Chapter 36
Blum

In the previous chapter a procedure has been presented for the determination of the minimum length of a sheet pile wall, needed to ensure equilibrium. This method is such that whenever the wall is shorter than that minimum length, no equilibrium is possible, and the wall will certainly fail. This suggests that it is advisable to choose the length of the wall somewhat larger than the minimum length, as a total failure of the wall would be disastrous. If the length is taken somewhat larger than required, the bending moments may perhaps be somewhat reduced. A method of analyzing the deformation and bending of the wall has been developed in the 1950s by the German engineer H. Blum. This method is presented in this chapter, including a simple computer program.

36.1   Blum's Schematization

If the length of the sheet pile wall is somewhat larger than strictly necessary to ensure equilibrium, the passive earth pressure need not be developed over the entire length of the embedded part of the wall. It may be expected that the pressures against the wall will be of the form shown in Fig. 36.1. Because of the extra length of the sheet pile wall the toe will act as a clamped edge, in which the lowest part may have a tendency to move to the right, building up a pressure towards the left. Together with the incomplete passive pressure towards the right this will constitute the clamping moment. Blum suggested to schematize the loads on the wall as shown in the right half of the figure. The force R (the *Ersatzkraft*) is equivalent to the pressure to the right at the extreme lower part of the wall. Its precise distribution is left undetermined. The toe of the sheet pile wall is now assumed to be a clamped edge, and it is also assumed that at the toe the bending moment is zero, but a shear force (of magnitude R) is allowed. In order that this force may indeed develop, and that there is enough material to form a clamped boundary, the actual length should be somewhat larger than assumed in the schematization: usually the embedment depth is taken 20% larger than calculated.
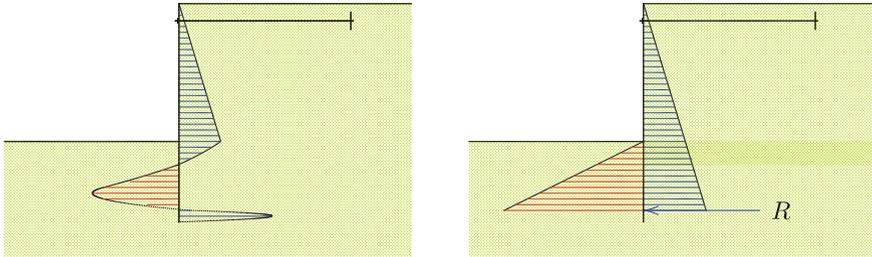
Fig. 36.1 Blum's schematization

One of the ideas behind Blum's schematization is that the clamping moment will probably lead to a reduction of the bending moments in the sheet pile wall, so that a lighter profile may be used. Thus the additional costs involved by taking a longer sheet pile wall is balanced by a lighter profile. That this is acceptable can be argued by noting that a failure by a wall that is too short is indeed disastrous, but that in case of failure by exceeding the maximum bending moment, some additional strength is available beyond the onset of plastic deformation of the steel. If a plastic deformation in bending is developed, the bending moment will at least be constant, and may even increase somewhat. Also, the soil pressures may be redistributed by the large deformations.

36.2   Blum's Method

The basic principle of Blum's method of analysis is that the sheet pile wall is considered as fully clamped at its toe, with the additional condition that the bending moment at the toe is zero. The shear force, however, in general will be unequal to zero. This shear force is supposed to be the resultant force of the stresses in the vicinity of the toe, including some length below the toe. The clamping of the edge is supposed to be so strong that the displacement and the rotation (that is the first derivative of the displacement) are zero, and even the second derivative is zero, so that the bending moment is zero. The length of the wall will be determined by the conditions of equilibrium, with active soil stresses on the high side and full passive stresses on the low side, and the condition that the horizontal displacement is zero at the level of the anchor. The procedure can best be illustrated by means of an elementary example.

The example refers to a sheet pile wall retaining a height h of homogeneous saturated soil, see Fig. 36.2. To enable an analytical solution it is assumed that on the two sides of the wall the groundwater table coincides with the soil surface. To further simplify the problem the anchor is supposed to be acting at the top of the wall. The embedment depth d is unknown. This is one of the parameters that have to be determined by the analysis.

Fig. 36.2 Example
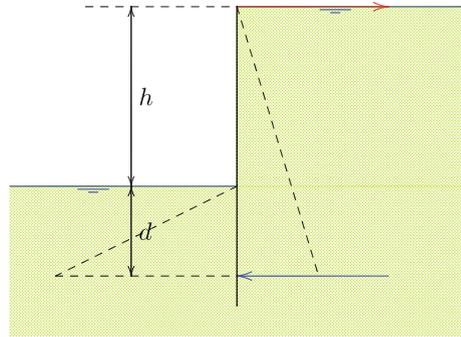
At the active side of the wall the vertical total stress is

$$\sigma_{zz} = \gamma z,$$

in which γ is the volumetric weight of the saturated soil. The pore pressures are

$$p = \gamma_w z,$$

so that the effective stresses are

$$\sigma'_{zz} = (\gamma - \gamma_w)z.$$

The horizontal effective stresses now are, for a cohesionless soil with $c = 0$,

$$\sigma'_{xx} = K_a(\gamma - \gamma_w)z.$$

The horizontal total stresses are obtained by adding the pore pressures,

$$\sigma_{xx} = [K_a(\gamma - \gamma_w) + \gamma_w]z.$$

This can also be written as

$$\sigma_{xx} = K_a^* \gamma z, \tag{36.1}$$

where

$$K_a^* = K_a(1 - \gamma_w/\gamma) + \gamma_w/\gamma. \tag{36.2}$$

If $K_a = 0.3333$ and $\gamma_w/\gamma = 0.5$, then $K_a^* = 0.6667$. It should be noted that the simple expression (36.1), linear in z, is valid only if the soil is homogeneous, with $c = 0$, and if the groundwater table coincides with the soil surface. In a more general case the computation of the horizontal total stresses proceeds in exactly the same way, but the result can not be expressed in the simple form of Eq. (36.1).

In the same way the horizontal stresses at the passive side, for $z > h$, can be determined. The result is

$$\sigma_{xx} = K_p^* \gamma (z - h), \tag{36.3}$$

where

$$K_p^* = K_p(1 - \gamma_w/\gamma) + \gamma_w/\gamma. \tag{36.4}$$

If $K_p = 3.0$ and $\gamma_w/\gamma = 0.5$, then $K_p^* = 2.0$.

The resulting active and passive forces are

$$F_a = \frac{1}{2} K_a^* \gamma (h + d)^2,$$

$$F_p = \frac{1}{2} K_p^* \gamma d^2.$$

The condition that the bending moment at the toe of the sheet pile wall must be zero, at the depth of the clamped edge, i.e. the point of application of the force R, gives

$$T(h + d) = \frac{1}{6} K_a^* \gamma (h + d)^3 - \frac{1}{6} K_p^* \gamma d^3. \tag{36.5}$$

For the computation of the horizontal displacement of the top of the sheet pile wall (which must be zero), the contribution of the three forms of loading can best be considered separately, see Fig. 36.3.

The first loading case is the anchor force T, acting at the top of the sheet pile wall. This force leads to a displacement of the top of magnitude

$$u_1 = \frac{T(h + d)^3}{3\,EI}. \tag{36.6}$$

This is a well known basic problem from applied mechanics.

For the case of a triangular load $f = az$ on a clamped beam of length l, the loading case in the central part of Fig. 36.3, the displacements can be found using the classical theory of bending of beams, from applied mechanics. By integrating the differential

Fig. 36.3 Loads on the clamped wall in Blum's schematization
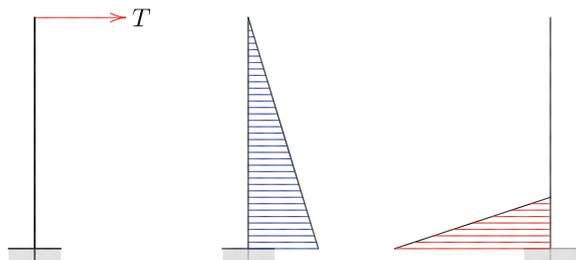
equation $EId^4u/dz^4 = f$, with the boundary conditions that at the top the bending moment and the shear force are zero, whereas at the toe the horizontal displacement u and its first derivative (the rotation) are zero, the displacement of the top can be obtained as

$$u_0 = \frac{al^5}{30\,EI}. \tag{36.7}$$

The rotation of the top is found to be

$$\varphi_0 = \frac{al^4}{24\,EI}. \tag{36.8}$$

Using these formulas the horizontal displacement of the top of the sheet pile wall caused by the active soil pressure on the right side is, with (36.1) and (36.7),

$$u_2 = -\frac{K_a^* \gamma (h + d)^5}{30\,EI}. \tag{36.9}$$

The minus sign indicates that this displacement is directed towards the left.

The displacement caused by the passive soil pressures at the left side of the sheet pile wall, as described by (36.3), is found to be

$$u_3 = \frac{K_p^* \gamma d^5}{30\,EI} + \frac{K_p^* \gamma d^4 h}{24\,EI}. \tag{36.10}$$

The first term in this expression is the displacement at the top of the load, the second term is the additional displacement due to the rotation at the top of the load. Together these two quantities constitute the displacement at the top of the sheet pile wall. The upper, unloaded part of the wall, does not deform in this loading case.

The sum of the three displacements (36.6), (36.9) and (36.10) must be zero. This gives, with (36.5), and after multiplication by $EI/K_p^* \gamma$,

$$\frac{K_a^*}{K_p^*} \frac{(h + d)^5}{18} - \frac{d^3 (h + d)^2}{18} - \frac{K_a^*}{K_p^*} \frac{(h + d)^5}{30} + \frac{d^5}{30} + \frac{d^4 h}{24} = 0,$$

or, after some rearranging of terms,

$$\left(\frac{d}{h}\right)^3 = \frac{8\,(K_a^*/K_p^*)\,(1 + d/h)^5}{20\,(1 + d/h)^2 - 15\,d/h - 12\,(d/h)^2}. \tag{36.11}$$

From this equation the value of d/h can be solved iteratively, using an initial estimate, possibly simply $d/h = 0.0$.

The computations can be made using the Program 36.1. The program only requests the input of the volumetric weights of water and (saturated) soil, and the values of the active and passive pressure coefficients, and then computes the values of

```
100 CLS:PRINT "Sheet pile wall in homogeneous saturated soil"
110 PRINT "Blum":PRINT:A$="& ####.###"
120 INPUT "Volumetric weight of water ...... ";GW
130 INPUT "Volumetric weight of soil ....... ";GG
140 INPUT "Active stress coefficient ....... ";KA
150 INPUT "Passive stress coefficient ...... ";KP
160 KSA=KA*(1-GW/GG)+GW/GG:KSP=KP*(1-GW/GG)+GW/GG:D=0
170 C=8*(KSA/KSP)*(1+D)^5/(20*(1+D)^2-15*D-12*D*D)
180 IF C<0 THEN PRINT "No solution":END
190 C=C^(1/3):E=ABS(C-D):D=C:IF E>0.000001 THEN 170
200 PRINT USING A$;"d/h   = ";D
210 T=(KSA*(1+D)^3-KSP*D^3)/(6*(1+D))
220 PRINT USING A$;"T/ghh = ";T
230 END
```

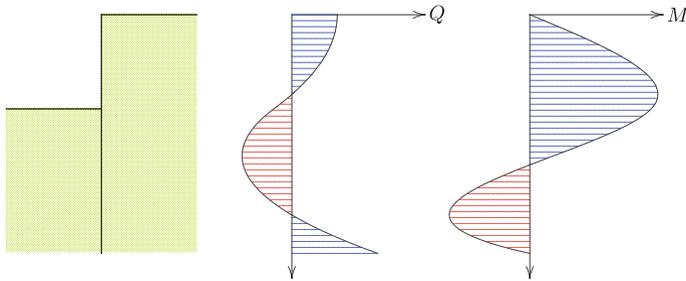**Program 36.1:** Blum's method for saturated soil

Fig. 36.4 Shear force and bending moment

d/h and $T/\gamma h^2$, using the Eqs. (36.11) and (36.5). For the case that $\mathsf{GW} = 10$, $\mathsf{GG} = 20$, $\mathsf{CA} = 0.3333$ and $\mathsf{CP} = 3.0$ the result of the program is $d/h = 1.534$ and $T/\gamma h^2 = 0.239$. It appears that in this case the sheet pile wall needs a rather long embedment depth (more than 1.5 times the retaining height). This is the price that has to be paid for a more favorable distribution of the bending moments. The profile of the steel elements can be somewhat lighter, but the length is considerably larger than in the simple method of the previous chapter.

The distribution of the shear force and the bending moment is shown in Fig. 36.4. The shear force at the top is the anchor force. The value at the toe is Blum's concentrated force R. It appears that this force results in a reduction of the bending moments in the sheet pile wall, as mentioned before. For the determination of the profile of the wall it is favorable that the positive and negative bending moments are of the same order of magnitude.

The results of the computations for a number of values of the earth pressure coefficients K_a and K_p are given in Table 36.1. It has been assumed that the volumetric weight of the water is $\gamma_w = 10$ kN/m^3, and that the volumetric weight of the saturated soil is $\gamma = 20$ kN/m^3, a common value.

The concentrated force R is an essential element in Blum's method. It should be remembered that this force actually represents the distributed load at the extreme toe

Table 36.1 Blum's method for homogeneous soil

ϕ	K_a	K_p	d/h	$T/\gamma h^2$
10°	0.7041	1.4203	5.228	0.881
15°	0.5888	1.6984	3.406	0.554
20°	0.4903	2.0396	2.481	0.394
25°	0.4059	2.4639	1.917	0.300
30°	0.3333	3.0000	1.534	0.239
35°	0.2710	3.6902	1.255	0.196
40°	0.2174	4.5989	1.040	0.165
45°	0.1716	5.8284	0.868	0.141

of the sheet pile wall, which is produced by the deformation of the sheet pile wall. For the generation of this concentrated force the wall should be given some additional length, by choosing the length of the wall somewhat larger than the theoretical value computed in the analysis. It is often assumed that the length of the embedment depth (the distance d in the example) should be taken 10% or 20% larger than computed. All this leads to a wall of considerable length. This is the price that has to be paid for the advantages of Blum's analysis: a lighter profile, and small displacements.

It may be noted that the example considered in this chapter is perhaps a very unfavorable case: the level of groundwater at the right side is very high, and on the left side it is very low. In the next chapter a more general method will be described. It must be admitted, however, that in many cases Blum's method leads to rather long sheet pile walls. The safety is perhaps large, but at a price.

Problem 36.1 Verify a number of values in Table 36.1 by substitution into Eq. (36.11), or by a computation using Program 36.1.

Problem 36.2 A sheet pile wall is used to construct a building pit in a polder. The depth of the pit is 5 m, and on both sides the groundwater level coincides with the soil surface. The sheet pile wall is supported by a strut connecting to an identical wall at the other side of the building pit. Determine the necessary length of the sheet pile wall, assuming that $c = 0$ and $\phi = 30°$.

Problem 36.3 It has been found that the friction angle in the previous problem should be 40° instead of 30°. Determine the length of the sheet pile wall for this case.

Problem 36.4 Equation (36.11) applies to saturated soil, with the groundwater level coinciding with the soil surface. Derive a similar equation for homogeneous dry soil. Then compute the value of d/h for dry soil, with $\gamma = 16$ kN/m³, $c = 0$ and $\phi = 30°$.

Problem 36.5 Verify the formulas (36.7) and (36.8) for the displacement and the rotation of the free end of a clamped beam loaded by a triangular stress.