# Chapter 22
# Shear Test

The notion that failure of a soil occurs by sliding along a plane on which the shear stress reaches a certain maximum value has lead to the development of *shear tests*. In such tests a sample is loaded such that it is expected that one part of the sample slides over another part, along a given sliding plane. It is often assumed that the sliding plane is fixed and given by the geometry of the equipment used, but it will appear that the deformation mode may be more complicated.

## 22.1 Direct Shear Test

The simplest apparatus is shown in Fig. 22.1. It consists of a box (the *shear box*) of which the upper half can be moved with respect to the lower half, by means of a motor which pushes the lower part away from the upper part, which is fixed in horizontal direction. The cross section of the container usually is rectangular, but circular versions have also been developed. The soil sample is loaded initially by a vertical force only, applied by the dead weight of a loading plate and some additional weights on it, through the intermediary of a small steel plate on top of the sample. Because of this plate the sample is free to deform in vertical direction during the test. The actual test consists of the lateral movement of the lower half of the box with respect to the upper half, at a constant (small) speed, with a horizontal force acting in the plane between the two halves. This force gradually increases, as the box moves, and is measured by a pressure ring or a strain gauge. The horizontal force reaches a maximum value after some time, and the force remains more or less constant afterwards, or it may slowly increase or decrease. It seems logical to assume that the maximum value of the horizontal force ($T_f$) is related to the vertical force $N$ by a relation of the form

$$T_f = cA + N \tan \phi, \tag{22.1}$$



A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_22

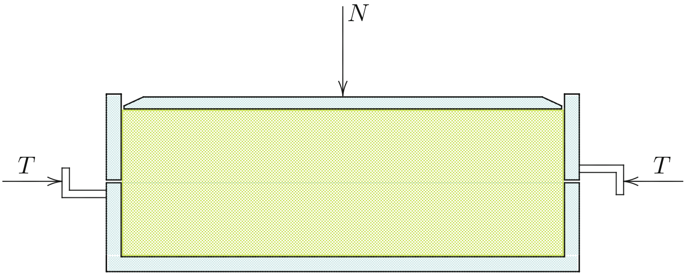


**Fig. 22.1** Direct shear test

where $A$ is the area of the sample, $c$ is the cohesion of the material, and $\phi$ its friction angle. For simplicity it is assumed that the soil is dry sand, with $c = 0$. This means that a single test is sufficient to determine the friction angle $\phi$.

Many investigators have found that the test results of shear tests lead to values for the shear strength that are considerably lower than the values obtained from triaxial tests. Furthermore, it has sometimes been found that the reproducibility of the results of shear tests is not so good. To explain the relatively large scatter in the results of shear tests it may be noted that in a shear test the horizontal stress is not imposed, and may vary from test to test. This may influence the test results, especially because it may be argued that it is not so certain that the stresses on a horizontal plane are indeed the critical stresses, as is assumed in Eq. (22.1). It may well be that there is some other plane on which the critical state of stress is reached earlier. A likely candidate for this possibility is the vertical plane, on which the normal stress may well be smaller than on a horizontal plane, whereas the shear stress on a vertical plane is equal to the shear stress on a horizontal plane because of equilibrium of moments, $\sigma_{xz} = \sigma_{zx}$. In such a case the soil may fail according to the mechanism of the toppling row of books suggested by De Josselin de Jong (1971), see Fig. 22.2.

It seems very likely that in a shear test the horizontal normal stress $\sigma_{xx}$ is smaller than the vertical normal stress $\sigma_{zz}$. If the sand has been poured into the shear box, and the vertical load has been applied by gradually increasing the load, it seems likely that the horizontal stress is smaller than the vertical stress. In an elastic material, for instance, the ratio of horizontal to vertical stress would be $\sigma_{xx}/\sigma_{zz} = \nu/(1-\nu)$, where $\nu$ is Poisson's ratio, which must be smaller than $\frac{1}{2}$. If the shear stress now is gradually increased, the maximum possible shear stress on a vertical plane is smaller than the maximum possible shear stress on a horizontal plane. Thus it can be expected that the maximum possible shear stress is reached first on a vertical plane, so that failure may occur by sliding along a vertical plane, combined with a certain rotation in order to satisfy the boundary condition on the lower and upper horizontal boundaries. The stresses are indicated in the Mohr circle that is also drawn in Fig. 22.2. It should be noted that in this case the shear stresses $\sigma_{xz}$ and $\sigma_{zx}$, in the coordinate system assumed, will be negative. In the Mohr circle it has been assumed that $\sigma_{xx} < \sigma_{zz}$. Because the point with coordinates $\sigma_{xx}$ and $\sigma_{xz}$ is located to the left

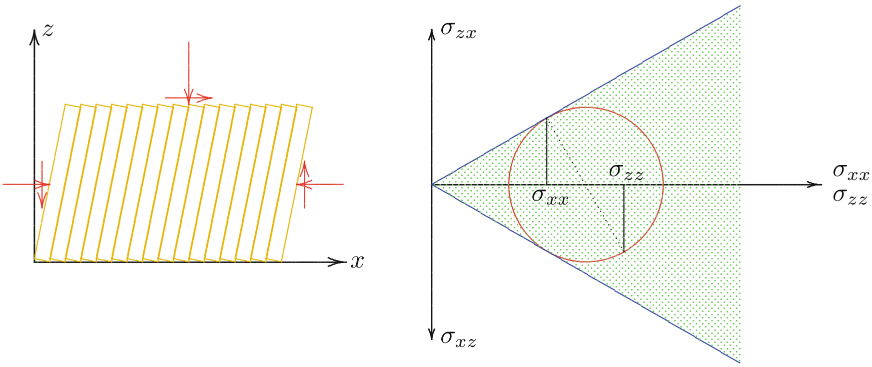


**Fig. 22.2** Toppling bookrow mechanism

of the point with coordinates $\sigma_{zz}$ and $\sigma_{zx}$, sliding will occur first along the planes on which the normal stress $\sigma_{xx}$ is acting, i.e. the vertical planes. On the horizontal planes, i.e. the planes on which the normal stress is $\sigma_{zz}$, failure will not be reached, so that no sliding along these planes is to be expected. With the shear stresses acting in the direction indicated in the figure this means that the soil to the right of a vertical plane will slide in upwards direction with respect to the soil at the left side of that plane. In Fig. 22.2 it has been assumed that such sliding occurs along a great number of vertical planes. In order to conform to the restrictions imposed by the deformation of the walls of the shear box, an additional rotation must be superimposed onto the sliding mechanism. This can be done without change of stress, as a rigid body rotation can occur without any deformation, and therefore requires no stresses. Thus the mechanism of a toppling book row is produced, just as a row of books in a book case will topple if there is insufficient lateral support.

If it is desired that the mechanism of toppling of a row of books is prevented, a large lateral stress must be applied, which may be generated by two heavy bookends, or by clamping the books between the two sides of the book case. Using this analogy it may be considered that the mechanism of Fig. 22.2 can be prevented by applying a high horizontal stress. If the horizontal normal stress is larger than the vertical normal stress, for instance because the sand has been densified by strong vibration, the state of stress on a horizontal plane will become critical before a vertical plane. The stresses $\sigma_{zx}$ and $\sigma_{zz}$, acting on a horizontal plane, will reach the critical ratio $\tan\phi$ before the stresses $\sigma_{xz}$ and $\sigma_{xx}$, acting on a vertical plane.

This means that sliding along horizontal planes can be expected if the horizontal stress is larger than the vertical stress. The situation is shown in Fig. 22.3. The Mohr circle for this case is also shown in the figure.

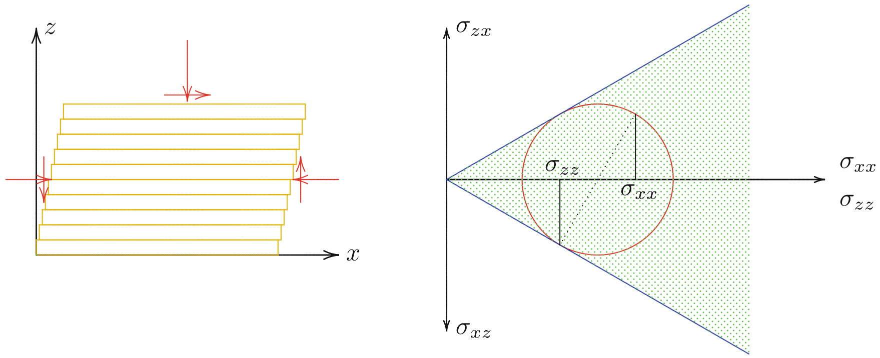


**Fig. 22.3** Sliding on horizontal planes

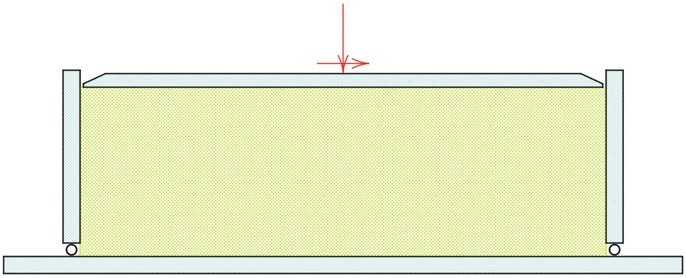

**Fig. 22.4** Simple shear test

## 22.2 Simple Shear Test

Apart from the difficulty that the state of stress is not completely given in a shear test, the direct shear test suffers from the disadvantage that the deformation is strongly inhomogeneous, because the deformations are concentrated in a zone in the center of the shear box. An improved shear box has been developed in Cambridge (England), in which the deformation is practically homogeneous. The apparatus has been constructed with rotating side walls, so that a uniform shear deformation can be imposed on the sample, see Fig. 22.4.

This is denoted as the *simple shear* apparatus. As in the direct shear box, the cross section in the horizontal plane is rectangular. The improvement is that the hinges at the top and the bottom of the side walls enable a uniform shear deformation of the sample. In Norway a variant of this apparatus has been developed, with a circular cross section. A uniform deformation is then ensured by constructing the box using a system of stiff metal rings, that can slide over each other.

Although the simple shear test is a definite improvement with respect to the direct shear test, because the deformations are much more homogeneous, it is still not certain that sliding will occur only along horizontal planes. This would be the case only if the state of stress on a horizontal plane would become critical first, which would require that the horizontal stress is larger than the vertical stress. It is doubtful whether this will always be the case. When preparing the sample for testing it seems more likely that the horizontal stress is smaller than the vertical stress, so that it is to be expected that failure will occur by sliding along vertical planes, with a simultaneous rotation.

It may be interesting to investigate the influence of the toppling book row mechanism on the critical stresses, see Fig. 22.2. Because in this case the stress combination on a vertical plane is critical, it follows from the Mohr circle that

$$\sigma_{xx} + c\cot\phi = (\sigma_{zz} + c\cot\phi)\,\frac{1-\sin^2\phi}{1+\sin^2\phi},$$

and

$$\sigma_{zx} = c + \sigma_{xx}\tan\phi.$$

Because $\sigma_{zz} = N/A$ and $\sigma_{zx} = T/A$, it follows that

$$T_f = cA + N\tan\phi\,\frac{1-\sin^2\phi}{1+\sin^2\phi}. \tag{22.2}$$

This value is smaller than the one following from Eq. (22.1). It seems reasonable to assume that the soil will fail according to the weakest mechanism, so that Eq. (22.2) applies. This means that in a test with a small horizontal stress the critical shear stress is smaller than in a test with a high horizontal stress. If the test result in a test with a small horizontal stress is interpreted in the traditional manner, using Eq. (22.1), this leads to a value of $\phi$ that is smaller than the true value. This explains why the strength determined in a shear test is often lower than the strength in a triaxial test.

In the two failure mechanisms considered the horizontal stress is the basic difference, and this suggests that the occurrence of one or the other mechanism (the toppling book row, or the sliding planks) will depend upon the relative magnitude of the horizontal stress in the test. This horizontal stress depends upon the material properties, but also on the method of installation of the sample. In general it is very difficult to say what the magnitude of the horizontal stress in a shear box is. This uncertainty in the state of stress is a disadvantage of the shear test, especially when compared to the triaxial test, in which the stresses in the three coordinate directions are well known.

It may be concluded that the shear test is not very well suited for an accurate determination of the shear strength parameters of a soil, because the state of stress is not fully known. The scatter in the results, and the relatively low values that are sometimes obtained, may well be a result of the unknown horizontal stress. The triaxial test does not suffer from this defect, as in this test the horizontal stress and the vertical stress can both be measured accurately.

It may be mentioned that in soil mechanics practice laboratory tests can sometimes be considered as scale tests of the behavior in the field. The oedometer test can be considered as such, when the initial stresses and the incremental stresses are taken equal to those in the field. For the problem of the shearing resistance of a large concrete offshore caisson, loaded by wave forces on the caisson, a shear test may be used if the vertical normal stress and the shear stress on the sample simulate the stresses to be expected in the field, and the sample has been carefully taken from the field to the shear box. Possible errors or inaccuracies may have the same effect in the laboratory and in the field, so that they do not invalidate the applicability of the test. But in this case it is also important to ensure that the horizontal stress in the sample is of the same order of magnitude as the horizontal stress in the field.

*Example 22.1* A shear test is performed on a sample of sand. It is known from previous triaxial tests that for this sand $c = 0$ and $\phi = 40^\circ$. The sand has been poured very carefully into the shear box, so that it can be expected that the horizontal stress in the sample is very low. The vertical normal stress is 100 kPa. What is the maximum shear stress that can be applied onto the the sample?

### Solution

If it is assumed that the horizontal stress is at its lowest possible value, Eq. (22.2) applies. With $c = 0$ and $\phi = 40^\circ$ one now obtains $\tau_{max} = 34.84$ kPa.

If the knowledge of the low horizontal stress is ignored, and the test is interpreted in the classical way, using Eq. (22.1), the apparent value of the friction angle would follow from $\tan\phi = 0.3484$, i.e. $\phi = 19.2^\circ$, which is considerably lower than the value obtained in the triaxial test.

## Reference

G. De Josselin de Jong, The double sliding, free rotating model for granular assemblies. Géotechnique **20**, 155–163 (1971)