Chapter 48
Pile Foundations

In Deltaic areas in the world, for instance the western part of the Netherlands, the soil consists of layers of soft soil (clay and peat), on a rather stiff sand layer, of pleistocene origin. The bearing capacity of the sand layer below the soft soil is derived for a large part from its deep location, with the soft layers acting as a surcharge. And the properties of the sand itself, a relatively high density, and a high friction angle, also help to give this sand layer a good bearing capacity. The system of soft soils and a deeper stiff sand layer is very suitable for a pile foundation. In this chapter a number of important soil mechanics aspects of such pile foundations are briefly discussed.

48.1 Bearing Capacity of a Pile

For the determination of the bearing capacity of a foundation pile it is possible to use a theoretical analysis, on the basis of Brinch Hansen's general bearing capacity formula (see Chap. 42). In this analysis the basic parameters are the shear strength of the sand layer (characterized by its cohesion c and its friction angle ϕ), and the weight of the soft layers, which can be taken into account as a surcharge q. In engineering practice a simpler, more practical and more reliable method has been developed, on the basis of a cone penetration test, considering this as a model test. It would be even better to perform a pile loading test on the pile, in which the pile is loaded, for instance by concrete blocks on a steel frame, with a test load approaching its maximum bearing capacity. This is very expensive, however, and the CPT is usually considered reliable enough. In a homogeneous soil it can be assumed that under static conditions the failure load of a long pile (expressed as a pressure) is independent of the diameter of the pile. This means that the cone resistance measured in a CPT can be considered to be equal to the bearing capacity of the pile point. A possible theoretical foundation behind this statement is that the failure is produced by shear deformations in a zone around the pile, the dimensions of which are determined by

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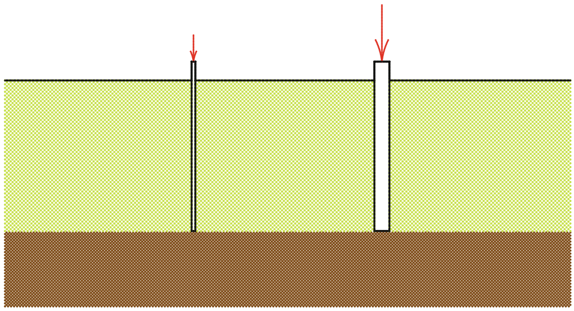

Fig. 48.1 CPT and pile

the only dimension in the problem, the diameter of the pile. If the pile diameter is taken twice as large, the dimensions of the failure zone around the pile will also be twice as large. The total force (stress times area) then is four times as large, see Fig. 48.1. This is also in agreement with the theory behind Brinch Hansen's formula, provided that the third term (representing the weight of the soil below the foundation level, and the width of the foundation) is small. This will be the case if the pile diameter is small compared to its length.

In reality the soil around the pile point usually is not perfectly homogeneous. Very often the soil consists of layers having different properties. For this case practical design formulas have been developed, which take into account the different cone resistance below and above the level of the pile point. Moreover, in these design formulas the possibility that the failure mode will prefer the weakest soil can be accounted for. In the Netherlands the resistance of a pile is assumed to consist of three contributions,

$$p = \frac{1}{2}\left[\frac{1}{2}(p_1 + p_2) + p_3\right]. \tag{48.1}$$

In this equation p_1 is the smallest value of the cone resistance below the pile point, up to a depth of $4d$, where d is the diameter of the pile, p_2 is the average cone resistance in that zone, and p_3 is a representative low value of the cone resistance above the pile point, in a zone up to $8d$ above the pile point. In this way a representative average value of the cone resistance around the pile point is obtained, in which engineering judgement is combined with experience.

A pile may also have a bearing capacity due to friction along the length of the pile. This is very important for piles in sand layers. In applications in very soft soil (clay layers), the contribution of friction is generally very unreliable, because the soil may be subject to settlements, whereas the pile may be rigid, if it has been installed into a deep sand layer. It may even happen that the subsiding soil exerts a downward friction force on the pile, *negative skin friction*, which reduces the effective bearing capacity of the pile. Friction is of course very important for tension piles, for which it is the only contributing mechanism.

The maximum value of the skin friction can be determined very well using a friction cone, that is a penetration test in which the sleeve friction is also measured.

The local values are often very small, however, so that the measured data are not very accurate. For sandy soils the friction therefore is often correlated to the cone resistance.

48.2 Statically Determinate Pile Foundation

If the maximum allowable load on a single pile is known, from a theoretical analysis, or from the interpretation of a cone penetration test, or from a pile loading test, the number of piles in the foundation of a large structure can be determined from the total load of the structure, including its weight. In this process a sufficiently large safety coefficient (e.g. 1.5 or 1.8) must be taken into account, to avoid possible failure. If all the loads are vertical the piles may all be vertical as well. The installation is then also simplest, as driving a vertical pile is easier than driving a tilting pile. A small horizontal force may be transmitted by a vertical pile, by bending of the pile, but if large horizontal forces must be transferred to the soil (due to wind or waves), it is better to use some tilted piles, so that the pile forces can all be axial, and the deformations of the piles remain small. The analysis of the pile forces deserves some special attention.

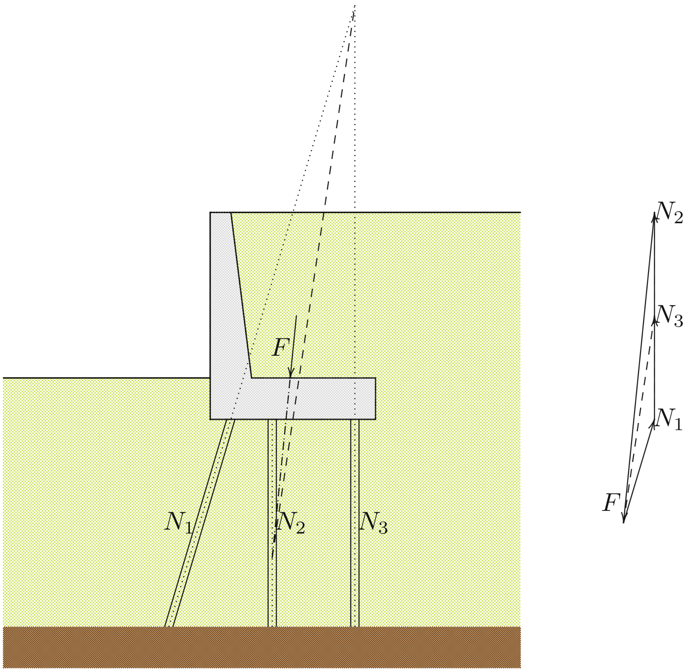

Fig. 48.2 Statically determinate pile foundation

As an example a retaining wall may be considered, see Fig. 48.2. In this case there is a considerable horizontal force, which can most easily be transferred to the ground by using a tilted pile. For this foundation system it may be assumed that the force in each pile is directed along its axis. The reason for that is that a pile is much stiffer in axial loading than it is in lateral loading. In the case shown in Fig. 48.2, with three rows of piles, the force in each row can be determined from the equilibrium equations alone. This is called a *statically determinate system*. The analysis can be performed graphically. The starting point is that the loading force F must be equilibrated by the sum of the forces in the piles, N_1, N_2 and N_3. Because N_2 and N_3 are vertical the force diagram shown in the right part of Fig. 48.2 can be constructed. The precise contributions of N_2 and N_3 is still unknown in the first stage. However, because the resulting force of F and N_2 must be in equilibrium with the resulting force of N_1 and N_3 these two resultants must have the same line of action, and they must be of equal magnitude, in opposite direction. The resulting force of N_1 and N_3 should pass through the intersection point of these two forces, and similarly the resulting force of F and N_2 passes through the intersection of these two forces. Thereby the line of action of these resultants is known. In the force diagram this line of action can then be drawn as well, as its direction is known. The three pile forces have now been determined, and the problem is solved.

48.3 Statically Indeterminate Pile Foundation

If there are more than three rows of piles, the problem of determining the individual pile forces is statically indeterminate. The solution then depends upon the flexibility of each of the piles, and of the superstructure. A well known procedure is to assume that the pile forces are directed along their axes (which means that the bending resistance of the piles is neglected with respect to their axial stiffness), and then to consider the piles as linear springs. For each pile one may write

$$N_i = k_i u_i. \tag{48.2}$$

in which N_i is the force in pile i, u_i the displacement of the pile top, and k_i the spring constant of the pile. This spring constant could be taken as the stiffness of the pile EA/l, but that would be valid only if the pile point is fully fixed. In reality the soil surrounding the pile point will also somewhat deform if the pile is loaded, so that the value of the spring constant k_i should be reduced. In the absence of further information about the stiffness of the soil it is sometimes assumed, as a first estimate, that the deformation of the pile top is twice as large as the deformation of the pile itself, leading to a value $k_i = \frac{1}{2}EA/l$. It can also be argued, however, that the pile force will not be constant along the pile, due to friction, which would lead to a larger value of k_i. In general, it is recommended to try to determine the spring constants by a careful analysis of the load transfer from the foundation to the soil.

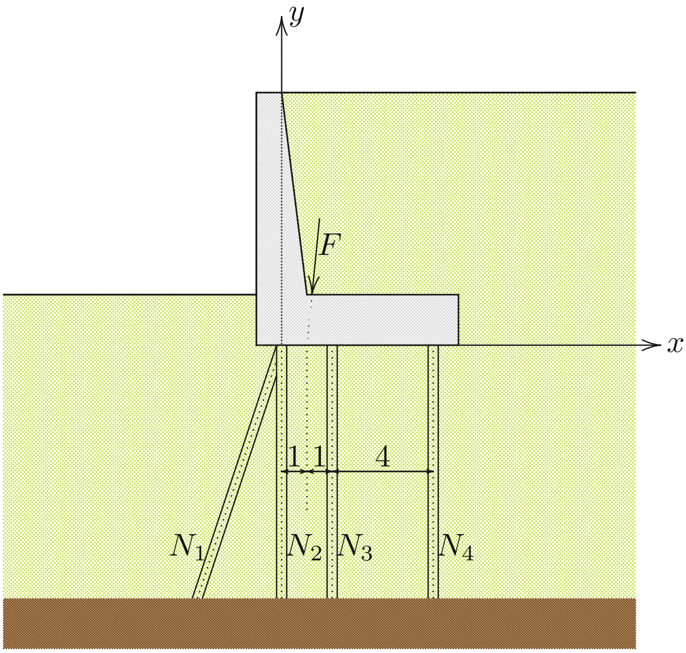

Fig. 48.3 Statically indeterminate pile foundation

If the superstructure can be considered as infinitely stiff, the computations can be performed using the displacement method. This will be illustrated by considering an example, see Fig. 48.3. In this two-dimensional case there are three basic parameters to describe the displacement of the foundation: the horizontal and vertical displacements, and the rotation. It is assumed that all pile rows have the same stiffness (k). The load is supposed to consist of a vertical component of 2000 kN and a horizontal component of 200 kN. The line of action of this force is supposed to pass through the point $x = 1$ m, $y = 0$, see Fig. 48.3. The slope of row 1 is 3:1 (vertical to horizontal).

The solution of the problem of determining the forces in each row of piles can be obtained by the standard procedure of the displacement method. This procedure is: first determine the basic displacement parameters (in this case the two displacements and the rotation), then express the internal forces into these parameters, and finally formulate the equations of equilibrium. In this case the procedure is as follows.
In case of a horizontal displacement u the forces in the pile rows are:

$$N_1 = \frac{1}{\sqrt{10}}\,ku, \quad N_2 = 0, \quad N_3 = 0, \quad N_4 = 0.$$

For a vertical displacement v the forces are:

$$N_1 = \frac{3}{\sqrt{10}}kv, \quad N_2 = kv, \quad N_3 = kv, \quad N_4 = kv.$$

For a rotation around the origin, of magnitude $\theta = w/1$ m the forces are:

$$N_1 = 0, \quad N_2 = 0, \quad N_3 = 2kw, \quad N_4 = 6kw.$$

Addition of these forces gives

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 6 \end{pmatrix} \begin{pmatrix} ku \\ kv \\ kw \end{pmatrix} \tag{48.3}$$

The forces in the piles have been considered positive for tension.

The equations of equilibrium of the foundation plate are that the sum of the horizontal forces should be −200 kN, the sum of the vertical forces should be −2000 kN, and the sum of the moments with respect to the origin should be −2000 kNm. These equations can be written as

$$\begin{pmatrix} \frac{1}{\sqrt{10}} & 0 & 0 & 0 \\ \frac{3}{\sqrt{10}} & 1 & 1 & 1 \\ 0 & 0 & 2 & 6 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} = \begin{pmatrix} -200 \text{ kN} \\ -2000 \text{ kN} \\ -2000 \text{ kN} \end{pmatrix} \tag{48.4}$$

Substitution of (48.3) into (48.4) yields the equilibrium equations expressed into the displacements,

$$\begin{pmatrix} \frac{1}{10} & \frac{3}{10} & 0 \\ \frac{3}{10} & \frac{39}{10} & 8 \\ 0 & 8 & 40 \end{pmatrix} \begin{pmatrix} ku \\ kv \\ kw \end{pmatrix} = \begin{pmatrix} -200 \text{ kN} \\ -2000 \text{ kN} \\ -2000 \text{ kN} \end{pmatrix} \tag{48.5}$$

This is a system of three equations with three unknowns. The solution is a simple mathematical problem. The result is

$$\begin{aligned} ku &= 143 \text{ kN}, \\ kv &= -714 \text{ kN}, \\ kw &= 93 \text{ kN}. \end{aligned} \tag{48.6}$$

The pile forces then are

$$\begin{aligned} N_1 &= -632\,\text{kN}, \\ N_2 &= -714\,\text{kN}, \\ N_3 &= -529\,\text{kN}, \\ N_4 &= -157\,\text{kN}. \end{aligned} \tag{48.7}$$

The vertical component of the force in row 1 is 600 kN, and its horizontal component is 200 kN. That result could have been obtained immediately, as this is the only pile that can transfer a horizontal load.

The distribution of the pile forces appears not to be uniform. The force in row 4 appears to be considerably smaller than in the other rows. If this is the only load that the foundation must carry, it may be considered to place the piles in row 4 at larger mutual distances (in y-direction). This would mean that the stiffness in that row would be smaller, and the computations should be repeated for the new stiffness parameters.

The procedure illustrated here can easily be generalized to the three-dimensional case. Then there are six degrees of freedom (three displacements and three rotations), and six equations of equilibrium. The number of piles may be very large. The procedure is very well suited for numerical analysis, using a simple computer program.

Example 48.1 Repeat the computation of the pile forces, see Fig. 48.3, for the case that the stiffness of pile row 4 is half the stiffness of the other pile rows. Predict the pile force in row 1.

Solution

The computation follows the lines given in the example considered above, but now in the expressions for the pile force N_4 the value k must be replaced by $\frac{1}{2}k$. The final result will be $N_1 = 632\,\text{kN}$. It should be noted that this happens to be the same value as in the original example. The reason is, of course, that the first row of piles is the only one that can carry the horizontal force.

Problem 48.1 Can a computer program for the analysis of space frames be used for the computation of pile forces in a pile foundation?