Chapter 21
Triaxial Test

The failure of a soil sample under shear could perhaps best be investigated in a laboratory test in which the sample is subjected to pure distortion, at constant volume. The volume could be kept constant by taking care that the isotropic stress $\sigma_0 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ remains constant during the test, or, better still, by using a test setup in which the volume change can be measured and controlled very accurately, so that the volume change can be zero. In principle such a test is possible, but it is much simpler to perform a test in which the lateral stress is kept constant, the *triaxial test*. In order to avoid the complications caused by pore pressure generation, it will first be assumed that the soil is dry sand. The influence of pore water pressures will be considered later.

21.1 The Triaxial Test

In the triaxial test, see Fig. 21.1 (Bishop and Henkel 1962), a cylindrical soil sample is placed in a glass or plastic cell, with the sample being enclosed in a rubber membrane. The membrane is connected to circular plates at the top and the bottom of the sample, with two o-rings ensuring a water tight connection. The cell is filled with water, with the pressure in the water (the *cell pressure*) being controlled by a pressure unit, usually by a connection to a tank in which the pressure can be controlled. Because the sample is completely surrounded by water, at its cylindrical surface and at the top, a pressure equal to the cell pressure is generated in the sample. The usual, and simplest, test procedure is to keep the cell pressure constant during the test.

In addition to the lateral (and vertical) loading by the cell pressure, the sample can also be loaded by a vertical force, by means of a steel rod that passes through the top cap of the cell. The usual procedure is that in the second stage of the test the rod is being pushed down, at a constant rate, by an electric motor. This means that the vertical deformation rate is constant, and that the force on the sample gradually

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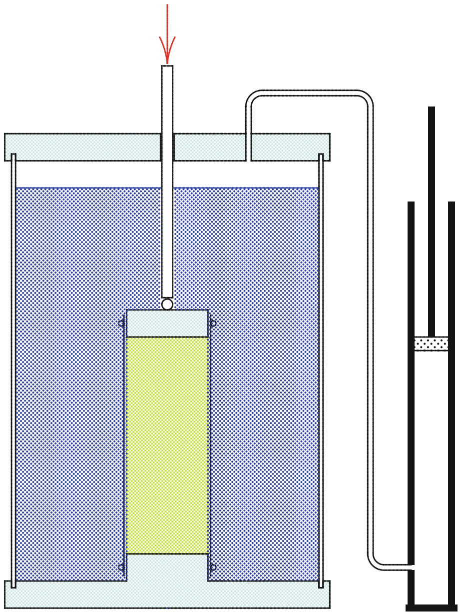

Fig. 21.1 Triaxial test

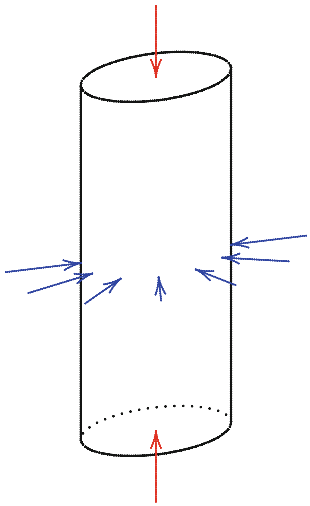

Fig. 21.2 Cell pressure

increases. The force can be measured using a strain gauge or a compression ring, and the vertical movement of the top of the sample is measured by a mechanical or an electronic measuring device (Fig. 21.2).

During the test the vertical displacement of the top of the sample increases gradually as a function of time, because the motor drives the steel rod at a very small constant velocity downwards. The vertical force on the sample will also gradually

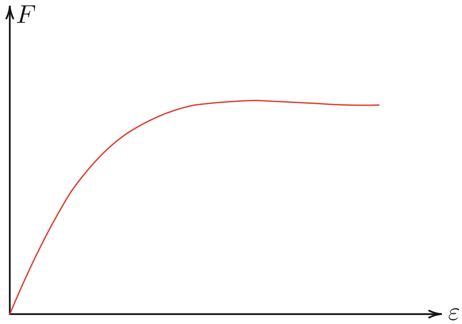

Fig. 21.3 Test result

increase, so that the difference of the vertical stress and the horizontal stress gradually increases, but after some time this reaches a maximum, and remains constant afterwards, or shows some small additional increase, or decreases somewhat. The maximum of the vertical force indicates that the sample starts to fail. Usually the test is continued up to a level where it is quite clear that the sample has failed, by recording large deformations, up to 5% or 10%. This can often be observed in the shape of the sample too, with the occurrence of some distinct sliding planes. It may also be, however, that the deformation of the sample remains practically uniform, with a considerable shortening and at the same time a lateral extension of the sample. In the interior of the sample many sliding planes may have formed, but these may not be observed at its surface.

The test is called the *triaxial test* because stresses are imposed in three directions. This can be accomplished in many different ways, however, and there even exist tests in which the stresses applied in three orthogonal directions onto a cubical soil sample (enclosed in a rubber membrane) can all be different, the *true triaxial test*. This gives many more possibilities, but it is a much more complex apparatus, and the testing procedures are more complex as well (Fig. 21.3).

In the normal triaxial test the sample is of cylindrical shape, and the two horizontal stresses are identical. The usual diameter of the sample is 3.8 cm (or 1.5 in, as the test was developed in England), but there also exist triaxial cells in which larger size samples can be tested. For tests on gravel a diameter of 3.8 cm seems to be insufficient to achieve a uniform state of stress. For clay and sand it is sufficient to guarantee that in every cross section there is a sufficient number of particles for the stress to be well defined.

If the cell pressure is denoted by σ_c, and the vertical axis is the z-axis, then the lateral stresses in the test are

$$\sigma_{xx} = \sigma_{yy} = \sigma_c, \tag{21.1}$$

and the vertical stress is

$$\sigma_{zz} = \sigma_c + \frac{F}{A}, \tag{21.2}$$

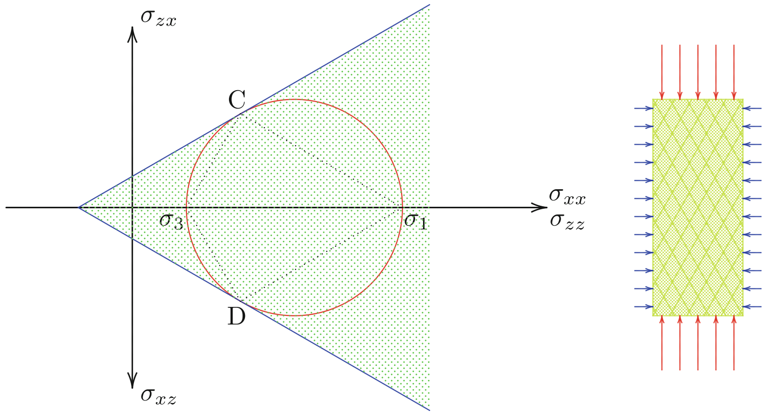

Fig. 21.4 Mohr's circle for the triaxial test

in which F is the vertical force, and A is the cross sectional area of the sample. Because the soil has been supposed to be dry sand, so that there are no pore pressures, these are effective stresses as well as total stresses.

In this case the vertical stress is the major principal stress, and the horizontal stress is the minor principal stress,

$$\sigma_1 = \sigma_c + \frac{F}{A}. \tag{21.3}$$

$$\sigma_3 = \sigma_c, \tag{21.4}$$

It should be noted that the stresses in the sample are assumed to be uniformly distributed. This will be the case only if the sample is of homogeneous composition. Furthermore, it has been assumed that there are no shear stresses on the upper and lower planes of the sample. This requires that the loading plates are very smooth. This can be accomplished by using special materials (e.g. Teflon) or by applying a thin smearing layer.

The stresses on planes having an inclined orientation with respect to the vertical axis, can be determined using Mohr's circle, see Fig. 21.4. The *pole* for the normal directions coincides with the rightmost point of the circle. On a horizontal plane and on a vertical plane the shear stresses are zero, but on all other planes there are certain shear stresses. If the vertical force F gradually increases during the test, the size of the circle will gradually increase, and if the force is sufficiently large the circle will touch the straight lines indicating the Coulomb criterion, the *Mohr-Coulomb envelope*. In that situation there are two planes on which the combination of shear stress and normal stress is such that the maximum shear stress, according to (20.1) is reached. These are the planes for which the stress points are indicated by C and D in the figure. The direction of the normals to these planes can be found by connecting the points C and D with the pole. The orientation of the planes themselves is perpendicular to

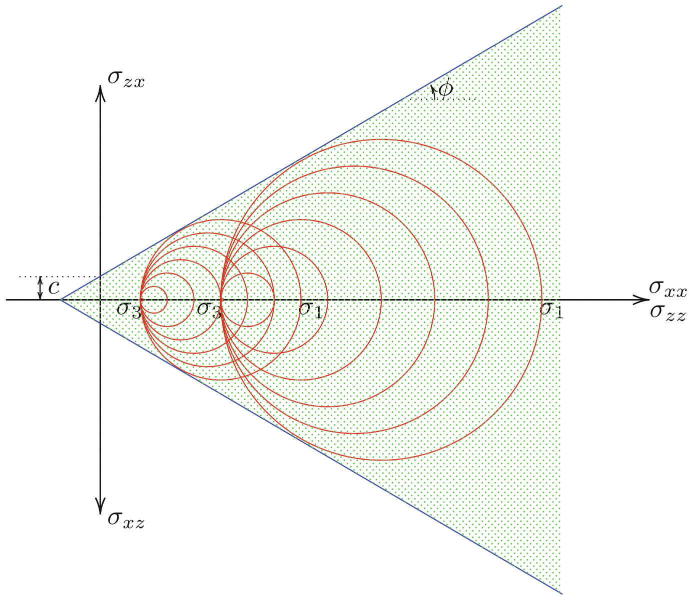

Fig. 21.5 Determination of c and ϕ from two tests

these normals. In the right half of Fig. 21.4 these planes have been indicated by the sloping lines.

When several tests are performed on the same material, but at different cell pressures, the various critical circles define the envelope, so that the values of the cohesion c and the friction angle ϕ can be determined. The usual practice is to do two tests, on the same material, at clearly different cell pressures. In each of the tests a value of the major principal stress σ_1 is found, at a certain value of the lateral stress σ_3. The two critical circles can be drawn in a Mohr diagram, and the Mohr-Coulomb envelope can then be determined by drawing straight lines touching these two circles, see Fig. 21.5. In this way the values of c and ϕ can be determined. When doing more than two tests the accuracy of the basic assumption that the envelope is a straight line can be tested. It is often found that for high stresses the value of the friction angle ϕ somewhat decreases.

For sands the tests usually give that the cohesion c is practically zero, and that the friction angle ϕ varies from about 30° to 45°, depending on the type of sand, and its packing. Sharp sand, i.e. sand with many sharp angles, usually has a much higher friction angle than sand consisting of rounded particles. And densely packed sand has a higher friction angle than loosely packed sand. For clay the cohesion may be of the order of magnitude of 5 to 50 kPa, or even higher, whereas ϕ may vary from 15° to 30°. For the determination of c and ϕ of clay care must be taken that the influence of pore pressures is accounted for, see Chap. 23.

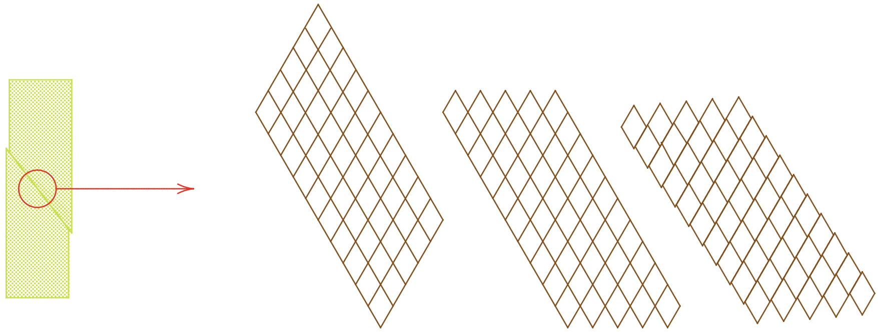

Fig. 21.6 Apparent shear plane in triaxial test

It may be mentioned that the strength of rock can also be determined by triaxial tests. The pressures then are much higher, and the cell wall usually is made of steel rather than glass. In petroleum engineering, where the properties of deep layers of rock are of paramount importance, rock samples are often tested by triaxial tests.

From Mohr's circle, see Fig. 21.4, it can be seen that the critical planes are inclined at angles of $\pi/4 - \frac{1}{2}\phi$ with the vertical direction. If the failure mechanism would consist only of sliding along one of these planes the test would result in a discontinuity in the deformation pattern in the direction of that plane. This is indeed sometimes found, for rather loose sands, but very often the deformation pattern is disturbed by more or less simultaneous sliding along different planes, by rotations, and by elastic deformations. Even when a clear sliding surface seems to appear, it is not recommended to try to determine the friction angle by measuring the angle of that surface with the vertical direction, and equating it to $\pi/4 - \frac{1}{2}\phi$. This often leads to significant errors, as angles between $\pi/4$ and $\pi/4 - \frac{1}{2}\phi$ may be observed, and repetition of the test may lead to a different direction. This can be explained by considering a thin zone in which failure occurs, with sliding along different sliding planes in the interior of that zone. The macroscopic (apparent) sliding angle depends on the relative contribution of each of the two sliding directions. Figure 21.6 shows an example with possible sliding planes at angles of 30° with the vertical direction. The case represented in the figure consists of a combination of a large shearing of the right hand side with respect to the left hand side along one set of planes, and a small shearing of the left hand side with respect to the right hand side along the other set of planes. The result appears to be that an apparent shearing takes place over an angle with the vertical direction.

The case represented in the figure consists of a combination of a large shearing of the right hand side with respect to the left hand side along one set of planes, and a small shearing of the left hand side with respect to the right hand side along the other set of planes. The result appears to be that an apparent shearing takes place over an angle with the vertical direction that is considerably larger than 30°, that is less steep. If one would consider that angle to be $\pi/4 - \frac{1}{2}\phi$, the friction angle ϕ would be underestimated.

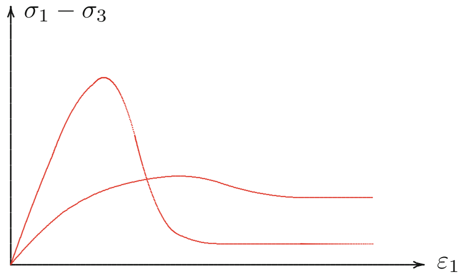

Fig. 21.7 Some test results

It should be noted that there is absolutely no need to determine the friction angle ϕ from the direction of a possible sliding plane. The merit of the triaxial test is that it provides a relatively simple and accurate method for the determination of the strength parameters c and ϕ from two tests, because in both tests the critical stresses are very accurately measured. The cell pressure and the vertical force can easily be controlled and measured, and therefore the determination of the critical stress states is very accurate. In other tests this may not be the case. It may be mentioned that very often laboratory tests are being used to determine the relation between stress and strain for the entire range of strains, from the small deformations in the early stages, up to the large deformations at failure, and perhaps beyond, see Fig. 21.7. If the vertical load is applied by imposing the strain (or the strain rate) a possible decrease of the stress after reaching the maximum stress can also be detected. The maximum strength is called the *peak strength*, and the final strength, at very large strains, is called the *residual strength*. For certain types of soils the residual strength is much lower than the peak strength, for instance the calcareous sands that occur in offshore coastal zones of Western Australia and Brazil. An example of such a result is shown in Fig. 21.7. In this type of material the peak strength is so high, with respect to the residual strength, because the sand particles have been cemented together. The sand will become very stiff, but brittle. It appears to be very strong, and it is, but as soon as the structure has been broken, the strength falls down to a much lower value. In the construction of offshore platforms near the coast of Western Australia this has caused large problems, because the shear strength of the soil was reduced very severely after the driving of the foundation piles through the soil.

Example 21.1 On two soil samples, having a diameter of 3.8 cm, triaxial tests are performed, at cell pressures of 10 and 20 kPa, respectively. In the first test failure occurs for an axial force of 22.7 N, and in the second test for an axial force of 44.9 N. Determine c and ϕ of this soil, assuming that there are no pore pressures.

Solution

The solution can be obtained using Eq. (20.12). In the absence of pore pressures the effective stresses are equal to the total stresses, so that this equation can be written as $\frac{1}{2}(\sigma_1 - \sigma_3) - \frac{1}{2}(\sigma_1 + \sigma_3)\sin\phi - c\cos\phi = 0$.

In the first test $\sigma_3 = 10$ kPa and $\sigma_1 = 10 + 22.7 \times 10^{-3}/(\pi \times 0.019 \times 0.019) = 30.02$ kPa, and in the second test $\sigma_3 = 20$ kPa and $\sigma_1 = 20 + 44.9 \times 10^{-3}/(\pi \times 0.019 \times 0.019) = 59.59$ kPa. This leads to the two equations $10.01 - 20.01 \sin\phi - c\cos\phi = 0$, and $19.79 - 39.79 \sin\phi - c\cos\phi = 0$. Subtraction of the first equation from the second one gives $\sin\phi = 0.4944$, or $\phi = 29.6°$. It now follows that $\cos\phi = 0.8692$, and then substitution into one of the equations gives $c = 0.13$ kPa. Of course the solution can also be obtained by drawing the two critical Mohr circles, and then measuring the values of c and ϕ in the figure.

Problem 21.1 It is given that for a certain sand $c = 0$ and $\phi = 30°$. A triaxial test is done on this sand, using a cell pressure of 100 kPa. The diameter of the sample is 3.8 cm. What is the axial force at the moment of failure?

Problem 21.2 Is it technically possible to perform a test on a sample in a triaxial apparatus such that the vertical stress is smaller than the horizontal stress, which is always equal to the cell pressure?

Reference

A.W. Bishop, D.J. Henkel, *The Measurement of Soil Properties in the Triaxial Test*, 2nd edn. (Edward Arnold, London, 1962)