Chapter 9
Flotation

In the previous chapter it has been seen that under certain conditions the effective stresses in the soil may be reduced to zero, so that the soil looses its coherence, and a structure may fail. Even a small additional load, if it has to be supported by shear stresses, can lead to a calamity. Many examples of failures of this type can be given: the bursting of the bottom of excavation pits, and the uplift or flotation of basements, tunnels and pipelines. The conditions for uplift or flotation of structures are discussed in this chapter.

9.1 Archimedes

The basic principle of the uplift force on a body submerged in a fluid is due to Archimedes. This principle can best be explained by first considering a small rectangular element, which is at rest in a fluid, see the left half of Fig. 9.1. The material of the block is irrelevant, but it must be given to be at rest, perhaps by the action of some external forces.

The pressure in the fluid is a function of depth only, and in a homogeneous fluid the pressure distribution is

$$p = \rho g z, \tag{9.1}$$

where ρ is the density of the fluid, g the acceleration of gravity, and z the depth below the fluid surface.

The pressures on the left hand side and the right hand side are equal, but act in opposite direction, and therefore are in equilibrium. The pressure below the element is greater than the pressure above it. The resultant force is equal to the difference in pressure, multiplied by the area of the upper or lower surfaces. Because the pressure difference is just $\rho g h$, where h is the height of the element, the upward force equals ρg times the volume of the element. That is just the volumetric weight of the water

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_9

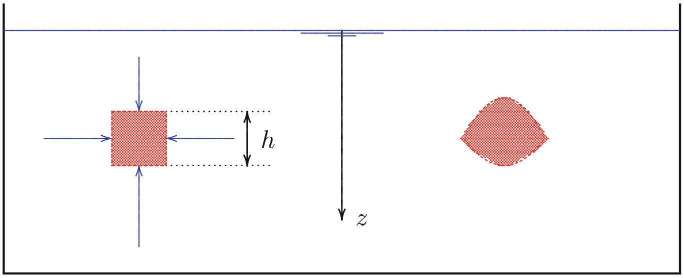


Fig. 9.1 Archimedes' principle

multiplied by the volume of the element. Because any body can be constructed from a number of such elementary blocks, the general applicability of Archimedes' principle (a submerged body experiences an upward force equal to the weight of the displaced fluid) follows.

A different argument, that immediately applies to a body of arbitrary shape, is that in a state of equilibrium the precise composition of a body is irrelevant for the force acting upon it. This means that the force on a body of water must be the same as the force on a body of some other substance, that then perhaps must be kept in equilibrium by some additional force. Because the body when composed of water is in equilibrium it follows that the upward force must be equal to the weight of the water in the volume. On a body of some other substance the resultant force of the water pressures must be the same, i.e. an upward force equal to the weight of the water in the volume. This is the proof that is given in most textbooks on elementary physics. The upward force is often denoted as the buoyant force, and the effect is denoted as *buoyancy*.

The buoyancy force on a body in a fluid may have as a result that the body floats on the water, if the weight of the body is smaller than the upward force. Floatation will happen if the body on the average is lighter than water. More generally, flotation may occur if the buoyancy force is larger than the sum of all downward forces together. This may happen in the case of basements, tunnels, or pipelines. In principle flotation can easily be prevented: the body must be heavy enough, and may have to be ballasted.

The problem of possible flotation of a foundation is that care must be taken that the effective stresses are always positive, taking into account a certain margin of safety. In practice this may be more difficult than imagined, because perhaps not all conditions have been foreseen. Some examples may illustrate the analysis.

9.2 A Concrete Floor Under Water

As a first example the concrete floor of an excavation is considered. Such structures are often used as foundations of basements, or as the pavement of the access road of a tunnel. One of the functions of the concrete plate is to give additional weight to

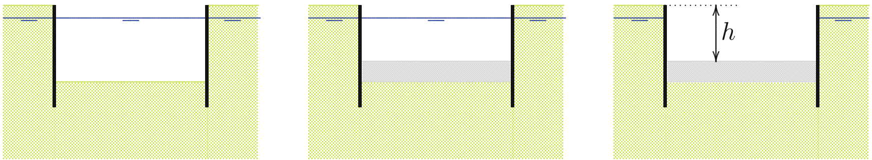


Fig. 9.2 Excavation with concrete floor under water

the soil, so that it will not float. Care must be taken that the water table is lowered only after the construction of the concrete plate. Therefore a convenient procedure is to build the concrete plate under water, before lowering the water table, see Fig. 9.2. After excavation of the building pit, under water, perhaps using dredging equipment, the concrete floor can be constructed, taking great care of the continuity of the floor and the vertical walls of the excavation. When the concrete structure has been finished, the water level can be lowered. In this stage the weight of the concrete is needed to prevent flotation.

There are two possible methods to perform the stability analysis. The best method is to determine the effective stresses just below the concrete floor. If these are always positive, in every stage of the building process, a compressive stress is being transferred in all stages, and the structure is safe. Whenever tensile stresses are obtained, even in a situation that is only temporary, the design must be modified, because the structure is not always in equilibrium, and will float or break. It is assumed that in the case shown in Fig. 9.2 the groundwater level is at a depth $d = 1$ m below the soil surface, and that the depth of the top of the concrete floor should be located at a depth $h = 5$ m below the soil surface. Furthermore the thickness of the concrete layer (which is to be determined) is denoted as D. The total stress just below the concrete floor now is

$$\sigma = \gamma_c D, \tag{9.2}$$

where γ_c is the volumetric weight of the concrete, say $\gamma_c = 25\,\text{kN/m}^3$. The pore pressure just below the concrete floor is

$$p = (h - d + D)\gamma_w, \tag{9.3}$$

so that the effective stress is

$$\sigma'_{zz} = \sigma_{zz} - p = \gamma_c D - \gamma_w(h - d + D) = (\gamma_c - \gamma_w)D - \gamma_w(h - d). \tag{9.4}$$

The requirement that this must be positive gives

$$D > \frac{\gamma_w(h - d)}{\gamma_c - \gamma_w}. \tag{9.5}$$

The effective stress will be positive if the thickness of the concrete floor is larger than the critical value. In the example, with $h - d = 4$ m and the concrete being a factor 2.5 heavier than water, it follows that the thickness of the floor must be at least 2.67 m.

It may be noted that the required thickness of the concrete floor should be even larger if the groundwater level may also coincide with the soil surface, namely 3.33 m. One must be very certain that this condition cannot occur if the concrete plate is thinner than 3.33 m.

It may also be noted that in time of danger, perhaps when the groundwater pressures rise beyond the design level because of some emergency or because of some human error, the foundation can often be saved by submerging it with water. The damage to a basement or a tunnel due to a temporary layer of water on it is usually less than the damage if the concrete floor is cracked and has to be replaced.

The analysis can be done somewhat faster by directly requiring that the weight of the concrete must be sufficient to balance the upward force acting upon it from below. This leads to the same result. The analysis using the somewhat elaborate process of calculating the effective stresses may take some more time, but it can more easily be generalized, for instance in case of a groundwater flow, when the groundwater pressures are not hydrostatic.

The concrete floor in a structure as shown in Fig. 9.2 may have to be rather thick, which requires a deep excavation and large amounts of concrete. In engineering practice more advanced solutions have been developed, such as a thin concrete floor, combined with tension piles. It should be noted that this requires a careful (and safe) determination of the tensile capacity of the piles. A heavy concrete floor may be expensive, its weight is always acting.

9.3 Flotation of a Pipe

The second example is concerned with a pipeline in the bottom of the sea, or a circular tunnel under a river, see Fig. 9.3. The pipeline is supposed to consist of steel, with a concrete lining, having a diameter $2R$ and a total weight (above water) G, in kN/m. This weight consists of the weight of the steel and the concrete lining, per unit length of the pipe. For the risk of flotation the most dangerous situation will be when the pipe is empty.

For the analysis of the stability of the pipeline it is convenient to express its weight as an average volumetric weight γ_p, defined as the total weight of the pipeline divided by its volume. In the most critical case of an empty pipeline this is

$$\gamma_p = G/\pi R^2. \tag{9.6}$$

The buoyant force F on the pipeline is, in accordance with Archimedes' principle,

$$F = \gamma_w \pi R^2, \tag{9.7}$$

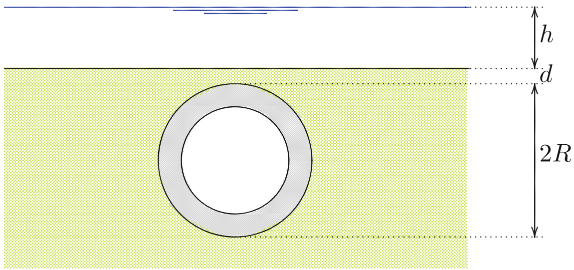


Fig. 9.3 A pipe in the ground

where γ_w is the volumetric weight of water. If the upward force F is smaller than the weight G there will be no risk of flotation. The pipeline then sinks in open water. This will be the case if $\gamma_p > \gamma_w$. For a pipeline on the bottom of the sea this is a very practical criterion. If one would have to rely on the weight of the soil above the pipeline for its stability, flotation might occur if the soil above the pipeline is taken away by erosion, which is not unlikely. The pipeline then might float to the sea surface, and that should be avoided.

In case of a tunnel buried under a river there seems to be more certainty that the soil above the tunnel remains in place. Then the weight of the soil above the tunnel may prevent flotation even if the tunnel is lighter than water ($\gamma_p < \gamma_w$). The weight W of the soil above the tunnel is

$$W = \gamma_s[2Rd + (2 - \pi/2)R^2], \tag{9.8}$$

where γ_s is the volumetric weight of the soil, and d is the cover thickness, the thickness of the soil at the top of the tunnel. It is now essential to realize, in accordance with Archimedes' principle that for the stability of the tunnel the soil above it only contributes insofar as it is heavier than water. The water above the tunnel does not contribute, of course. A block of wood will float in water, even if the water is very deep. This means that the effective downward force of the soil above the tunnel is

$$W' = (\gamma_s - \gamma_w)[2Rd + (2 - \pi/2)R^2], \tag{9.9}$$

the difference of the weight of the soil and the weight of the water in the same volume. The amount of soil that is minimally needed now follows from the condition

$$W' + G - F > 0. \tag{9.10}$$

This gives

$$(\gamma - \gamma_w)[2Rd + (2 - \pi/2)R^2] > (\gamma_w - \gamma_p)\pi R^2, \tag{9.11}$$

from which the ground cover d can be calculated. There still is some additional safety, because when the tunnel moves upward the soil above it must shear along the soil next to it, and the friction force along that plane has been disregarded. It is recommended to keep that as a hidden reserve, because flotation is such a serious calamity.

The analysis can, of course, also be performed in the more standard way of soil mechanics stress analysis: determine the effective stress as the difference of the total stress and the pore pressure. The procedure is as follows.

The average total stress below the tunnel is (averaged over its width $2R$)

$$\sigma = \gamma_w h + W/2R + G/2R = \gamma_w h + \gamma_s[d + (1 - \pi/4)R] + \gamma_p \pi R/2, \quad (9.12)$$

where h is the depth of the water in the river. The average pore pressure below the tunnel is determined by the volume of the space occupied by the tunnel and everything above it, up to the water surface,

$$p = \gamma_w h + \gamma_w[d + (1 - \pi/4)R] + \gamma_w \pi R/2. \quad (9.13)$$

The average effective stress below the tunnel now is

$$\sigma' = (\gamma_s - \gamma_w)[d + (1 - \pi/4)R] + (\gamma_p - \gamma_w)\pi R/2. \quad (9.14)$$

The condition that this must be positive, because the particles can not transmit any tensile force, leads again to the criterion (9.11).

Example 9.1 A block of wood, having a volume of 0.1 m^3, is kept in equilibrium below water in a basin of water by a cord attached to the bottom of the basin, see Fig. 9.4. The volumetric weight of the wood is 9.0 kN/m^3, and the volumetric weight of the water is 10.0 kN/m^3. Calculate the force in the cord.

Solution

The weight of the block leads to a downward force of 0.9 kN. The upward force due to buoyancy is the weight of the displaced fluid: 1.0 kN. The difference must be the force in the cord, i.e. 0.1 kN.

Example 9.2 The basin next is filled with two fluids: salt water (volumetric weight 10.2 kN/m^3) and fresh water above it. The separation level of the salt and the fresh water coincides with the top of the block of wood, see Fig. 9.5. Again calculate the force in the cord.

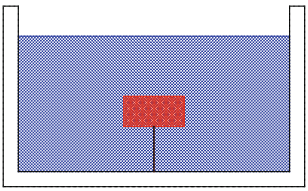
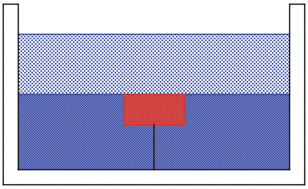


Fig. 9.4 Block in water

Fig. 9.5 Block in two fluids



Solution

The weight of the block remains 0.9 kN. The upward force now is determined by the heavier fluid, which completely surrounds the block. The resulting upward force now is 1.02 kN. The difference is the force in the cord, i.e. 0.12 kN.

Example 9.3 A tunnel of square cross section, 8 m $\times$ 8 m, has a weight (above water) of 50 ton per meter length. The tunnel is being floated to its destination, with its two ends closed by temporary sheets. Calculate the draught (the depth of the bottom below water) (Fig. 9.6).

Solution

The weight of the tunnel per meter length is 500 kN. If the depth of the floating tunnel below water is denoted by d, the upward force per unit meter is $d \times 8\,\text{m} \times 10\,\text{kN/m}^3$. These two forces cancel if $d = 6.25\,\text{m}$.

Example 9.4 The tunnel of the previous problem is sunk into a trench that has been dredged in the river bottom, and then covered with sand. The volumetric weight of the sand is $20\,\text{kN/m}^3$. Determine the minimum cover of sand necessary to prevent flotation of the tunnel (Fig. 9.7).

Fig. 9.6 Floating tunnel

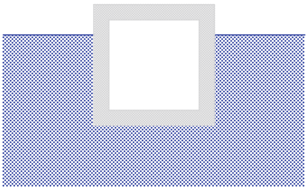
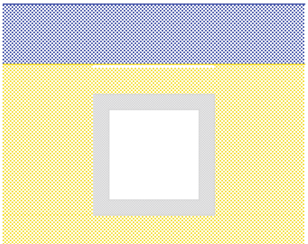


Fig. 9.7 Tunnel at rest



Solution

As seen before the weight of the tunnel per meter length is 500 kN. If the depth of the sand above the tunnel is denoted by h, the total force at the bottom of the tunnel (expressed in kN/m) is $F = 500 + 8 \times 20 \times h = 500 + 160 \times h$. The upward force due to the water pressure just below the tunnel is $P = (h + 8) \times 10 \times 8 = 640 + 80 \times h$. In order that the effective stress below the tunnel remains positive, the condition is that $F > P$. The critical situation is when $F = P$, which will be the case for $h = 1.75$ m. This is the minimum value of the sand cover to prevent flotation. Of course in reality a larger value should be taken, to avoid uplift if some of the sand is eroded.

It may be noted that the depth of the water above the sand has been disregarded, but this would add an equal value to both F and P, and thus would have no effect on the outcome.

It may also be noted that many engineers would prefer to design the tunnel so that its weight is at least equal to the buoyancy force. Then there is never any risk of uplift.