Chapter 14
One-Dimensional Compression

In the previous chapters the deformation of soils has been separated into pure compression and pure shear. Pure compression is a change of volume in the absence of any change of shape, whereas pure shear is a change of shape, at constant volume. Ideally laboratory tests should be of constant shape or constant volume type, but that is not so simple. An ideal compression test would require isotropic loading of a sample, that should be free to deform in all directions. Although tests on spherical samples are indeed possible, it is more common to perform a compression test in which no horizontal deformation is allowed, by enclosing the sample in a rigid steel ring, and then deform the sample in vertical direction. In such a test the deformation consists mainly of a change of volume, but some change of shape also occurs. The main mode of deformation is compression, however.

14.1 Confined Compression Test

In the confined compression test, or *oedometer test*, a cylindrical soil sample is enclosed in a very stiff steel ring, and loaded through a porous plate at the at the top, see Fig. 14.1. The equipment is usually placed in a somewhat larger container, filled with water. Pore water may be drained from the sample through porous stones at the bottom and the top of the sample. The load is usually applied by a dead weight pressing on the top of the sample. This load can be increased in steps, by adding weights. The ring usually has a sharp edge at its top, which enables to cut the sample from a larger soil body.

In this case there can be no horizontal deformations, by the confining ring, so that

$$\varepsilon_{xx} = \varepsilon_{yy} = 0. \tag{14.1}$$

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A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_14

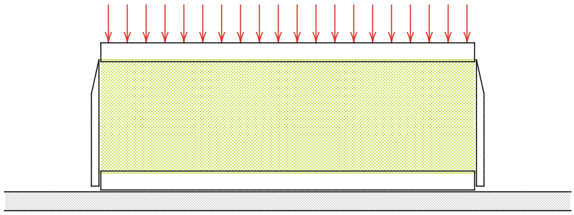

Fig. 14.1 Confined compression test

This means that the only non-zero strain is a vertical strain. The volume strain will be equal to that strain,

$$\varepsilon_{vol} = \varepsilon = \varepsilon_{zz}. \tag{14.2}$$

For convenience this strain will be denoted simply as ε. The load of the sample is a vertical stress σ_{zz}, which will be denoted as σ,

$$\sigma = \sigma_{zz}. \tag{14.3}$$

When performing the test, it is observed, as expected, that the increase of vertical stress caused by a loading from say 10 to 20 kPa leads to a larger deformation than a loading from 20 to 30 kPa. The sample becomes gradually stiffer, when the load increases. Often it is observed that an increase from 20 to 40 kPa leads to the same incremental deformation as an increase from 10 to 20 kPa. And increasing the load from 40 to 80 kPa gives the same additional deformation. Each doubling of the load has about the same effect. This suggests to plot the data on a semi-logarithmic scale, see Fig. 14.2. In this figure $\log(\sigma/\sigma_0)$ has been plotted against ε, where σ_0 denotes the initial stress. The test results appear to form a straight line, approximately, on this scale. The logarithmic relation between vertical stress and strain has been found first by Terzaghi, around 1930.

It means that the test results can be described reasonably well by the formula

$$\varepsilon = -\frac{1}{C}\ln\left(\frac{\sigma}{\sigma_0}\right). \tag{14.4}$$

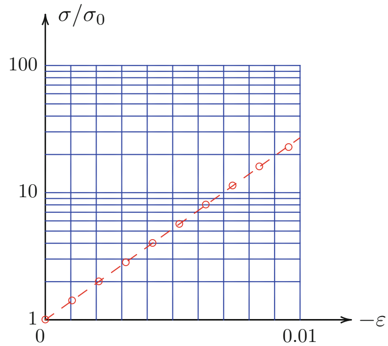

Fig. 14.2 Results

Table 14.1 Compression constants

Type of soil	C	C_{10}
Sand	50–500	20–200
Silt	25–125	10–50
Clay	10–100	4–40
Peat	2–25	1–10

Using this formula each doubling of the load, i.e. loadings following the series $1, 2, 4, 8, 16, \ldots,$ gives the same strain. The relation (14.4) is often denoted as Terzaghi's logarithmic formula. Its approximate validity has been verified by many laboratory tests.

In engineering practice the formula is sometimes slightly modified by using the common logarithm (of base 10), rather than the natural logarithm (of base e), perhaps because of the easy availability of semi-logarithmic paper on the basis of the common logarithm. The formula then is

$$\varepsilon = -\frac{1}{C_{10}} \log(\frac{\sigma}{\sigma_0}). \tag{14.5}$$

Because $\log(x) = \ln(x)/2.3$ the relation between the constants is

$$C_{10} = \frac{C}{2.3}, \tag{14.6}$$

or

$$C = 2.3 \times C_{10}. \tag{14.7}$$

The *compression constants* C and C_{10} are dimensionless parameters. Some average values are shown in Table 14.1.

The large variation in the compressibility suggests that the table has only limited value. The compression test is a simple test, however, and the constants can easily be determined for a particular soil, in the laboratory. The circumstance that there are two forms of the formula, with a factor 2.3 between the values of the constants, means that great care must be taken that the same logarithm is being used by the laboratory and the consultant or the design engineer.

The values in Table 14.1 refer to *virgin loading*, i.e. cases in which the load on the soil is larger than the previous maximum load. If the soil is first loaded, then unloaded, and next is loaded again, the results, when plotted on a logarithmic scale for the stresses, are as shown in Fig. 14.3. Just as in loading, a straight line is obtained during the unloading branch of the test, but the stiffness is much larger, by a factor of about 10. When a soil is loaded below its preconsolidation load the stress strain relation can best be described by a logarithmic formula similar to the ones presented above, but using a coefficient A rather than C, where the values of A are about a

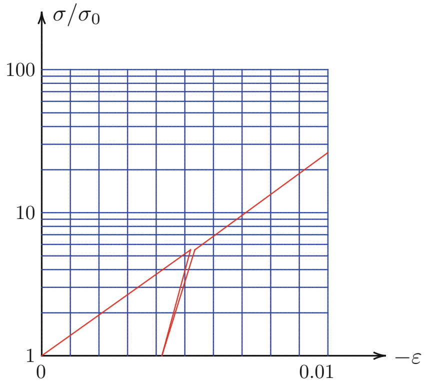

Fig. 14.3 Loading, unloading, and cyclic loading

factor 10 larger than the values given in Table 14.1. Such large values can also be used in cyclic loading. A typical response curve for cyclic loading is shown in the right part of Fig. 14.3. After each full cycle there will be a small permanent deformation. When loading the soil beyond the previous maximum loading the response is again much softer.

In many countries, such as the Scandinavian countries and the USA, the results of a confined compression test are often described in a slightly different form, using the void ratio e to express the deformation, rather than the strain ε. The formula then used is

$$e_1 - e = C_c \log(\frac{\sigma}{\sigma_0}), \tag{14.8}$$

where e_1 represents the void ratio at the initial stress σ_0. In this representation the test results also lead to a straight line, when using a logarithmic scale for the stresses. The formula indicates that the void ratio decreases when the stress increases, which corresponds to a compression of the soil. The coefficient C_c is denoted as the *compression index*. A highly compressible soil will have a large value of C_c. As seen before the behavior in unloading and reloading is much stiffer. The compression index is then much smaller (by about a factor 10). Three typical branches of the response are shown in Fig. 14.4. The relationship shown in the figure is often denoted as an $e - \log(p)$ diagram, where the notation p has been used to indicate the effective stress.

To demonstrate that Eq. (14.8) is in agreement with the formula (14.5), given before, it may be noted that the strain ε has been defined as $\varepsilon = \Delta V/V$, where V is the volume of the soil. This can be expressed as $V = (1+e)V_p$, where e is the void ratio, and V_p is the volume of the particles. Because the particle volume is constant (the particles are practically incompressible) it follows that $\Delta V = \Delta e\, V_k$, so that

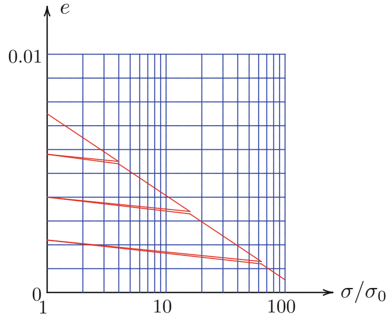

Fig. 14.4 $e-\log p$

$$\varepsilon = \frac{\Delta e}{1+e}. \tag{14.9}$$

Equation (14.8) therefore can also be written as

$$\varepsilon = -\frac{C_c}{1+e}\log(\frac{\sigma}{\sigma_0}), \tag{14.10}$$

Comparison with Eq. (14.5) shows that the relation between C_c and C_{10} is

$$\frac{1}{C_{10}} = \frac{C_c}{1+e}. \tag{14.11}$$

It is of course unfortunate that different coefficients are being used to describe the same phenomenon. This can only be explained by the historical developments in different parts of the world. It is especially inconvenient that in both formulas the constant is denoted by the character C, but in one form it appears in the numerator, and in the other one in the denominator. A large value for C_{10} corresponds to a small value for C_c. It can be expected that the compression index C_c will prevail in the future, as this has been standardized by ISO, the International Organization for Standardization.

It may also be noted that in a well known model for elasto-plastic analysis of deformations of soils, the Cam clay model, developed at Cambridge University, the compression of soils is described in yet another somewhat different form,

$$\varepsilon = -\lambda \ln(\frac{\sigma}{\sigma_0}). \tag{14.12}$$

The difference with Eq. (14.8) is that a natural logarithm is used rather than the common logarithm (the difference being a factor 2.3), and that the deformation is expressed by the strain ε rather than the void ratio e. The difference between these two quantities is a factor $1+e$.

The logarithmic relations given in this chapter should not be considered as fundamental physical laws. Many non-linear phenomena in physics produce a straight line when plotted on semi-logarithmic paper, or if that does not work, on double logarithmic paper. This may lead to very useful formulas, but they need not have much fundamental meaning. The error may well be about 1 to 5%. It should be noted that the approximation in Terzaghi's logarithmic compression formula is of a different nature than the approximation in Newton's laws. These last are basic physical laws (even though Einstein has introduced a small correction). The logarithmic compression formula is not much more than a convenient approximation of test results.

14.2 Elastic Analysis

In a confined compression test on a sample of an isotropic linear elastic material, the lateral stresses are, using (13.8), and noting that $\varepsilon_{xx} = \varepsilon_{yy} = 0$,

$$\sigma_{xx} = \sigma_{yy} = \frac{\nu}{1-\nu}\,\sigma_{zz}. \tag{14.13}$$

From the last equation of the system (13.8) it now follows that

$$\varepsilon_{zz} = -\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\,\sigma_{zz}. \tag{14.14}$$

When expressed into the constants K and G this can also be written as

$$\sigma_{zz} = -(K + \frac{4}{3}G)\,\varepsilon_{zz}. \tag{14.15}$$

The elastic coefficient for one dimensional confined compression appears to be $K + \frac{4}{3}G$. This is sometimes denoted as D, the *constrained modulus*,

$$D = K + \frac{4}{3}G = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = 3K(\frac{1-\nu}{1+\nu}). \tag{14.16}$$

When $\nu = 0$ it follows that $D = E$; if $\nu > 0 : D > E$. In the extreme case that $\nu = \frac{1}{2}$ the value of $D \to \infty$. Such a material is indeed incompressible.

Similar to the considerations in the previous chapter on tangent moduli the logarithmic relationship (14.4) may be approximated for small stress increments. The relation can be linearized by differentiation. This gives

$$\frac{d\varepsilon}{d\sigma} = -\frac{1}{C\sigma}. \tag{14.17}$$

so that

$$\Delta\sigma = -C\sigma\,\Delta\varepsilon. \tag{14.18}$$

Comparing Eqs. (14.15) and (14.18) it follows that for small incremental stresses and strains one write, approximately,

$$D = K + \frac{4}{3}G = C\sigma. \tag{14.19}$$

This means that the stiffness increases linearly with the stress, and that is in agreement with many test results (and with earlier remarks).

The formula (14.19) is of considerable value to estimate the elastic modulus of a soil. Many computational methods use the concepts and equations of elasticity theory, even when it is acknowledged that soil is not a linear elastic material. On the basis of Eq. (14.19) it is possible to estimate an elastic "constant". For a layer of sand at 20 m depth, for instance, it can be estimated that the effective stress will be about 170 kPa (assuming that the soil above the sand is clay, and that the water table is very high). For sand the value of C_{10} is about 100, and thus $C \approx 230$. This means that the elastic modulus is about 40,000 kPa = 40 MPa. This is a useful first estimate of the elastic modulus for virgin loading. As stated before, the soil will be about a factor 10 stiffer for cyclic loading. This means that for problems of wave propagation the elastic modulus to be used may be about 400 MPa. It should be noted that these are only first estimates. The true values may be larger or smaller by a factor 2, or even more. And nothing can beat measuring the stiffness in a laboratory test or a field test, of course.

Example 14.1 In a confined compression test, see Fig. 14.1, a soil sample of 2 cm thickness has been preloaded by a stress of 100 kPa. An additional load of 20 kPa leads to a vertical displacement of 0.030 mm. Determine the value of the compression constant C_{10}.

Solution

The formula to be applied is Eq. (14.5), where now $\sigma_0 = 100$ kPa and $\sigma = 120$ kPa, so that $\log(\sigma/\sigma_0) = 0.0792$. The strain has been measured as $\varepsilon = -0.030/20 = -0.0015$. It follows that $C_{10} = 52.8$.

If the test is continued with a next loading step of 20 kPa, the additional strain in that step will be $\varepsilon = -\log(140/120)/C_{10} = -0.00127$. This means that the additional displacement will be 0.025 mm.

The total strain after the two steps can be calculated as $\varepsilon = -\log(140/100)/C_{10} = -0.00277$, which is precisely the sum of the two values in each step, as one would expect from a consistent theory. Mathematically speaking it is a consequence of the property of the logarithmic function that $\log(ab) = \log(a) + \log(b)$.

Problem 14.1 A clay layer of 4 m thickness is located below a sand layer of 10 m thickness. The volumetric weights are all 20 kN/m^3, and the groundwater table coincides with the soil surface. The compression constant of the clay is $C_{10} = 20$.

Predict the settlement of the soil by compression of the clay layer due to an additional load of 40 kPa.

Problem 14.2 A sand layer is located below a road construction of total weight 20 kPa. The sand has been densified by vibration before the road was built. Estimate the order of magnitude of the elastic modulus of the soil that can be used for the analysis of traffic vibrations in the soil.

Problem 14.3 The book Soil Mechanics by Lambe and Whitman (1969) gives the value $C_c = 0.47$ for the compression index of a certain clay, see page 319. The void ratio, given in Fig. E22.1 of that book, is about 0.95. Estimate C_{10}, and verify whether this value is in agreement with Table 14.1.

Reference

T.W. Lambe, R.V. Whitman, *Soil Mechanics* (Wiley, New York, 1969)