Chapter 7
Permeability

In this chapter the determination of the permeability of a soil sample by laboratory tests is presented. The two tests considered are Darcy's original test and the falling head test, which is better suited for soils of small permeability.

7.1 Permeability Test

In the previous chapter Darcy's law for the flow of a fluid through a porous medium has been formulated, in its simplest form, as

$$q = -k\frac{dh}{ds}. \tag{7.1}$$

This means that the hydraulic conductivity k can be determined if the specific discharge q can be measured in a test in which the gradient dh/ds is known. An example of a test setup is shown in Fig. 7.1. It consists of a glass tube, filled with soil. The two ends are connected to small reservoirs of water, the height of which can be adjusted. In these reservoirs a constant water level can be maintained. Under the influence of a difference in head Δh between the two reservoirs, water will flow through the soil. The total discharge Q can be measured by collecting the volume of water in a certain time interval. If the area of the tube is A, and the length of the soil sample is ΔL, then Darcy's law gives

$$Q = kA\frac{\Delta h}{\Delta L}. \tag{7.2}$$

Because $Q = qA$ this formula is in agreement with (7.1). Darcy performed tests as shown in Fig. 7.1 to verify his formula (7.2). For this purpose he performed tests

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_7

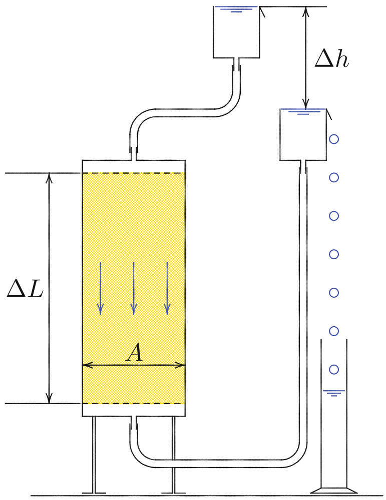


Fig. 7.1 Permeability test

with various values of Δh, and indeed found a linear relation between Q and Δh. The same test is still used very often to determine the hydraulic conductivity (coefficient of permeability) k.

For sand normal values of the hydraulic conductivity k range from 10^{-6} to 10^{-3} m/s. For clay the hydraulic conductivity usually is several orders of magnitude smaller, for instance $k = 10^{-9}$ m/s, or even smaller. This is because the permeability is approximately proportional to the square of the grain size of the material, and the particles of clay are about 100 or 1000 times smaller than those of sand. An indication of the hydraulic conductivity of various soils is given in Table 7.1.

As mentioned before, the permeability also depends upon properties of the fluid. Water will flow more easily through the soil than a thick oil. This is expressed in the formula (6.11),

$$k = \frac{\kappa \gamma_w}{\mu}, \tag{7.3}$$

where μ is the dynamic viscosity of the fluid. The quantity κ (the intrinsic permeability) depends upon the geometry of the grain skeleton only. A useful relation is given by the formula of Kozeny–Carman,

Table 7.1 Hydraulic conductivity k

Type of soil	k (m/s)
Gravel	10^{-3}–10^{-1}
Sand	10^{-6}–10^{-3}
Silt	10^{-8}–10^{-6}
Clay	10^{-10}–10^{-8}

Fig. 7.2 Failure of Teton Dam

$$\kappa = cd^2 \frac{n^3}{(1-n)^2}. \tag{7.4}$$

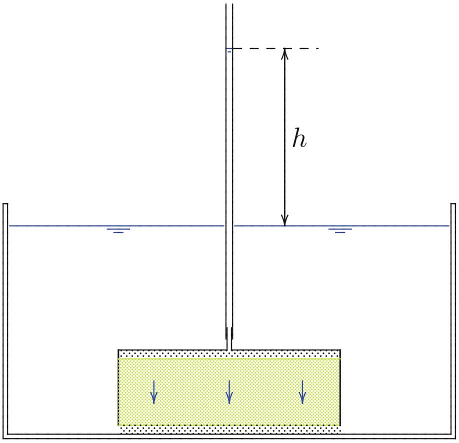
Here d is a measure for the grain size, and c is a coefficient, that now only depends upon the tortuosity of the pore system, as determined by the shape of the particles. Its value is about 1/200 or 1/100. Equation (7.4) is of little value for the actual determination of the value of the permeability κ, because the value of the coefficient c is still unknown, and because the hydraulic conductivity can easily be determined directly from a permeability test. The Kozeny–Carman formula (7.4) is of great value, however, because it indicates the dependence of the permeability on the grain size and on the porosity. The dependence on d^2 indicates, for instance, that two soils for which the grain size differs by a factor 1000 (sand and clay) may have a difference in permeability of a factor 10^6. Such differences are indeed realistic.

The large variability of the permeability indicates that this may be a very important parameter. In constructing a large dam, for instance, the dam is often built from highly permeable material, with a core of clay. This clay core has the purpose to restrict water losses from the reservoir behind the dam. If the core is not very homogeneous, and contains thin layers of sand, or if the clay core is not well encased into the rock bottom, the function of the clay core is disturbed to a high degree, and large amounts of water may be leaking through the dam. Severe accidents of this type have happened, see for instance Fig. 7.2, which shows the collapse of the Teton Dam, in Idaho, USA, in 1976.

7.2 Falling Head Test

For soils of low permeability, such as clay, the normal permeability test shown in Fig. 7.1 is not suitable, because only very small quantities of fluid are flowing through the soil, and it would take very long to collect an appreciable volume of water. For such soils a test set up as illustrated in Fig. 7.3, the *falling head test*, is more suitable. In this apparatus a clay sample is enclosed by a circular ring, placed in a container filled with water. The lower end of the sample is in open connection with the water

Fig. 7.3 Falling head test



in the container, through a porous stone below the sample. At the top of the sample it is connected to a thin glass tube, in which the water level is higher than the constant water level in the container. Because of this difference in water level, water will flow through the sample, in very small quantities, but sufficient to be observed by the lowering of the water level in the thin tube.

In this case the head difference h is not constant, because no water is added to the system, and the level h is gradually reduced. This water level is observed as a function of time. On the basis of Darcy's law the discharge is

$$Q = \frac{kAh}{L}. \tag{7.5}$$

If the cross sectional area of the glass tube is a it follows that

$$Q = -a\frac{dh}{dt}. \tag{7.6}$$

Elimination of Q from these two equations gives

$$\frac{dh}{dt} = -\frac{kA}{aL}\,h. \tag{7.7}$$

This is a differential equation for h, that can easily be solved,

$$h = h_0 \exp(-kAt/aL). \tag{7.8}$$

where h_0 is the value of the head difference h at time $t = 0$. If the head difference at time t is h, the hydraulic conductivity k can be calculated from the relation

$$k = \frac{aL}{At} \ln(\frac{h_0}{h}). \tag{7.9}$$

If the area of the tube a is very small compared to the area A of the sample, it is possible to measure relatively small values of k with sufficient accuracy. The advantage of this test is that very small quantities of flowing water can be measured.

It may be remarked that the determination of the hydraulic conductivity of a sample in a laboratory is relatively easy, and very accurate, but large errors may occur during sampling of the soil in the field, and perhaps during the transportation from the field to the laboratory. Furthermore, the measured value only applies to that particular sample, having small dimensions. This value may not be representative for the hydraulic conductivity in the field. In particular, if a thin layer of clay has been overlooked, the permeability of the soil for vertical flow may be much smaller than follows from the measurements. On the other hand, if it is not known that a clay layer contains pockets of sand, the flow in the field may be much larger than expected on the basis of the permeability test on the clay. It is often advisable to measure the permeability in the field (in situ), measuring the average permeability of a sufficiently large region.

Example 7.1 In a permeability test (see Fig. 7.1) a head difference of 20 cm is being maintained between the top and bottom ends of a sample of 40 cm height. The inner diameter of the circular tube is 10 cm. It has been measured that in 1 min an amount of water of 35 cm^3 is collected in a measuring glass. What is the value of the hydraulic conductivity k?

Solution

In this case the gradient is $i = -20/40 = -0.5$. The discharge is $Q = 35\ \text{cm}^3/60\ \text{s}$ $= 0.5833\ \text{cm}^3/\text{s}$. The area of a cross section of the tube is $\pi \times (5\ \text{cm})^2 = 78.54\ \text{cm}^2$. This means that the specific discharge is $q = 7.426 \times 10^{-3}$ cm/s. Because $q = -ki$ it follows that $k = 0.0148$ cm/s.

Example 7.2 A circular glass tube is filled with 20 cm of sand, having a hydraulic conductivity of 10^{-5} m/s, and on top of that 20 cm sand having a hydraulic conductivity that is a factor 4 larger, see Fig. 7.4. The inner diameter of the circular tube is 10 cm. Calculate the discharge Q through this layered sample, if the head difference between the top and bottom of the sample is 20 cm.

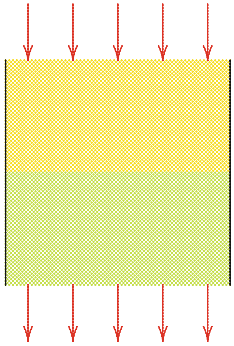
Solution

In this case the water must flow through two media, in series,

$$Q_1 = k_1 A_1 \Delta h_1/\Delta s_1, \quad Q_2 = k_2 A_2 \Delta h_2/\Delta s_2.$$

Continuity of flow requires that $Q_1 = Q_2 = Q$. Furthermore it is given that $A_1 = A_2 = A = 78.54\ \text{cm}^2$, $\Delta s_2 = \Delta s_1 = \Delta s = 20$ cm and $k_2 = 4k_1 = 4 \times 10^{-3}$ cm/s, and it is also given that $\Delta h_1 + \Delta h_2 = 20$ cm.

Fig. 7.4 Two-layered soil



The simplest way to solve this problem is to express the total head difference Δh as

$$\Delta h = \Delta h_1 + \Delta h_2 = \frac{Q}{A}(\frac{\Delta s_1}{k_1} + \frac{\Delta s_2}{k_2}),$$

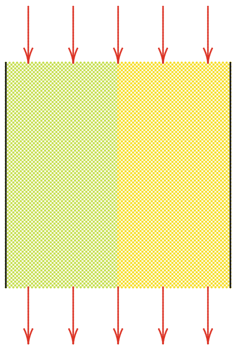
or

$$Q = \frac{(\Delta h_1 + \Delta h_2)A}{\Delta s_1/k_1 + \Delta s_2/k_2}.$$

Using the given data it now follows that $Q = 0.0628\ \text{cm}^3/\text{s}$.

Example 7.3 A similar circular glass tube is filled over one half of its area with 40 cm of sand, having a hydraulic conductivity of 10^{-5} m/s, and over the other half of its area with 40 cm sand having a hydraulic conductivity that is a factor 4 larger, see Fig. 7.5. The inner diameter of the circular tube is 10 cm. Calculate the discharge Q through this layered sample, if the head difference between the top and bottom of the sample is 20 cm.

Fig. 7.5 Two-layered soil



Solution

In this case the water must flow through the two media in parallel. The discharge through the system now is the sum of the discharges through the two media, $Q = Q_1 + Q_2$, with

$$Q_1 = \tfrac{1}{2}k_1 A_1 \Delta h/\Delta s, \quad Q_2 = \tfrac{1}{2}k_2 A_2 \Delta h/\Delta s,$$

It is given that $A_1 = A_2 = \frac{1}{2}A = 39.27\ \text{cm}^2$, and for both parts $\Delta h/\Delta s = 20/40 = 0.5$.

It then follows that $Q = 0.0982\ \text{cm}^3/\text{s}$.

Problem 7.1 In Darcy's test, see Fig. 7.1, the fluid flows through the soil in vertical direction. In principle the tube can also be placed horizontally. The formulas then remain the same, and the measurement of the head difference is simpler. The test is usually not done in this way, however. Why not?

Problem 7.2 An engineer must give a quick estimate of the permeability of a certain sand. He remembers that the hydraulic conductivity of the sand in a previous project was 8 m/d. The sand in the current project seems to have particles that are about $\frac{1}{4}$ times as large. What is his estimate?