Chapter 18
Consolidation Coefficient

In this chapter two methods to determine the coefficient of consolidation c_v are described. They are based on measurements in a one-dimensional test. However, inaccuracies in the description of the deformations in such tests require modifications in the measured displacements.

18.1 Theory Versus Test Results

If the theory of consolidation, presented in the previous chapters, were a perfect description of the physical behavior of soils, it should be rather simple to determine the value of the coefficient of consolidation c_v from the data obtained in a consolidation test. For instance, one could measure the time at which 50% of the final deformation has taken place. From the theory it follows that this is reached when $c_v t/h^2 = 0.197$, because then the value of $U = 0.5$, see formula (16.21). As the values of time t and the sample thickness h are known, it is then possible to determine the value of c_v. Unfortunately, there are some practical and some theoretical difficulties. The procedure would require an accurate determination of the initial deformation and of the ultimate deformation, and that is not so simple as it may seem. The initial deformation of the sample, Δh_0, is the deformation at the moment of application of the load, and at the moment of loading the indicator of the deformation will suddenly start to move, with a sudden jump followed by a continuous increase. It is difficult to decide what the value at the exact moment of loading is, as the moment is gone when the indicator starts to move. Also, it usually appears that no final constant value of the deformation, Δh_∞ is reached, as the deformation seems to continue, even when the pore water pressures have been dissipated completely. For these reasons somewhat modified procedures have been developed to define the *initial deformation* and the *final deformation*. In this chapter the two most common procedures are presented.

© Springer International Publishing AG 2018

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_18

18.2 Log(T)-Method

A first method to overcome the difficulties of determining the initial value and the final value of the deformation has been proposed by Professor Arthur Casagrande of Harvard University, Cambridge (USA), see Taylor (1948). In this method the deformation of the sample, as measured as a function of time in a consolidation test, is plotted against the logarithm of time, see Fig. 18.1. It usually appears that there is no horizontal asymptote of the curve, as the classical theory predicts, but for very large values of time a straight line is obtained, see also the next chapter. It is now postulated, somewhat arbitrarily, that the intersection point of the straight line asymptote for very large values of time, with the straight line that can be drawn tangent to the measurement curve at the inflection point (that is the steepest possible tangent), is considered to determine the final deformation of the primary consolidation process. The continuing deformation beyond that deformation is denoted as secondary consolidation, representing deformation at practically zero pore pressures. This procedure is indicated in Fig. 18.1, leading to the value Δh_∞ for the final deformation.

In order to define the initial settlement of the loaded sample use is made of the knowledge, see Chap. 16, that in the beginning of the consolidation process the degree of consolidation increases proportional to $\sqrt{t}$. This means that between $t = 0$ and $t = t_1$ the deformation will be equal to the deformation between $t = t_1$ and $t = 4t_1$. If the deformation is measured after 1 min and after 4 mins, it can be assumed that between $t = 0$ and $t = 1$ min the deformation would have been the same as the deformation between $t = 1$ min and $t = 4$ mins. This procedure has been indicated in Fig. 18.1, leading to the value Δh_0 for the initial deformation.

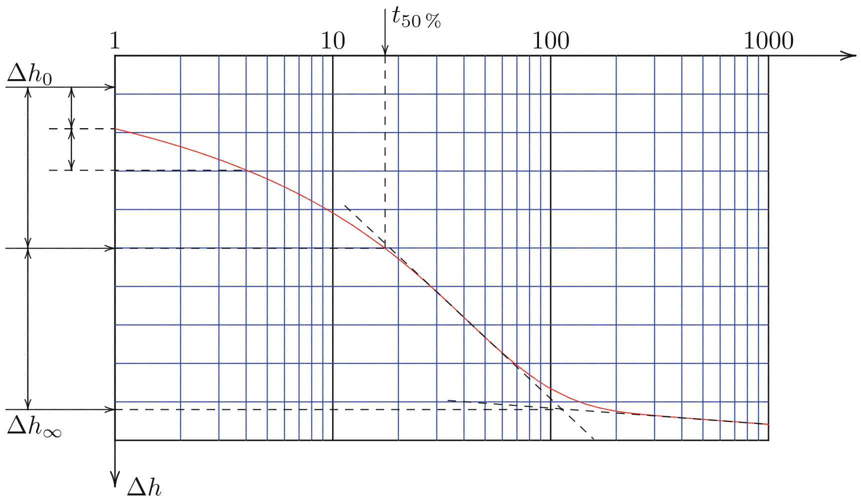


Fig. 18.1 Log(t)-method

From the values of the initial deformation Δh_0 and the final deformation Δh_∞, it is simple to determine the moment at which the degree of consolidation is just between these two values, which would mean that $U = 0.5$. This is also indicated in Fig. 18.1, giving a value for $t_{50\%}$. The value of the coefficient of consolidation then follows from $c_v t_{50\%}/h^2 = 0.197$, or

$$c_v = 0.197 \frac{h^2}{t_{50\%}}. \tag{18.1}$$

It should be noted that the quantity h in this expression represents the thickness of the sample, for the case of a sample drained on one side only. The consolidation process would be the same in a sample of thickness $2h$ and drainage to both sides. The original solution of Terzaghi considers that case, and the solution of the consolidation problem is given in that form in many textbooks. Because of the symmetry of that problem there is no difference with the problem and the solution considered here.

18.3 $\sqrt{t}$-Method

A second method to determine the value of the coefficient of consolidation, proposed by Taylor (1948), is to use only the results of a consolidation test for small values of time, and to use the fact that in the beginning of the process its progress is proportional to the square root of time. In this method the measurement data are plotted against $\sqrt{t}$, see Fig. 18.2. The basic formula is, see (16.24),

$$\Delta h - \Delta h_0 = (\Delta h_\infty - \Delta h_0) \frac{2}{\sqrt{\pi}} \sqrt{\frac{c_v t}{h^2}}. \tag{18.2}$$

In theory the value of the coefficient of consolidation c_v could be determined from the slope of the straight line in the figure, but this again requires the value of the initial deformation and the final deformation, as these appear in the formula (18.2). The value of the initial deformation Δh_0 can be determined from the intersection point of the straight tangent to the curve with the axis $\sqrt{t} = 0$. The final deformation Δh_∞, however, can not be obtained directly from the data. In order to circumvent this difficulty Taylor has suggested to use the result following from the theoretical curve and its approximation that for $U = 0.90$, i.e. for 90% of the consolidation, the value of $\sqrt{t}$ according to the exact solution is 15% larger that the value given by the approximate formula (18.2). The exact formula (16.13) gives that $U = 0.90$ if $c_v t/h^2 = 0.8481$, and the approximate formula (18.2) gives that $U = 0.90$ for $c_v t/h^2 = 0.6362$. The ratio of these two values is 1.333, which is the square of 1.154. This means that if in Fig. 18.2 a straight line is plotted at a slope that is 15% smaller than the tangent to the measurement data for small values of time, this line should intersect the measured curve in the point for which $U = 0.90$. The corresponding value of the time parameter $c_v t/h^2$ is 0.848, and therefore the

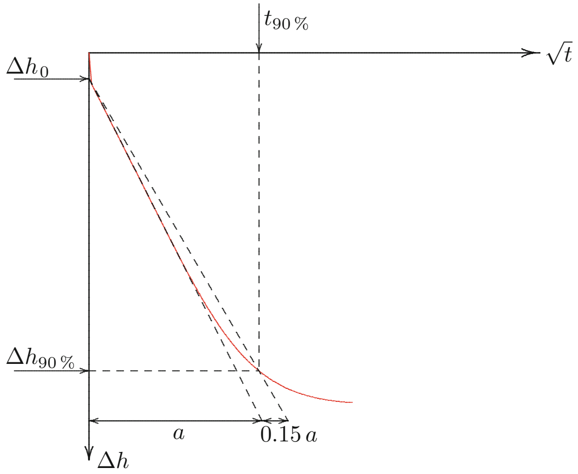


Fig. 18.2 $\sqrt{t}$-method

consolidation coefficient can be determined as

$$c_v = 0.848 \frac{h^2}{t_{90\%}}. \tag{18.3}$$

If the theory of consolidation were an exact description of the real behavior of soils, the two methods described above should lead to precisely the same value for the coefficient of consolidation c_v. Usually this appears to be not the case, however, with errors of the order of magnitude up to 10 or 20%. This indicates that the measurement data may be imprecise, especially when the deformations are very small, or that the theory is less than perfect. Perhaps the weakest point in the theory is the assumption of a linear relation between stress and strain.

18.4 Determination of m_v and k

In both of the two methods, the $\log(t)$-method and the $\sqrt{t}$-method, the procedure includes a value for the *final* consolidation settlement of the sample, even though it is realized that the deformations may continue beyond that value. In the $\log(t)$-method this final value forms part of the analysis, in the $\sqrt{t}$-method the final value of the deformation can be determined by adding 10% to the difference of the level of 90% consolidation and the initial deformation,

$$\Delta h_\infty = \Delta h_0 + \frac{10}{9}\,(\Delta h_{90\%} - \Delta h_0). \tag{18.4}$$

In general the final deformation is

$$\Delta h_\infty = h_{\text{tot}} m_v q, \tag{18.5}$$

where h_{tot} is the total thickness of the sample. The value of the compressibility m_v follows from

$$m_v = \frac{\Delta h_\infty}{h_{\text{tot}}\, q}. \tag{18.6}$$

Because the coefficient of consolidation c_v has been determined before, it follows that the permeability k can be determined as

$$k = \gamma_w m_v c_v. \tag{18.7}$$

The determination of the permeability k and the compressibility m_v may be theoretically unique, but because of approximations in the theory and inaccuracies in the measurement data the accuracy of the calculated values may not be very large.

Example 18.1 A consolidation test, on a sample of 2 cm thickness, with drainage on both sides, has resulted in the following deformations, under a load of 10 kPa. Determine the coefficient of consolidation, using the log(t)-method.

t (s)	10	20	30	40	60	120
Δh (mm)	0.070	0.082	0.089	0.094	0.105	0.127
t (s)	240	600	1200	1800	3600	7200
Δh (mm)	0.157	0.201	0.230	0.240	0.258	0.275

Solution

The graphical construction as described in this chapter, see Fig. 18.1, is shown in Fig. 18.3. The measured data have been plotted using a logarithmic scale for the values of time. Two lines have been drawn (in green): the line with the steepest slope and the asymptote for large values of time, and the intersection point of these two lines has been determined. This defines the value of the displacement at the end of the consolidation process, Δh_∞. Furthermore the initial displacement Δh_0 has been determined from the data for $t = 10$ and 40 s, assuming that initially the displacements vary proportional to $\sqrt{t}$. From the graph it can then be observed that $t_{50\%} = 155$ s. With Eq. (18.1) it now follows that $c_v = 0.127 \times 10^{-6}$ m^2/s, taking into account that in this case of drainage on both sides $h = 1$ cm.

The method also enables to determine an estimation of the vertical displacement at the end of the consolidation process, Δh_∞, ignoring the creep following thereafter. This value appears to be $\Delta h_\infty = 0.226$ mm. Using the given value of the load q, and taking into account that the total thickness of the sample in this case is $h_{\text{tot}} = 2$ cm, it now follows from Eq. (18.6) that $m_v = 0.00113$ m^2/kN. Using the relation $c_v = k/m_v\gamma_w$ the hydraulic conductivity k can now also be determined. The result is, assuming that $\gamma_w = 10$ kN/m^3, $k = 1.44 \times 10^{-9}$ m/s.

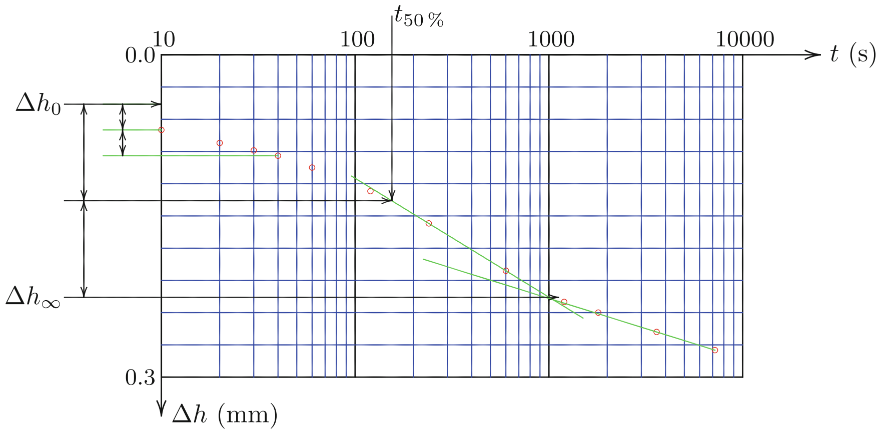
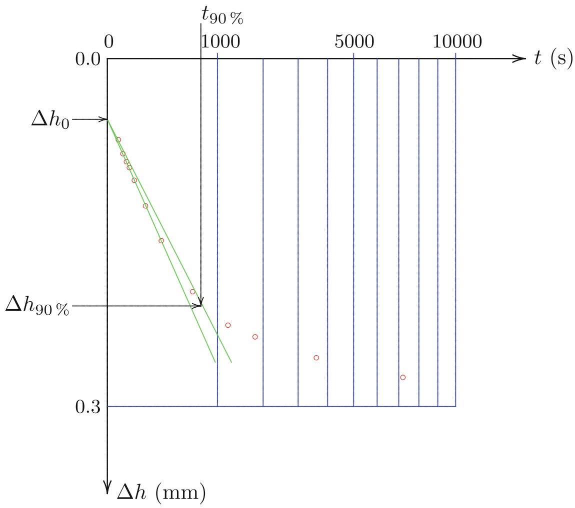


Fig. 18.3 Log(t)-method

Fig. 18.4 $\sqrt{t}$-method



Example 18.2 Verify that the $\sqrt{t}$-method leads to (approximately) the same values for the soil parameters as the log(t)-method in the case of the test described above.

Solution

The graphical construction for this method is shown in Fig. 18.4. The experimental data have been plotted using a scale of $\sqrt{t}$ on the horizontal axis. The data for small values of time have been used to draw the $\sqrt{t}$-approximation as a green line. Also, a line at a 15% smaller slope has been drawn, and the intersection point with the experimental curve has been determined (approximately). The location of this point indicates that $t_{90\%} = 721$ s. From Eq. (18.3) it then follows that

$c_v = 0.118 \times 10^{-6}$ m^2/s, which is about 7% smaller than the value obtained earlier using the log(t)-method. This gives an indication of the accuracy of the methods.

From the figure the displacement at the end of the consolidation process can be determined by adding 10% to the value of $\Delta h_{90\%} - \Delta h_0$. With $\Delta h_0 = 0.052$ mm and $\Delta h_{90\%} = 0.213$ mm this gives $\Delta h_\infty = 0.229$ mm. This value compares well with the value $\Delta h_\infty = 0.226$ mm obtained using the log(t)-method. It follows that the values of m_v and k that are obtained using the $\sqrt{t}$-method will also be close to the values obtained by the log(t)-method.

Reference

D.W. Taylor, *Fundamentals of Soil Mechanics* (Wiley, New York, 1948)