Chapter 19
Creep

As mentioned in the previous chapter, in a one dimensional compression test on clay, under a constant load, the deformation usually appears to continue practically forever, even if the pore pressures have long been reduced to zero. Similar types of behavior are found in other materials, such as plastics and concrete. The phenomenon is usually denoted as *creep*.

19.1 Keverling Buisman

For many materials creep can be modeled reasonably well by the theories of visco-elasticity or visco-plasticity. In such models the creep is represented by a viscous element, in which part of the stress is related to the rate of deformation of the material. Although the behavior of soils may contain such a viscous component, the creep behavior of soils is usually modeled by a special type of model, that has been based upon the observations in laboratory testing and in field observations. In 1936 Keverling Buisman, the first professor of soil mechanics at the Delft University, found, by doing long-duration tests, that the deformations of clay in a consolidation test did not approach a constant final value, but that the deformations continued very long. On a semi-logarithmic scale the deformations can be approximated very well by a straight line, see Fig. 19.1.

This suggests that the relation between strain and stress increment, after very long values of time, can be written as

$$\varepsilon = \varepsilon_p + \varepsilon_s \log\left(\frac{t}{t_0}\right). \tag{19.1}$$

Here ε_p is the *primary strain*, and ε_s is the *secondary strain*, or the *creep*. The quantity t_0 is a reference time, usually chosen to be 1 day. Keverling Buisman denoted the

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_19

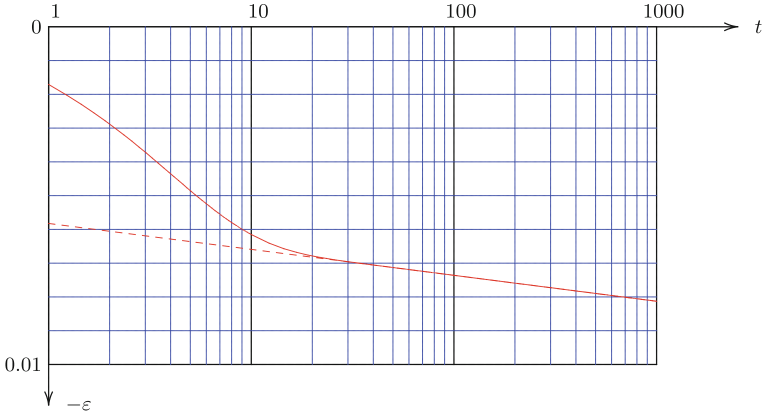

Fig. 19.1 Primary and secondary consolidation

continuing deformations after the dissipation of the pore pressures as the *secular effect*, with reference to the Latin word seculum (for century). In most international literature it is denoted as creep or secondary consolidation, the primary consolidation being Terzaghi's pore pressure dissipation process.

The primary strain ε_p is the deformation due to the consolidation of the soil. This is being retarded by the outflow of groundwater from the soil, as described in Terzaghi's theory of consolidation. Afterwards the deformation continues, and this additional deformation can be described, in a first approximation, by a semi-logarithmic relation, see Fig. 19.1, using the secular strain parameter ε_s. The phenomenon can be modeled at the microscopic level by the outflow of water from micro pores to a system of larger pores, or by a slow creeping deformation of clay elements (clay plates) under the influence of elementary forces at the microscopic level.

From a theoretical point of view the formula (19.1) is somewhat peculiar, because for $t \to \infty$ the strain would become infinitely large. It seems as if one can calculate the time span after which the thickness of the sample will have been reduced to zero, when the deformation becomes as large as the original thickness of the sample. For $t < t_0$ the behavior of the formula is also peculiar, because then the strain would be negative. Attempts have been made to adjust the formula for very large values of time, but in engineering practice the original formula, in its simple form (19.1) is perfectly usable, as long as it is assumed that $t \geq t_0$ and that the values of time in practice will be limited to say a few thousands (or perhaps millions) of years.

The magnitude of the parameters ε_p and ε_s can be determined from the data of a compression test at two different values of time, for instance at time $t = t_0$ (=1 day) and $t = 10\,t_0$ (=10 days). In the case illustrated in Fig. 19.1 this gives $\varepsilon_p = -0.0058$ and $\varepsilon_p + \varepsilon_s = -0.0066$ (the values are negative because the sample becomes thinner), so that $\varepsilon_s = -0.0008$. If the results are extrapolated to a value of $t = 100$ years the strain will be, after 100 years, $\varepsilon = -0.0094$. And after 1000 years the strain is $\varepsilon = -0.0102$. Predictions over longer periods of time are unusual in civil engineering practice. The time span of a structure is usually considered to be several hundreds of years.

In many countries the secondary strain is often denoted by C_α, the *secondary compression index*, although various researchers have proposed slightly different formulas and parameters.

Both the primary strain ε_p and the secondary strain ε_s can, of course, depend upon the magnitude of the applied load. For this reason Keverling Buisman wrote his formula in the form

$$\varepsilon = -\sigma' \Big[\alpha_p + \alpha_s \log \Big(\frac{t}{t_0} \Big) \Big], \tag{19.2}$$

in which σ' represents the load increment. This may suggest that the relation between stress and strain is linear, which in general is not the case. The coefficients α_p and α_s therefore depend upon the stress, and on the stress history.

The dependence of the stiffness has been considered earlier in the discussion on Terzaghi's logarithmic compression formula, see Chap. 14. It can be considered that the deformation considered in that chapter (for sandy soils) is a special case of the more general case considered here, in the absence of creep, i.e. with $\varepsilon_s = 0$. It then appears that the primary strain ε_p is proportional to the logarithm of the stress, with proportionality constants that are different for virgin loading and for unloading and reloading. It has been suggested to combine the formulas of Terzaghi and Keverling Buisman to

$$\varepsilon = -\Big[\frac{1}{C_p} + \frac{1}{C_s} \log \Big(\frac{t}{t_0} \Big) \Big] \ln \Big(\frac{\sigma}{\sigma_0} \Big). \tag{19.3}$$

The coefficients C_p and C_s should be understood to have quite different values for virgin loading and for unloading and reloading.

Den Haan (1994) found that the time dependent term is practically independent of the actual magnitude of the load, and therefore proposed the formula

$$\varepsilon = -a \ \ln \Big(\frac{\sigma}{\sigma_0} \Big) - b \ \ln \Big(\frac{\sigma}{\sigma_0} \Big) H(\sigma - \sigma_0) - c \ \ln \Big(\frac{t}{t_0} \Big), \tag{19.4}$$

where the function $H(x)$ represents Heaviside's step function,

$$H(x) = \begin{cases} 0 \text{ if } x < 0, \\ 1 \text{ if } x > 0. \end{cases} \tag{19.5}$$

This means that the second term of the formula (19.4) applies only if $\sigma > \sigma_0$, i.e. when the stress is larger than the largest stress ever experienced before. In unloading and reloading $\sigma < \sigma_0$, and then this term vanishes. Thus the second term represents the irreversible component of the deformation. The first term represents the reversible part of the deformation. It should be noted that in this formula the natural logarithm is used, whereas in other forms of stress-strain-relations the logarithm of base 10 may be used.

In many countries the deformation is often expressed into the void ratio e. A familiar form of the compression formula is Bjerrum's relation

$$e_0 - e = C_c \log\left(\frac{\sigma}{\sigma_0}\right) + C_\alpha \log\left(\frac{t}{t_0}\right), \tag{19.6}$$

in which e_0 is the void ratio at the initial stress σ_0, for $t = t_0$. In this form the relation has been incorporated in the international standards.

In Chap. 14 the relation between the change of the void ratio e and the strain ε has been shown to be

$$\varepsilon = \frac{\Delta e}{1+e}, \tag{19.7}$$

See Eq. (14.9). Using this relation the various expressions given in this chapter can be shown to be equivalent, and the various coefficients can be expressed into each other.

It is, of course, regrettable that so many different formulas and different constants are being used for the same phenomenon, especially as there is general agreement on the basic form of relationships, with a logarithm of time. This is mainly a consequence of national traditions and experiences. In engineering practice care must be taken to translate local experience with certain constants into a formula using different constants from literature or reports from another area. The conversion is simple, however.

One of the main applications in engineering practice is the prediction of the settlement of a layered soil due to an applied load. The standard procedure is to collect a sample of each of the soil layers, to apply the initial load to each of the samples, and then to load each sample by an additional load corresponding to the load in the field. In this way the stress dependence of the stiffness is taken into account by subjecting each sample to the same stress increment in the laboratory and in the field. In general the settlement appears to increase with the logarithm of time after application of the load, in agreement with the formula (19.1). The deformation in the field can then be predicted using this formula. The contribution of each layer to the total settlement is obtained by multiplying the strain of the layer by its thickness. The total settlement is obtained by adding the deformations of all layers.

The prediction of the deformations can be complicated because the stiffness of the soil depends on the stress history. In an area with a complex stress history (for instance a terrain that has been used for different purposes in history, or a field that has been subject to high preloading in an earlier geologic period) this means that the behavior of the soil may be quite different below an unknown earlier stress level and above that stress level. Extrapolation of laboratory results may be inaccurate if the stress history is unknown. For this purpose it is advisable to estimate or simulate the actual stress level and its proposed increase in the field in the laboratory tests. In that case the laboratory tests will be a good representation of the behavior in the field. As

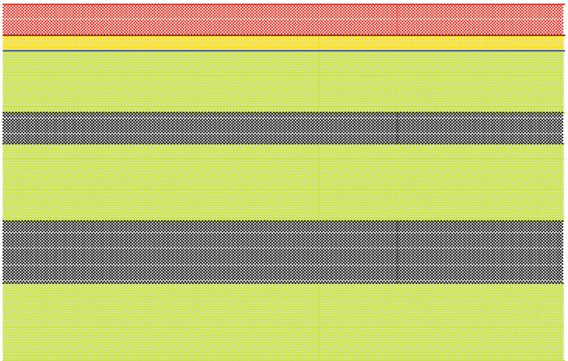

Fig. 19.2 Layered soil

the logarithmic time behavior is generally observed, the duration of the tests need not be very long. Extrapolation in time is usually sufficiently accurate.

It should be mentioned that all the considerations in this chapter refer only to one-dimensional compression. This means that they apply only if in the field there are no horizontal deformations. In case of a local load it can be expected that there will be lateral deformation as well as vertical deformation. In such cases consolidation and creep should be considered as three-dimensional phenomena. These are considerably more complicated than the one-dimensional case considered here (Fig. 19.2).

Example 19.1 A terrain consists (from top to bottom) of 1 m dry sand ($\gamma = 17\,\text{kN/m}^3$), 4 m saturated sand ($\gamma = 20\,\text{kN/m}^3$), 2 m clay ($\gamma = 18\,\text{kN/m}^3$), 5 m sand ($\gamma = 20\,\text{kN/m}^3$), 4 m clay ($\gamma = 19\,\text{kN/m}^3$), and finally a thick sand layer. The terrain is loaded by an additional layer of 2 m dry sand ($\gamma = 17\,\text{kN/m}^3$). The deformations of the clay layers will be analyzed by performing oedometer tests on samples from each clay layer. What should be the initial load on each of the two samples, and what should be the additional load?

Solution

The most accurate results will be obtained if the initial stress in the clay samples is equal to the average initial effective stress in the field. For the sample representing the upper clay layer this means that the initial stress should be $\sigma_0 = 17+4\times10+1\times8 = 65\,\text{kPa}$. For the sample representing the deeper clay layer the initial stress should be $\sigma_0 = 17 + 4 \times 10 + 2 \times 8 + 5 \times 10 + 2 \times 9 = 141\,\text{kPa}$. For both samples the additional load should be 34 kPa.

Example 19.2 Suppose that in the tests mentioned above, the test results are that after one day a strain of 2% is observed, and after 10 days a strain of 3%, for both clay layers. If it is assumed that the deformation of the sand layers can be neglected, predict the total settlement of the terrain after 1, 10, and 100 years.

Solution

Using equation (19.1), with $t_0 = 1$ day, it follows that $\varepsilon_p = 0.02$ and $\varepsilon_s = 0.01$. The primary settlement now is $\Delta h_p = 0.02 \times 6$ m $= 0.120$ m. Because 1 year equals 365 days and log(365/1) = 2.562, it follows that after 1 year the secondary settlement is $\Delta h_s = 0.01 \times 2.562 \times 6$ m $= 0.154$ m. Because log(3650/1) = 3.562 the secondary settlement after 10 years is 0.214 m, and because log(36500/1) = 4.562 the secondary settlement after 100 years is 0.274 m.

The total settlement after 1, 10, and 100 years is 0.274, 0.334, and 0.394 m.

Example 19.3 In a certain town it is required that in a period of 20 years after the sale of a terrain the deformation may not be more than 20 cm. For a terrain that has been prepared by the application of a sand layer on a soft soil layer of 7.6 m thickness, it has been found from tests on the soft soil that the deformation after one day is 1.1%, and after 10 days 2.4%. How long should the town wait after the application of the sand layer before the terrain can be sold?

Solution

In this case $\varepsilon_p = 0.011$ and $\varepsilon_s = 0.013$. The deformation in a period of 20 years (=7300 days) after the time of sale, say t_s, is $\varepsilon_s\, h \log\{(t_s + 7300)/t_s\}$. The requirement that this must be less than 0.02 m leads to the condition that $t_s > 70$ days.