Chapter 47
Model Tests

A useful tool in engineering is the analysis of the behavior of a structure by doing a model test, at a reduced scale. The purpose of the test may be just to investigate a phenomenon in a qualitative way, but more often its purpose is to obtain quantitative information. In that case the scale rules must be known. For a soil a special difficulty is that the mechanical properties often depend upon the state of stress, which is determined to a large extent by the weight of the soil itself. This means that in a scale model the soil properties are not well represented, because in the model the stresses are much smaller than in reality (the *prototype*).

47.1 Types of Models

An ingenious way to simulate the stresses in a model is to increase gravity, by placing the scale model in a geotechnical centrifuge, in which the model is rotated at high speed. The principles of this method are briefly presented in this chapter. Some attention is also paid to $1g$-testing, the testing of a model without scaling gravity. It will appear that in some situations this can be useful method of model testing.

The scale rules of a certain field in physics can usually be derived by considering the basic equations that fully describe a certain process, and then taking care that all relevant terms in each of the equations are scaled by the same factor. The equations describing the process may be partly symbolic, if a detailed description can not be given, but the character of the relations is known. It is essential that all important factors are taken into account. Less important factors may be disregarded, if their small influence can be demonstrated.

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47.2 Simple Scale Models

One of the most important properties of soils is that it may shear, possibly up to very large deformations, and that this shear is caused by the relative magnitude of the shear stress, compared to the normal stress. In Coulomb's failure criterion

$$\tau_{max} = c + \sigma' \tan\phi, \tag{47.1}$$

this appears if the first term, the cohesion c, is very small. This is the case for sand. In that case one may write

$$c = 0: \quad \frac{\tau_{max}}{\sigma'} = \tan\phi. \tag{47.2}$$

It appears that failure is determined only by a ratio of the stresses, not by their magnitude. This does not necessarily mean that the ratio of shear stress to normal stress determines the soil behavior throughout the entire range from zero deformation to failure. For very small deformations the behavior is more or less elastic, and it is not certain that in that range the ratio τ/σ is the only parameter that governs the deformations. However, there is much evidence that the stiffness of soils increases with the stress level, both in shear as in compression (compare Terzaghi's logarithmic compression formula). Thus, it is not unreasonable to assume, at least for sandy soils, that the deformations can be described by a formula of the character

$$\varepsilon_{ij} = f\left(\frac{\sigma'_{ij}}{\sigma'_o}\right), \tag{47.3}$$

where σ'_o is an invariant of the stress tensor, say the isotropic stress.

This means that the deformations are determined only by the ratio of the shear stresses and a characteristic normal stress, say the isotropic stress. For sands this is a useful approximation. It may be noted that in compression the deformation is also determined by a stress ratio, in this case the ratio of the stress to the initial stress. The assumption excludes effects as consolidation, creep and dilatancy. These must be small compared to shear and primary compression for the assumption (47.3) to be valid. Examples of problems for which the assumption is valid are a laterally loaded pile, or a caisson loaded by cyclic forces.

If all the spatial dimensions are scaled down by a factor n_L, i.e.

$$x_{i-m} = x_{i-p}/n_L, \tag{47.4}$$

the equations of equilibrium, including the term representing the weight of the material, are satisfied if the scale factor for the stresses is also n_L,

$$\sigma_{ij-m} = \sigma_{ij-p}/n_L. \tag{47.5}$$

This can be verified by noting that the equations of equilibrium consist of terms of type $\partial\sigma_{xx}/\partial x$, and the gravity term γ. All these terms now are identical in the model and in the prototype.

If the relation between stresses and strain is of the form (47.3), the deformations are represented at scale 1,

$$\varepsilon_{ij-m} = \varepsilon_{ij-p}. \tag{47.6}$$

Because the deformations are related to the displacements by derivatives with respect to the spatial coordinates (for example $\varepsilon_{xx} = \partial u_x/\partial x$), the displacements are at the same scale as a length,

$$u_{i-m} = u_{i-p}/n_L, \tag{47.7}$$

In each of the relevant equations (equilibrium, compatibility and constitutive equations) the ratio of all terms in the model is the same as the corresponding terms in the prototype. This means that it is indeed possible to study the behavior of the prototype in a scale model. The boundary values of stress and deformations must also be applied using the scale n_L.

A problem that can be studied in this way is a laterally loaded pile, see Fig. 47.1. Compression is not important in this case, so that it is unlikely that pore water pressures will be generated. The determining factor for the deformations is the ratio of shear stress to normal stress. In the model these ratios will be the same as in the prototype if the material is the same. The deformations then are at scale 1. Similarly, problems of sheet pile walls, or retaining walls, can be studied by $1g$-models, if the material is non-cohesive, i.e. sand.

Even dynamic problems may be studied by such a scale model, by noting that in that case the equations of motion contain terms of the type $\rho\,\partial^2 u_i/\partial t^2$. These terms will be the same in the model and in the prototype if the time is scaled according to the square root of the length scale,

$$t_m = t_p/\sqrt{n_L}. \tag{47.8}$$

Here it has been assumed that the density ρ is the same in the model as in the prototype, which is easy to accomplish, by using the same material. It may be noted that dynamic effects are important only for special problems, such as earthquakes

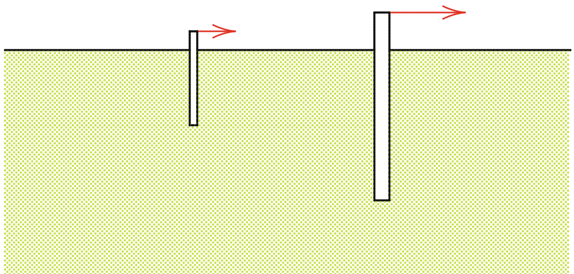

Fig. 47.1 Model test

and high speed trains. In the standard engineering problems dynamic effects usually play a minor role. Even in the cyclic loading of an offshore platform the dynamic effects are small because the period of the cycles (about 10 s) is so large.

Problems of consolidation can also be studied in $1g$-models, at least in a first approximation. In the consolidation equation,

$$\frac{\partial e}{\partial t} = -n\beta\frac{\partial p}{\partial t} + \frac{k}{\gamma_w}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right), \tag{47.9}$$

all terms should then be scaled by the same factor. If all the stresses are scaled on the same scale (n_L) as a length, in order to model equilibrium, and the deformations on scale 1, the term in the left hand side of the equation can be in agreement with the other terms only if time is scaled on the length scale,

$$t_m = t_p/n_L. \tag{47.10}$$

The first term in the right hand side of the equation then is not scaled correctly, because this term consists of a ratio of two factors at length scale. But in many cases this is a small term anyway, as the compressibility of the water (β) is very small. This means that the error in scaling the consolidation process will be very small.

It follows from the considerations given above that it is impossible to take both consolidation and dynamic effects into account, as these two phenomena lead to different requirements for the time scale. An ingenious way to solve this difficulty is to scale the permeability, without changing the porous material, by using a different fluid in the model, having a different viscosity, such that the two terms scale in the same way.

As mentioned before, all this does not apply if the material behavior is more complex than is indicated by Eq. (47.3). This will be so in the majority of problems, for instance in case of simultaneous elastic and plastic deformations, or in case of a cohesive material. This means that simple scale tests on clays are not representative for the behavior in the prototype. They can be used only if friction is the dominant property in the mechanical behavior, and the plastic deformations are relatively large.

47.3 Centrifuge Testing

A general way of describing the relation between stresses and strains in a soil is

$$\Delta\varepsilon_{ij} = f(\sigma'_{ij}, \Delta\sigma'_{ij}, h_k), \tag{47.11}$$

where f is an arbitrary function, and h_k indicates that there may be some other physical parameters involved in the functional relationship, such as the cohesion c, or the stiffness parameters K and G. Equation (47.11) states that the incremental strains are determined by the stresses and the incremental stresses, in a not yet specified manner. Various types of behavior can be described by relations of the type (47.11), such as elastic and plastic deformations. Of particular importance is that the incremental strains depend upon the actual stresses. This means that the stiffness may depend upon the stresses, which is a typical property of many soils. Dilatancy and contractancy can also be described by the general relation (47.11). And elastic deformations, in which the incremental strains are fully determined by the incremental stresses can also be described by (47.11), of course.

Assuming the validity of the general relationship (47.11), model testing is possible only if the stresses and the strains are all modelled at scale 1, and that the same soil is used, to ensure that the properties are the same. This implies that the stresses caused by the weight of the material must also be modelled at scale 1. In the equations of equilibrium terms of the type $\partial\sigma_{xx}/\partial x$ appear with a term $\gamma = \rho g$. In order to model both these terms at the same scale, the volumetric weight γ must be inversely proportional to the length scale,

$$\gamma_m = \gamma_p \times n_L. \tag{47.12}$$

This can be realized by rotating the model very fast, in a *geotechnical centrifuge*. Gravity then appears to be magnified, see Fig. 47.2. The facility consists of an arm that can be rotated around a central axis. At the two ends of the arm containers are placed, one containing the model, and the other containing a counter weight (or another model), to balance the arm. If the arm rotates a centrifugal force acts on the material in the two containers, which will rotate around a hinge. If the rotation is very fast the bottom of the two containers will be practically vertical.

For safety of people and the surroundings, the centrifuge must be protected by heavy steel plates and concrete walls, to prevent damage in case of failure of a part of the system. For this reason the centrifuge is often located in the basement of a geotechnical laboratory.

An elementary consideration of the motion of a body moving along a circular path, of radius R, indicates that an acceleration perpendicular to the path occurs, of magnitude

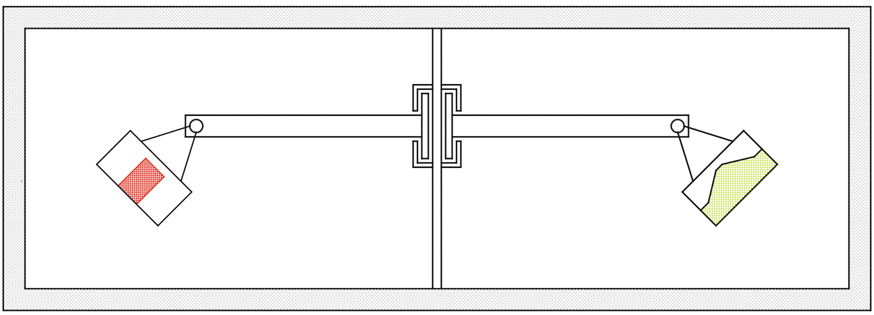

Fig. 47.2 Geotechnical centrifuge

$$a = \frac{v^2}{R}. \tag{47.13}$$

This is called the *centripetal acceleration*. In the case of a container filled with soil that rotates in a centrifuge this acceleration is caused by the force from the container on the soil, and transmitted through the soil, in upward direction. If the soil were not contained by the container, it would fly on, in a straight path, but it is retained in its circular path by the container. This requires a very large force, and this force is larger if the velocity is larger, or the radius smaller (at the same velocity). The stress state would be the same if the container were at rest, and a volumetric force would act upon the soil. If this volumetric force is denoted by g_m, we have

$$g_m = \frac{v^2}{R} = \omega^2 R, \tag{47.14}$$

in which ω is the angular velocity (or the frequency) of the centrifuge. Many geotechnical centrifuges have an arm length of about 5 m. This means that an acceleration of $100\,\text{g} = 1000\,\text{m/s}^2$ is achieved if the velocity of the container is 71 m/s, or 254 km/h. The angular velocity then is 14.14 rad/s, which means that the container flies by every 0.444 s. This corresponds to 2.25 revolutions per second, or 135 revolutions per minute.

The major principle of centrifuge testing is that all stresses in the model are the same as the stresses in the prototype, so that it is practically guaranteed that soil will behave in the same way as in reality. A geotechnical centrifuge is a reasonably complex machine, however, and it generates large forces in its parts. Furthermore, observing deformations and measuring stresses is not a simple matter. Electronic measuring devices may be built in, but these should be very small, and the measuring signals must be transmitted to the outside world, through the central axis. An alternative registration method is to record the measurements on a data recorder that is attached to the arm itself, and to read the data later. A video signal can be used to observe deformations in flight. Preparation of the samples also requires much attention, as the sample must be a good representation of the prototype, at a small scale. A small disturbance in the model corresponds to a large disturbance in reality.

Consolidation problems, in which time is important, can be studied in a centrifuge if the terms $\partial e/\partial t$ and $\partial^2 p/\partial x^2$ are scaled in the same way. Because stresses and strains are at scale 1, it follows that the time scale must be the square of the length scale,

$$t_m = t_p/n_L^2. \tag{47.15}$$

If the time scale is determined by inertia effects (in dynamic problems) the terms $\rho\, \partial^2 u/\partial t^2$ must be scaled by the same factor as the derivatives of the stresses, $\partial\sigma_{xx}/\partial x$. That will be the case if the time scale equals the length scale,

$$t_m = t_p/n_L. \tag{47.16}$$

Again it is not easily possible to scale consolidation in combination with dynamic effects, except by using a fluid of different viscosity.

Example 47.1 The arm of a certain centrifuge is of 6 m length, and the machine is designed for testing at a maximum acceleration of 300 g. What is then its number of rotations per minute? And the velocity of the container?

Solution

It follows from Eq. (47.14), with $g_m = 300\ g = 3000\,\text{m/s}^2$, that the velocity v of the container at the maximum acceleration is $v = 134\,\text{m/s}$, which is equivalent to $v = 483\ \text{km/h}$.

The angular velocity $\omega = v/R$. In this case this is $\omega = 22.33\ \text{s}^{-1}$. This means that a full circle (an angle 2π) is completed in 0.28 s. This means that in 1 min, or 60 s, the number of rotations is 214.

Problem 47.1 Is it possible to study the problem of slope stability of a sand dyke in a 1g-model test? And in a centrifuge?

Problem 47.2 Is it possible to study the problem of slope stability of a clay dyke in a 1g-model test? And in a centrifuge?

Problem 47.3 At a fair one may see a large rotating cylinder, in which people remain hanging against the wall if the bottom moves down. If it is supposed that the friction coefficient between man and steel wall is about 0.2, the radial acceleration must be about 0.2 g. If the radius of the cylinder is 4.5 m, then what is the velocity of the people, in km/h?