larger than the vertical stress, which is equal to the surcharge q. This horizontal

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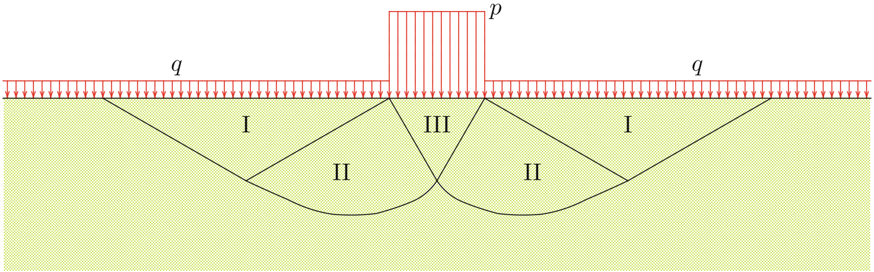


Fig. 42.1 Strip foundation

stress is then the passive lateral stress corresponding to the vertical stress q. In zone III the vertical normal stress is supposed to be the largest stress, and its value is equal to the unknown load p. The transition is formed by the wedge shaped zone II (*Prandtl's wedge*), which is bounded below by a logarithmic spiral. The results of the analysis can be written as

$$p = cN_c + qN_q, \tag{42.1}$$

where the coefficients N_c and N_q are dimensionless constants, for which Prandtl obtained the following expressions,

$$N_q = \frac{1+\sin\phi}{1-\sin\phi}\exp(\pi\tan\phi), \tag{42.2}$$

$$N_c = (N_q - 1)\cot\phi. \tag{42.3}$$

In Table 42.1 the values of N_c and N_q are given, as a function of the friction angle ϕ. In the limiting case $\phi = 0$ the value of $N_c = 2 + \pi$, as found in Chap. 40. If $c = 0$ and $\phi = 0$ the bearing capacity must be equal to the surcharge, i.e. $p = q$. Even a layer of mud can support a certain load, provided that it is the same all over its surface. This is expressed by the value $N_q = 1$ for $\phi = 0$.

Prandtl's formula (42.1) has been extended by Keverling Buisman (1940), Terzaghi (1943), Meyerhof (1952), and Brinch Hansen (1970). Various refinements were introduced, such as an inclined load, the influence of the shape of the load. The complete formula is written in the form

$$p = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma, \tag{42.4}$$

where B is the total width of the loaded strip, and γ is the volumetric weight of the soil. That all effects may be superimposed, as has been assumed in Eq. (42.4), has been confirmed by various investigations, but has never been proved rigorously. For the coefficient N_γ various suggestions have been made, on the basis of theoretical analysis or experimental evidence, for instance

Table 42.1 Bearing capacity coefficients

ϕ	N_c	N_q	N_γ
0	5.142	1.000	0.000
1	5.379	1.094	0.003
2	5.632	1.197	0.014
3	5.900	1.309	0.032
4	6.185	1.433	0.060
5	6.489	1.568	0.099
6	6.813	1.716	0.151
7	7.158	1.879	0.216
8	7.527	2.058	0.297
9	7.922	2.255	0.397
10	8.345	2.471	0.519
11	8.798	2.710	0.665
12	9.285	2.974	0.839
13	9.807	3.264	1.045
14	10.370	3.586	1.289
15	10.977	3.941	1.576
16	11.631	4.335	1.913
17	12.338	4.772	2.307
18	13.104	5.258	2.767
19	13.934	5.798	3.304
20	14.835	6.399	3.930
21	15.815	7.071	4.661
22	16.833	7.821	5.512
23	18.049	8.661	6.504
24	19.324	9.603	7.661
25	20.721	10.662	9.011
26	22.254	11.854	10.558
27	23.942	13.199	12.432
28	25.803	14.720	14.590
29	27.860	16.443	17.121
30	30.140	18.401	20.093
31	32.671	20.631	23.591
32	35.490	23.177	27.715
33	38.638	26.092	32.590
34	42.164	29.440	38.366
35	46.124	33.296	45.228
36	50.586	37.753	53.404
37	55.630	42.920	63.178
38	61.352	48.933	74.899
39	67.867	55.957	89.007
40	75.313	64.195	106.054

$$N_\gamma = 2(N_q - 1)\tan\phi. \tag{42.5}$$

There appears to be general agreement on the character of this expression, but various researchers have proposed different values for the constant factor. Brinch Hansen used a factor $\frac{3}{2}$ rather than a factor 2, probably to avoid an overestimation, and therefore including some extra safety. In modern engineering it is considered that safety factors should be kept apart from the theoretical formulas, so that it was agreed that the best value of the multiplication factor is 2. A safety factor must be taken into account explicitly, in the design stage, by reducing the soil strength, or as a load factor.

Later the formula (42.4) has been further extended with various correction coefficients, in order to take into account the shape of the loaded area, the inclination of the load, a possible inclined soil surface, and a possible inclined loading area. Most of these effects were assembled into a single formula by Brinch Hansen,

$$p = i_c s_c c N_c + i_q s_q q N_q + i_\gamma s_\gamma \frac{1}{2}\gamma B N_\gamma. \tag{42.6}$$

In this equation the coefficients i_c and i_q are correction factors for a possible inclination of the load (*inclination factors*), and s_c and s_q are correction factors for the shape of the loaded area (*shape factors*). Some other factors may be used (for a sloping soil surface, or a sloping foundation plate), but these are not considered here.

42.2 Inclination Factors

In case of an inclined load, i.e. loading by a vertical force and a horizontal load, see Fig. 42.2, the bearing capacity is considerably reduced. This can be understood by noting that sliding would occur if the horizontal force approaches the maximum possible shear force on the foundation surface,

$$t_{\max} = c + p\tan\phi.$$

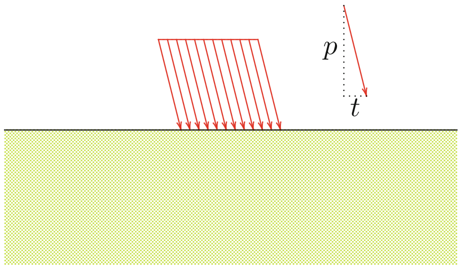
The formulas should be such that for this limiting value of the shear stress t (with respect to the constant value of the vertical stress p) the bearing capacity reduces to zero.

For cases in which the shear force is smaller than its maximum possible value, the correction factors for the inclination of the load are usually expressed as

$$i_c = 1 - \frac{t}{c + p\tan\phi}, \tag{42.7}$$

$$i_q = i_c^2, \tag{42.8}$$

Fig. 42.2 Inclined load



$$i_\gamma = i_c^3. \tag{42.9}$$

There is no general agreement on the precise value of these reduction factors, on an international level and even on a national level. Various researchers prefer slightly different values, and even the national standards may give different values.

The formulas given above at least are in agreement with certain special cases. The coefficients approach 0 if the shear stress approaches the maximum value $t_{\max} = c + p\tan\phi$. The other extreme case is when the load is vertical ($t = 0$). Then all factors reduce to 1, as required.

42.3 Shape Factors

If the shape of the foundation area is not an infinitely long strip, but a rectangular area, of width B and length L (where it is assumed, for definiteness, that the width is the smallest dimension, i.e, $L \geq B$), the usual correction factors are of the form

$$s_c = 1 + 0.2\,\frac{B}{L}, \tag{42.10}$$

$$s_q = 1 + \frac{B}{L}\,\sin\phi, \tag{42.11}$$

$$s_\gamma = 1 - 0.3\,\frac{B}{L}. \tag{42.12}$$

There is no international agreement on the precise values of these correction factors either. Some consultants prefer to take $s_q = 1$, for all values of ϕ, and some use coefficients with slightly different values for the factors 0.2 and 0.3. It may be noted that for $B/L = 0$, the formulas all give a factor 1, in agreement with the basic results for an infinite strip. It should also be remembered that $B/L \leq 1$, by definition (Fig. 42.3).

Some justification for values of the coefficients s_c and s_q larger than 1 is that when loading a rectangular plate, some of the soil surrounding the plate will also deform

Fig. 42.3 Rectangular area

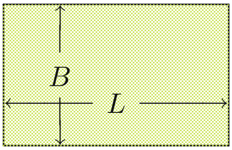
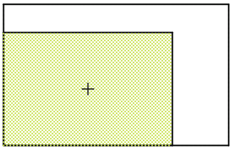


Fig. 42.4 Eccentric load



with the plate, so that a larger area contributes to the bearing capacity of the plate. The coefficient s_γ is smaller than 1 because tests indicate that for a rectangular plate in sand a sliding surface may occur that is less deep than the sliding surface for a long strip.

In case of an eccentric resultant force of the load, the width B and the length L may be reduced such that the resulting force does apply in the center of the reduced area, see Fig. 42.4. Part of the foundation plate then does not contribute to the bearing capacity, at least for this loading case. It may, of course, give a contribution to the bearing capacity of other loading cases.

As mentioned before, there is no general agreement about the values of many of the correction factors, because the results obtained by researchers in different countries, from theoretical or experimental studies, appear to give different results. Great care is needed when using data from literature. When a certain value has been obtained by one single researcher, and deviates from the results of many others, that value may well be in error.

It is also very inconvenient that there is sometimes no agreement about the basic formula (42.4). In some older publications the factor $\frac{1}{2}$ is omitted. Then the values of N_γ are (approximately) half as large, so that the final result is the same, but it may give rise to some confusion when using a formula from one publication, and taking the coefficients from another publication. In this book Terzaghi's original formula has been used, as is common practice internationally.

The formulas presented in this chapter have originally been derived for foundations on land, with relatively modest dimensions, say a few square meters. The third term in Brinch Hansen's formula (42.6) then is rather small, because of the small value of B, and it is often omitted. In offshore engineering the development of *gravity foundations* has meant that production platforms may be founded on huge concrete caissons that are placed on the sea bottom, in deep water. The surface area may be up to 80 m $\times$ 80 m. For the design of such structures the bearing capacity of the foundation is of great importance, and then the third term in the formulas (42.4) or (42.6), which describes the influence of the unit weight of the soil (i.e. the *gravity* term), is the most important term of all, giving the major contribution to the bearing capacity, especially in sand. This is why considerable attention has been paid to a more accurate definition of this term.

It must be emphasized that all the considerations of this chapter are restricted to dry soils, in which there is no difference between effective stresses and total stresses. For saturated soils the formulas should be expressed in terms of effective stresses. Usually this can be accomplished simply by replacing the volumetric weight γ by the effective volumetric weight $\gamma_s - \gamma_w$. That is a simple, but very fundamental adjustment.

Example 42.1 A gravity foundation, with surface dimensions of 60 m $\times$ 60 m, is being considered as the foundation of an offshore platform, on a sandy soil, under water. The sea at the location is 200 m deep. A first estimate of the bearing capacity of the foundation is to be determined.

In the absence of detailed soil data it may be assumed that the friction angle of the sand is $\phi = 30°$. In this case there is no contribution to the bearing capacity of the soil cohesion c or the surcharge q. For a vertical load the Brinch Hansen formula (42.6) reduces to $p = s_\gamma \frac{1}{2}\gamma B N_\gamma$, where now $N_\gamma = 20.093$, see Table 42.1. Furthermore, from Eq. (42.12) it follows that for a square foundation $s_\gamma = 0.7$. Assuming that the effective volumetric weight of the sand (under water) is $\gamma = 10$ kN/m^3, the formula now gives $p = 4220$ kPa. The total bearing capacity is obtained by multiplication with the surface of 3600 m^2. This gives $F = 15190$ MN, a very large value, but the weight of the structure and the foundation itself will probably also be very large.

Example 42.2 In some countries the soil consists of a thick deposit of soft soil (clay or peat), above a layer of sand. A popular foundation method then is to use prefabricated concrete piles. Estimate the bearing capacity of a pile, if the thickness of the soft soil is 20 m, so that the pile is 20 m long. The cross section of the pile is assumed to be 40 cm $\times$ 40 cm.

A first estimate of the bearing capacity of the pile can be made using Brinch Hansen's formula, in which now the surcharge term is most important, i.e. $p = s_q q N_q$. The surcharge q now is, assuming that the effective weight of the soft soil is $\gamma = 9$ kN/m^3, $q = 180$ kN/m^2. For a square pile $s_q = 1.5$, assuming that for the sand (which determines the actual bearing capacity) $\phi = 30°$. Furthermore, from Table 42.1 it follows that $N_q = 18.4$. It now follows that $p = 4968$ kPa. The bearing capacity of the pile then is $F = 795$ kN.

An additional contribution to the bearing capacity of the pile may be from the weight of the sand below the point of the pile. This can be estimated by the term $p = s_\gamma \frac{1}{2}\gamma B N_\gamma$, with now $s_\gamma = 0.7$, $\frac{1}{2}\gamma B = 2$ kN/m^2 and $N_\gamma = 20.093$. This gives $p = 28$ kPa, which is only a small contribution, compared to the contribution of the surcharge. A pile in the sand appears to derive its bearing capacity mainly from the weight of the soft soil.

Problem 42.1 Show that the expression (42.3) tends to $\pi + 2$ if $\phi \rightarrow 0$, confirming Prandtl's original formula.

Problem 42.2 In the examples presented above it was assumed that the value of the friction angle of sand is 30°. If there is some indication that the value is larger, say $\phi = 40°$, the bearing capacity will probably be larger. Investigate this effect, using the values from Table 42.1.

References

J. Brinch Hansen, A revised and extended formula for bearing capacity, *Bulletin No. 28* (Danish Geotechnical Institute, Copenhagen, 1970)

A.S. Keverling Buisman, *Soil Mechanics (in Dutch)* (Waltman, Delft, 1940)

G.G. Meyerhof, The ultimate bearing capacity of foundations. Géotechnique **2**, 301–332 (1952)

L. Prandtl, Über die Härte plastischer Körper. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse **1920**, 74–85 (1920)

K. Terzaghi, *Theoretical Soil Mechanics* (Wiley, New York, 1943)