Chapter 23
Pore Pressures

In a previous chapter the main principles of triaxial tests have been presented. For simplicity it was assumed that the material was dry soil, so that there were no pore pressures, and the effective stresses were equal to the applied stresses. In reality, especially for clay soils, the sample usually contains water in its pores, and loading the soil may give rise to the development of additional pore pressures. The influence of these pore pressures will be described in this chapter.

23.1 Measuring the Pore Pressure

There are two possibilities to controll the pore pressures in a triaxial test: either execute the test, on a drained sample, at a very low deformation rate, so that no pore pressures are developed at all, or measure the pore pressures during the test. In the first case the drainage of the sample can be ensured by filter paper applied at the top and/or bottom ends of the sample, together with a drainage connection to a water reservoir, and taking care that the duration of the test is so long that consolidation has been completed during the test. The consolidation time should be estimated, using estimated values of the permeability and the compressibility of the sample, and the duration of the test should be large compared to that consolidation time. This may mean that the test will take very long, and usually this is impractical. A better option is to measure the pore pressures in the sample, for instance by means of an electrical pore pressure meter. This is a pressure deducer in which the pressure is measured on the basis of the deflection of a thin steel membrane, using a strain gauge on the membrane. The pore pressure meter is connected to the top or the bottom of the sample, see Fig. 23.1. An alternative is to measure the pore pressure in the interior of the sample, using a thin needle. Whatever the precise system is, care should be taken that the measuring device is very stiff, i.e. that it requires only a very small amount of water to record a pore pressure increment. Otherwise a considerable time lag

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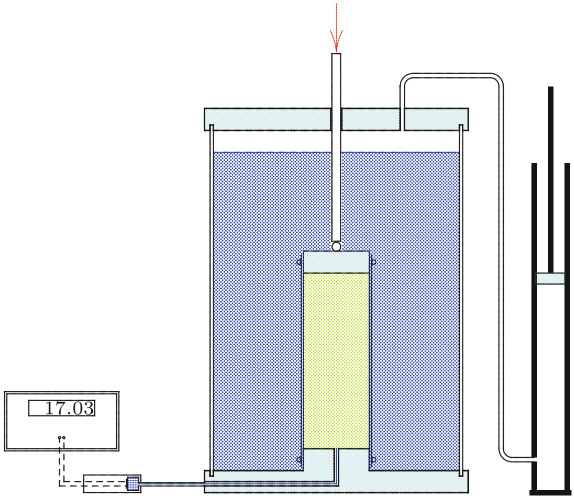

Fig. 23.1 Triaxial test with measurement of pore pressures

between the sample and the measuring device would occur, and the measurements would be unreliable, as they may not be representative of the pore pressures in the sample. An electrical pore pressure meter usually is very stiff: it may require only 1 mm^3 of water to record a pressure increment of 100 kPa. The response of such a stiff instrument is very fast, but it is very sensitive to the inclusion of air bubbles, because air is very compressible. Great care should be taken to avoid the presence of air in the system. If the pore pressures during the test are known it is simple to determine the effective stresses from the measured total stresses, by subtraction of the pore pressures. Because failure of the soil is determined by the critical values of the effective stresses the shear strength parameters c and ϕ can then be determined. In the stress diagrams the effective stresses must be plotted, and the envelope of the Mohr circles yields the cohesion c and the friction angle ϕ. The procedure is illustrated in Fig. 23.2, for the test results shown in Table 23.1. The table refers to two tests, performed at cell pressures of 40 and 95 kPa. In both tests the pore pressures developed in the first stage of loading, by the application of the cell pressure, have been reduced to zero, by waiting sufficiently long for complete drainage to have taken place. In the second stage of the tests the vertical force has been increased, at a fairly rapid rate, measuring the pore pressures during the test. The total stresses have all been represented in Fig. 23.2 by the Mohr circles. For the last circles, corresponding to the maximum values of the vertical load, the effective stress circles have also been drawn, indicated by the dotted circles. On the basis of the two critical effective stress circles the Mohr–Coulomb envelope can be drawn, and the values of the cohesion c and the friction angle ϕ can be determined. In the case of Fig. 23.2 the result obtained is $c = 9$ kPa and $\phi = 30°$.

It should be emphasized that the strength parameters c and ϕ should be determined on the basis of critical states of stress for the *effective stresses*. If in the test described above some drainage would have occurred, and the pore pressures would have been

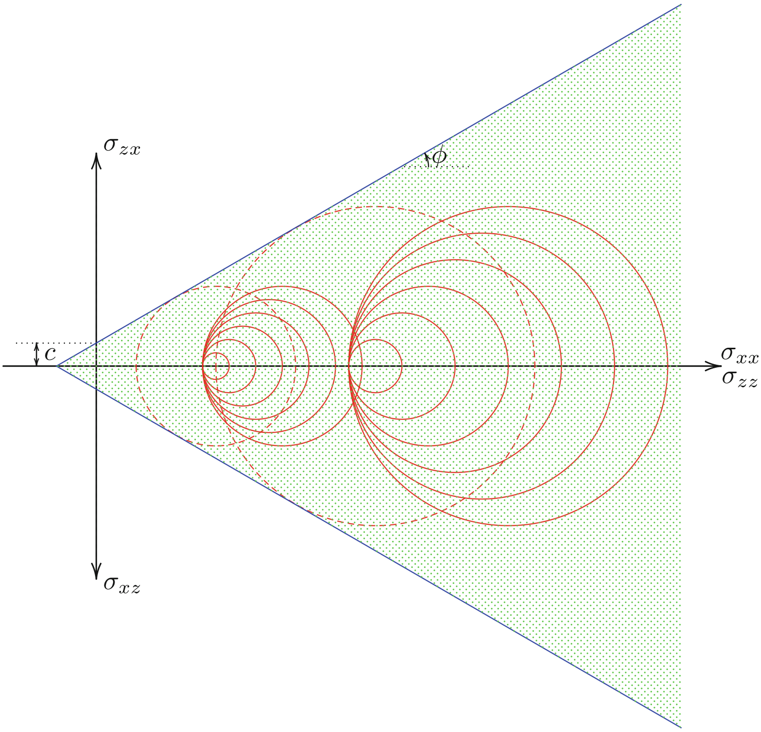

Fig. 23.2 Determination of c and ϕ from two tests

Table 23.1 Test results

Test	σ_3	$\sigma_1 - \sigma_3$	p	σ_3'	σ_1'
1	40	0	0	40	40
	40	10	4	36	46
	40	20	9	31	51
	40	30	13	27	57
	40	40	17	23	63
	40	50	21	19	69
	40	60	25	15	75
2	95	0	0	95	95
	95	20	8	87	107
	95	40	17	78	118
	95	60	25	70	130
	95	80	33	62	142
	95	100	42	53	153
	95	120	50	45	165

smaller, the critical total stresses would also have been different (smaller). Only if the test results are represented in terms of effective stresses they will lead to the same values of c and ϕ, as they should.

23.2 Types of Triaxial Tests

The procedure in the tests described above, with the results given in Table 23.1, is that in the first stage of the tests, the application of the cell pressure, the soil is free to consolidate, and sufficient time is taken to allow for complete consolidation, i.e. the excess pore pressures are reduced to zero. In the second stage of the test, however, no consolidation is allowed, by closing the tap to the drainage reservoir. Such a test is denoted as a *Consolidated Undrained* test, or a CU-test. This is a common procedure, but several other procedures exist.

If in the second stage, the vertical loading of the sample, pore pressures are again avoided by allowing for drainage, and by a very slow execution of the test (a very small loading rate), the test is denoted as a *Consolidated Drained* test, or a CD-test. Such a test takes a rather long time, which is expensive, and sometimes impractical.

A further possibility is to never allow for drainage in the test, not even in the first stage of the test, by sealing off the sample. This is an *Unconsolidated Undrained* test, or a UU-test.

23.3 Elastic Response

It may be illustrative to try to predict the pore pressures developed in a triaxial test using basic theory. This will appear to be not very accurate and reliable, but it may give some insight into the various mechanisms that govern the generation of pore pressures.

The basic notion is that the presence of water in the pores obstructs a volume change of the sample. The presence of water in no way hinders the shear deformation of a soil element, but a volume change is possible only if water is drained from the sample or if the water itself is compressed. The particles are assumed to be so stiff that their volume is constant. At the moment of loading drainage can not yet have lead to a volume change, and thus the only possibility for an immediate volume change is a compression of the fluid itself. This can be described by

$$\Delta V_w = -n\beta V \Delta p, \tag{23.1}$$

where V is the volume of the sample, Δp is the increment of the pore pressure, and β is the compressibility of the water, see also Chap. 15. The instantaneous volume strain is

$$\varepsilon_{vol} = \frac{\Delta V}{V} = -n\beta\Delta p. \tag{23.2}$$

Because the compressibility of the water (β) is very small, this is a very small quantity.

On the other hand, if the soil skeleton is assumed to deform elastically, the volume strain can be expressed as

$$\varepsilon_{vol} = -\frac{\Delta\sigma'}{K}, \tag{23.3}$$

where K is the compression modulus of the soil, and $\Delta\sigma'$ is the increment of the isotropic effective stress. Because the volume strain is so small, the increment of the isotropic effective stress will also be very small. It can be expressed as the increment of the average of the three principal stresses,

$$\Delta\sigma' = \frac{1}{3}(\Delta\sigma'_1 + \Delta\sigma'_2 + \Delta\sigma'_3). \tag{23.4}$$

It follows from (23.3), with $\sigma' = \sigma - p$, that

$$\varepsilon_{vol} = -\frac{\Delta\sigma - \Delta p}{K}. \tag{23.5}$$

From (23.2) to (23.5) it finally follows that

$$\Delta p = \frac{\Delta\sigma}{1 + n\beta K}. \tag{23.6}$$

This formula expresses the increment of the pore water pressure into the increment of the isotropic total stress. If the water is incompressible ($\beta = 0$), the increment of the pore pressure is equal to the increment of the isotropic total stress. All this is in complete agreement with the considerations in Chap. 15 on consolidation. The relation (23.6), with $\beta = 0$ can directly be obtained by noting that in a very short time there can be no volume change if the water is incompressible. Hence there can be no change in the isotropic effective stress, and thus the pore pressure must be equal to the isotropic total stress. Only if the water is somewhat compressible there can be a small instantaneous volume change, so that there can be a small increment of the effective stress, and thus the pore pressure is somewhat smaller than the isotropic total stress.

In general Eq. (23.6) can also be written as

$$\Delta p = \frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3(1 + n\beta K)}. \tag{23.7}$$

In a triaxial test $\Delta\sigma_2 = \Delta\sigma_3$, and in such tests the basic stress parameters are the cell pressure $\Delta\sigma_3$ and the additional vertical stress, produced by the axial load, $\Delta\sigma_1 - \Delta\sigma_3$. This suggests to write Eq. (23.7) in the form

$$\Delta p = \frac{1}{1 + n\beta K}[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)]. \tag{23.8}$$

In an undrained triaxial test it can be expected that increasing the cell pressure leads to an increment of the pore pressure practically equal to the increment of the cell pressure, assuming that $n\beta K \ll 1$. Furthermore, if the cell pressure remains constant, and the vertical load increases, the increment of the pore pressure will be about $\frac{1}{3}$ of the additional vertical stress. Indeed, such values are sometimes measured, approximately, see for instance the test results given in Table 23.1. Very often the results show considerable deviations from these theoretical results, because the water may not be incompressible (perhaps due to the presence of air bubbles in the soil), or because the sample is not isotropic, or because the sample exhibits non-linear properties, such as dilatancy. Furthermore, the measurements may be disturbed by inaccuracies in the measurement system, such as air bubbles in the pore pressure meter.

23.4 Dilatancy

The analysis of the previous section may be generalized by taking dilatancy into account. The basic idea remains that at the moment of loading there can not yet have been any drainage, so that the only possibility for a volume change is the compression of the water in the pores. This can be expressed by Eq. (23.2),

$$\varepsilon_{vol} = \frac{\Delta V}{V} = -n\beta\Delta p. \tag{23.9}$$

It is now postulated that the volume change of the pore skeleton is related to the stress changes by

$$\varepsilon_{vol} = -\frac{\Delta\sigma'}{K} + \frac{\Delta\tau}{M}. \tag{23.10}$$

The first term is the volume change due to the average compressive stress, which is determined by the isotropic effective stress σ'. The second term in Eq. (23.10) is the volume change caused by the shear stresses. It has been assumed that this is determined by some measure for the deviatoric stresses, indicated as τ, and as a first approximation it has been assumed that this volume change is proportional to the increment of τ, with a modulus M. That is a simplification of the real behavior, but at least it gives the possibility to investigate the effect of dilatancy, because this term expresses that shear stresses lead to a volume increase, if $M > 0$, which indicates a densely packed soil. If $M < 0$ there would be a volume decrease due to an increment of the shear stresses. Such a behavior can be expected in a loose material.

Because $\sigma' = \sigma - p$ it follows that

$$\Delta p = \frac{1}{1 + n\beta K}\left(\Delta\sigma - \frac{K}{M}\Delta\tau\right). \tag{23.11}$$

This is a generalization of the expression (23.6). For the conditions in a triaxial test one may write

$$\Delta\sigma = \frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) = \Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3). \tag{23.12}$$

The deviator stress τ is assumed to be

$$\Delta\tau = \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3). \tag{23.13}$$

This means that the radius of the Mohr circle is used as the measure for the deviator stress τ.

The final result is

$$\Delta p = \frac{1}{1 + n\beta K}[\Delta\sigma_3 + \left(\frac{1}{3} - \frac{1}{2}\frac{K}{M}\right)(\Delta\sigma_1 - \Delta\sigma_3)]. \tag{23.14}$$

This is a generalization of Eq. (23.8). Dilatancy does not appear to have any influence in the first stage of a triaxial test, when the isotropic stress is increased. In the second stage of a triaxial test, during the application of the vertical load, the generation of pore pressures is determined by the factor $\frac{1}{3} - \frac{1}{2}K/M$. The first term is a result of compression, the second term is a consequence of the dilatancy (or contractancy, when $M < 0$).

In a dilatant material, with $M > 0$, the pore water pressure will be larger than in a material without dilatancy. This is caused by the tendency of the densely packed material to expand, which reduces the compression due to the isotropic loading. If the dilatancy effect (here expressed by the parameter M) is very large, the pore pressure may even become negative. In a very dense material the tendency for expansion will lead to a suction of water.

In a contractant material, with $M < 0$, the pore pressures will become larger due to the tendency of the material to contract. The loosely packed soil will tend to contract as a result of shear stresses, thus enlarging the volume decrease due to the isotropic stress increment. The water in the pores opposes such a volume change.

23.5 Skempton's Coefficients

Skempton (1954) has suggested to write the relation between the incremental pore water pressure and the increments of the total stress in the form

$$\Delta p = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]. \tag{23.15}$$

The idea is that the coefficients A and B are measured in an undrained triaxial test.

The relations given in this section would mean that

$$B = \frac{1}{1 + n\beta K}, \tag{23.16}$$

and

$$A = \frac{1}{3} - \frac{1}{2}\frac{K}{M}. \tag{23.17}$$

Indeed, the values of B observed in tests are usually somewhat smaller than 1, and for the coefficient A various values, usually between 0 and $\frac{1}{2}$ are found, but sometimes even negative values have been obtained.

Skempton's coefficients A and B have been found to be useful in many practical problems, but it should be noted that they have limited physical significance, because they are based upon a rather special description of the deformation process of a soil, see Eq. (23.10). When their values are measured in a triaxial test, they may be influenced by partial saturation, by anisotropy, and by the stiffness of the pore pressure meter. It should also be noted that the values of the coefficients depends upon the stress level. It is therefore suggested to determine the values of A and B in tests in which the stress changes simulate the real stress changes in the field.

Example 23.1 On a number of samples from the same soil three CU-triaxial tests are being performed. The cell pressure is applied, then consolidation is allowed to reduce the pore water pressures to zero, and in the second stage the sample is very quickly brought to failure, undrained. The pore pressures are measured. The results are given in the following table (all stresses in kPa).

Test	σ_3	$\sigma_1 - \sigma_3$	p
1	20	40.94	8.19
2	40	69.52	13.90
3	60	98.09	19.62

Determine the values of the cohesion c and the friction angle ϕ.

Solution

From the test results the values of the effective stresses in each test can be determined. This gives

Test	σ'_3	σ'_1	$\frac{1}{2}(\sigma'_1 + \sigma'_3)$
1	11.81	52.75	32.28
2	26.10	95.62	60.86
3	40.38	138.47	89.24

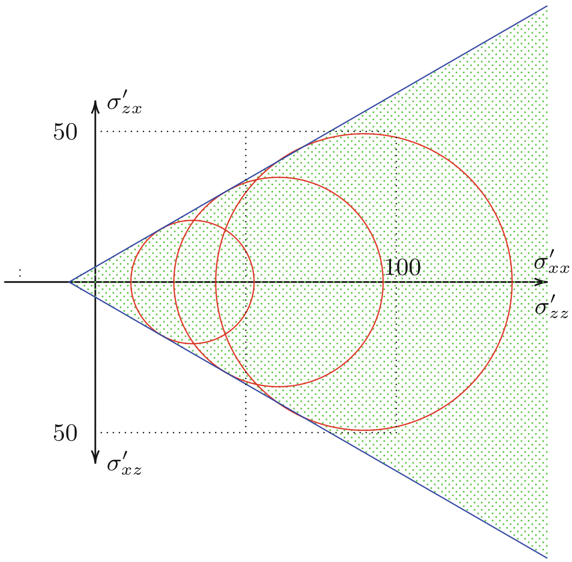

Fig. 23.3 Example 23.1

Using these data the critical Mohr circles from each test can be drawn, see Fig. 23.3. From the figure the values of c and ϕ can be measured. The result is $c = 5$ kPa and $\phi = 30°$.

In this case the value of the pore pressure p is considerably less than $\frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)$. When compared to the theoretical formula (23.15) this indicates that Skempton's parameter $A < \frac{1}{3}$. This suggests that the parameter M in formula (23.14) is positive, indicating a dilatant material, or a relatively dense material, that expands under shear.

Example 23.2 A completely saturated clay sample is loaded in a cell test by a vertical stress of 80 kPa. Due to this load the cell pressure is found to increase by 20 kPa. If the soil were perfectly elastic, what would then be the increment of the pore pressure?

Solution

For an elastic sample Eq. (23.8) applies. In this case $\Delta\sigma_1 = 80$ kPa and $\Delta\sigma_3 = 20$ kPa. Assuming that the fluid is incompressible ($\beta = 0$) it follows that $\Delta p = 40$ kPa.

Reference

A.W. Skempton, The pore pressure coefficients A and B. Géotechnique **4**, 143–147 (1954)