Chapter 25
Stress Paths

A convenient way to represent test results, and their correspondence with the stresses in the field, is to use a graph of the stresses. In this technique the stresses in a point are represented by two (perhaps three) characteristic parameters, and they are plotted in a diagram. This diagram is called a *stress path*.

25.1 Parameters

In some presentations of books on soil mechanics, for instance from the University of Cambridge (UK), see the books by Schofield and Wroth (1968) and by Wood (1990), it is assumed that the state of stress in a point can be characterized by the average stress (the isotropic stress), $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$, and the difference of the major and minor principal stresses, $\sigma_1 - \sigma_3$. Alternatively, for instance in the book by Lambe and Whitman (1969), the average value of the major and minor principal stresses, $\frac{1}{2}(\sigma_1 + \sigma_3)$, may be used rather than the average stress, whereas the second parameter is chosen as one half of the difference of the major and minor prinicipal stresses, $\frac{1}{2}(\sigma_1 - \sigma_3)$. Although the first pair of parameters may be prefered for theoretical reasons, here the second pair will be used, because of its close correspondence to the Mohr circle concept.

The two basic variables are denoted by σ and τ,

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3), \tag{25.1}$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3). \tag{25.2}$$

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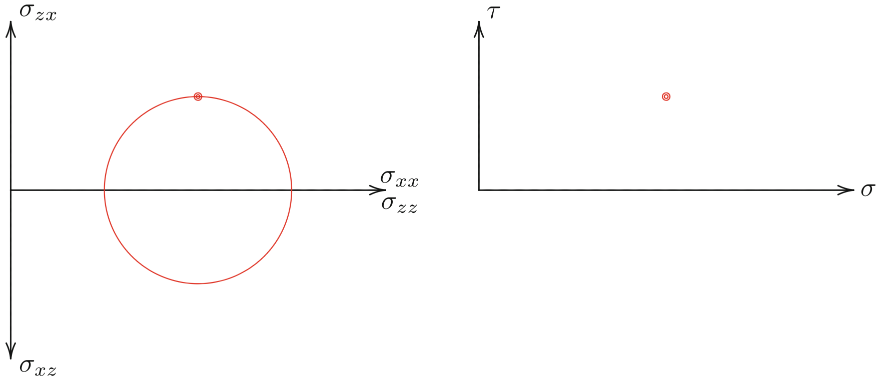


Fig. 25.1 Mohr's *circle* and stress *point*

The introduction of the factor $\frac{1}{2}$ in the two definitions results in σ and τ being the location of the center, and the radius of the circle in Mohr's diagram, see Fig. 25.1. By choosing these parameters it is implicitly assumed that other parameters are unimportant for the description of the material behavior of the soil. It is assumed, for instance, that the intermediate principal stress is unimportant, as is the orientation of the principal stresses. This is approximately correct for the failure state of a soil, because the Mohr–Coulomb failure criterion can be formulated in σ and τ, but for smaller stresses it may be a first approximation only. Actually, even the failure criterion of a soil is often found to be dependent on other parameters (such as the value of σ_2) too, so that the Mohr–Coulomb failure criterion should be considered as merely a first approximation of real soil behavior.

It should be mentioned that in many publications the symbols p and q are used, rather than σ and τ, and the diagram is denoted as a p, q-diagram. This convention will not be followed here, as the notation p has been reserved for the pore pressure.

The state of stress is represented in the right half of Fig. 25.1 in the σ, τ-diagram. The basic principle is that the Mohr circle is characterized by the location of its top only. If the state of stress changes, the values of σ and τ will be different, so that the location of the stress point changes. The path of the stress point is called the *stress path*.

A stress path can be drawn for the total stresses as well as the effective stresses, in the same diagram. The difference is the pore pressure, see Fig. 25.2. The total stress path is indicated by TSP, and the effective stress path by ESP.

The possible states of stress are limited by the Mohr–Coulomb failure criterion, see Eq. (20.12). In a diagram of Mohr circles this is a straight line, limiting the stress circles, see the left half of Fig. 25.2. This limit is described by

$$\left(\frac{\sigma_1' - \sigma_3'}{2}\right) - \left(\frac{\sigma_1' + \sigma_3'}{2}\right) \sin\phi - c\cos\phi = 0. \tag{25.3}$$

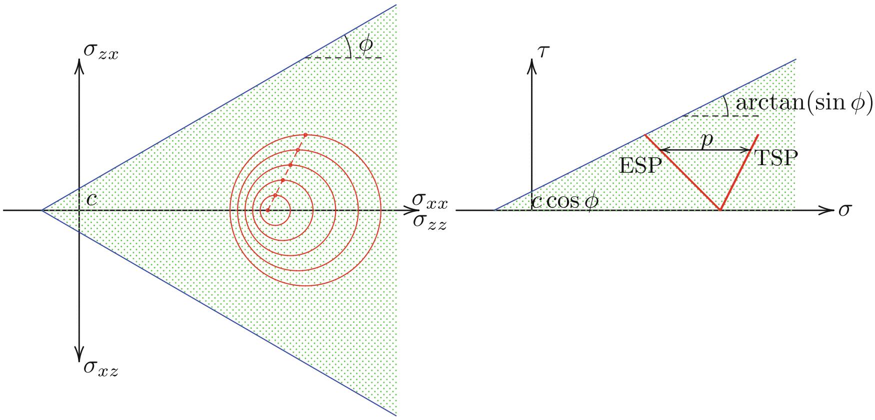


Fig. 25.2 Stress paths

Expressed in terms of σ and τ this is

$$\tau' = \sigma' \sin\phi + c\cos\phi. \tag{25.4}$$

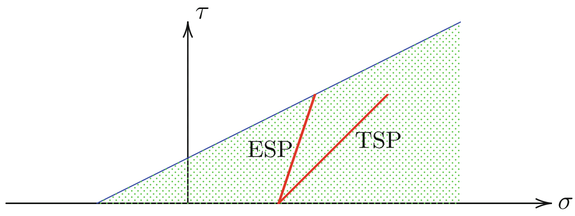
This describes a straight line in the σ, τ -diagram. This straight line has been indicated in the right half of Fig. 25.2. The slope of this line is $\sin\phi$, which is slightly less steep than the envelope in the diagram of Mohr circles (which is $\tan\phi$). The intersection with the vertical axis is $c\cos\phi$.

As mentioned before, many researchers use different parameters to characterize the stresses in soils, because they are claimed to provide a better approximation of the behavior of soils in certain tests. Actually, any combination of stress invariants may be used, for instance the three principal stresses. The parameters σ and τ used here are convenient because the Mohr–Coulomb failure criterion can so easily be formulated in terms of σ and τ. This criterion is not a basic physical principle, however, but rather a simple way to represent some test results. Other failure criteria, perhaps involving more parameters (such as the intermediate principal stress), may be formulated, and these may give a better approximation of a wider class of test results. In conclusion, the choice of stress path parameters is based upon considerations of convenience and experience as well as pure science.

25.2 Triaxial Test

In the second stage of a normal triaxial test the cell pressure is kept constant, and the vertical stress is increased. The cell pressure then is the minor principal stress, σ_3, which is kept constant. During the test the value of σ_1 increases. The total stress path is a straight line, with a slope of 45°, see Fig. 25.3. Its mathematical description is

Fig. 25.3 Stress path in triaxial test



$$\Delta\sigma_3 = 0 : \quad \Delta\tau = \Delta\sigma. \tag{25.5}$$

The course of the effective stress path depends upon the pore pressures. In Chap. 23 it was postulated that these may be expressed by Skempton's formula,

$$\Delta p = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]. \tag{25.6}$$

This formula can also be written as

$$\Delta p = B[\Delta\sigma - (1 - 2A)\Delta\tau]. \tag{25.7}$$

In the case of a triaxial test the pore pressure is

$$\Delta\sigma_3 = 0 : \quad \Delta p = 2BA\Delta\sigma. \tag{25.8}$$

For a completely saturated isotropically elastic material the values of the coefficients A and B are, if the compressibility of the water is neglected (see Chap. 23): $B = 1$ and $A = \frac{1}{3}$. It then follows from (25.8) that the pore pressure increment will be $\frac{2}{3}$ of the increment of σ, $\Delta p = \frac{2}{3}\Delta\sigma$, see also Eq. (23.7). For such an idealized material behavior the effective stress path will be a straight line at a slope of 3 : 1, see Fig. 25.3.

Figure 25.4 shows the stress paths for a dilatant material and for a contractant material. When the material is dilatant, it will tend to expand during shear, so that the pore pressures will be reduced (the volume expansion results in suction). In a contractant material the volume will tend to decrease, so that the pore pressures are increased. It can be seen from the figure that in a contractant material failure will be reached much faster than in a non-contractant or dilatant material. The two mechanisms of

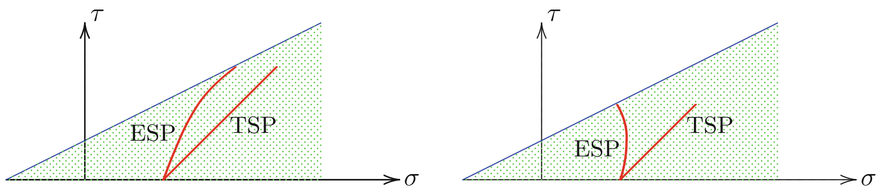


Fig. 25.4 Stress paths in triaxial tests on dilatant and contractant material

Table 25.1 Test results

Test	σ_3	$\sigma_1 - \sigma_3$	p	σ_1	σ	σ'	τ
1	40	0	0	40	40	40	0
	40	10	4	50	45	41	5
	40	20	9	60	50	41	10
	40	30	13	70	55	42	15
	40	40	17	80	60	43	20
	40	50	21	90	65	44	25
	40	60	25	100	70	45	30
2	95	0	0	95	95	95	0
	95	20	8	115	105	97	10
	95	40	17	135	115	98	20
	95	60	25	155	125	100	30
	95	80	33	175	135	102	40
	95	100	42	195	145	103	50
	95	120	50	215	155	105	60

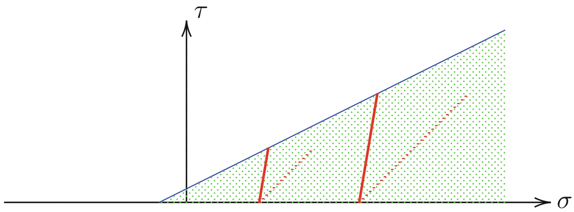


Fig. 25.5 Stress paths in triaxial tests

pore pressure development, increasing the isotropic total stress (i.e. compression) and shear deformation, add up to a relatively large pore pressure increase, so that the isotropic effective stress σ' decreases, and this may result in rapid failure. In a dilatant material the two phenomena (compression and shear) counteract. The compression tends to increase the pore pressure, whereas the shear tends to decrease the pore pressure. The effective stress path will be located to the right of the path for a non-dilatant material. In a triaxial test this will result in a large apparent strength, as the vertical load can be very high before failure is reached.

The practical implications of dilatancy and contractancy may be very important. A dilatant soil (i.e. a very dense material) may create additional strength by shearing, through the negative pore pressures that are developed. On the other hand, in a very loose material (a contractant soil) extra pore pressures may be generated by shearing, thus reducing the effective stresses and the strength. The soil may well collapse.

As a further illustration the triaxial tests presented in Chap. 23 will be further elaborated, using stress paths. The test results have been taken from Table 23.1, but they have been elaborated some more, to calculate the values of σ, σ' and τ, see Table 25.1. The stress paths for the two tests are shown in Fig. 25.5. The paths for the total stresses have been indicated by dotted lines, the effective stress paths have been indicated by fully drawn lines. The two end points of the effective stress paths determine the critical envelope. According to Eq. (25.4) the critical points of the effective stress paths are located on the straight line

$$\tau' = a\sigma' + b, \tag{25.9}$$

where $a = \sin\phi$ and $b = c\cos\phi$. In this case there are two critical points: $\sigma' = 45$ kPa, $\tau = 30$ kPa and $\sigma' = 105$ kPa, $\tau = 60$ kPa. Substitution of these two pairs of values into (25.9) leads to two equations with two unknowns, a and b. This gives $a = 0.5$ and $b = 7.5$ kPa. Because $a = \sin\phi$ it follows that $\phi = 30°$, and because $b = c\cos\phi$ it follows that $c = 8.7$ kPa. These results are in agreement with the values obtained in Chap. 23.

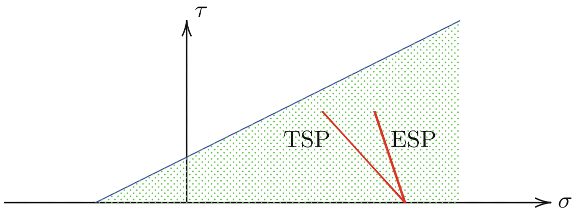
Example 25.1 In a triaxial apparatus it is also possible to apply a negative value of the axial force, by pulling on the steel rod (the plunger), at constant cell pressure. This is called a *triaxial extension test*. Draw the total stress path for such a test, and the effective stress path, assuming isotropic elastic behaviour, and assuming that the test is a CU test, i.e. full drainage in the first stage of the test, and no drainage in the second stage.

Solution

In this case the horizontal stress (the cell pressure) will be larger than the vertical stress. Thus the horizontal stress will be the largest principal stress, to be denoted by σ_1, and the vertical stress will be the smallest principal stress, to be denoted by σ_3. The total stress path can be constructed by first considering the initial isotropic state of stress, produced by the cell pressure, so that $\sigma = \sigma_1$ and $\tau = 0$. In the second stage of the test $\sigma_3 = \sigma_1 - t$, where t is the tensile stress in the plunger, so that then $\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) = \sigma_1 - \frac{1}{2}t$, and $\tau = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}t$. This total stress path (TSP) is shown in Fig. 25.6.

For the construction of the effective stress path, assuming isotropic elastic behaviour, and no drainage in the second stage of the test, it must be noted that the formulas given in Chap. 23, such as the Skempton relations, were derived for triaxial conditions in which the isotropic stress is $\frac{1}{3}(\sigma_1 + 2\sigma_3$, whereas in this case the isotropic stress is $\frac{1}{3}(2\sigma_1 + \sigma_3$. It may be assumed that the relation (23.7) remains valid, because that relation was based upon the idea that there is no drainage, and thus the volume change is just the compression of the fluid. This relation now becomes, however,

Fig. 25.6 Stress path in triaxial extension test



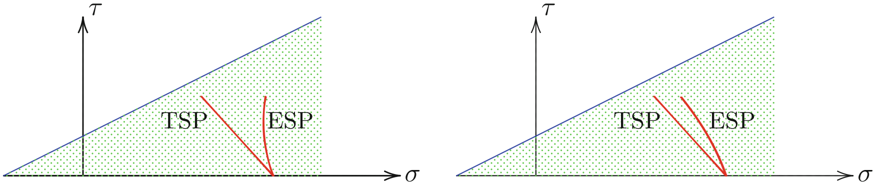


Fig. 25.7 Stress paths in extension tests on dilatant and contractant material

with $\sigma_2 = \sigma_1$,

$$\Delta p = \frac{2\Delta\sigma_1 + \Delta\sigma_3}{3(1+n\beta K)} = \frac{1}{1+n\beta K}[\Delta\sigma_1 - \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)],$$

Assuming that the fluid is incompressible ($\beta = 0$) it follows that an increment of the cell pressure, with $\Delta\sigma_1 = \Delta\sigma_3$, will lead to a pore pressure $\Delta p = \Delta\sigma_1$ in the first stage of the test, but in a CU-test this will be reduced to zero by consolidation. In the second stage of the test, a reduction of the vertical stress by pulling the plunger will result in a positive value for $\Delta\sigma_1 - \Delta\sigma_3$, and thus the pore pressure will be changed by an amount $\Delta p = -\frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3) = -\frac{1}{3}t$. This is a negative value, indicating tension, which seems in agreement with the fact that tension is applied to the sample. In this case of a negative pore pressure, the effective stress will be larger than the total stress. Actually, because $\sigma = \sigma_1 - \frac{1}{2}t$, it now follows that $\sigma' = \sigma - \Delta p = \sigma_1 - \frac{1}{6}t$. The effective stress path (ESP) is also shown in Fig. 25.6. It can be expected that in a dilatant material (a dense sand) the shear deformations will be accompanied by a tendency for volume extension. In undrained conditions this will lead to a further decrease of the pore pressure, and thus to a further increase of the strength. This is illustrated in Fig. 25.7. In a contractant material (a loose sand) the shear deformations will be accompanied by a tendency for volume compaction. In undrained condition this will lead to an increase of the pore pressure, and thus to a reduction of the effective stress and a reduction of the strength.

References

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