Chapter 28
Newmark

This chapter presents an ingenious method for the determination of the vertical normal stresses at a certain depth, caused by some arbitrary load distribution on the surface, developed by Nathan M. Newmark, Professor at the University of Illinois.

28.1 Newmark's Problem

The basis of Newmark's analysis is Eq. (27.13), which gives the vertical normal stress at a depth z below the center of a uniform load p on a circular area, see Fig. 28.1. This equation can also be written as

$$r = 0 : \quad \frac{\sigma_{zz}}{p} = 1 - \frac{1}{\sqrt{(1 + a^2/z^2)^3}}. \tag{28.1}$$

It will appear that this formula can be used to develop a technique to calculate the vertical normal stress σ_{zz} below a load over an area of arbitrary shape. The method can be constructed in the following way.

Equation (28.1) gives the value of the vertical normal stress σ_{zz} for a given value of a/z. Conversely, it is also possible, of course, to calculate the value of a/z for which certain values of σ_{zz}/p occur, in multiples of 0.1. For instance, by taking $\sigma_{zz}/p = 0.5$, it follows from Eq. (28.1) that a/z should be equal to 0.7664. In this way the values in Table 28.1 have been calculated.

All the values given in the table can easily be verified using Eq. 28.1. The first value expresses that a ring of radius 0 leads to a stress $\sigma_{zz} = 0$, and the last value expresses that in case of a uniform load over the entire surface the vertical stress σ_{zz} at a depth z always is p.

It also follows from the table that for a load on a circle of radius 0.7664 z the stress σ_{zz} at a depth is 0.5 p, and that for a load on a circle of radius 0.9176 z the

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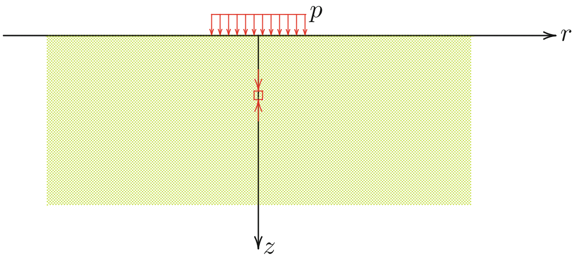

Fig. 28.1 Uniform load on circular area

Table 28.1 Vertical stresses below a circular load

a/z	σ_{zz}/p
0.0000	0.0
0.2698	0.1
0.4005	0.2
0.5181	0.3
0.6370	0.4
0.7664	0.5
0.9176	0.6
0.1097	0.7
1.3871	0.8
1.9083	0.9
∞	1.0

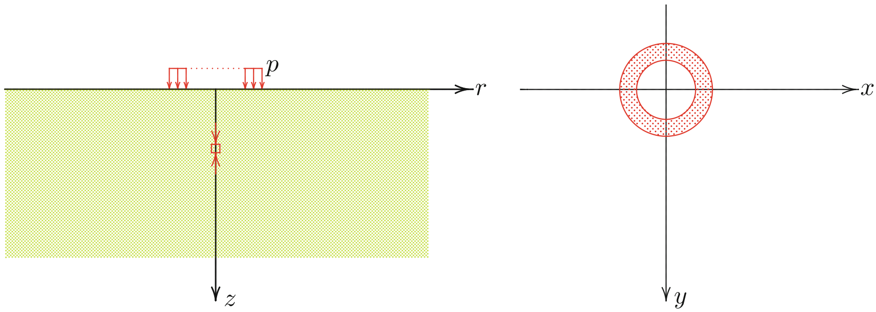

Fig. 28.2 Uniform load on a ring

stress is 0.6 p. The increase by 0.1 p must be due to the additional load, which is a uniform load over a circular ring, between the circles of radius 0.7664 z and the circle of radius 0.9176 z, see Fig. 28.2. It can be concluded that each ring shaped load, between two successive circles from Table 28.1 leads to a stress $\sigma_{zz} = 0.1\ p$ in a point at depth z just between the center of the circles and rings. If each ring is

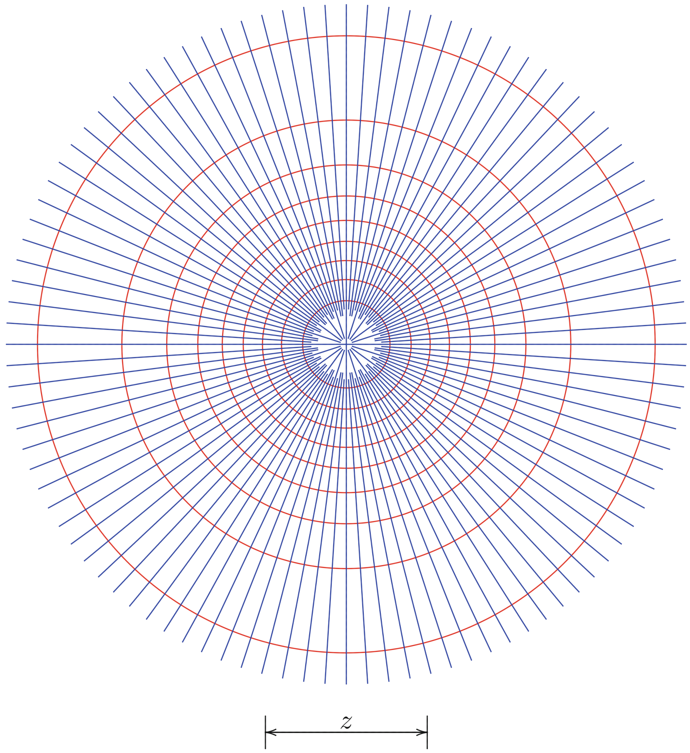

Fig. 28.3 Newmark's diagram

further subdivided into 100 equal sectors it follows that the load on every individual segment leads to a contribution to the stress σ_{zz} of magnitude $0.001\ p$.

Using these properties, Newmark's diagram can be constructed, see Fig. 28.3. A load of magnitude p on each of the 1000 elementary rectangles in this diagram gives rise to a stress $\sigma_{zz} = 0.001\ p$, in the point at a depth z below the origin.

For clarity, not all the radial lines inside the smallest circle in Fig. 28.3 have been fully continued to the center, because they all intersect in that point, and drawing all lines would result in a big blue spot. It should be remembered that in reality the inner ring, inside the smallest circle, contains 100 elementary rectangles, as each of the other rings. In the outer ring all radial lines should be extended towards infinity.

The principle of the application of Newmark's diagram is that a load of magnitude p on any one of the 1000 elementary rectangles leads to a stress $\sigma_{zz} = 0.001\ p$ in a point at depth z just below the center of the circles. This is valid for each of the rectangles, and because the problem is linear, the stress caused by various loads may be superimposed. This means that a load of magnitude p on a surface that covers n rectangles, leads to a stress σ_{zz} at a depth z below the origin of magnitude $\sigma_{zz} = n \times 0.001\ p$.

The value of the depth z plays an important role in the method. It actually determines the scale of the problem in the horizontal plane. To determine the stress at a deeper point the value of z is somewhat larger, and the size of the loaded area in the diagram will be somewhat smaller. This smaller area then covers a smaller number of rectangles, so that the stress will be smaller, as expected. This will be illustrated by an example.

The method can also be used for non-uniform loads. As a load p on any rectangle leads to a stress $\sigma_{zz} = 0.001\ p$, it follows that a load kp on a rectangle results in a contribution to the stress of magnitude $\sigma_{zz} = k \times 0.001\ p$. This will also be illustrated in the example.

It may be mentioned that similar diagrams can be developed for other stress components and for displacement components, sometimes for a given value of Poisson's ratio ν. In geotechnical engineering the most important stress component is the vertical normal stress σ_{zz}.

Example 28.1 As an example the problem of a load on an L-shaped region is considered, see Fig. 28.4. On the short leg the load is 15 kPa, on the larger leg the load is 5 kPa. The problem is to determine the vertical normal stress at a depth of 8 m below the point A. The first step in the solution is to draw the loaded area on such a scale that the reference length z in Newmark's diagram corresponds to 8 m, see Fig. 28.5, and such that the point A is located in the origin of the circles. The short leg of the loaded area now covers about 7 rectangles, and the long leg covers about 34 rectangles. This means that the stress is

$$\sigma_{zz} = 7 \times 0.001 \times 15\ \text{kPa} + 34 \times 0.001 \times 5\ \text{kPa} = 0.275\ \text{kPa}.$$

In order to determine the stress at a greater depth, say at a depth of 16 m, the loaded area should be drawn half as large. This then covers a smaller number of rectangles, so that the stress will be smaller.

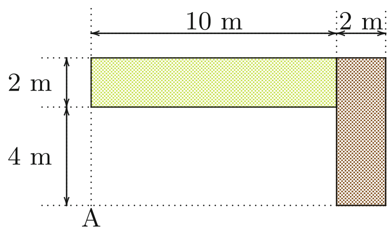

Fig. 28.4 Example 28.1

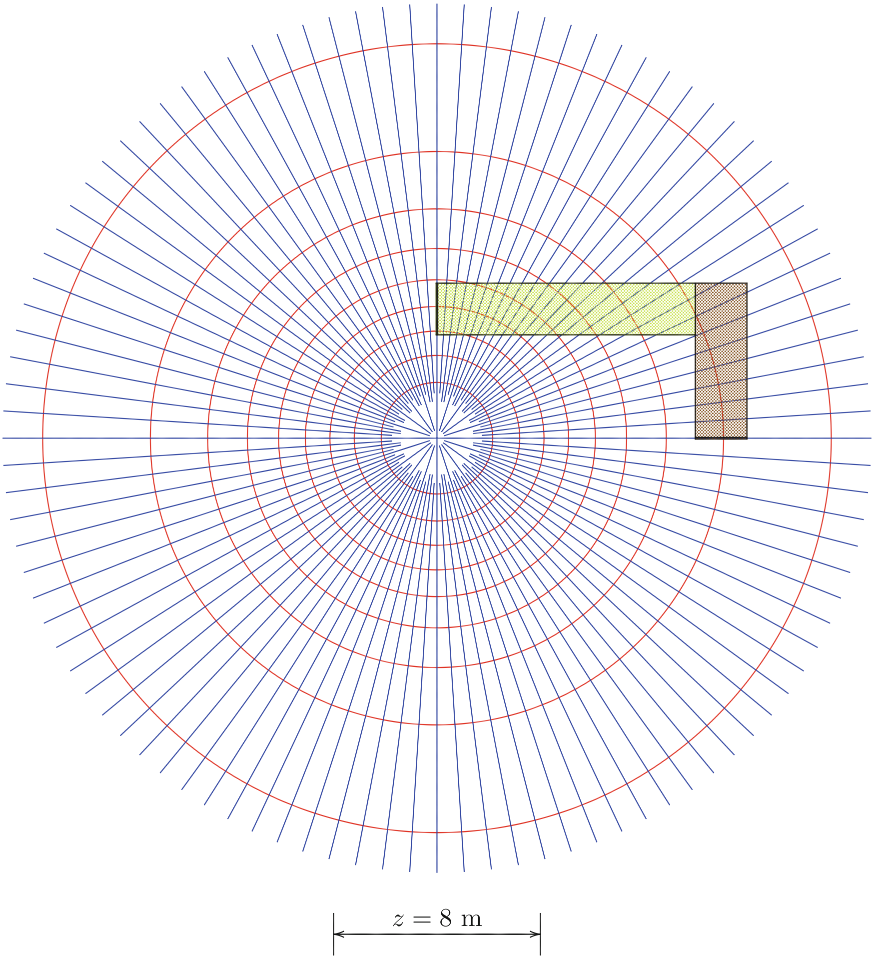

Fig. 28.5 Example 28.1

Problem 28.1 In the example considered above, determine the vertical stress σ_{zz} at a depth of 8 m below the corner point in the upper right corner in the plan.

Problem 28.2 Determine the stress in that corner point at the surface (for $z = 0$). Also determine the stress in the point A at the surface. And finally, also calculate the stress at a depth of 8000 m.

Problem 28.3 A square region of dimensions 4 m times 4 m is loaded by a uniform load 10 kPa. Determine the vertical normal stress at a depth of 4 m in a number of characteristic points : the center, one of the corners, and a point in the center of one of the sides.