Chapter 5
Stresses in a Layer

This chapter presents some examples for the determination of the vertical stresses (effective stresses and pore pressures) in a layer with a horizontal surface.

5.1 Vertical Stresses

In many places on earth the soil consists of practically horizontal layers. If such a soil does not carry a local surface load, and if the groundwater is at rest, the vertical stresses can be determined directly from a consideration of vertical equilibrium. The procedure is illustrated in this chapter.

A simple case is a homogeneous layer, completely saturated with water, see Fig. 5.1. The pressure in the water is determined by the location of the *phreatic surface*. This is defined as the plane where the pressure in the groundwater is equal to the atmospheric pressure. If the atmospheric pressure is taken as the zero level of pressures, as is usual, it follows that $p = 0$ at the phreatic surface. If there are no capillary effects in the soil, this is also the upper boundary of the water, which is denoted as the *groundwater table*. In the example it is assumed that the phreatic surface coincides with the soil surface, see Fig. 5.1. The volumetric weight of the saturated soil is supposed to be $\gamma = 20\ \mathrm{kN/m^3}$. The vertical normal stress in the soil now increases linearly with depth,

$$\sigma_{zz} = \gamma d. \tag{5.1}$$

This is a consequence of vertical equilibrium of a column of soil of height d. It has been assumed that there are no shear stresses on the vertical planes bounding the column in horizontal direction. That seems to be a reasonable assumption if the terrain is homogeneous and very large, with a single geological history. Often this is assumed, even when there are no data.

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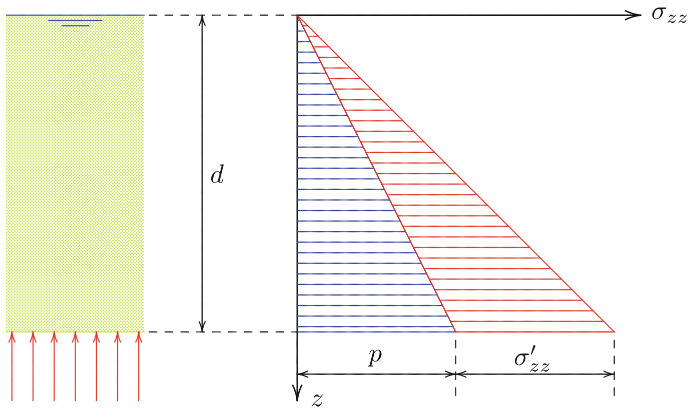


Fig. 5.1 Stresses in a homogeneous layer

At a depth of 10 m, for instance, the vertical total stress is 200 kN/m^2 = 200 kPa. Because the groundwater is at rest, the pressures in the water will be hydrostatic. The soil can be considered to be a container of water of very complex shape, bounded by all the particles, but that is irrelevant for the actual pressure in the water. This means that the pressure in the water at a depth d will be equal to the weight of the water in a column of unit area, see also Fig. 4.3,

$$p = \gamma_w d, \tag{5.2}$$

where γ_w is the volumetric weight of water, usually $\gamma_w = 10$ kN/m^3. It now follows that a depth of 10 m the effective stress is 200 – 100 kPa = 100 kPa.

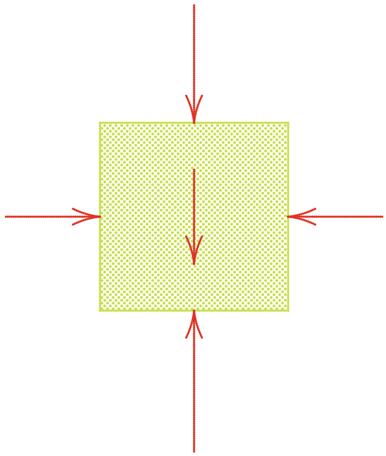
Formally, the distribution of the effective stress can be found from the basic equation $\sigma'_{zz} = \sigma_{zz} - p$, or, with (5.1) and (5.2),

$$\sigma'_{zz} = (\gamma - \gamma_w)d. \tag{5.3}$$

The vertical effective stresses appear to be linear with depth. That is a consequence of the linear distribution of the total stresses and the pore pressures, with both of them being zero at the same level, the soil surface.

It should be noted that the vertical stress components, both the total stress and the effective stress, can be found using the condition of vertical equilibrium only, together with the assumption that the shear stresses are zero on vertical planes. The horizontal normal stresses remain undetermined at this stage. Even by also considering horizontal equilibrium these horizontal stresses can not be determined. A consideration of horizontal equilibrium, see Fig. 5.2, does give some additional information, namely that the horizontal normal stresses on the two vertical planes at the left and at the right must be equal, but their magnitude remains unknown. The determination of horizontal (or *lateral*) stresses is one of the essential difficulties of soil mechanics.

Fig. 5.2 Equilibrium



Because the horizontal stresses can not be determined from equilibrium conditions they often remain unknown. It will be shown later that even when also considering the deformations, the determination of the horizontal stresses remains very difficult, as this requires detailed knowledge of the geological history, which is usually not available. Perhaps the best way to determine the horizontal stresses is by direct or indirect measurement in the field. The problem will be discussed further in later chapters.

The simple example of Fig. 5.1 may be used as the starting point for more complex cases. As a second example the situation of a somewhat lower phreatic surface is considered, say when it is lowered by 2 m. This may be caused by the action of a pumping station in the area, such that the water level in the canals and the ditches in a polder is to be kept at a level of 2 m below the soil surface. In this case there are two possibilities, depending upon the size of the particles in the soil. If the soil consists of very coarse material, the groundwater level in the soil will coincide with the phreatic surface (the level where $p = 0$), which will be equal to the water level in the open water, the ditches. However, when the soil is very fine (for instance clay), it is possible that the top of the groundwater in the soil (the groundwater level) is considerably higher than the phreatic level, because of the effect of *capillarity*. In the fine pores of the soil the water may rise to a level above the phreatic level due to the suction caused by the surface tension at the interface of particles, water and air. This surface tension may lead to pressures in the water below atmospheric pressure, i.e. negative water pressures. The zone above the phreatic level is denoted as the *capillary zone*. The maximum height of the groundwater above the phreatic level is denoted as h_c, the *capillary rise* (Fig. 5.3).

If the capillary rise h_c in the example is larger than 2 m, the soil in the polder will remain saturated when the water table is lowered by 2 m. The total stresses will not change, because the weight of the soil remains the same, but the pore pressures throughout the soil are reduced by $\gamma_w \times 2\text{ m} = 20\text{ kN/m}^2$. This means that the effective stresses are increased everywhere by the same amount, see Fig. 5.4.

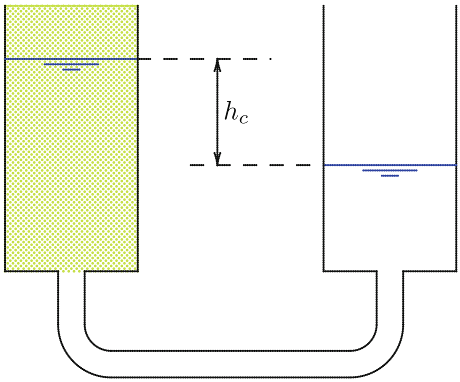


Fig. 5.3 Capillary rise

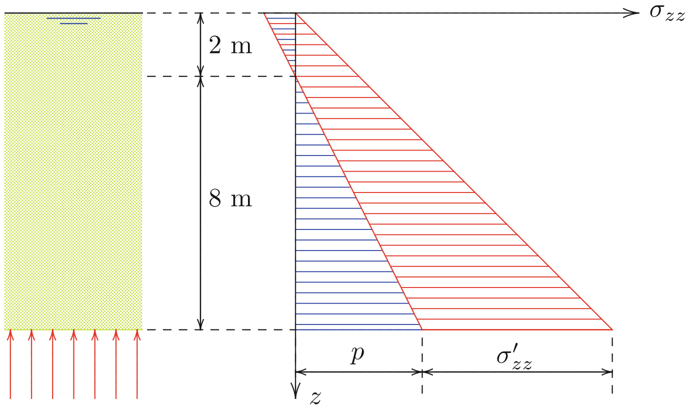


Fig. 5.4 Lowering the phreatic surface by 2 m, with capillary rise

Lowering the phreatic level appears to lead to an increase of the effective stresses. In practice this will cause deformations, which will be manifest by a subsidence of the ground level. This indeed occurs very often, wherever the groundwater table is lowered. Lowering the water table to construct a dry building pit, or lowering the groundwater table in a newly reclaimed polder, leads to higher effective stresses, and therefore settlements. This may be accompanied by severe damage to buildings and houses, especially if the settlements are not uniform. If the subsidence is uniform there is less risk for damage to structures founded on the soil in that area. Lowering the phreatic level may also have some positive consequences. For instance, the increase of the effective stresses at the soil surface makes the soil much stiffer and stronger, so that heavier vehicles (tractors or other agricultural machines) can be supported. In case of a very high phreatic surface, coinciding with the soil surface, as illustrated in Fig. 5.1, the effective stresses at the surface are zero, which means that there is no force between the soil particles. Man, animal and machine then can not find support on the soil, and they may sink into it. The soil is called soggy or swampy. It seems natural that in such cases people will be motivated to lower the water table. This will result in some subsidence, and thus part of the effect of the lower groundwater table is lost. This can be restored by a further lowering of the water table, which in turn will lead to further subsidence. In some places on earth the process has had almost catastrophic consequences (Venice, Bangkok). The subsidence of Venice, for instance, was found to be caused for a large part by the production of ever increasing amounts of drinking water from the soil in the immediate vicinity of the city. Further subsidence has been reduced by finding a water supply farther from the city.

When the soil consists of very coarse material, there will practically be no capillarity. In that case lowering the phreatic level by 2 m will cause the top 2 m of the soil to become dry, see Fig. 5.5. The upper 2 m of soil then will become lighter. A reasonable value for the dry volumetric weight is $\gamma_d = 16$ kN/m^3. At a depth of 2 m the vertical effective stress now is $\sigma'_{zz} = 32$ kPa, and at a depth of 10 m the effective stress is $\sigma'_{zz} = 112$ kPa. It appears that in this case the effective stresses increase by 12 kPa, compared to the case of a water table coinciding with the ground surface. The distribution of total stresses, effective stresses and pore pressures is shown in Fig. 5.5. Again there will be a tendency for settlement of the soil. In later chapters a procedure for the calculation of these settlements will be presented. For this purpose first the relation between effective stress and deformation must be considered.

Subsidence of the soil can also be caused by the extraction of gas or oil from soil layers. The reservoirs containing oil and gas are often located at substantial depth (in Groningen at 3000 m depth). These reservoirs usually consist of porous rock, that have been consolidated through the ages by the weight of the soil layers above it, but some porosity (say 10 or 20%) remains, filled with gas or oil. When the gas or oil is extracted from the reservoir, by reducing the pressure in the fluid, the effective stresses increase, and the thickness of the reservoir will be reduced. This will cause the soil layers above the reservoir to settle, and it will eventually give rise to subsidence

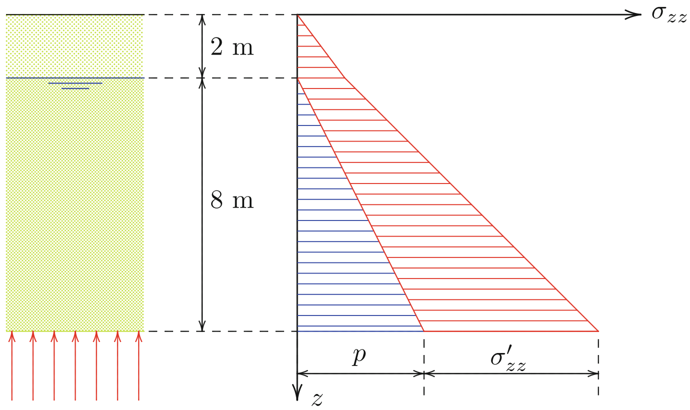


Fig. 5.5 Lowering of the phreatic surface by 2 m, no capillarity

of the soil surface. In Groningen the subsidence above the large gas reservoir is estimated to reach about 50 cm, over a very large area. All structures subside with the soil, with a risk of damage. Also, because the soil surface is below sea level, great care must be taken to maintain the drainage capacity of the hydraulic infrastructure. Sluices may have to be renewed because they subside, whereas water levels must be maintained. The dikes also have to be raised to balance the subsidence due to gas production. Also, because the pressures in the gas are reduced by a large amount, the effective stresses in the rock will be very much increased, which is causing earth quakes because of inhomogeneities in the rock. As a result many buildings show considerable damage.

In some parts of the world subsidence may have very serious consequences, for instance in areas of coal mining activities. In mining the entire soil is being removed, and sudden collapse of a mine gallery may cause great damage to the structures above it.

5.2 The General Procedure

It has been indicated in the examples given above how the total stresses, the effective stresses and the pore pressures can be determined on a horizontal plane in a soil consisting of practically horizontal layers. In most cases the best general procedure is that first the total stresses are determined, from the vertical equilibrium of a column of soil. The total stress then is determined by the total weight of the column (particles and water), plus an eventual surcharge caused by a structure. In the next step the pore pressures are determined, from the hydraulic conditions. If the groundwater is at rest it is sufficient to determine the location of the phreatic surface. The pore pressures then are hydrostatic, starting from zero at the level of the phreatic surface, i.e. linear with the depth below the phreatic surface. When the soil is very fine a capillary zone may develop above the phreatic surface, in which the pore pressures are negative. The maximum negative pore pressure depends upon the size of the pores, and can be measured in the laboratory. Assuming that there are sufficient data to determine the pore pressures, the effective stresses can be determined as the difference of the total stresses and the pore pressures.

A final example is shown in Fig. 5.6. This concerns a layer of 10 m thickness, carrying a surcharge of 50 kPa. The phreatic level is located at a depth of 5 m, and it has been measured that in this soil the capillary rise is 2 m. The volumetric weight of the soil when dry is 16 kN/m^3, and when saturated it is 20 kN/m^3. Using these data it can be concluded that the top 3 m of the soil will be dry, and that the lower 7 m will be saturated with water. The total stress at a depth of 10 m then is 50 kPa $+$ 3 m $\times$ 16 kN/m^3 $+$ 7 m $\times$ 20 kN/m^3 $=$ 238 kPa. At that depth the pore pressure is 5 m $\times$ 10 kN/m^3 $=$ 50 kPa. It follows that the effective stress at 10 m depth is 188 kPa. The distribution of total stresses, effective stresses and pore pressures is shown in Fig. 5.6.

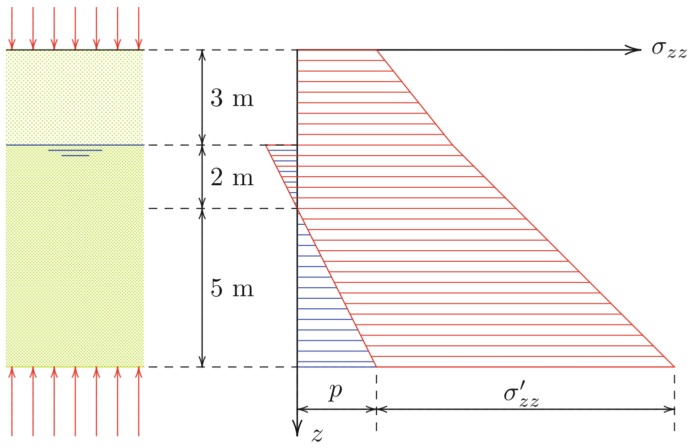


Fig. 5.6 Example of the general procedure

It should be noted that throughout this chapter it has been assumed that the groundwater is at rest, so that the pressure in the groundwater is hydrostatic. When the groundwater is flowing this is not so, and more data are needed to determine the pore pressures. For this purpose the flow of groundwater is considered in the next chapters.

Example 5.1 A lake is being reclaimed by lowering the water table below the bottom of the lake, see Fig. 5.7. The soil consists of 10 m of homogeneous clay, having a saturated volumetric weight of 18 kN/m^3. Below the clay the soil is sand. Initially the water level is 2 m above the soil surface, after the reclamation the phreatic level is at 2 m below the soil surface, and it is assumed that soil remains saturated. Construct graphs of total stresses, effective stresses and pore pressures before and after the reclamation.

Solution

The stresses in the initial state are shown in the left half of Fig. 5.7. The stresses in the final state are shown in the right half of the figure.

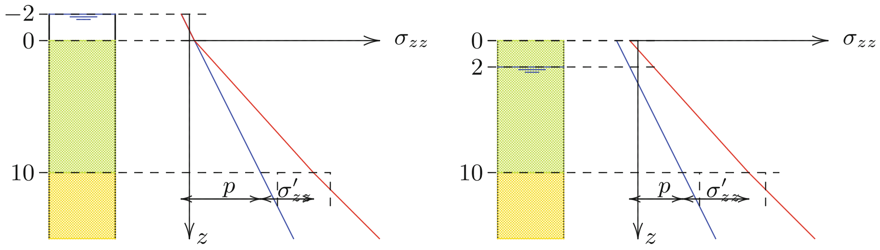


Fig. 5.7 Stresses before and after lowering the water table

For the stresses in the initial state the total stresses can best be calculated first. At a depth of −2 m: $\sigma_{zz} = 0$, and at a depth of 0 m: $\sigma_{zz} = \gamma_w \times 2$ m $= 20$ kPa. The top layer is 10 m of clay, with a unit weight of 18 kN/m^3. This means that at a depth of 10 m: $\sigma_{zz} = 200$ kPa. Below that level the soil is sand, with a unit weight of 20 kN/m^3, so that at a depth of 15 m: $\sigma_{zz} = 300$ kPa.

Next the pore pressures can be calculated. At a depth of −2 m: $p = 0$, and then the pore pressure increases hydrostatically with depth, so that for instance at a depth of 10 m: $p = 120$ kPa.

The effective stresses can finally be determined using the relation $\sigma'_{zz} = \sigma_{zz} - p$. At a depth of 10 m: $\sigma'_{zz} = 80$ kPa.

In the final state the total stresses start at the surface $z = 0$, and then at a depth of 10 m: $\sigma_{zz} = 180$ kPa, and at a depth of 15 m: $\sigma_{zz} = 280$ kPa.

The pore pressures now are zero at a depth of 2 m, but above that level it is given that the soil remains saturated, so that negative pore pressures will be developed. The distribution of the pore pressures in the final state will again by hydrostatic. This means that at a depth of 10 m: $p = 80$ kPa. At the soil surface the pore pressure will be $p = -20$ kPa.

Again the effective stresses can be determined as the difference of the total stresses and the pore pressures, $\sigma'_{zz} = \sigma_{zz} - p$. At a depth of 10 m: $\sigma'_{zz} = 100$ kPa.

It may be noted that the total stresses decrease, but the pore pressures decrease even more, so that the effective stresses increase. For instance at a depth of 10 m the initial effective stress is 80 kPa, and the final effective stress is 100 kPa. This means that the soil will be compressed, and subsidence of the soil surface can be expected.

Example 5.2 A concrete caisson having a mass of 5000 ton, a foundation surface of 20×20 m, and a height of 10 m, is being placed on dry sand. Calculate the average total stress and the average effective stress just below the caisson.

Solution

The total force on the soil is $F = M \times g$, where M is the mass of the caisson, and g is the gravity constant, which is approximately $g = 10$ N/kg. In this case it follows that $F = 5000 \times 1000 \times 10 = 50 \times 10^6$ N $= 50,000$ kN. Because the area of the bottom of the caisson is 400 m^2 the average total stress is $\sigma_{zz} = 125$ kPa. There is no water in the soil, so that the pore pressure is zero, and the effective stress is equal to the total stress.

Example 5.3 A similar caisson is placed in open water, on a layer of sand. The water level is 5 m above the top of the sand, so that the top of the caisson is 5 m above water. Again calculate the average total stress and the average effective stress just below the caisson.

Solution

In this case the total stress is the same as in the previous case, $\sigma_{zz} = 125$ kPa. The pore pressure in the soil just below the caisson is $p = 50$ kPa (the pressure caused by 5 m of water). The effective stress now is $\sigma'_{zz} = \sigma_{zz} - p = 75$ kPa.

It may be noted that this last answer can also be obtained directly by subtracting the upward buoyancy force on the caisson from its weight, i.e. $F' = 50,000 - 20,000 = 30,000$ kPa, and then dividing this by the area of the caisson. This may be faster, but it is recommended to always determine the effective stress as the difference of the total stress and the pore pressure, because this can be more easily generalized, for instance to problems involving flowing groundwater.

Problem 5.1 A certain soil has a dry volumetric weight of 15.7 kN/m^3, and a saturated volumetric weight of 21.4 kN/m^3. The phreatic level is at 2.5 m below the soil surface, and the capillary rise is 1.3 m. Calculate the vertical effective stress at a depth of 6.0 m, in kPa.

Problem 5.2 A layer of saturated clay has a thickness of 4 m, and a volumetric weight of 18 kN/m^3. Above this layer a sand layer is located, having a dry volumetric weight of 16 kN/m^3 and a saturated volumetric weight of 20 kN/m^3. The groundwater level is at a depth of 1 m below soil surface, which is the top of the sand layer. There is no capillary rise in the sand, and the pore pressures are hydrostatic. Calculate the average effective stress in the clay, in kPa.

Problem 5.3 The soil in the previous problem is loaded by a surcharge of 2 m of the same sand. The groundwater level is maintained. Calculate the increase of the average effective stress in the clay, in kPa.