Chapter 3
Particles, Water, Air

Soils usually consist of particles, water and air. In order to describe a soil various parameters are used to describe the distribution of these three components, and their relative contribution to the volume of a soil. These are also useful to determine other parameters, such as the weight of the soil. They are defined in this chapter.

3.1 Porosity

An important basic parameter is the *porosity* n, defined as the ratio of the volume of the pore space and the total volume of the soil,

$$n = V_p / V_t. \tag{3.1}$$

For most soils the porosity is a number between 0.30 and 0.45 (or, as it is usually expressed as a percentage, between 30 and 45%). When the porosity is small the soil is called densely packed, when the porosity is large it is loosely packed.

It may be interesting to calculate the porosities for two particular cases. The first case is a very loose packing of spherical particles, in which the contacts between the spheres occur in three mutually orthogonal directions only. This is called a *cubic array* of particles, see Fig. 3.1. If the diameter of the spheres is D, each sphere occupies a volume $\pi D^3/6$ in space. The ratio of the volume of the solids to the total volume then is $V_p/V_t = \pi/6 = 0.5236$, and the porosity of this assembly thus is $n = 0.4764$. This is the loosest packing of spherical particles that seems possible. Of course, it is not stable: any small disturbance will make the assembly collapse.

A very dense packing of spheres can be constructed by starting from layers in which the spheres form a pattern of equilateral triangles, see Fig. 3.2. The packing is constructed by packing the layers such that the spheres of the next layer just fit in the hollow space between three spheres of the previous layer. The axial lines from a

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_3

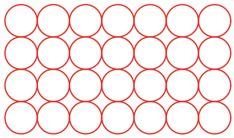

Fig. 3.1 Cubic array

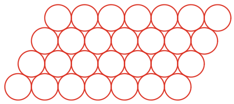

Fig. 3.2 Densest array

sphere with the three spheres that support it from below form an regular tetrahedron, having sides of magnitude D. The height of each tetrahedron is $D\sqrt{2/3}$. Each sphere of the assembly, with its neighboring part of the voids, occupies a volume in space of magnitude $D \times (D\sqrt{3/4}) \times (D\sqrt{2/3}) = D^3\sqrt{1/2}$. Because the volume of the sphere itself is $\pi D^3/6$, the porosity of this assembly is $n = 1 - \pi/\sqrt{18} = 0.2595$. This seems to be the most dense packing of a set of spherical particles.

Although soils never consist of spherical particles, and the values calculated above have no real meaning for actual soils, they may give a certain indication of what the porosity of real soils may be. It can thus be expected that the porosity n of a granular material may have a value somewhere in the range from 0.25 to 0.45. Practical experience confirms this statement.

The amount of pores can also be expressed by the *void ratio* e, defined as the ratio of the volume of the pores to the volume of the solids,

$$e = V_p/V_s. \tag{3.2}$$

In many countries this quantity is preferred to the porosity, because it expresses the pore volume with respect to a fixed volume (the volume of the solids). Because the total volume of the soil is the sum of the volume of the pores and the volume of the solids, $V_t = V_p + V_s$, the porosity and the void ratio can easily be related,

$$e = n/(1-n), \qquad n = e/(1+e). \tag{3.3}$$

The porosity can not be smaller than 0, and can not be greater than 1. The void ratio can be greater than 1.

The void ratio is also used in combination with the *relative density*. This quantity is defined as

$$RD = \frac{e_{\max} - e}{e_{\max} - e_{\min}}. \tag{3.4}$$

Here e_{max} is the maximum possible void ratio, and e_{min} the minimum possible value. These values may be determined in the laboratory. The densest packing of the soil can be obtained by strong vibration of a sample, which then gives $e_{\min}$. The loosest packing can be achieved by carefully pouring the soil into a container, or by letting the material subside under water, avoiding all disturbances, which gives $e_{\max}$. The accuracy of the determination of these two values is not very good. After some more vibration the sample may become even denser, and the slightest disturbance may influence a loose packing.

It follows from Eq. (3.4) that the relative density varies between 0 and 1. A small value, say $RD < 0.5$, means that the soil can easily be densified. Such a densification can occur in the field rather unexpectedly, for instance in case of a sudden shock (an earthquake), with dire consequences.

Of course, the relative density can also be expressed in terms of the porosity, using Eq. (3.3), but this leads to an inconvenient formula, and therefore this is unusual.

3.2 Degree of Saturation

The pores of a soil may contain water and air. To describe the ratio of these two the *degree of saturation* S is introduced as

$$S = V_w / V_p. \tag{3.5}$$

Here V_w is the volume of the water, and V_p is the total volume of the pore space. The volume of air (or any other gas) per unit pore space then is $1 - S$. If $S = 1$ the soil is completely saturated, if $S = 0$ the soil is perfectly dry.

3.3 Density

For the description of the density and the volumetric weight of a soil, the densities of the various components are needed. The *density* of a substance is the mass per unit volume of that substance. For water this is denoted by ρ_w, and its value is about 1000 kg/m^3. Small deviations from this value may occur due to temperature differences or variations in salt content. In soil mechanics these are often of minor importance, and it is often considered accurate enough to assume that

$$\rho_w = 1000\ \text{kg/m}^3. \tag{3.6}$$

For the analysis of soil mechanics problems the density of air can usually be disregarded.

The density of the solid particles depends upon the actual composition of the solid material. In many cases, especially for quartz sands, its value is about

$$\rho_p = 2650\ \text{kg/m}^3. \tag{3.7}$$

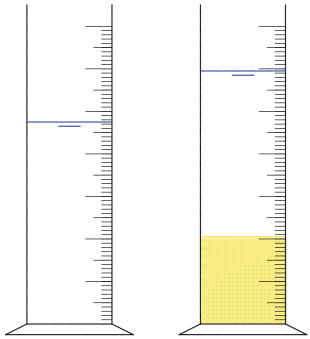

Fig. 3.3 Measuring the density of solid particles

This value can be determined by carefully dropping a certain mass of particles (say W_p) in a container partially filled with water, see Fig. 3.3. The precise volume of the particles can be measured by observing the rise of the water table in the glass. This is particularly easy when using a graduated measuring glass. The rising of the water table indicates the volume of the particles, V_p. Their mass W_p can be measured most easily by measuring the weight of the glass before and after dropping the particles into it. The density of the particle material then follows immediately from its definition,

$$\rho_p = W_p/V_p. \tag{3.8}$$

The principle of this simple test, in which the volume of a body having a very irregular shape (a number of sand particles) is measured, is due to Archimedes. He had been asked to check the composition of a golden crown, of which it was suspected that it contained silver (which is cheaper). He realized that this could be achieved by comparing the density of the crown with the density of a piece of pure gold, but then he had to determine the precise volume of the crown. The legend has it that when stepping into his bath he discovered that the volume of a body submerged in water, whatever its precise shape, equals the volume of water above the original water table. While shouting "Eureka!" he ran into the street, according to the legend, to the surprise of the bystanders.

3.4 Volumetric Weight

In soil mechanics it is often required to determine the total weight of a soil body. This can be calculated if the porosity, the degree of saturation and the densities are known. The weight of the water in a volume V of soil is $Sn\rho_w gV$, and the weight of the particles in that volume is $(1-n)\rho_p gV$, where g is the strength of the gravity field, or the acceleration of gravity. The value of the gravity constant is about $g = 9.8$ N/kg, or, approximately, $g = 10$ N/kg. Thus the total weight W is

$$W = [Sn\rho_w g + (1-n)\rho_p g]V. \tag{3.9}$$

This means that the *volumetric weight* γ, defined as the weight per unit volume, is

$$\gamma = W/V = Sn\rho_w g + (1-n)\rho_p g. \tag{3.10}$$

This formula indicates that the volumetric weight is determined by a large number of soil parameters: the degree of saturation, the porosity, the densities of water and soil particles, and the gravity constant. In reality it is much simpler to determine the volumetric weight (often also denoted as the *unit weight*) directly by measuring the weight W of a volume V of soil. It is then not necessary to determine the contribution of each of the components.

If the soil is completely dry the dry volumetric weight is

$$\gamma_d = W_d/V = (1-n)\rho_p g. \tag{3.11}$$

This value can also be determined directly by weighing a volume of dry soil. In order to dry the soil a sample may be placed in an oven. The temperature in such an oven is usually close to 100°, so that the water will evaporate quickly. At a much higher temperature there would be a risk that organic parts of the soil would be burned.

From the dry volumetric weight the porosity n can be determined, see Eq. (3.11), provided that the density of the particle material is known. This is a common method to determine the porosity in a laboratory.

If both the original volumetric weight γ and the dry volumetric weight γ_d are known, by measuring the weight and volumes both in the original state and after drying, the porosity n may be determined from Eq. (3.11), and then the degree of saturation S may be determined using Eq. (3.10). Unfortunately, this procedure is not very accurate for soils that are almost completely saturated, because a small error in the measurements may cause that one obtains, for example, $S = 0.97$ rather than the true value $S = 0.99$. In itself this is rather accurate, but the error in the air volume is then 300%. In some cases, this may lead to large errors, for instance when the compressibility of the water-air-mixture in the pores must be determined.

3.5 Water Content

The *water content* is another useful parameter, especially for clays. It has been used in the previous chapter. By definition the water content w is the ratio of the weight (or mass) of the water and the solids,

$$w = W_w/W_p. \tag{3.12}$$

It may be noted that this is not a new independent parameter, because

$$w = S\frac{n}{1-n}\frac{\rho_w}{\rho_p} = Se\frac{\rho_w}{\rho_p}. \tag{3.13}$$

For a completely saturated soil ($S = 1$) and assuming that $\rho_p/\rho_w = 2.65$, it follows that void ratio e is about 2.65 times the water content.

A normal value for the porosity is $n = 0.40$. Assuming that $\rho_k = 2650$ kg/m^3 it then follows from Eq. (3.11) that $\gamma_d = 15900$ N/m^3, or $\gamma_d = 15.9$ kN/m^3. Values of the order of magnitude of 16 kN/m^3 are indeed common for dry sand. If the material is completely saturated it follows from Eq. (3.10) that $\gamma \approx 20$ kN/m^3. For saturated sand this is a common value. The volumetric weight of clay soils may also be about 20 kN/m^3, but smaller values are very well possible, especially when the water content is small, of course. Peat is often much lighter, sometimes hardly heavier than water.

Example 3.1 A glass is initially filled with some water, see Fig. 3.4. The volume of the water is measured to be 240 cm^3. Some sand particles are carefully poured into the water, avoiding the formation of air bubbles. The water table in the glass then rises to indicate a volume of 320 cm^3. The sand particles come to rest at the bottom of the glass, indicating a total volume of 144 cm^3. Calculate the porosity n of the sand. Also calculate the void ratio e.

Solution

The rise of the water level after pouring the sand particles indicates that the volume of the water plus the volume of the solid particles is 320 cm^3. Because the volume of the water is 240 cm^3 it follows that the volume of the solids is $V_s = 80$ cm^3. The level of the sand at the bottom of the glass indicates that the volume of the sand particles plus the volume of the water in the pores is $V_t = 144$ cm^3. It follows that the volume of the water in the pores is 64 cm^3. Because there is no air in the water or the sand it follows that the volume of the pores is $V_p = 64$ cm^3. The porosity now is $n = V_p/V_t = 0.44$, or $n = 44\%$. The void ratio is $e = V_p/V_s = 0.80$, or $e = 80\%$. Note that $e = n/(1-n)$ and $n = e/(1+e)$.

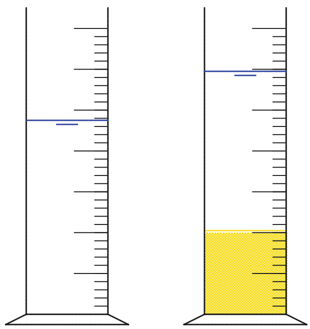

Fig. 3.4 Measuring the porosity

Example 3.2 If the glas is shaken, it will be observed that the water level remains the same, but the level of the sand decreases. If this level now indicates a total volume of 128 cm^3, calculate the porosity and the void ratio after shaking.

Solution

After shaking the volume of the sand (including the water in the pores) is $V_t = 128$ cm^3, of which the solid particles occupy a volume $V_s = 80$ cm^3 (as before), so that the volume of the pores is $V_p = 48$ cm^3. It follows that $n = V_p/V_t = 0.375$, or $n = 37.5\%$. The void ratio now is $e = V_p/V_s = 0.60$, or $e = 60\%$.

Example 3.3 A test such as shown in Fig. 3.4 can also be used to determine the density of the particle material, if not only the volumes are measured but also the weights. Let the initial water level in the glass indicate a volume of 312 cm^3, and the weight of glass and water be 568 g. After carefully pouring some sand particles into the glass, the water level rises to indicate a volume of 400 cm^3. The weight of the glass (with the water and the sand) now appears to be 800 g. Determine the density ρ_s of the particle material, in g/cm^3, or in kg/m^3.

Solution

The volume of the sand particles is $400 - 312 = 88$ cm^3, and the weight of these particles is $800 - 568 = 232$ g. This means that the density of the particle material is $\rho_s = 2.64$ g/cm^3, or $\rho_s = 2640$ kg/m^3

Example 3.4 A steel ring contains a sample of natural soil. The total weight of the ring and the soil appears to be 490 g. The ring is placed in an oven, in order to let the water evaporate. Then the weight of the ring and the dry soil is found to be 380 g. The ring itself (empty and dry) weighs 210 g. What is the water content of the soil? (Fig. 3.5).

Solution

The weight of the soil in its natural condition is 490−210 = 280 g, and the weight of the water initially was 490−380 = 110 g. This means that the water content is $w = W_w/W_p = 110/170 = 0.65$, or $w = 65\%$.

Problem 3.1 A truck loaded with 2 m^3 dry sand appears to weigh "3 tons" more than the weight of the empty truck. What is the meaning of the term "3 tons", and what is the volumetric weight of the sand?

Problem 3.2 If it is known that the density of the sand particles in the material of the previous problem is 2600 kg/m^3, then what is the porosity n? And the void ratio e?

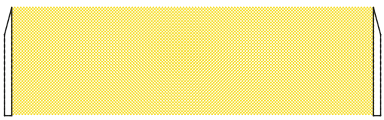

Fig. 3.5 Soil sample in ring

Problem 3.3 It would be possible to fill the pores of the dry sand of the previous problems with water. What is the volume of the water that the sand could contain, and then what is the volumetric weight of the saturated sand?

Problem 3.4 The soil in a polder consists of a clay layer of 5 m thickness, with a porosity of 50%, on top of a deep layer of stiff sand. The water level in the clay is lowered by 1.5 m. Experience indicates that then the porosity of the clay is reduced to 40%. What is the subsidence of the soil?

Problem 3.5 The particle size of sand is about 1 mm. Gravel particles are much larger, of the order of magnitude of 1 cm, a factor 10 larger. The shape of gravel particles is about the same as that of sand particles. What is the influence of the particle size on the porosity?