

Chapter 12

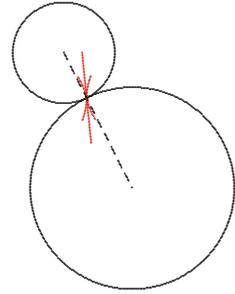
Stress Strain Relations

As stated in previous chapters, the deformations of soils are determined by the *effective stresses*, which are a measure for the contact forces transmitted between the particles. The soil deformations are a consequence of the local displacements at the level of individual particles. In this chapter some of the main aspects of these deformations will be discussed, and this will lead to qualitative properties of the relations between stress and strain. In later chapters these relations will be formulated in a quantitative sense.

12.1 Compression and Distortion

In the contact point of two particles a normal force and a shear force can be transmitted, see Fig. 12.1. The normal force can only be a compressive force. Tension can not be transmitted, unless the soil particles are glued together. Such soils do exist (e.g. calcareous soils near the coast of Brazil or Australia), but they are not considered here. The magnitude of the shear force that can be transmitted depends upon the magnitude of the normal force. It can be expected that if the ratio of shear force and normal force exceeds a certain value (the *friction coefficient* of the material of the particles), the particles will start to slide over each other, which will lead to relatively large deformations. The deformations of the particles caused by their compression can be disregarded compared to these sliding deformations. The particles might as well be considered as incompressible.

This can be further clarified by comparing the usual deformations of soils with the possible elastic deformations of the individual particles. Consider a layer of soil of a normal thickness, say 20 m, that is being loaded by a surcharge of 5 m dry sand. The additional stress caused by the weight of the sand is about 100 kN/m^2 , or 0.1 MPa. Deformations of the order of magnitude of 0.1% or even 1% are not uncommon for soils. For a layer of 20 m thickness a deformation of 0.1% means a settlement

Fig. 12.1 Particle contact

of 2 cm, and that is quite normal. Many soil bodies show such settlements, or even much more, for instance when a new embankment has been built. Settlements of 20cm may well be observed, corresponding to a strain of 1%. If one writes, as a first approximation $\sigma = E\varepsilon$, a stress of 0.1 MPa and a strain of 0.1% suggests a deformation modulus $E \approx 100$ MPa. For a strain of 1% this would be $E \approx 10$ MPa. The modulus of elasticity of the particle material can be found in an encyclopedia or handbook. This gives about 20 GPa, about one tenth of the modulus of elasticity of steel, and about the same order of magnitude as concrete. That value is a factor 200 or 2000 as large as the value of the soil body as a whole. It can be concluded that the deformations of soils are not caused by deformations of the individual particles, but rather by a rearrangement of the system of particles, with the particles rolling and sliding with respect to each other.

On the basis of this principle many aspects of the behavior of soil can be explained. It can, for instance, be expected that there will be a great difference between the behavior in compression and the behavior in shear. Compression is a deformation of an element in which the volume is changing, while the shape remains the same. In pure compression the deformation in all directions is equal, see Fig. 12.2. It can be expected that such compression will occur if a soil element is loaded isotropically, i.e. by a uniform normal stress in all directions, and no shear stresses. In Fig. 12.2 the load has been indicated on the original element, in the left part of the figure. The deformed element is shown in the right part of the figure.

With such a type of loading, there will be little cause for a change of direction of the forces in the particle contacts. Because of the irregular character of the grain skeleton there may be local shear forces, but these need not to increase to carry increasing compressive forces. If all forces, normal forces and shear forces, increase proportionally, an ever larger compressive external pressure can be transmitted. If the particles were completely incompressible there would be no deformation in that case. In reality the particles do have a small compressibility, and the forces transmitted by the particle contacts are not distributed homogeneously. For these reasons there may be some local sliding and rolling even in pure compression. But it is to be expected that the soil will react much stiffer in compression than in shear when shear stresses are applied. When external shear stresses are applied to a soil mass, the local shear forces must increase on the average, and this will lead to considerable deformations.

Fig. 12.2 Compression

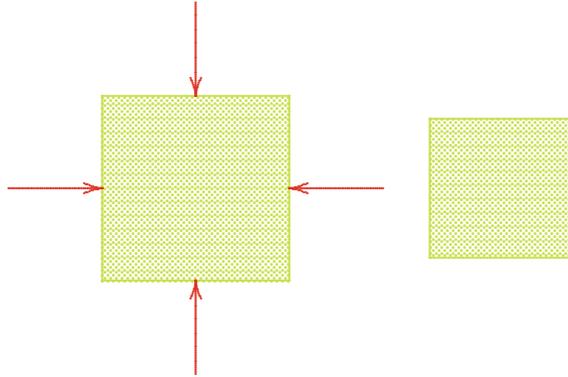
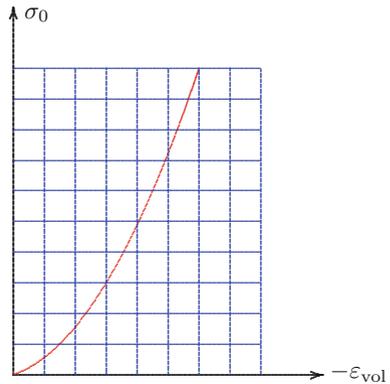


Fig. 12.3 Stiffness in compression



In tests it appears that soils are indeed relatively stiff under pure compression, at least when compared to the stiffness in shear. When compared to materials such as steel, soils are highly deformable, even in pure compression.

It can also be expected that in a continuing process of compression the particles will come closer together, increasing the number of contacts, and enlarging the areas of contact. This suggests that a soil will become gradually stiffer when compressed. Compression means that the porosity decreases, and it can be expected that a soil with a smaller porosity will be stiffer than the same assembly of particles in a structure with a larger porosity. It can be concluded that in compression a relation between stress and strain can be expected as shown in Fig. 12.3. The quantity σ_0 is the normal stress, acting in all three directions. This is often denoted as the *isotropic stress*. The quantity ϵ_{vol} is the *volume strain*, the relative change of volume (the change of the volume divided by the original volume).

$$\epsilon_{vol} = \frac{\Delta V}{V}. \tag{12.1}$$

Fig. 12.4 Distortion

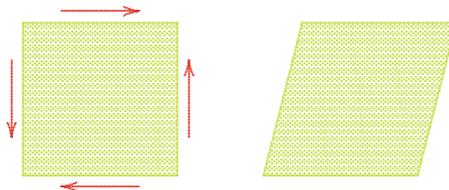
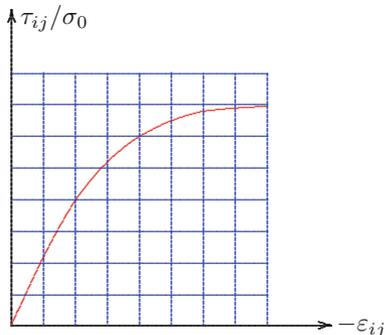


Fig. 12.5 Stiffness in distortion

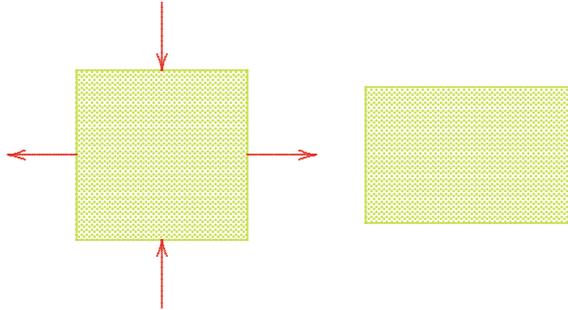


Because the volume will, of course, decrease when the isotropic stress increases the quantity on the horizontal axis in Fig. 12.3 has been indicated as $-\epsilon_{vol}$.

It may be concluded that the stiffness of soils will increase with continuing compression, or with increasing all round stress. Because in the field the stresses usually increase with depth, this means that in nature it can be expected that the stiffness of soils increases with depth. All these effects are indeed observed in nature, and in the laboratory.

Quite a different type of loading is pure distortion, or pure shear: a change of shape at constant volume, see Fig. 12.4. When a soil is loaded by increasing shear stresses it can be expected that in the contact points between the particles the shear forces will increase, whereas the normal forces may remain the same, on the average. This leads to a tendency for sliding in the contact points, and thus there will be considerable deformations. It is even possible that the sliding in one contact point leads to a larger shear force in a neighboring contact point, and this may slide in its turn. All this means that there is more cause for deformation than in compression. There may even be a limit to the shear force that can be transmitted, because in each contact point the ratio of shear force to normal force can not be larger than the friction angle of the particle material.

During distortion of a soil a relation between stresses and strains as shown in Fig. 12.5 can be expected. In this figure the quantity on the vertical axis is a shear stress, indicated as τ_{ij} , divided by the isotropic stress σ_0 . The idea is that the friction character of the basic mechanism of sliding in the contact points will lead to a maximum for the ratio of shearing force to normal force, and that as a consequence for

Fig. 12.6 Distortion

the limiting state of shear stress the determining quantity will be the ratio of average shear stress to the isotropic stress. Tests on dry sand confirm that large deformations, and possible failure, at higher isotropic stresses indeed require proportionally higher shear stresses. By plotting the relative shear stress (i.e. τ_{ij} divided by the isotropic stress σ_0) against the shear deformation, the results of various tests, at different average stress levels, can be represented by a single curve. It should be noted that this is a first approximation only, but it is much better than simply plotting the shear stress against the shear deformation. In daily life the proportionality of maximum shear stress to isotropic can be verified by trying to deform a package of coffee, sealed under vacuum, and to compare that with the deformability of the same package when the seal has been broken.

It must be noted that Fig. 12.4 represents only one possible form of distortion. A similar deformation can, of course, also occur in the two other planes of a three dimensional soil sample. Moreover, the definition of distortion as change of shape at constant volume means that a deformation in which the width of a sample increases and the height decreases, is also a form of distortion, see Fig. 12.6, because in this case the volume is also constant. That there is no fundamental difference with the shear deformation of Fig. 12.4 can be seen by connecting the centers of the four sides in Fig. 12.6, before and after the deformation. It will appear that again a square is deformed into a diamond, just as in Fig. 12.4, but rotated over an angle of 45° .

Conclusions

In the relations between stresses and strains, as described above, it is of great importance to distinguish between compression and distortion. The behavior in these two modes of deformation is completely different. The deformations in distortion (or shear) are usually much larger than the deformations in compression. Also, in compression the material becomes gradually stiffer, whereas in shear it becomes gradually softer.

12.2 Unloading and Reloading

Because the deformations of soils are mostly due to changes in the particle assembly, by sliding and rolling of particles, it can be expected that after unloading a soil will not return to its original state. Sliding of particles with respect to each other is an irreversible process, in which mechanical energy is dissipated, into heat. It is to be expected that after a full cycle of loading and unloading of a soil a permanent deformation is observed. Tests indeed confirm this.

When reloading a soil there is probably less occasion for further sliding of the particles, so that the soil will be much stiffer in reloading than it was in the first loading (*virgin loading*). The behavior in unloading and reloading, below the maximum load sustained before, often seems practically elastic, see Fig. 12.7, although there usually is some additional plastic deformation after each cycle. In the figure this is illustrated for shear loading.

A good example of irreversible deformations of soils from engineering practice is the deformation of guard rails along highways. After a collision the guard rail will have been deformed, and has absorbed the kinetic energy of the vehicle. The energy is dissipated by the rotation of the foundation pile through the soil. After removal of the damaged vehicle the rail will not rotate back to its original position, but it can easily be restored by pulling it back. That is the principle of the structure: kinetic energy is dissipated into heat, by the plastic deformation of the soil. That seems much better than to transfer the kinetic energy of the vehicle into damage of the vehicle and its passengers. The dissipated energy can be observed in the figure as the area enclosed by the branches of loading and unloading, respectively.

It is interesting to note that after unloading and subsequent reloading, the deformations again are much larger if the stresses are increased beyond the previous maximum stress, see Fig. 12.8. This is of great practical importance when a soil layer that in earlier times has been loaded and unloaded, is loaded again. If the final load is higher than the maximum load experienced before, a relation such as indicated in Fig. 12.8 may be observed, with the discontinuity in the curve indicating the level of the previous maximum load, the *preload*. The soil is said to be *overconsolidated*.

Fig. 12.7 Unloading and reloading

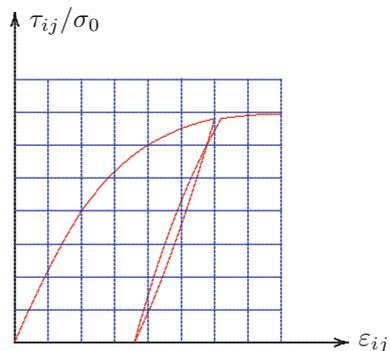
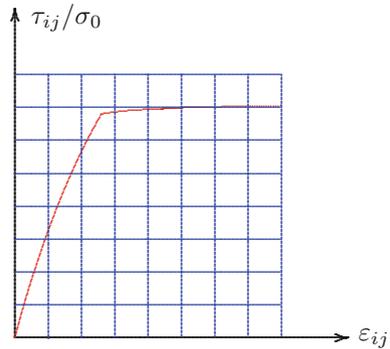


Fig. 12.8 Preload

As long as the stresses remain below the preconsolidation load the soil is reasonably stiff, but beyond the preconsolidation load the behavior will be much softer. This type of behavior is often observed in soils that have been covered in earlier times (an ice age) by a thick layer of ice.

12.3 Dilatancy

One of the most characteristic phenomena in granular soils is *dilatancy*, first reported by Reynolds around 1885. Dilatancy is the volume increase that may occur during shear. In most engineering materials (such as metals) a volume change is produced by an all round (isotropic) stress, and shear deformations are produced by shear stresses, and these two types of response are independent. The mechanical behavior of soils is more complex. This can most conveniently be illustrated by considering a densely packed sand, see Fig. 12.9. Each particle is well packed in the space formed by its neighbors. When such a soil is made to shear, by shear stresses, the only possible mode of deformation is when the particles slide and roll over each other, thereby creating some moving space between them. Such a dense material is denoted as *dilatant*.

Dilatancy may have some unexpected results, especially when the soil is saturated with water. A densely packed sand loaded by shear stresses can only sustain these shear stresses by a shear deformation. Through dilatancy this can only occur if it is accompanied by a volume increase, i.e. by an increase of the porosity. In a saturated

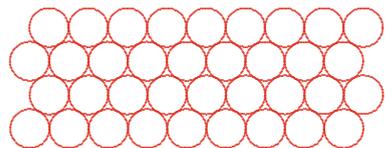
Fig. 12.9 Densely packed sand

Fig. 12.10 Dilatancy on the beach

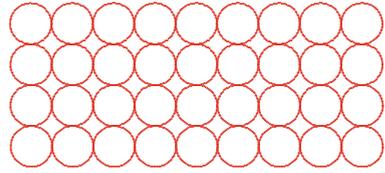


soil this means that water must be attracted to fill the additional pore space. This phenomenon can be observed on the beach when walking on the sand in the area flooded by the waves. The soil surrounding the foot may be dried by the suction of the soil next to and below the foot, which must carry the load, see Fig. 12.10. For sand at greater depth, for instance the sand below the foundation of an offshore platform, the water needed to fill the pore space can not be attracted in a short time, and this means that an under pressure in the water is being produced. After a certain time this will disappear, when sufficient amounts of water have been supplied. For short values of time the soil is almost incompressible, because it takes time for the water to be supplied, and the shear deformation will lead to a decrease of the pore water pressure. This will be accompanied by an increase of the effective stress, as the total stress remains approximately constant, because the total load must be carried. The soil appears to be very stiff and strong, at least for short values of time. That may be interpreted as a positive effect, but it should be noted that the effect disappears at later times, when the water has flowed into the pores.

The phenomenon that in densely packed saturated sand the effective stresses tend to increase during shear is of great importance for the dredging process. When cutting densely packed strata of sand under water an under pressure is generated in the pore water, and this will lead to increasing effective stresses. This increases the resistance of the sand to cutting. A cutting dredger may have great difficulty in removing the sand. The effect can be avoided when the velocity of the cutting process is very small, but then the production is also small. Large production velocities will require large cutting forces.

The reverse effect can occur in case of very loosely packed sand, see Fig. 12.11. When an assembly of particles in a very loose packing is being loaded by shear stresses, there will be a tendency for volume decrease. This is called *contractancy*. The assembly may collapse, as a kind of card house structure. Again the effect is most dramatic when the soil is saturated with water. The volume decrease means that there is less space available for the pore water. This has to flow out of the soil, but that takes some time, and in the case of very rapid loading the tendency for volume

Fig. 12.11 Loosely packed sand



decrease will lead to an increasing pore pressure in the water. The effective stresses will decrease, and the soil will become weaker and softer. It can even happen that the effective stresses are reduced to zero, so that the soil loses all of its coherence. This is called *liquefaction* of the soil. The soil then behaves as a heavy fluid (*quick sand*), having a volumetric weight about twice as large as water. A person will sink into the liquefied soil, to the waist.

The phenomenon of increasing pore pressures caused by contractancy of loose soils can have serious consequences for the stability of the foundation of structures. For example, the sand in the estuaries in the South West of the Netherlands is loosely packed because of the ever continuing process of erosion by tidal currents and deposition of the sand at the turning of the tide. For the construction of the storm surge barrier in the Eastern Scheldt the soil has been densified by vibration before the structure could safely be built upon it. For this purpose a special vessel was constructed, the *Mytilus*, see Fig. 12.12, containing a row of vibrating needles. Other examples are the soils in certain areas in Japan, for instance the soil in the artificial Port Island in the bay near Kobe. During the earthquake of 1995 the loosely packed sand liquefied, causing great damage to the quay walls and to many buildings. In the area where the soil had previously been densified the damage was much less. For the Chek Lap Kok airport of Hong Kong, an artificial sand island has been constructed in the sea, and to prevent damage by earthquakes the soil has been densified by vibration, at large cost.

It can be concluded that the density of granular soils can be of great importance for the mechanical behavior, especially when saturated with water, and especially

Fig. 12.12 *Mytilus*



for short term effects. Densely packed sand will have a tendency to expand (dilatancy), and loosely packed sand will have a tendency to contract (contractancy). At continuing deformations both dense and loose sand will tend towards a state of average density, sometimes denoted as the *critical density*. This is not a uniquely defined value of the density, however, as it also depends upon the isotropic stress. At high stresses the critical density is somewhat smaller than at small stress. The branch of soil mechanics studying these relations is *critical state soil mechanics*.

It may be interesting to mention that during *cyclic loads* soils usually tend to contract after each cycle, whatever the original density is. It seems that in a full cycle of loading a few particles may find a more dense packing than before, resulting in a continuing volume decrease. The effect becomes smaller and smaller if the number of cycles increases, but it seems to continue practically forever. It can be compared to the situation in a full train, where there seems to be no limit to the number of passengers that can be transported. By some more pressing a full train can always accommodate another passenger. The cyclic effect is of great importance for the foundation of offshore structures, which may be loaded by a large number of wave loads. During a severe storm each wave may generate a small densification, or a small increase of the pore pressure, if the permeability of the soil is small. After a great many of these wave loads the build up of pore pressures may be so large that the stability of the structure is endangered. Again previous densification of the soil reduces the risk.

Example 12.1 In road building one may observe that before the final layer of the pavement is placed heavy equipment, such as a road roller, is being used to further densify the soil, see Fig. 12.13.

This makes good sense, taking into account the influence of a pre-loading on the stiffness of a soil, as shown in Fig. 12.8. Due to the passage of the heavy roller the soil will deform, and some additional soil may be needed to raise the top surface to the desired level. The result will be that when the road is completed the surface

Fig. 12.13 A road roller



deformations will remain small. The initial cost is larger, but maintenance costs will be smaller.

It may be noted that the stresses near the soil surface can be further increased by constructing picks on the surface of the roller, as shown in the figure.

Example 12.2 A rubber ball contains saturated sand, and water reaching into the neck of the glass tube inserted into the ball, above the sand. When squeezing the ball between two fingers, the water level appears to go down.

This phenomenon, which is often used in geotechnical laboratories to demonstrate the special properties of soils, can be explained by assuming that the initial density of the sand is very high (if this not the case the sand may be densified before the demonstration by knocking the ball several times onto the table, preferably before the arrival of the spectators). The squeezing of the sand results in a shear deformation, and in this dilatant material this is accompanied by a volume expansion of the assembly of granular particles. This means that the pore volume increases, and thus water is drawn into the ball from the glass tube.

Problem 12.1 A soil sample is loaded in a laboratory test, by an isotropic stress. If the stress is increased from 100 to 200 kPa, the volume decrease is 0.1%. Suppose that the stress is further increased to 300 kPa. Will the additional volume decrease then be smaller than, larger than, or equal to 0.1%?

Problem 12.2 A part of a guard rail along a highway has been tested by pulling sideways. A lateral force of 10 kN has been found to lead to a lateral displacement of 1 cm. The force is next increased to 20 kN. Will the additional displacement then be more than or less than 1 cm?

Problem 12.3 Is the sand used in the experiment shown in Fig. 12.14 suitable for the foundation of a bridge pier?

Problem 12.4 In a laboratory quicksand is being produced in a large cylindrical tank filled with sand, by pumping water into it from below, while the excess of water flows over the top of the tank, back into the reservoir. (The experiment is shown in the YouTube film “mythbusters quicksand”). How deep will a student sink into the fluidized mixture of sand and water?

Fig. 12.14 Sand in a rubber ball

