Chapter 45
Slope Stability

For the analysis of the stability of slopes of arbitrary shape and composition various approximate methods have been developed. Many of these assume a circular slip surface. Using a number of simplifying assumptions a value for the safety factor F, the ratio of strength and load, is determined. The circle giving the smallest value of F is considered to be critical. The multitude of methods (developed by Fellenius, Taylor, Bishop, Morgenstern-Price, Spencer, among others) in itself illustrates that none of them is exact. The results should always be handled with care. A value $F = 1.05$ gives no absolute certainty that the slope will stand. In this chapter two of the simplest methods will be presented.

45.1 Circular Slip Surface

Most methods assume that the soil fails along a circular slip surface, see Fig. 45.1. The soil above the slip surface is subdivided into a number of *slices*, bounded by vertical interfaces. At the slip surface the shear stress is τ, which is assumed to be a factor F smaller than the maximum possible shear stress, i.e.

$$\tau = \frac{1}{F}\,(c + \sigma'_n \tan\phi). \tag{45.1}$$

The factor F is assumed to be the same for all slices, an assumption that is common to all methods.

The equilibrium equation to be used in conjunction with a circular slip surface is the equation of equilibrium of moments with respect to the center of the circle. This equation gives

$$\sum \gamma h b R \sin\alpha = \sum \frac{\tau b R}{\cos\alpha}. \tag{45.2}$$

A. Verruijt, *An Introduction to Soil Mechanics*, Theory and Applications of Transport in Porous Media 30, DOI 10.1007/978-3-319-61185-3_45

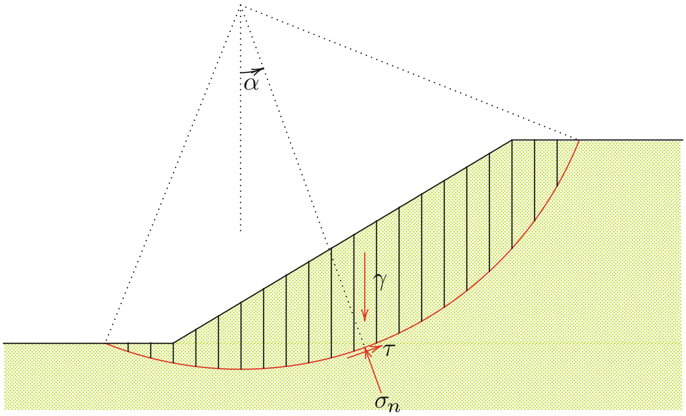


Fig. 45.1 Circular slip surface

Here h is the height of a slice, b its width, γ the volumetric weight of the soil in the slice, and R is the radius of the circle. More generally it can be defined that γbh is the weight of the slice, possibly consisting of a sum of parts with different unit weight.

If all slices have the same width, it now follows from (45.1) and (45.2) that

$$F = \frac{\sum[(c + \sigma_n' \tan\phi)/\cos\alpha]}{\sum \gamma h \sin\alpha}. \tag{45.3}$$

This is the basic formula for many computation methods. The various methods usually differ in the method of calculating the normal effective stress σ_n'.

45.2 Fellenius

In the method of Fellenius (1926), the oldest method for the analysis of slope stability, it is assumed that there are no forces between the slices. The only remaining forces acting on a slice, see Fig. 45.2, then are the weight γhb, a normal stress σ_n and a shear stress τ at the bottom of the slice. The normal stress σ_n can most conveniently be expressed into the known weight by considering the equilibrium of the slice in the direction perpendicular to the slip surface. This gives

$$\sigma_n = \gamma h \cos^2\alpha, \tag{45.4}$$

and, because $\sigma_n = \sigma_n' + p$,

$$\sigma_n' = \gamma h \cos^2\alpha - p. \tag{45.5}$$

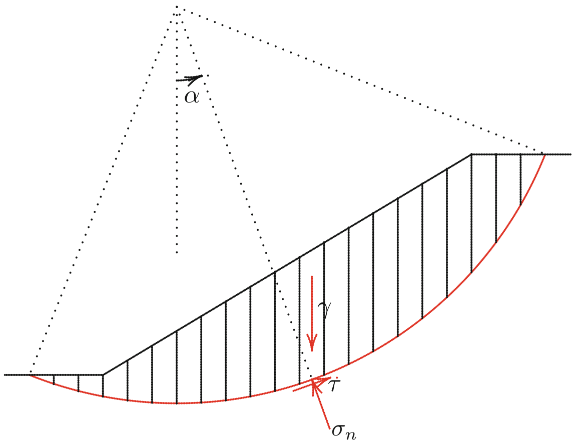


Fig. 45.2 Fellenius

Substitution into (45.3) finally gives

$$F = \frac{\sum\{[c + (\gamma h \cos^2\alpha - p)\tan\phi]/\cos\alpha\}}{\sum \gamma h \sin\alpha}. \tag{45.6}$$

This is the Fellenius formula.

For a slope in homogeneous soil the computation can be executed by assuming a certain location of the center of the circle and its radius, and subdividing the sliding soil wedge into 10 or 20 slices. By measuring the values of the angle α and the height h for each slice the value of the stability factor F can be determined. This must be repeated for a large number of circles, to determine the smallest value of F. In a non-homogeneous soil the computation is somewhat more complicated because for each slice the value of γh must be determined as the sum of the contributions of a number of layers in the slice.

Several objections can be made against this method. To begin with, a sound fundamental base lacks for all slip surface methods for materials with internal friction, as seen before (see Chap. 41). But there are other objections as well. Disregarding the forces transmitted between the slices is a severe approximation, and vertical equilibrium is violated. Furthermore, there is an internal inconsistency in stating on the one hand that sliding occurs along the circle, and on the other hand stating that the horizontal and vertical directions are the directions of principal stress (as it is assumed that there are no shear stresses on vertical planes). This inconsistency can best be seen by considering the slice in the center, for which $\alpha = 0$. At that slice $\sigma_n = \gamma h$, and it is assumed that there is a shear stress $(\sigma_n - p)/F$ on that slice. This violates the assumption that the vertical direction is a direction of principal stress. Horizontal equilibrium of that slice is also clearly violated. For other slices vertical equilibrium is violated, as only the condition of equilibrium perpendicular to the slip surface is taken into account.

Fellenius' method has the property that in a number of special cases it confirms certain limiting values. For instance, for an infinite slope in a dry frictional material without cohesion, one obtains from (45.6), assuming a straight slip surface at a depth d below the slope, and taking $p = c = 0$,

$$F = \frac{\sum \gamma d \cos\alpha \tan\phi}{\sum \gamma d \sin\alpha} = \frac{\tan\phi}{\tan\alpha}.$$

This is in perfect agreement with formula (44.7) in the previous chapter.

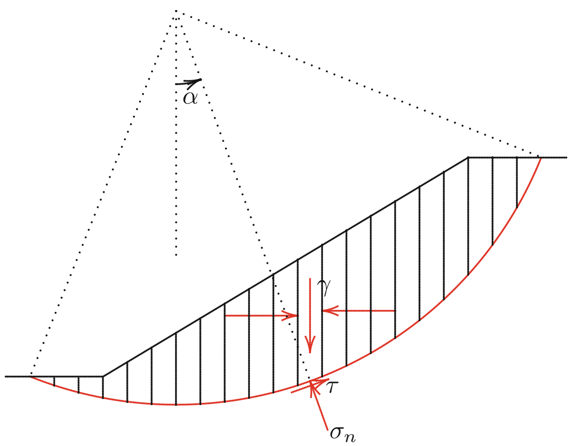
In the case of a slope under water, in the absence of groundwater flow, see Fig. 44.2, the limiting value (44.14) is not immediately recovered. For such problems the Fellenius formula might be modified by using the volumetric weight under water, $(\gamma - \gamma_w)h$ rather than γh, and using the excess water pressure with respect to the hydrostatic water pressure for p. This is somewhat artificial, however, and for this reason and the objections formulated above, the Fellenius method is rarely used.

45.3 Bishop

A method that is frequently used in engineering practice is the method proposed by Bishop (Bishop, 1954). In this method the forces between the slices are not neglected, but it is assumed that the resultant force is horizontal, see Fig. 45.3. By considering the vertical equilibrium of each slice only, the horizontal forces do not enter into the computations, however.

The basic equation again is the equation of moment equilibrium, Eq. (45.3). Vertical equilibrium of a slice now requires that

Fig. 45.3 Bishop



$$\gamma h = \sigma_n + \tau \frac{\sin\alpha}{\cos\alpha}, \tag{45.7}$$

or, because $\sigma_n = \sigma'_n + p$,

$$\gamma h = \sigma'_n + p + \tau \frac{\sin\alpha}{\cos\alpha}. \tag{45.8}$$

If in this equation the value of τ is written, in agreement with Eq. (45.1), as $\tau = (c + \sigma'_n \tan\phi)/F$, the result is

$$\sigma'_n \left(1 + \frac{\tan\alpha\, \tan\phi}{F}\right) = \gamma h - p - \frac{c}{F}\tan\alpha. \tag{45.9}$$

Substitution of σ'_n into (45.3) now leads to the final equation for Bishop's method,

$$F = \frac{\sum \dfrac{c + (\gamma h - p)\tan\phi}{\cos\alpha(1 + \tan\alpha\, \tan\phi/F)}}{\sum \gamma h \sin\alpha}. \tag{45.10}$$

Because the stability factor F also appears in the right hand side, it must be determined iteratively, by starting from an initial estimate (for instance $F = 1$), and then calculating an updated value using (45.10). This must be repeated until the value of F no longer changes. In general the procedure converges rather fast. As the computations must be executed by a computer program anyway (many circles have to be investigated) the iterations can easily be incorporated into the program. Computer programs are available on the internet (search for *geotechnical software*).

If $\phi = 0$ the Bishop and Fellenius methods are identical. If $\phi > 0$ Bishop's method usually gives somewhat smaller values. Because Bishop's method is more consistent (vertical equilibrium is satisfied), and it confirms known results for special cases, it is often used in geotechnical engineering. Various other methods have been developed, for instance using forces between the slices, but their results often differ only slightly from those obtained by Bishop's method. That may explain its popularity.

Example 45.1 A slope that has been designed using one of the standard static methods may be subject to a more critical condition in case of an earthquake. A simple method to include the shaking forces produced by an earthquake is to assume that the earthquake is equivalent to a horizontal force of a certain magnitude, say 10% of the weight of the sliding soil mass. Such a horizontal force can be included in the basic equation of equilibrium of moments as an additional driving moment. Another method, which is about as elementary and simple, would be to tilt the entire soil mass over an angle 1:10.

Experience shows that this method, in which the horizontal force continues indefinitely, may too quickly lead to the conclusion that the slope is unsafe, and that the slope must be redesigned. On the other hand, in many areas the possibility of an earthquake may be remote, and a more refined method may be considered. These have been developed, for instance by Newmark, who proposed to take into account the limited duration of the earthquake, and to allow for the limited deformations that may occur in case of an earthquake.

Problem 45.1 Verify that Fellenius' method gives the correct limiting value for an infinite slope with a groundwater flow parallel to the slope.

Problem 45.2 Verify that Bishop's method gives the correct limiting values for the special cases considered in the previous chapter.

References

A.W. Bishop The use of the slip circle in the stability of slopes. Proc. Eur. Conf. Stability Earth Slopes **1**, 1–13 (1954)
W. Fellenius, *Erdstatische Berechnungen* (Ernst & Sohn, Berlin, 1926)